passing through $w$. Now the minimum amount of material required to sustain the weight with a given stress proportioned to the cross section, is when $acb$ is a right angle. To demonstrate this, join $ab$, and draw the perpendicular $cd$. Then, the thrust on $ac$ and $bc$ each, is to $\frac{1}{2}w$, as the line $ac$ (or $bc$) is to $cd$, that is, equal to $\frac{1}{2}w\frac{ac}{p}$, making $p=cd$. But making $ad=h, ac=\sqrt{\left(h^2 + p^2\right)}$ and $\frac{1}{2}w\frac{ac}{p}=\frac{1}{2}w\sqrt{\left(h^2 + p^2\right)}$ and this multiplied by $ac$ (or $\sqrt{\left(h^2 + p^2\right)}$) is proportional to the amount of material required to sustain the weight $w$, upon the supposition above laid down, being the stress multiplied by the length of the piece. Therefore, when $p$ has such a value that the product $\frac{1}{2}w\frac{\sqrt{\left(h^2 + p^2\right)}}{p} \times \sqrt{\left(h^2 + p^2\right)}=\frac{1}{2}w\left(\frac{h^2}{p} + p\right)$ is a minimum, the least possible amount of material will support the weight.

Then, differentiating the function $\frac{h^2}{p} + p$ in which $p$ is the variable, and deducing the value of $p$ when the differential $=0$ we obtain $p=h$, and consequently $acb$ is a right angle, and $ac$ and $cb$ each incline $45^\circ$ to the horizon.

XXVI. A. But a more important problem is, having the length and height of the truss given, to determine the horizontal reach of the diagonal which will give the greatest degree of economy.

This will present two cases, according as the diagonals act by tension or thrust. We will first consider them as acting by tension.

Now, the greatest economy in the use of a given amount of material in a diagonal acting by tension, is, manifestly, when the weight it can sustain, multiplied by the horizontal reach, gives the greatest product. This may be called