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Visualizing Concrete Economy in Terms of Strength

Straight-line relation between strength and cement-water ratio permits direct measure of cost in terms of strength—Cost per unit of strength decreases with an increase in strength

By Inge Lyse

Research Assistant Professor of Engineering Materials, Fritz Engineering Laboratory, Lehigh University, Bethlehem, Pa.

IN PREVIOUS articles the writer has shown that the strength of concrete is determined by the concentration of cement particles in the cement paste (*ENR*, Nov. 5, 1931, p. 723), and that the design and control of concrete mixes are greatly simplified by this relation (*ENR*, Feb. 18, 1932, p. 248). Data given in these articles show that, for concretes containing aggregates of a given type and gradation, the consistency remains constant regardless of the richness of the mix if the water content per unit of concrete remains constant. Thus the straight-line relationship between the strength and the cement-water ratio of the concrete generates into a straight-line relation between strength and cement content, expressed as follows:

$S = A + B\,(c/w) = A + (B/w)\,c = A + Kc$ where S is strength of concrete, A and B are constants depending upon the materials and conditions of test, and c and w are amounts of cement and water in a cubic yard of fresh concrete. If the water content per unit of concrete is held constant, B/w becomes a new constant, K. Thus for given materials and conditions of test the strength of the concrete is proportional to the cement content.

The total cost of the concrete is composed of the cost of the materials plus the cost of formwork, mixing, placing, curing, etc. Since the costs of formwork, mixing and placing are practically the same for concrete mixes of different richness as long as their consistencies are nearly the same, the variation in the cost of concretes of different richness is mainly due to the difference in the cost of the materials—aggregates and cement. For 1 cu.yd. of concrete the cost of the materials may be expressed by: $P = p_1 a + p_2 c$ where p_1 and p_2 are unit prices of aggregates and cement, and a and c are aggregate and cement contents per cubic yard of concrete. Since the total volume of properly placed concrete is the sum of the absolute volumes of aggregates, cement and water, the volume of the freshly placed concrete may be expressed by: $V = V_a + V_c + V_w$. The absolute volumes are obtained by dividing the weight of the materials by their specific weights, so that the above relation becomes:

$$V = \frac{1}{27\times 62.4}\left(\frac{a}{ga}+\frac{c}{gc}+\frac{w}{1.0}\right)$$

$$= \frac{1}{27\times 62.4}\left(\frac{a}{2.65}+\frac{c}{3.10}+\frac{w}{1.0}\right)$$

$$= D + \frac{w}{27\times 62.4}$$

where

$$D = \frac{1}{27\times 62.4}\left(\frac{a}{2.65}+\frac{c}{3.10}\right)$$

$$\text{or } a = 4460D - 0.85c$$

The value 27×62.4 represents the weight of 1 cu.yd. of water, ga and gc are specific gravities of aggregates and cement respectively, and D is density, or portion of solid materials in the concrete.

The cost of the materials is then:

$$P = p_1 a + p_2 c = p_1 4460D - p_1\times 0.85c + p_2 c = 4460Dp_1 + (p_2 - 0.85p_1)c$$

Thus the cost of the materials per cubic yard of concrete varies directly with the cement content for given unit prices of aggregate and cement and for a given consistency of concrete.

The cost of the concrete may also be expressed in terms of the strength of the concrete from the following relation:

$$S = A + Kc,\ \text{or}\ c = \frac{S-A}{K},$$

$$\text{so that } P = 4460D.\,p_1 + (p_2 - 0.85p_1)\left(\frac{S-A}{K}\right)$$

If the strength-giving qualities of the materials are known, it is a simple matter to compute the cost for any given strength of concrete. It follows that for concrete containing aggregates of a given type and gradation and containing a constant amount of water per cubic yard of concrete, the cost per unit of concrete varies with the strength of the concrete.

The following illustration will make the significance of this fact more apparent. Assume the use of aggregates requiring 303 lb. of water per cubic yard of concrete for the desired consistency. The volume of water in the concrete then is $303/(27 \times 62.4)$, or 18 per cent of the volume of the fresh concrete. Since the density, D, is the difference between the volume of concrete and the volume of water, $D = 1.0 - 0.18$, or 82 per cent of the volume of the concrete. With these units the cost equation becomes:

$$P = 4460\times 0.82\times p_1 + (p_2 - 0.85p_1)c.$$

If the price of aggregate is \$2 per ton or 10 cents per pound, and the price of cement is \$1.88 per barrel, or 50 cents per pound, the cost of materials for 1 cu.yd. of concrete is:

$$P = 4460\times 0.82\times 0.10 + (0.50 - 0.85\times 0.10)c = 3660 + 0.415c$$

cents, where c is given in pounds.

In Fig. 1 the relation between cost and cement content is given for a price of \$2 a ton for aggregates and prices of \$1.10, \$1.88 and \$2.60 per barrel of cement. In Fig. 2 the price of cement was \$1.88 per barrel, and the prices of

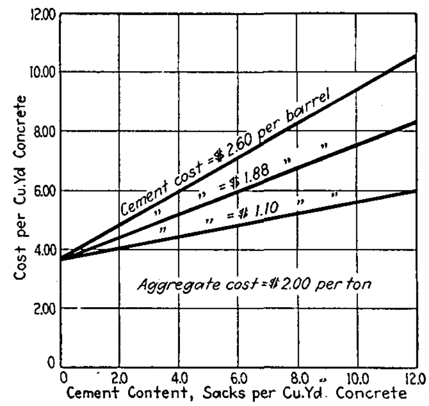

Fig. 1—Relation between cost and cement content of concrete mixes for different prices of cement and with aggregate cost constant.

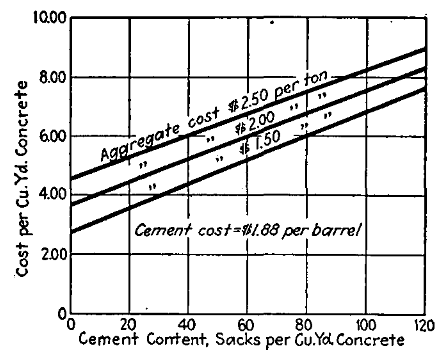

Fig. 2—Relation between cost and cement content of concrete mixes for different prices of aggregate with cement cost constant.

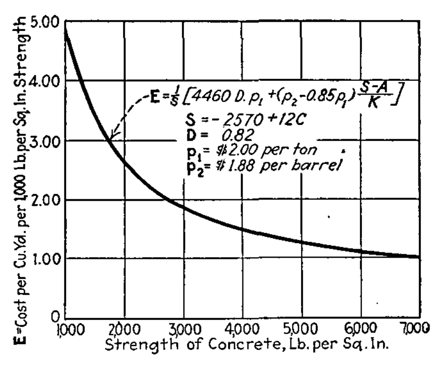

Fig. 3—Cost of concrete per unit of strength (1,000 lb. per sq.in.) decreases rapidly with an increase in concrete strength.

aggregate were \$1.50, \$2.00 and \$2.50 per ton.

In structural concrete members, where the total area is in compression, the load-carrying capacity of the member is directly proportional to the strength of the concrete. The cost of materials for 1 cu.yd. of concrete per pound of strength of the concrete is therefore a direct measure of the economy of the concrete mix. This cost per unit of strength is expressed by the formula:

$$E=\frac{P}{S}=\frac{1}{S}\left[4460Dp_1+(p_2-0.85p_1)\frac{S-A}{K}\right]$$

Since for given materials and conditions of construction all units on the right of the equation are given, the cost can be computed directly. The relation between cost of concrete per unit of strength and the strength of the concrete is shown in Fig. 3 for given materials and conditions of construction. It is seen that the cost per unit of strength decreases rapidly with an increase in strength of the concrete. Thus for concrete members loaded directly in compression the more economical mix is the one giving the higher strength.

The above relations are valuable aids for selecting the most economical type of aggregate and cement for use on a given job. The relations also give information as to the feasibility of using ordinary portland or high-early-strength cement. Similar relations may also be found for concrete subjected to flexural stresses. Thus by means of the constant water requirement for a given workability and the straight-line relation between strength and cement-water ratio, the most economical mix is readily determined for all types of plain and reinforced-concrete members.

In addition to the direct effect of the strength of the concrete on the economy of the mix, an additional important economic item is the reduction in size of the member. The benefit of the higher strength concrete is therefore magnified by the accompanied reduction of size of member for a given load. For columns the saving in floor space by the use of high-strength concrete becomes very important.

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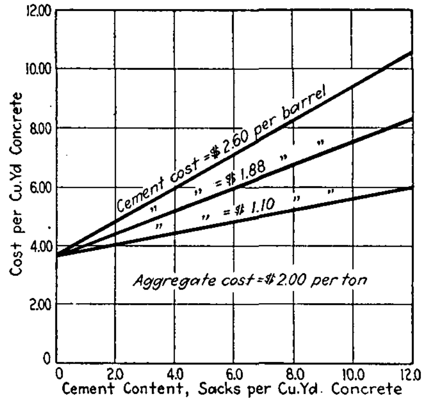

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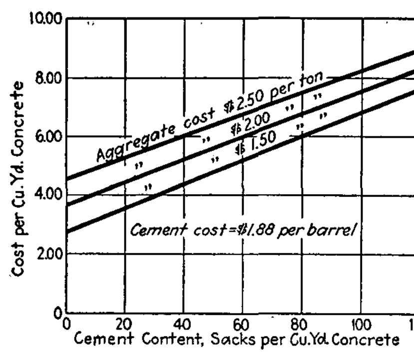
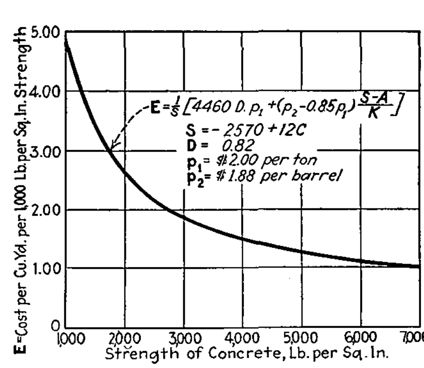


Fig. 2—Relation between cost and cement content of concrete mixes for different prices of aggregate with cement cost constant.



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