STRUCTURAL BEAMS IN TORSION

Discussion
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In Fig. 9 the authors evince the fact that there has been considerable diversity of opinion regarding the problem of stress concentration in the fillets of twisted bars. Several analytical and experimental solutions giving widely varying results have been proffered. An investigation of the effect of size and shape of structural members on torsional behavior was consequently carried out at the Fritz Engineering Laboratory of Lehigh University in an attempt to help in the final solution of this problem. Over eighty experimental solutions involving various structural shapes were found. The membrane analogy provided a fairly rapid method for determining the increase of shearing stress in fillets by means of soap films. Torsion constants were also obtained for most of the sections analyzed.

The following variables were studied: radius of fillet; length and thickness of components of member; and shape of member, angle or I- section. In each series the fillet varied from about 1/8-in. to 1-1/2 in. Eight series of angles having 1/2-in. to 1-in. thicknesses and 3-in. to 6-in. lengths and three I- wide flange series corresponding to three of the angle

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series were analyzed to determine the change of the hump at the junctures. An angle formed by taking half the I-section, viz. a T-section, and clipping one wing of the flange with the web intact will be said to correspond to that particular I-section. For the sake of clarity it is mentioned that an angle having long sides of 4 in. and 3 in. and constant thickness of 3/4-in. corresponds to an I- wide flange section having a depth of 8 in., a flange width of 5-1/4 in., and flange and web thicknesses of 3/4-in. Torsion constants were evaluated for all the I-sections and for six of the angle series.

The radius of the circular hole which is used for comparison in soap-film analysis must be approximately equal to twice the area divided by the perimeter of the test hole. Volume correction factors allowed for a slight deviation from this precaution. The errors due to the assumptions that the sine equals the tangent and that the pressure acts vertically instead of normal to the surface of the film are largely eliminated and pressure correction is achieved by resorting to the volume correction curve which was procured by testing several sections of known torsional properties against each other, two at a time. The initial height of water in the flask, an essential part of the volume displacement apparatus, was at all times kept approximately equal to that prevailing in the preliminary volume correction tests.

In contour work the height of the circular film was checked after every five or six points to insure that conditions remained constant. Elevations were correct to within a thousandth of an inch. It was found that a boundary angle ranging between

15 and 25 degrees was advisable for the circular film in order to obtain a precision of two per cent in the stress concentration tests. For satisfactory volume measurements the boundary angle for the test film should not exceed 35 degrees. The minimum radius of the circular section in this investigation was 1/2-in., a value which should not be lower for reliable results according to Taylor[1]. In the case of a symmetrical shape, such as the I-beam, only half of the section was studied. A vertical septum passing through the axis of symmetry provided the essential continuity. It was necessary to double the torsion constant of the reduced shape in order to get the desired constant.

After a few complete contour diagrams were made, it was thought best to diminish the amount of work required for a section by adopting the so-called cross-section method wherein points spaced from 0.03- to 0.1-in. apart along a normal to the boundary were located and their elevations recorded. Especial care and smaller spacings were taken immediately near the edge, since there lies the crux of the slope determination. Tangents to enlarged drawings of the films at the boundaries gave a direct measure of the border shearing stresses. Cross sections were taken in the case of angles at the center of the fillet and at the center of the inner straight portion of the flanges. For I-sections additional slope data were recorded for the web. The shearing stress in the fillet could thus be compared with that in the arms of angles and with the shear in the flange and web of I-sections.

1 Taylor, G.I. - The Mechanical Properties of Fluids, A Collective Work, 1924, p.236.

The effect of size and shape of members on stress concentration may be observed from Figs. 1 and 2, which give the results of a few series. It is apparent that the stress concentrations of I-sections are considerably higher than for their corresponding angles. An angle 4 by 3 by 1/2 is one having long sides 4 and 3 in., with the thickness of both legs 1/2 in. An I-section 8 by 5-1/4 by 3/4 by 3/4 has a depth of 8 in., a flange width of 5-1/4 in., with web and flange thicknesses of 3/4 in. That the stress varies directly with the thickness may be concluded from the fact that the fillet stress concentration factors are inversely related to the thickness. The lowest points on the curves are peculiar in two respects. As the thickness of the straight portion increases, the ratio of fillet to thickness for the minimum concentration decreases linearly according to Fig. 3, which also shows a fairly straight line variation of magnitude of minimum concentration factor. This figure was obtained by taking averages of all the angle series.

A comparison of the results of this investigation with those of Timoshenko's analytical solution and Taylor's soap-film tests is found in Fig. 2. For small fillets, including the usual radii, Timoshenko's formula proved to be superior to all others for angles. Timoshenko obtained his approximate theoretical solution[2] from the membrane analogy by assuming that

2 Timoshenko, S. - Theory of Elasticity, first edition, 1934, p. 258

the shearing stress in the fillet becomes zero at a point n/2 from the boundary, where n is the thickness of flange. Soap-film tests showed that this assumption is true for small radii of fillet. For larger fillet sizes there is fair agreement between the tests of Tayler and those of the present investigation.

Torsion constants are found in Figs. 4 and 5, from which it is evident that the thickness of angle has a considerable effect on the rigidity, whereas the effect of length of arm is comparatively small. Experimental results were checked analytically and a fair precision was noted.

The following are the principal deductions drawn from these tests:

1. The inherent relations of the membrane analogy have proved especially useful in determining the stress concentration factors at the fillets of a twisted rod.

2. Timoshenko's formula for stress concentration proved suitable for the usual fillet radii in the case of angles, whereas that proposed by Westergaard and Mindlin gave results having similar tendencies manifested in the present tests on I-sections.

3. Observations of the soap films in the case of I-sections confirmed the fact mentioned by the authors that the critical shearing stresses prevail at the fillets and at the center of the outer side of the flange.

4. There is a linear relation between the minimum percentage increase of stress in the fillets of angles and the thickness of the straight portion. The ratio of fillet size to flange thickness, f/n, for minimum concentration factor also shifts linearly with variation in thickness.

5. The shearing stress in general increases linearly with the thickness. In I-sections, however, the stress in the web is

slightly greater than in the flange for the same web and flange thickness. The stress falls off rapidly at the fillet, the rate being greater for small radii.

6. For ductile materials not subjected to alternating stress, the use of small fillets does not involve danger because of the redistribution of stress that follows local yielding. In the case of brittle materials, however, the weakening effect of the stress concentration should be mitigated by the use of greater fillet radii.