that the portion of the beam $D\,S$, with the applied weight at W, presses against the cross section at S, and must be resisted by the reaction at that point.

If the horizontal force $\frac{u\,w}{2\,d}$, acting with its leverage (d), was sufficient to sustain the part $D\,S$, the effect of the weight at W would be entirely overcome, and there would remain nothing to produce a downward strain upon the fibres at (S), or, in other words, the vertical force would be zero. That this is not the case, however, can be seen by estimating the force necessary to sustain $D\,S$ in equilibrium.

As (W) acts with a leverage $D\,W$ or $A\,n$, the equation of moments will be $H'\,d = \frac{w\,l}{2}$, or $H' = \frac{w\,l}{2\,d}$. But we have seen that the horizontal strain at S is actually $H = \frac{u\,w}{2\,d}$.

$$\text{The difference is } (H' - H) = \frac{w}{2\,d}\,(l - u).$$

As this expression cannot become zero for any point between W and C, it follows, that the horizontal force is not sufficient to sustain the weight, and there must consequently be a cross strain upon the fibres which must compensate for this deficiency, and be resisted by a vertical reaction.

Call this vertical force f, it acts with a leverage $= D\,S = (l - u)$. The difference of the horizontal forces, or $H' - H$, acts with a leverage $= d$. The equation of moments will therefore be $f\,(l - u) = d \cdot \frac{w}{2\,d}\,(l - u)$, from which, we obtain $f = \frac{w}{2}$, or, *the cross strain upon the fibres produced by a weight applied in the middle is constant at every point, and equal to one-half the weight.*

We can obtain the same result by another method. Using the same notation as before, we may suppose that the vertical force (f) [acting at S with a leverage $S\,D = (l - u)$,] and the horizontal strain at S, acting with a leverage d, sustain in equilibrium the weight (W) acting with a leverage $W\,D = \frac{l}{2}$