The following results, obtained by omitting for the present the consideration of the destructive force, agree with theirs.

Proposition 1. If a beam be supported at the ends, and the load applied at any point between the supports, the extended side being straight, the form of the compressed side will be that formed by two semi-parabolas.

Fig. 16.

Consider C, the point of application of the weight, as a fulcrum.

Let \( w \) represent the portion of the weight sustained by \( A \).

\( x \) = the distance to any section whose depth is \( y \).

The strain will be \( w x \), the resistance proportional to the square of the depth will be \( y^2 \); hence \( y^2 = w x \), which is the equation of a parabola.

Prop. 2. If the depth be constant, the horizontal section will be a trapezium. For in this case \( w x = d^2 y \), \( x = \frac{d^2}{w} y \) or \( y \) is proportional to \( x \), which is the property of a triangle.

Fig. 17.

Prop. 3. When a beam is regularly diminished towards the points that are least strained, so that all the sections will be circles, or other similar figures, the outline should be a cubic parabola.

For, in this case, if \( y \) represent the diameter, or side of the section, the resistance will be as \( y^3 \), and the equation of moments will be \( w x = y^3 \).

The same figure is proper for a beam fixed at one end and the force acting at the other.