wards, will be opposed by the weight upon \( x \) acting downward, and the difference of the moments will represent the strain.

For the first we have \( \frac{w}{3} \cdot x = \frac{wx}{3} \) = moment of force \( \frac{1}{3} w \).

For the second we have, since the weights will be as the square of the lengths, \( l^2 : x^2 :: w : \frac{wx^2}{l^2} \) = weight on \( x \). As the leverage is \( \frac{x}{3} \cdot \frac{wx^2}{l^2} \cdot \frac{1}{3} x = \frac{wx}{3 \cdot l^2} \) = moment of the weight on \( x \).

The difference will be \( \left( \frac{wx}{3} - \frac{wx^3}{3 \cdot l^2} \right) \) = strain on section.

By the principles of maxima and minima we have

\[
\left( \frac{w}{3} - \frac{wx^2}{l^2} \right) = 0. \quad x^2 = \frac{l^2}{3}. \quad x = l \sqrt{\frac{1}{3}}. *
\]

**Prop. 20.** To determine the extension of the fibres when a beam is supported at the ends and loaded in the middle.

A beam supported at the ends and loaded in the middle is in the same condition as a beam resting upon a fulcrum in the middle and loaded with equal weights at the ends.

**Fig. 37.**

Let \( l = \) one-half the whole length

\( w = \) the weight on \( A \)

\( e = \) the maximum extension, which will be at \( C \).

Now, as the extension at any distance is in proportion to the strain, it will evidently be in proportion to \( x \); and we have therefore, \( l : x :: e : \frac{ex}{l} \) = extension at the distance \( x \).

* Tredgold gives it \( \sqrt{\frac{1}{3}} l \).