Presented before the Structural Steel Committee of the American Welding Society on April 26, 1934.
Slides 1-10

FIRST PROGRESS REPORT OF SEAT ANGLE INVESTIGATION BY PHOTOELASTIC METHODS

by D.M. Stewart

April 1, 1934

FRITZ ENGINEERING LABORATORY
LEHIGH UNIVERSITY
BETHLEHEM, PENNSYLVANIA

A photoelastic study of the welded seat angle problem is being made in conjunction with Mr. Schreiner's research, and this report summarizes the results to date. A photoelastic investigation is restricted by the nature of the apparatus to two-dimensional problems under plane loading, and as a result this investigation is limited to a study of the lateral bending effect mentioned as the second factor in Mr. Schreiner's report. Furthermore, ideal welding conditions must be assumed and continuity of welding material and parent metal. Inasmuch as the pilot tests already conducted have indicated that, when the fillet weld is of sufficient size, failure of the angle in bending will occur first, this study of the bending effect becomes of importance. Furthermore, a model study will permit the study of other variables, and in this case, the effect of varying the size of the fillet in the angle as rolled was investigated as being of significance in causing bending rigidity.

Three models have been made and tested, these being a half-scale section of a pair of 4 x 4 x 1/2, 6 x 4 x 5/8 and 8 x 4 x 1 angles respectively. These models were accurately machined from a sheet of 1/4" thick transparent bakelite and carefully polished and annealed to remove initial stresses. At the same time a calibration beam was cut from the same sheet and subjected to identical conditions of annealing and storing. The seat angle models were then loaded thru rollers by dead weights in a manner identical with the steel angles.

In case some of you may not be familiar with photoelastic methods, I might explain them briefly. It is possible by passing plane polarized light thru the model to obtain lines from which the direction of the two principal stresses at any point, that is the stress trajectories, may be determined. Furthermore, each fringe as shown when the model is subjected to circularly polarized light , represents a definite value of the difference of these principal stresses, (P-Q), at that point. Since the maximum shear is also proportional to (P-Q) , these fringes are also lines of constant maximum shear. At the free boundaries of a model, where the stresses have in general their critical values, one of the principal stresses vanishes, and thus the fringe order is a direct measure of the stress along the boundaries. This latter fact was made use of in this investigation and from the calibration beam under uniform bending moment, the value of the stress difference for each fringe was determined to be 346 $\#/in^2$ x the number of the fringe. From the values of the extreme fiber stress, moment curves were then plotted, and a first integration of these gave the slope at any point, while a second integration gave the deflection curve of the outstanding leg.

It was evident that the 4 x 4 x 1/2 model was too small to give accurate results because of the high stresses developed.

However, the results obtained conform in general trends with later results on the other models. The 8 x 4 x 1 model is under tests at present. Consequently this report deals with the 6 x 4 x 5/8 model, tested with fillet sizes of 1",3/4" and 1/2" radius, the latter being a standard section.

The variables studied were as follows:

1) With the load at 2" from the back of angle, the load was varied. This was done for only one fillet size.

2) With the load constant (16# per angle) the lever arm was varied, the points selected being 1.2",2.0",2.5" and 3.0" from the back of angle. This was done for all fillet sizes.

3) From these results, a comparison was made of the three fillet sizes, all other variables remaining constant.

A discussion of these items follows.

1. The first slide shows the effect of the variation of load upon the maximum stress, on both the compression and tension sides. As to be expected, the relation is linear, and the accuracy of thenpoints gives an idea of the general accuracy of the stress values. It is to be noted that the compressive values exceed the tensile ones, indicating that the position of the neutral plane is below the center of the section.

The second slide shows the variation in end slope and deflection under the load with varying loads. In this case the values are more accurate, since the process of integration by which they were obtained tends to iron out any irregularities in the data. Both curves are linear in form, as is to be expected.

2. The constant load of 16# per angle was then applied to each of the four lever arms and the next slide shows a typical set of results for the 6 x 4 x 5/8 model with 1/2" fillet. The maximum stress, both in tension and compression, is seen to vary linearly down to the point where the influence of the fillet is felt. The curve for the end slope is a parabola, while that for the deflection under the load is a cubic parabola, both modified at low values by the restraint caused by the fillet.

3. The next slide shows the actual variation in fiber stress along the boundaries of the model for the three different fillet sizes. This is the 3" lever arm and the other lever arms show identical effects. On the tension side, we may note that all the curves practically coincide outside of the fillet. Also they reach a maximum value directly above the point where the fillet commences and this maximum value increases as the fillet size decreases. On the compression side, ordinates around the fillet have been plorted radially. Again we note that the curves coincide beyond the fillet, and that they reach zero values directly under the load. The maximum compressive stress occurs on the fillet at an angle of roughly 20°-30° with the vertical, and the values increase with decrease in filletcsize. It might be noted at this point that chipping of the whitewash in actual tests commences at this point on the fillet as nearly as can be observed. Finally, in the compression portion of the specimen, the stress falls off rapidly and the lower tip of the angle actually takes no stress at all.

The quantitative effect of the fillet size has been summarized in the next three slides. The first shows the variation in end slope for the four load points; it is seen that the relation is linear. The next shows variation in deflection under the load for the various loading points and again the relationship is linear. The third shows the variation in maximum stress, both tensile and compressive, with fillet raduis for the four leverarms. The relationship appears to be linear, at least in this portion of the curves, although we know that for zero radius we should obtain theoretically an infinite stress in compression. We may note that although the maximum tensile stress increases as the fillet grows smaller, it does not increase as fast as the compression. The table shown in the next slide presents a summary of the results on the preceding slide, expressing the maximum stress as a per cent of the stress with a 1/2" fillet. It will be seen that a 3/4" fillet reduces the compressive stress to an average of 75% and the tensile stress to an average of 88%. Also a 1" fillet reduces the compressive stress to an average of 56% and the tensile stress to an average of 74% of the stresses in a standard angle as rolled with a 1/2" fillet. Since the compressive stresses limit the strength of the angle in every case, it is these values that are of greatest significance, and the reduction in stress caused by the addition of only a small amount of material is remarkable. A 3/4" fillet would increase the weight of the section only 1.2% over the standard size, while a 1" fillet would increase it only 2.7%.

The next slide shows on the right side the results of a study made of the directions of the principal stresses, or stress trajectories, throughout the entire model. This consists of two sets of mutually perpendicular lines and at all free boundaries one set of lines is normal to the boundary. The crowding together of the lines around the fillet is well shown, as well as the almost pure tension existing just above the fillet and the pure compression in the main body of the model. On the left side of the figure are shown the directions of the maximum shearing stresses, or shearing trajectories, which are two sets of mutually perpendicular lines inclined at 45° to the stress trajectories. A singular point just over the fillet, shown in these two figures, has been found to be a point of zero stress, which may be said to correspond to the neutral axis. A similar study was made of each of the three fillet sizes, to determine whether this variable altered the stress directions much. Slight variations were noted, but the distribution of stress was not markedly altered.

The vertical shearing stress was then analyzed by use of Mohr's circle along sections A, B, and C as shown on this figure. The results are shown on the next slide. On section A we have the usual parabolic distribution of the vertical shear. At B, the stress trajectories are horizontal and vertical for a short distance down from the top; hence there is no vertical shear for a short distance. Also, since the lower edge of the section is curved, the vertical shear has a value at this point. At C, quite a different form of vertical shear distribution is observed. An explanation of this is seen by returning to the previous slide. At C, down to a certain point the lines of stress (in tension) are pulling up, instead of down as at A; in other words the sign of the shear is reversed.

Below this point the compressive forces are pushing down as at A, and the sign of the shear is again positive. Values of the average vertical shearing stress over the section and also the maximum values are shown on the other slide. The worst section is that at B, which is the transition point between distribution diagrams of types A and C. The area under each of these curves, representing the internal shear, has been found to check the external shear within about 10% in each case.

That concludes the summary of results obtained with the 6 x 4 x 5/8 specimen. Future work includes the comparison of stresses and deflections with those obtained by Mr. Schreiner in his tests, especially those with heavy fillet welds. An 8 x 4 x 1 model is being tested in an exactly similar manner, with three varying fillet sizes, and the results will be analyzed similarly. A further study contemplated on this model is the gradual cutting down of the vertical leg of the angle, observing the point at which the first change in stress values around the fillet is noted. This will give us some idea of the value of this vertical leg in bending, providing it is adequately strengthened to resist shear.

Timoshenko in his new book "Theory of Elasticity" gives the following general equation for the two-dimensional problem:

$$\frac{\partial^4 \phi}{\partial x^4} + 2\frac{\partial^4 \phi}{\partial x^2 \partial y^2} + \frac{\partial^4 \phi}{\partial y^4} = 0$$

where ϕ is a stress function which must be evaluated for any special case, subject to definite boundary conditions. This stress function is independent of the material being loaded, the equation containing no constant term to distinguish one material from another. A typical stress function given by Timoshenko is $\phi = \frac{a}{2}x^2 + b\,xy + \frac{c}{2}y^2$, where a, b and c depend on boundary conditions alone.

Since the photoelastic effect is caused by retardation of light waves in proportion to the stress difference, the fringes produced therefore are independent of the material used. Of course, when dealing with strains or deflections the modulus of elasticity of the material must be used in comparing two materials.