Feb. 22, 1951 Robert L. Ketter

RESEARCH SEMINAR

Spring 1951

Lehigh University

Development of the Column Program at Lehigh University

With Special Attention toward Interaction Curves

In 1946 several of Rube Goldberg's helpers at Lehigh University, edged on by the A. I. S. C., undertook the problem of designing a frame for testing columns. The resulting frustrations during each of the tests conducted since then have caused those testing to consider this frame a contrivance of the devil.

Research on the general column problem has been carried on at Fritz Laboratory for a number of years. In 1942 a study of the local buckling of flanges of WF Sections was completed. This was followed by an investigation of the behavior of eccentrically-loaded columns.

The present investigation of columns loaded with combined axial force and various end moment simulating the loads acting on a column in a rigid frame commenced in 1946.

The investigation had two principal objectives in mind:

1). to determine the ultimate strength of the column.

2). to determine the moment distribution carry-over factors experimentally.

The investigation was unique in several ways:

1). columns, equal in size to those encountered in practice, were used where the moment could be controlled independent of the axial load.

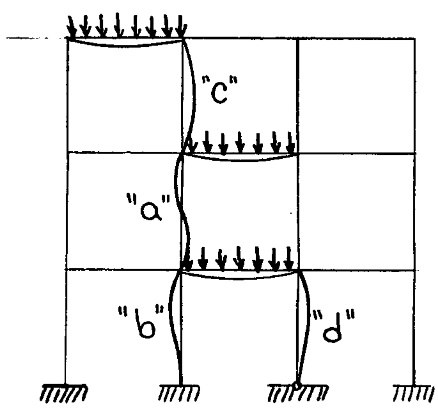
2). as-delivered rolled structural sections were used.

The program is now coordinated with a five-year investigation sponsored by the Welding Research Council on "The Strength of Welded Continuous Frames and Their Components".

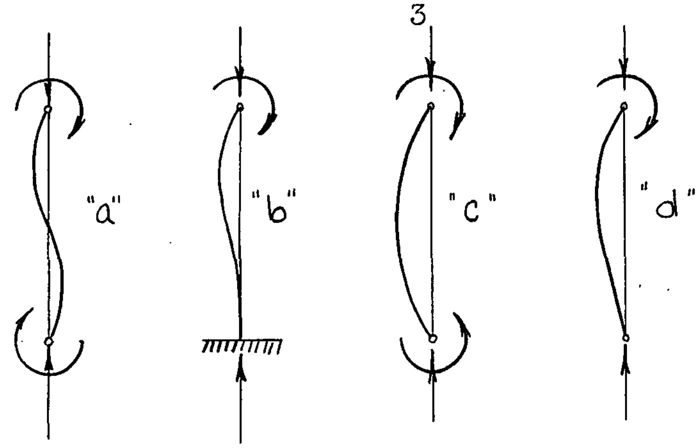
The eventual objective of the program is to develop practical design methods, based on elastic and plastic action. In order to accomplish this we are in the process of determining variables, such as: condition of loading (meaning combination of axial load and end moments), slenderness ratio, the ratio of P/P_y (where $P_y = \sigma_y A$), and the shape of cross-section.

To state the objective more specifically: To determine the influence on the interaction curve of each form of loading, relating it to existing specifications and thereby indicate the true load factor implied in present design methods.

In order that we may more clearly understand the possible load conditions that may be imposed on a column, consider the following tier building.



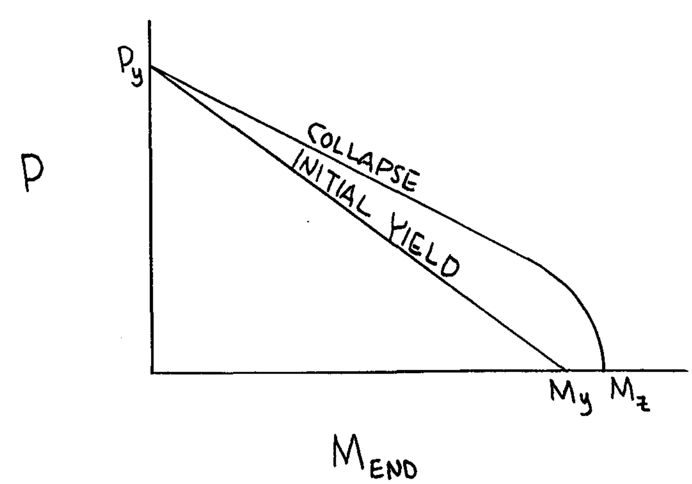
If the structure were loaded as shown we will have 4 different column loading conditions. These are:



We might also have the special case where the moment were equal to zero, thereby having a column with pure axial load.

In average design practice, all would be designed the same, providing the magnitude of moments and axial load were the same in all cases. Actually, the carrying capacities of these are quite different. For example, in condition "c" the maximum moment is at the center while in the others it will more than likely occur at the end of the column.

One method used in this investigation for showing the strength of the column is the interaction curve. This is a curve of Axial Load plotted against Moment. Since designers usually know the moment at the end of the column, and not necessarily the maximum moment along the column, we have chosen the moment at the end as our abscissa. Here is a typical interaction curve used in this investigation:



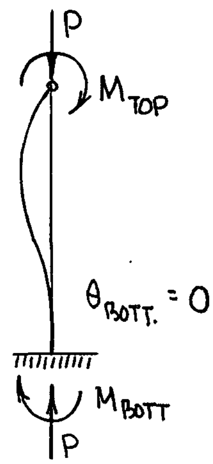
$P_y = \sigma_y A$

$M_y = \sigma_y$ (Section Modulus)

$M_z = \sigma_y$ (Moment of Area about Centroid)

Later, we will go into the method of obtaining these curves.

Another part of the investigation thus far, has been the determination of Moment Distribution Carry-over factors experimentally. This is made possible through use of loading condition "b".



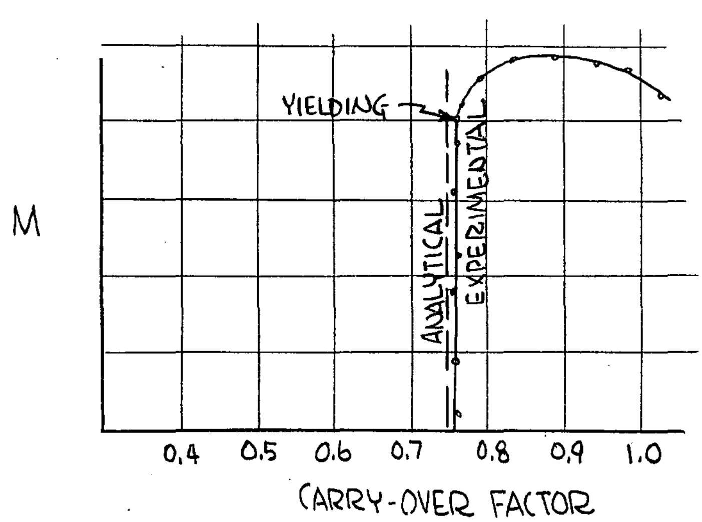
The method of obtaining these factors is:

1. Apply Moment at top.
2. Apply Moment at bottom until rotation = 0.
3. Divide resulting moment at the bottom by the applied moment at the top.

This shoudl be equal to the carry-over factor.

Since the carry-over factor varies with the axial load, it is an easy matter to compare the experimentally determined ones with that obtained analytically.

A typical resulting curve is shown below:



Other studies being made or to be made in the near future are:

1. Stiffness
2. Strain distribution
3. Lateral and Local Buckling

Having quickly covered the general aspects of the program, let us now consider the part on Interaction Curves more in detail.

As you remember, I stated earlier in this talk that the interaction curve with which we are concerned is that where axial load is to be plotted -vs- end moment.

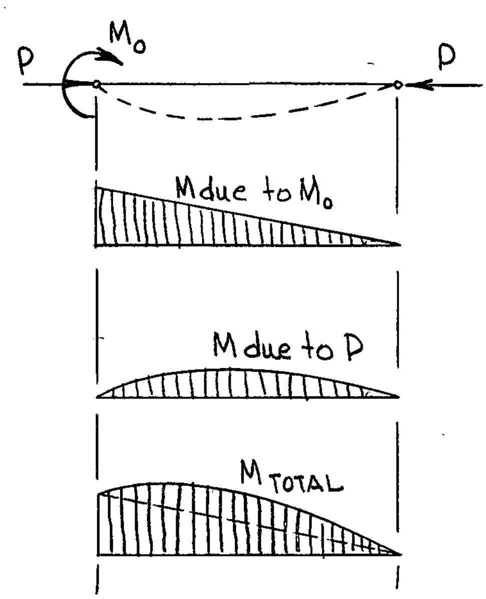
We will mainly be interested today, with the development of the initial yield interaction curve.

Our general plan of attack will be to first consider the simpler case and then if time permits go on to more complex ones.

Let us consider the problem qualitatively for a few minutes.

Consider the column shown at the right. The moment at any point along the column is composed of two parts. That caused by the end moment (M_o) and that by the axial load (P).

If we superimpose these moment diagrams, we get an M-diagram similar to that shown at the right.



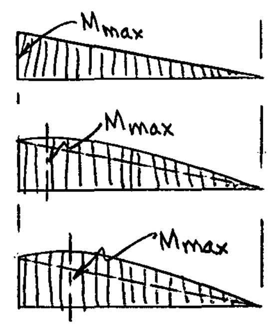
From this it is possible to see that there is a possibility that the maximum moment does not occur at the end.

Now consider the limiting cases:

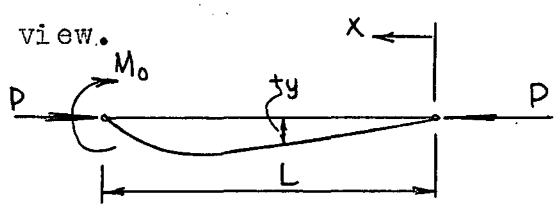
1). Let $P = 0$, then we have a case of a cantilever beam with the moment greatest at the applied point M_0.

2). Let $M_0 = 0$, then we have a column under pure axial load with the maximum moment occuring at the center upon buckling.

With these two limits in mind it is quite reasonable to assume that under a combination of end moment and axial load, the point of maximum moment along the column will move from the end toward the center as the axial load is increased.



Now let us approach the problem from a mathematical point of view.



The equation of the deflection curve is

$$y = \frac{M_o}{P}\left(\frac{\sin kx}{\sin kL} - \frac{x}{L}\right) \qquad \text{where} \quad k = \sqrt{\frac{P}{EI}}$$

If this is differentiated twice with respect to x we obtain the equation for curvature along the member

$$y' = \frac{M_o}{P}\left(k\,\frac{\cos kx}{\sin kL} - \frac{1}{L}\right)$$

$$y'' = \frac{M_o}{P}\left(-k^2\,\frac{\sin kx}{\sin kL}\right) = -\frac{\text{MOMENT}}{EI}$$

However,

$$K^2 = \frac{P}{EI} \qquad \text{(as previously defined)}$$

Therefore,
$$M = \frac{M_o}{P} \cdot \frac{P}{EI} \cdot EI \cdot \frac{\sin kx}{\sin kL}$$

or

$$M = M_o \frac{\sin kx}{\sin kL} \qquad \text{-------------------------(1)}$$

Since yielding occurs at the point of maximum moment, differentiate the moment with respect to x and set equal to zero; then solve for the point of maximum moment.

$$\frac{dM}{dx} = 0 = M_o K \frac{\cos kx}{\sin kL}$$

Therefore:
$$\cos kx = 0$$

This occurs when $kx = \frac{\pi}{2}$

Therefore,
$$x = \frac{\pi}{2} \cdot \frac{1}{k}$$

Since:
$$P_E = \frac{\pi^2 EI}{L^2} \qquad L\sqrt{\frac{P_E}{EI}} = \pi$$

then,
$$x = \frac{L}{2}\sqrt{\frac{P_E}{P}} \qquad \text{-------------------------(2)}$$

where P_E = Euler Buckling load

Now considering limits:

when $P = P_E$ $\qquad$ $x = \frac{1}{2} L$

Note that when:

$x = L$ $\qquad$ $P = \frac{1}{4} P_E$

This means that as long as the axial load is less than or equal to 1/4 Euler's Buckling Load, the maximum moment will occur at the end of the column. As P increases beyond this load, the point of maximum moment moves down the column.

Because our basis is to be the initial yield of the extreme fiber at any section along the column, we will write the equation of stress at any section in terms of moment and axial load.

$$\sigma = \frac{P}{A} + \frac{Mc}{I}$$

Since yielding occurs at the section of maximum moment,

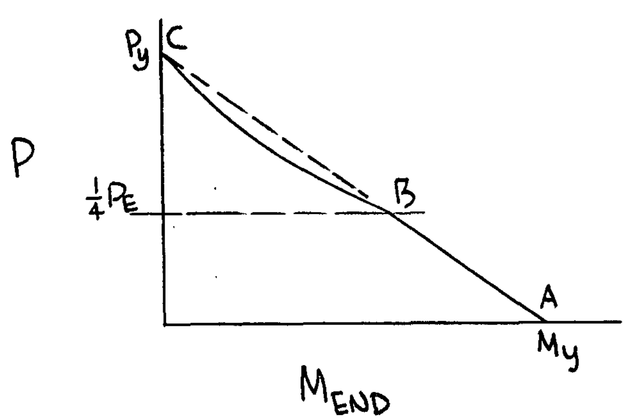
$$\sigma_y = \frac{P}{A} + \frac{M_{max}c}{I} \qquad \text{---------------(3)}$$

Note, a linear relation exists between P and M_{max} since all other quantities are constants.

Remember, we stated previously that our interaction curve would be P -vs- End Moment. Therefore, the general method of attack will be:

1. Assume a load P
2. Solve for M_{max} (Equation No. 3)
3. With P and P_E - locate point of maximum moment (Equation No. 2)
4. Compute M_o (Equation No. 1)
5. Plot the point
6. Pick a new P
7. Proceed as before

Thus, the final initial yield interaction curve will have an appearance as follows:

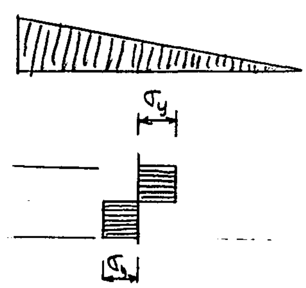


Since we know that the maximum moment between $P = 0$ and $P - 1/4\ P_E$ will occur at the end of the column we need only investigate the case where $P = 0$ (that corresponding) to a cantilever beam) and $P = 1/4\ P_E$. This will determine the portion of the curve between A and B. For points between B and C we must go through the process of selecting P and calculating M_o outlined above.

In the few remaining minutes, let us qualitatively consider the case of a collapse interaction curve.

When $P = 0$, we have the case of the cantilever beam again with M_{max} occuring at the end. If collapse is considered as taking place when the full plastic hinge value is reached, the moment causing collapse will be

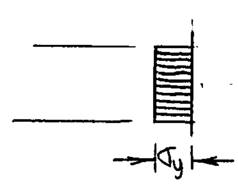
$$M_{COLLAPSE} = \sigma_y Z$$



where Z is the moment of the area of the section about its centroid. (The stress distribution for this case is shown at the right).

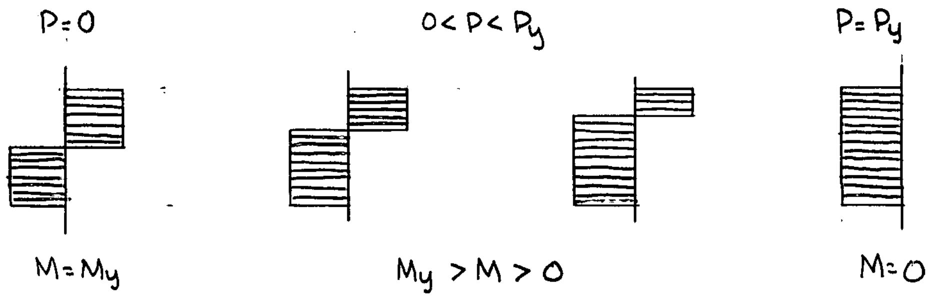
Now the case where $M = 0$ (in other words a case of pure axial load). Assuming buckling will not occur before this load is reached, the axial load causing collapse will be

$$P_{COLLAPSE} = \sigma_y A$$

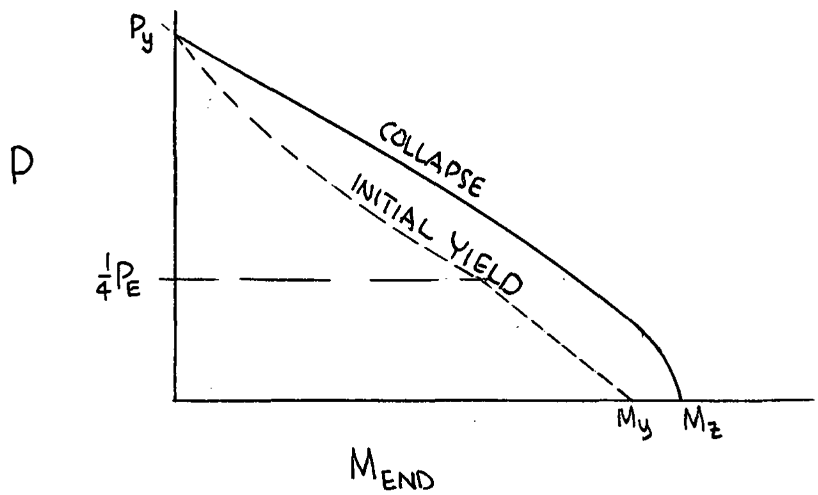


(The stress distribution is shown at the right).

Therefore, we can say that between these two limiting conditions, some stress distribution between these two cases will occur.



With this information and manipulation of a few formulas, we arrive at the following collapse interaction curve.* (The initial yield curve previously derived is shown in dotted Lines).



Having arrived at these curves, we have a means of Picturing how the strength of a column varies with different ratios of axial load to end moment for this one particular condition of loading. For other conditions of loading, the same general procedure is used.

Gentlemen, it has been a great pleasure to be here with you today. Thank you very much for your kind attention.

* -Concept arrived at by Baker at Cambridge.