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Design Economy by Connection Restraint

Bruce Johnston
Assistant Director
Fritz Engineering Laboratory

Robert A. Hechtman
American Institute of Steel Construction
Research Fellow

Lehigh University, Bethlehem, Pa.

Contents in Brief—*Heretofore continuity in building framing has been taken advantage of infrequently because of a lack of information on the restraint values of beam-column connections. Laboratory work has now provided much of this information and a rational and workable design procedure has been developed. Beams are first designed for maximum moment assuming simple supports. Then the ratio of beam stiffness to the sum of column stiffnesses at the joint is calculated. Charts equating this ratio to the percentage rigidity give a reduction factor F, which is applied to the section modulus of the simple beam. The beam corresponding to this reduced section modulus is the one to use. It will be 15 to 20 per cent lighter than if assumed to be simply supported.*

THE DESIGN OF THE BEAMS in multistoried steel building frames has usually been based on the simplifying assumption that the ends of the beam are freely supported. While this assumption leads to a safe design, economy is sacrificed since no account is taken of the reduction in maximum positive moment that results from the end restraint that is present even in the most flexible connections. It is notable that slight increases in the stiffness of standard types of end connections provide enough end restraint to reduce the average weight of beams in a building frame by 15 to 20 per cent.

Unfortunately the application to building frames of methods of analyzing continuous structures is exceedingly laborious, and furthermore, there has been considerable uncertainty as to just how dependable and to what degree a semi-rigid connection provides end restraint. Now, however, experimental evaluation of the behavior of various types of beam-column connections has furnished much of the information necessary for the design of rigid and semi-rigidly connected frames, and this article presents such a design method which may be applied to any building frame in which the minimum restraint values of the connections have been determined.

Background of the method

The possibilities of economy are greatest when there is a repetition of similar span lengths, load conditions and connection types. To demonstrate the possible economy, a study of 105 beam sizes for various uniform loads and degrees of restraint has been made by the writers, showing substantial saving in weight for handbook selection of beams, even for cases of small percentage rigidity. The beam sizes ranged from 12-in. 22-lb. sections to 21 in. 63-lb. sections, the spans from 16 to 24 ft., and the loads from 80 to 120 lb. per sq ft. of floor. Fig. 1 shows the average minimum savings for various percentage rigidities. The beams were designed by the method outlined in this article.

Correlated to the problem of beam design are the problems of connection design, column design, and analysis for wind stresses. Experimental work on connections and columns are now in progress at Lehigh with the specific problem of building design in mind. Experimental work on moment-resisting riveted and welded connections that has furnished much of the necessary information for this design method are listed in the bibliography at the end of this article.

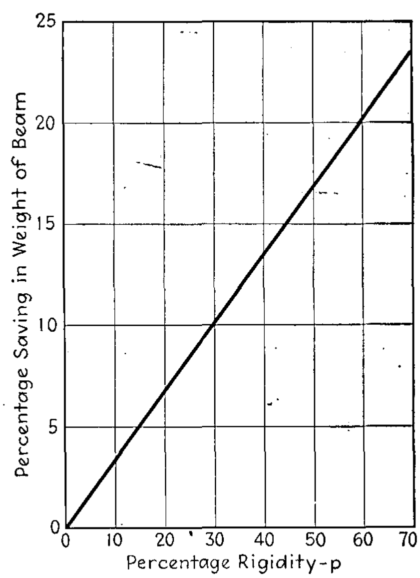

Fig. 1. Saving in weight of beams for various degrees of restraint from 105 trial designs.

The semi-rigid joint

Before discussing the design of beams, it is necessary to have a definition of the term, semi-rigid joint. If the beam-column connections of a building frame transmit bending moment without relative rotation between the end of the beam and the column, the connection and the structure are termed "rigid" (Fig. 2a). In such a case, the connections afford 100 per cent restraint or full continuity, and the maximum bending moments are at the ends of the beam. If the connections transmit bending moment with some relative rotation between the end of the beam and the column, the connections and the structure are termed "semi-rigid" (Fig. 2b). In such a structure, the connections resist bending moment to some degree less than in the case of full continuity, and the moment in the center of the span is always less than if the connection afforded no restraint, as in a simply supported beam (Fig. 2c).

The semi-rigid joint, such as the standard beam web connection, the top and seat angle connection, and the split-I connection thus results in a restraint somewhere between full fixity and full freedom of rotation. It is important to note that 100 per cent restraint does not afford the greatest possible economy in building construction, largely because the cost of making the rigid connection tends to overbalance the saving in beam cost. Maximum economy for beam and connections usually occurs at a degree of restraint somewhere between 40 and 75 per cent.

The joint constant

A typical graph of the test of a semi-rigid connection is shown in Fig. 3 in which applied connection moment is plotted against relative column-beam end rotation. The connection passes through three stages: first, an initial stage where moment is approximately proportional to rotation; second, a yielding of the connection; and third, a stage of accelerated rotation finally resulting either in failure or very excessive deformation.

The first stage is the useful design range of the connection. It is especially important that the connection also have a sufficient factor of safety with respect to rotation. The maximum rotation which a semi-rigid connection approaches is the simple-beam end slope, and this occurs well within the rotation at failure for all semi-rigid connections except a few having very high rigidities. In these few cases, the working moment must be based on the ultimate moment.

The experimental determination of one factor is necessary as a basis for the design method. This is the connection constant, J, which may be defined as:

$$J = \frac{M}{E\phi} \qquad (1)$$

where ϕ is the rotation due to an applied moment M and E is Young's modulus. Physically, the joint constant is the slope of the first stage of the moment-rotation curve divided by the modulus of elasticity of the material. It is a measure of the connection stiffness. A connection, therefore, whose joint constant J is large is more rigid, or has more moment-taking ability within its working capacity, than one whose joint constant is small.

The percentage rigidity, p, depends on the connection constant J and the stiffness of the beam K, which is the gross moment of inertia of the cross section divided by the span length.

$$p = \frac{100}{1 + 2\frac{K}{J}} \qquad (2)$$

The percentage rigidity, then, is fixed when the connection and the beam size are chosen.

Fig. 4 is plotted from Eq. 2 and shows the relation between the joint constant and the percentage rigidity for various values of beam stiffness in the case of a beam fastened to rigid walls by semi-rigid joints. Most building beams have a stiffness of 0.5 to 5.0. It may be seen that in the design range of p less than 70 per cent, a considerable variation in the connection constant J has little influence on the percentage rigidity p. For instance, in the case of a beam of stiffness $K = 1.5$ and $J = 10$, the reduction of the connection stiffness J by 100 per cent to 5 would change the span-end moment less than 20 per cent.

It follows that while differences in welding or riveting processes may affect the value of J considerably there will be relatively much less variation in the actual beam moment. It also follows that a range of permissible variation in connection behavior, as shown in Fig. 3, should be allowed for any typical connection to take care of variations in fabricating as well as non-uniform relationship between moment and relative angle change.

Proposed design method

The proposed method of design is one that proportions the connection for the semi-fixed end moment which would occur if the columns did not rotate, and proportions the beam for maximum center moment which occurs when the columns do rotate.

In order to develop a direct method of design the beams in spans adjacent to that under consideration will be neglected. The approximation is on the side of safety and also allows the method to be applied to outside panels which have no adjacent beams. Fig. 5a shows the most critical load condition for maximum moment in beam AB, and Fig. 5b shows the same beam with adjacent beams

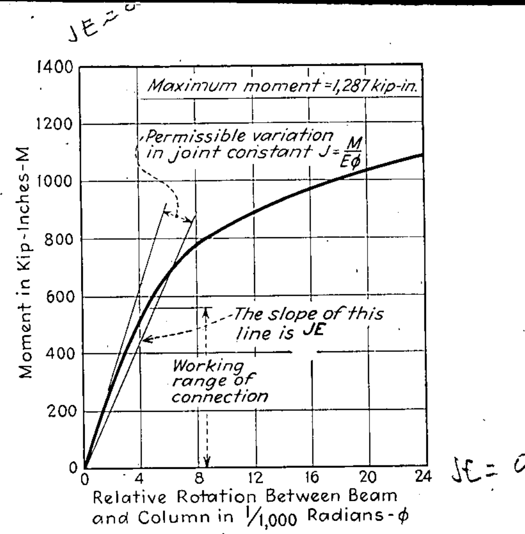

Fig. 3. Typical test curve of a semi-rigid connection. It will be noted that variation in joint constant does not greatly affect the moment that can be carried.

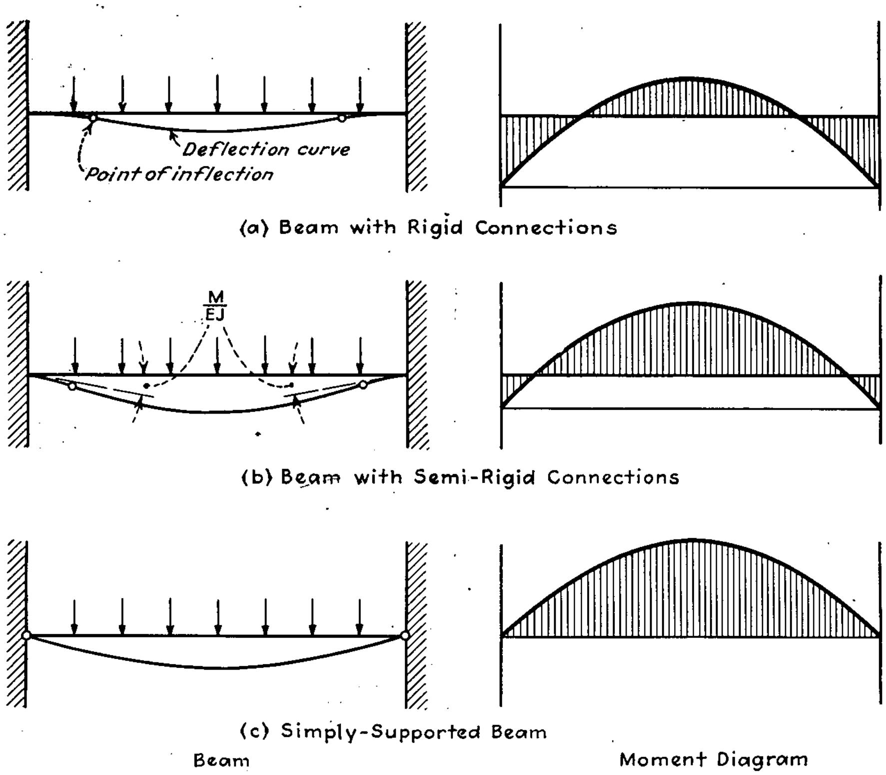

Fig. 2. Beam with different conditions of end restraint, showing how bending moments vary at the center and at the supports.

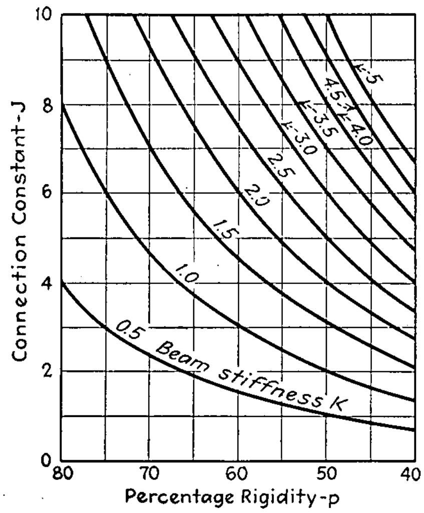

Fig. 4. Relation between connection constant and percentage rigidity.

omitted. Symmetrical conditions of load, connections, and adjacent columns are assumed to exist. Connections of 50 per cent rigidity are assumed in the following derivation.

The ordinary relation between the moment at the end A of a beam, AB, and the angle changes at its two ends is;

$$M_{AB} = 2EK\,(2\Theta_A + \Theta_B) \pm M_R \qquad (3)$$

where M_R is the fixed end moment in a fully rigid connection.

When semi-rigid connections providing 50 per cent rigidity are introduced the equation becomes:

$$M_{AB} = EK\,(1.25\Theta_A + 0.25\Theta_B) \pm \frac{M_R}{2} \qquad (4)$$

Due to symmetry, $\Theta_A = -\Theta_B = -\Theta_C$

Hence:

$$M_{AB} = EK\Theta_A - \frac{M_R}{2} \qquad (5)$$

The moments acting on the joint must be in static equilibrium, hence:

$$M_{AB} + 2M_{AC} = 0 \quad (\text{since } M_{AC} = M_{AE}) \qquad (6)$$

Substituting (4) and (5) into (6) there results

$$\Theta_A = \frac{M_R}{2}\left(\frac{1}{4\,EK_C + EK_B}\right)$$

$$= \frac{M_R}{2}\left(\frac{1}{2E\Sigma K_C + EK_B}\right) \qquad (7)$$

Subscripts C and B in Eq. 7 refer to columns and beam, respectively.

Substituting (7) into Eq. (5), there results

$$M_{AB} = -M_R\left(\frac{1}{2 + \frac{K_B}{\Sigma K_C}}\right) \qquad (8)$$

The moment at the center of the beam is given by

$$M_C = M_S + M_{AB}$$

$$= M_S - M_R\left(\frac{1}{2 + \frac{K_B}{\Sigma K_C}}\right) \qquad (9)$$

For rigidities equal to or less than 75 per cent the center moment is maximum and will govern the design of the beam. In the design procedure the beam is first designed as a simple beam, freely supported. The required simple beam section modulus is then multiplied by a reduction factor F which gives the section modulus required for the worst condition of loading but which takes advantage of the semi-rigid connections.

$$F = \frac{\text{Section modulus required by proposed method}}{\text{Section modulus required for simple beam}} = \frac{M_C}{M_S}$$

Hence:

$$F = \frac{M_C}{M_S} = 1 - \frac{M_R}{M_S}\left(\frac{1}{2 + \frac{K_B}{\Sigma K_C}}\right) \qquad (10)$$

The reduction factor F, then, is the factor by which the simple-beam section modulus is multiplied to obtain the required section modulus for the beam. The ratio M_R/M_S depends on the type of load.

Eq. (10) was evaluated for end connections providing 50 per cent rigidity; similar equations have been derived for other rigidities. In Figs. 6, 7, and 8, charts are shown which give the reduction factor for various types of loads and percentage rigidities.

The design procedure

The following design procedure is based upon the assumption that data are available which give the dependable end restraint value, or "percentage rigidity", of any standard connection. Such values have already been evaluated for a limited number of connection types. Tests now in progress at the Fritz Laboratory sponsored by the American Institute of Steel Construction will supplement previous work on riveted connections by J. Charles Rathbun[3] in this country and by J. F. Baker[2], C. Batho[2] and others in England. The combined results of these tests should establish dependable criteria for riveted connections. In the welding field, highly rigid connections have been tested[6], but in the semi-rigid class only the seat and top angle type has been studied in detail[7]. Further work is needed on various types of welded connections.

It is important to note that, in spite of the present lack of established standards, the application of this design procedure may be made to any particular building design through the expedient of actually testing typical proposed connections to be used in the structure.

The actual details of the design procedure may be outlined as follows:

1. Design the beams for maximum bending moment assuming simple supports.
2. Calculate $K_B = \frac{I_B}{l_B}$ for the beam and $\Sigma K_C = \Sigma\frac{I_C}{l_C}$ for the columns above and below one end of the beam.
3. Determine the ratio of $\frac{K_B}{\Sigma K_C}$

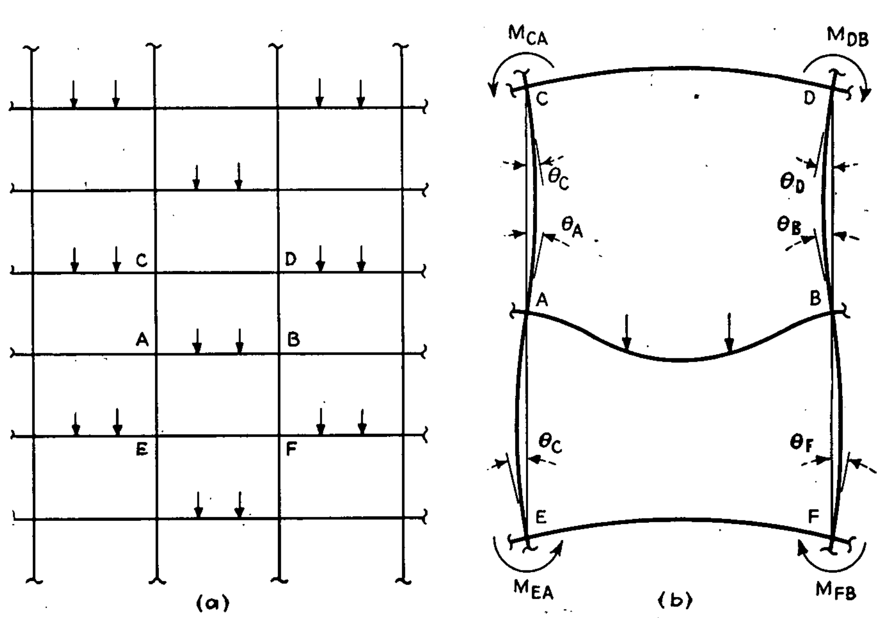

Fig. 5. Analysis of semi-rigidly connected frame for critical loading condition.

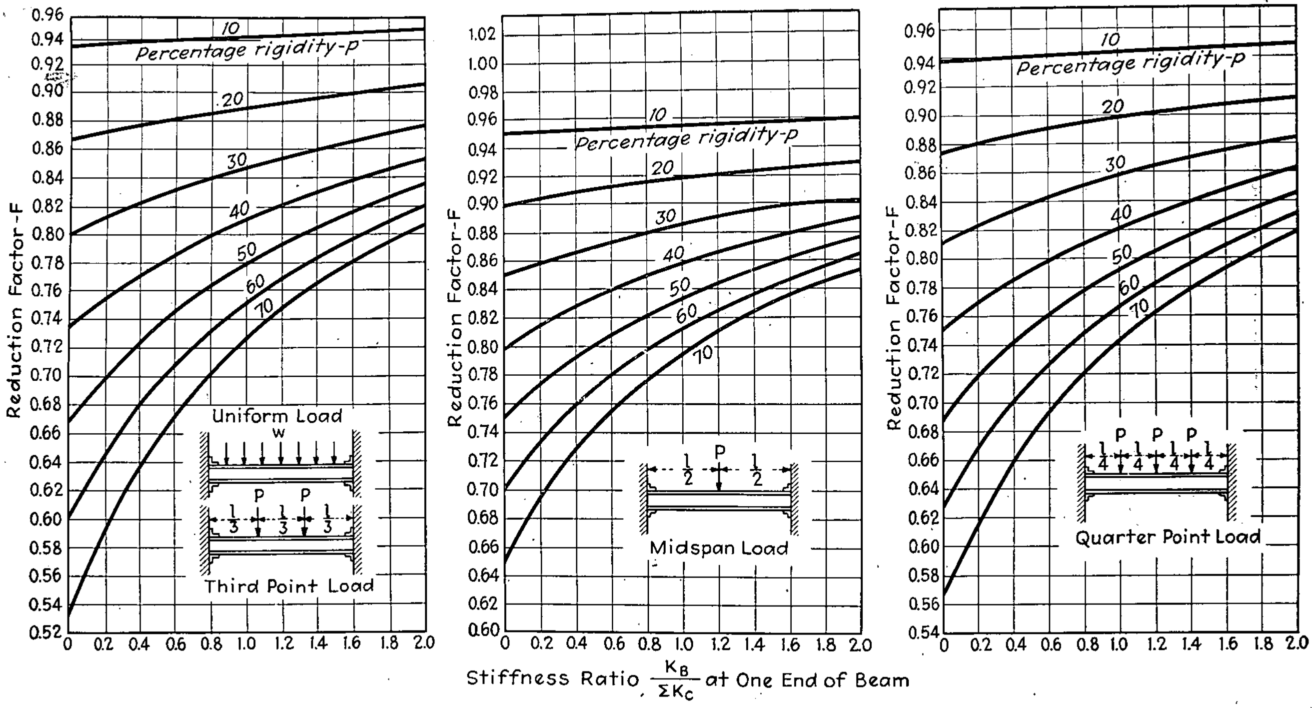

Fig. 6. Reduction factor F for four different types of loading. Abscissas are ratios of beam and column rigidities at a joint.

decide on the percentage rigidity to use in design, and determine from Figs. 6, 7, or 8 the reduction factor F for the existing load condition.

4. Multiply the section modulus required for simple-beam design by the reduction factor F, and redesign the beam on the basis of the reduced modulus.

5. Calculate the semi-rigid end moment for the condition of all beams loaded by multiplying the fixed end moment by the per cent rigidity assumed.

6. Select a connection on the basis of end reaction, semi-rigid end moment, and percentage rigidity assumed.

In step 2 the stiffness of the beam, K_B, is based on the simple beam design. K_B could be based upon the reduced I_B of the final design, but since this is not known the approximation provides a direct design procedure and is on the side of safety. However, if a particular beam size is repeated under identical loading conditions a great number of times a further economy would be introduced by estimating K_B as 80 to 85 per cent of K_B for simple beam moment and verifying the estimate after the beam is designed on the basis of reduced moment. One trial design would be the most that might be required.

If the column sizes are not the same at each end the design may be based on the more flexible end with the approximation again on the side of safety. If the loading condition is moderately unsymmetrical, the end moments may be approximated at each end by Eq. 8, and the approximate bending moment diagram for the beam constructed. The severe loading condition assumed in Fig. 5 and the extreme improbability of its occurrence renders meaningless small errors of a few per cent which might be introduced by applying Eq. 8 to unsymmetrical conditions.

In step 3 the decision regarding what per cent rigidity to use in design may be made arbitrarily, but after a little practice its selection will be based on questions of feasibility, economy, and preference for a particular connection type. The final design of the connection in step 6 ultimately may be made simply by reference to standardized connection tables which give safe values of shear, moment, and percentage rigidity. At present the selection must be based on existing experimental data available in the publications listed in items 2, 3, 4 and 7 of the accompanying bibliography.

The design procedure may be illustrated as far as beam selection is concerned by an example. It is desired to select a beam, having 50 per cent rigid connections for a uniform load of 2 kips per ft., a span of 20 ft., and framing into the flanges of 10-in 49-lb. WF 49 columns of 10 ft. story height.

The simple beam moment is:

$$M_S = \frac{(2)(20^2)}{8} = 100 \text{ kip-ft.}$$

$$= 1200 \text{ kip-in.}$$

$$S = \frac{M}{f} = \frac{1200 \text{ kip-in.}}{20} = 60 \text{ in.}^3$$

f = allowable working stress in kips per sq. in.

For a simple beam, a 16-in. 40-lb. WF beam would be required with I = 515.5 in.[4]

$$K_B = \frac{515.5\text{in.}^4}{240 \text{ in.}} = 2.15 \text{ in.}^3$$

$$\Sigma K_C = 2\,\frac{272.9\text{in.}^4}{120 \text{ in.}} = 4.55 \text{ in.}^3$$

$$\frac{K_B}{\Sigma K_C} = \frac{2.15}{4.55} = 0.47$$

From Fig. 6, for p=50 per cent. F=0.73
Required S=(0.73) (60 in.)=43.8 in.^{3}
Use a 15-in. 33-lb. M-beam
Saving = 40 − 33 = 7 lb. or 17.5 per cent

BIBLIOGRAPHY

1. Wilbur M. Wilson and H. F. Moore—"Tests to Determine the Rigidity of Riveted Joints of Steel Structures" University of Illinois Bulletin No. 104, 1917.
2. First, Second, and Final Reports of the Steel Structures Research Committee of the Department of Scientific and Industrial Research of Great Britain, 1931-1936.
3. J. Charles Rathbun—"Elastic Properties of Riveted Connections" *Transactions*, Am. Soc. C. E., Vol. 101, p. 524, 1936.
4. C. R. Young and K. B. Jackson—"The Relative Rigidity of Welded and Riveted Connections" Canadian Journal of Research, Vol. 11, p. 62, 1934.
5. Inge Lyse and G. J. Gibson—"Welded Beam-Column Connections", American Welding Society *Journal*, Supplement, pp. 34-40, Vol. 15, No. 10, October 1936; and pp. 2-9, Vol. 16, No. 10, October 1937.
6. Wilbur M. Wilson—"Tests to Determine the Feasibility of Welding the Steel Frames of Buildings for Complete Continuity" American Welding Society *Journal*, pp. 28-28, Vol. 15, No. 1, January 1936.
7. Bruce Johnston and E. H. Mount—"Designing Welded Frames for Continuity" American Welding Society *Journal*, Vol. 18, No. 10, October 1939; Research Supplement, pp. 355 to 374.