Note ON THE ULTIMATE STRENGTH OF CONTINUOUS BEAMS AND RIGID FRAMES

Report 205B.

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I. INTRODUCTION

Yield strength of a structure is defined as the load at which a part or several parts of the structure have reached their yield stress. It is predicted by the elastic structure theory and is usually used as the cryterion of strength of the structure in design. It has been pointed out in Progress Report 3[1] that most indeterminate structures will take further increase of loads beyond their initial yield strength. Hence, it is suggested in simple plastic theory a higher limit in design which is termed ultimate strength of the structure should be used instead to achieve further economy.

A section in a structural steel member under applied moment will give a moment curvature relation as shown in Fig. 1[2]. It is seen that resisting moment at the section gradually approaches a limit. The value of the resisting moment shortly after its yield strength will have very slight change with further rotation of the section. During loading the section can be regarded as a hinge with a constant moment in the plastic range. In the simple plastic theory ultimate strength is defined to be the load at which there are enough plastic hinges developed in the structure to make it becomes a statically unstable mechanism. Take for instance a fully fixed-end beam under uniform load. The ultimate strength of the beam is the load when three plastic hinges developed at the center and both ends as shown in Fig. 2.

~~However, the idea of a plastic hinge is under the assumption that at~~

However, the idea of a plastic hinge is under the assumption that structural steel has a hypothetical stress-strain relation as shown in Fig. 3. Actually the stress-strain relation under unaxial stress is as shown in Fig. 4. The material becomes strain hardened after a certain amount of plastic straining. Questions have been raised frequently on the effects of strain hardening of steel on the ultimate strength of structures. Whether, or not, steel has enough plasticity to develop all the plastic hinges in a structure before any rapture occurs, is also being questioned by some authors[2].

The estimation of ultimate strength by the simple plastic theory has, however, paid no attention to the deformation of the structure. As discussed in Progress Report 3, the deformations of the practical structures all have to be limited under certain limits. Then, it is doubtful whether all the possible plastic hinges will be able to develop before the deformation of the structure exceeds its limited value. It is the purpose of this paper to give a discussion of the above listed questions. Illustrations have also been made with calculated examples.

3. The effect of strain hardening of structural steel on the ultimate strength of structures.

According to the simple plastic theory when the ultimate strength of a structure is reached it will act like a mechanism i.e. the deformation of the structure can be increased without applying any additional load. Take the case of a cantilever beam with a concentrated load. The load-deflection relation will be a curve as shown in Fig. 5. The actual experimental load-deflection curve of a cantilever beam of an 8WF40 section from the continuous beam test program is shown in Fig. 6. The load rises and exceeds the predicted ultimate strength as deflection increases. It is apparent that the region near the support of the beam was strain hardened. The moment of strain hardeing introduced into a structural member is not only proportional to the amount of deformation but also depends on the type of loading and the geometric shape of section used. The segment of a structural member under a constant moment will secure more idealistic plastic hinge than under gradient moment diagrams. Illustrations are made between a fully fixed-end beam and elastically supported beam with one third point loading. ~~Suppose~~ Wide flange section are used for above beam. The thickness of web of these sections is thin and the thichness of the flanges is small as compared with the width.

For simplification of illustration M_y is taken equal to M_p for these sections. For a fully fixed-end beam the load-deflection curve willbe predicted by the simple plastic theory as in Fig. 7. As the load readhes P_1 the moment at the two supports reaches M_p and the moment at the central section is only one half of the value of M_p. Further increase of load will make the beam act as a simply supported beam providdd the two ends are free to rotate. Since the beam is built-in the plastic strain at both ends will have to give room for this rotation. Looking at the corresponding moment diagram in Fig. 7 it can be seen that there are only two very thin section which are under moment of the magnitude of M_p. The plastic strains on the top and lower flanges of these sections have to be extremely large to secure the rotations for further increase of load beyond P_1 on the beam. In the case of assuming $M_y = M_p$, the plastic zones at both ends are of infinitismal thickness . Any definite increase of load will cause a finite amount of rotation that will take the plastic strains on the flanges to infinity immediately. Actually the plastic strain will reach its strain hardening range, as we can see from Fig. 4. The rise of end moments exceed M_p due to strain hardening will consequently bring their sections into plastic range. The plastic zone will progress from two ends toward the center of the beam as increases. The extended plastic regions will then give the necessary rotation at the supports. Since the end moments are no longer a constant M_p, the ultimate strength of the beam is being raised.

A segment of a structural member under constant moment will however act more likely like a plastic hinge, as we can see from the following axample:

A beam is loaded with one third point loading as shown in Fig. 8. If we still let $M_y = M_p$ the moment curvature relation will be one as shown in Fig. 9. After the load on the beam reaches P_1 the fibres of the portion of the beam between two loads are all in plastic range. From Fig. 9 it is noted the central portion will be able to secure a rotation of angle of $\;_o$ without getting the beam fibre strain hardened.

Where $\;_o = \;_o \times \frac{1}{3}$

In this region the central portion acts like a hinge with a constant moment M_p on.

Contents

Reference

(1) P. R. 3

(2) Trends in Steel Design and Research. by George Winter Cornell Univ.

Fig 1

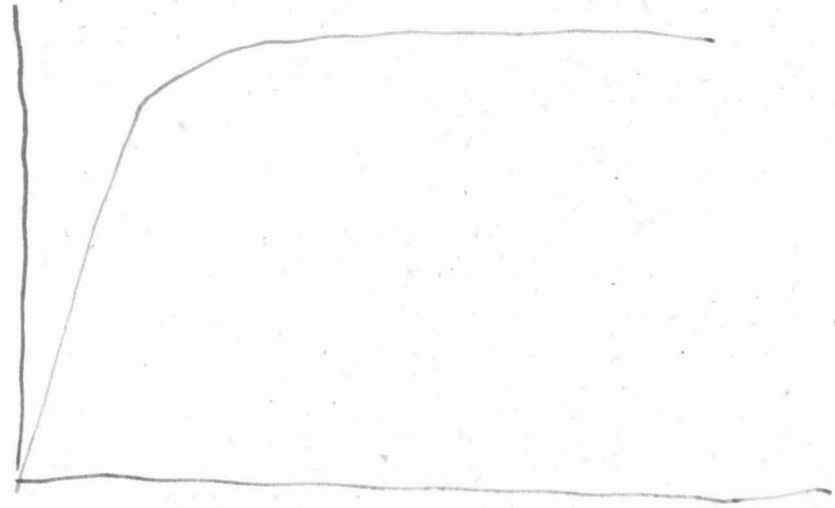
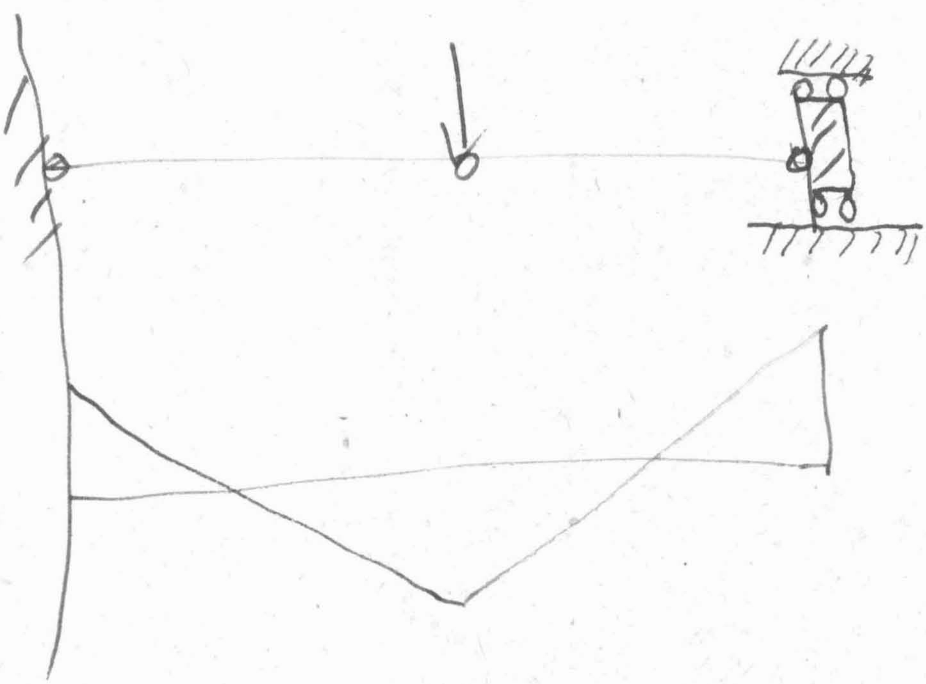


Fig 2



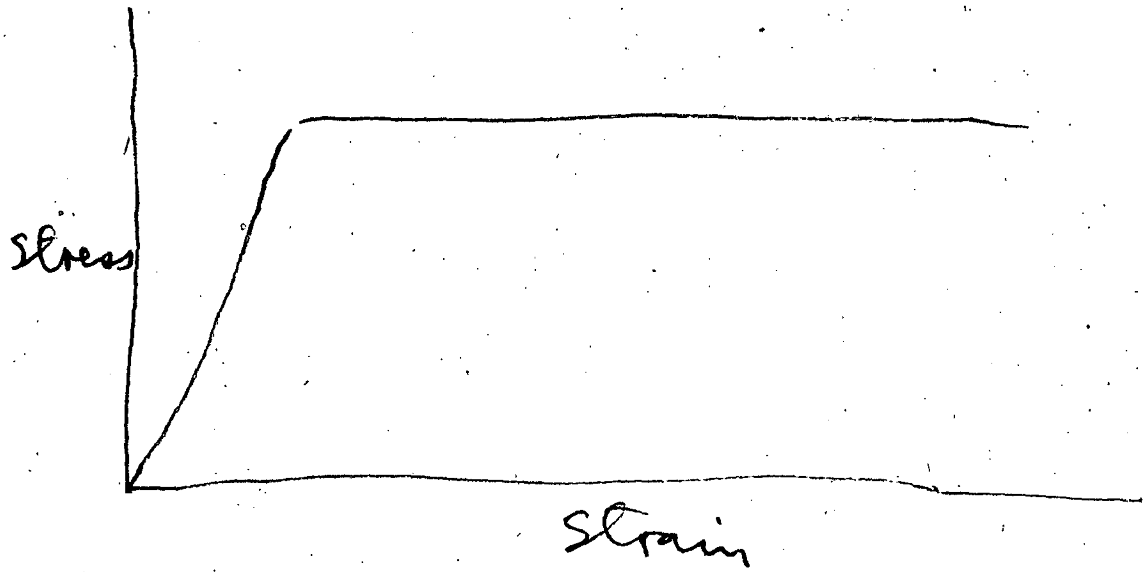


Fig 3

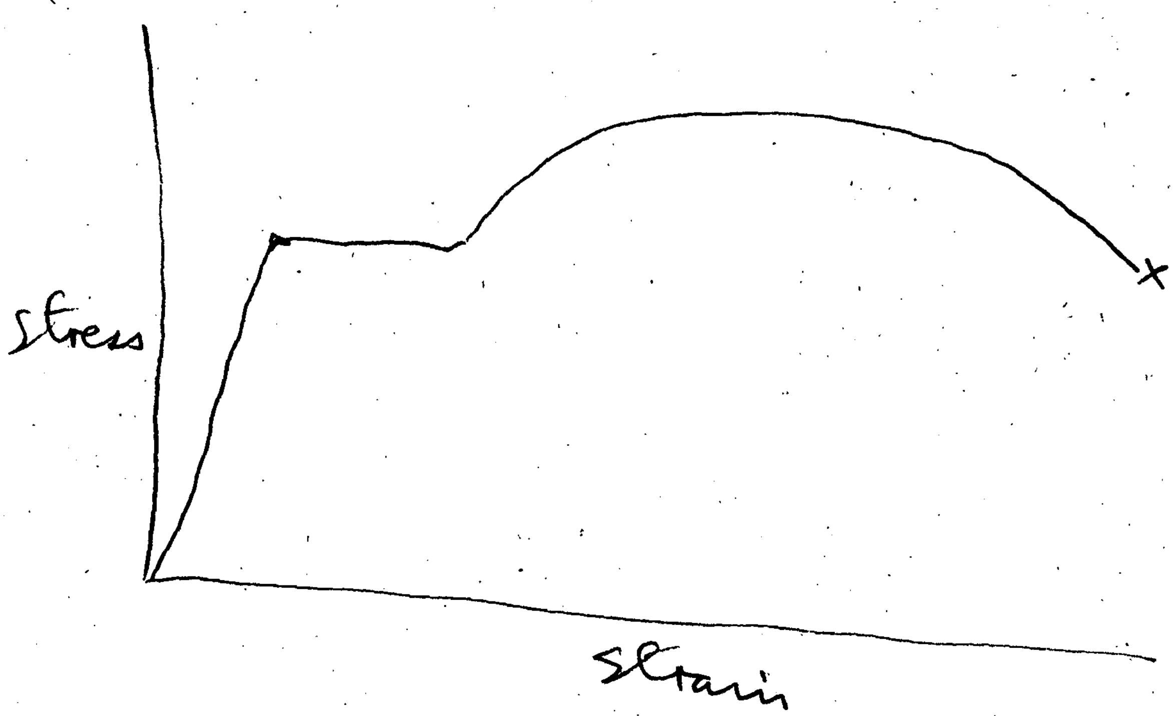
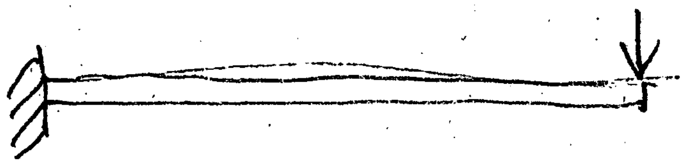


Fig 4



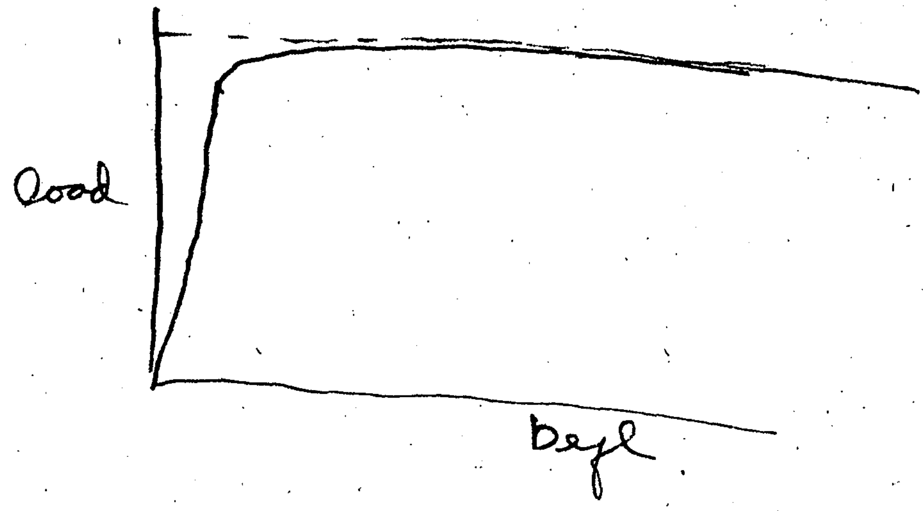


Fig 5

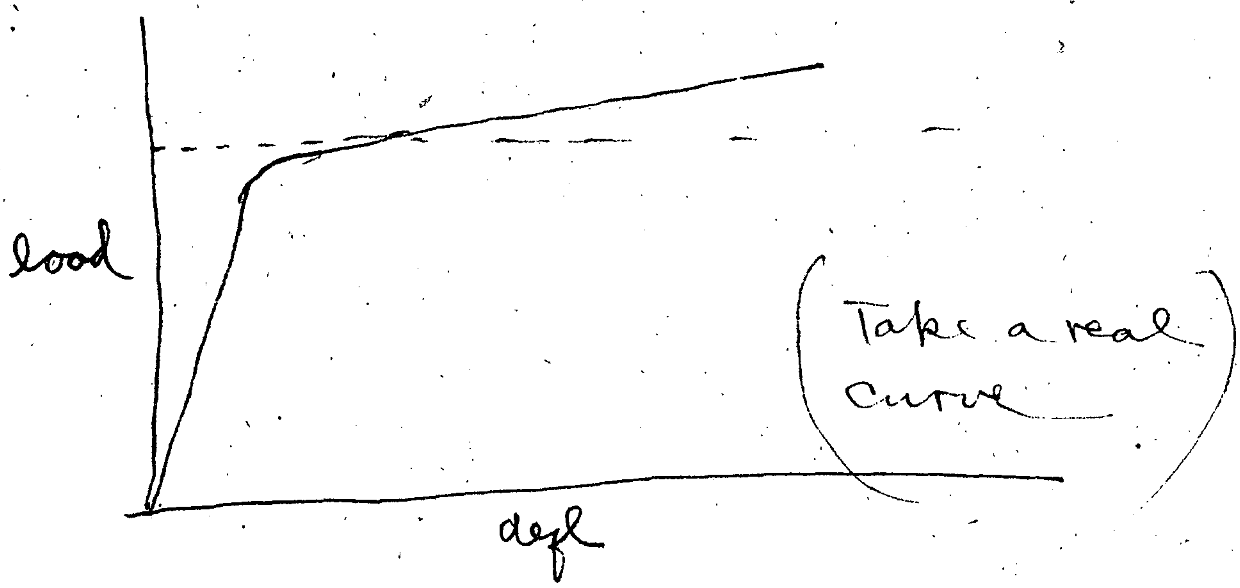


Fig 6

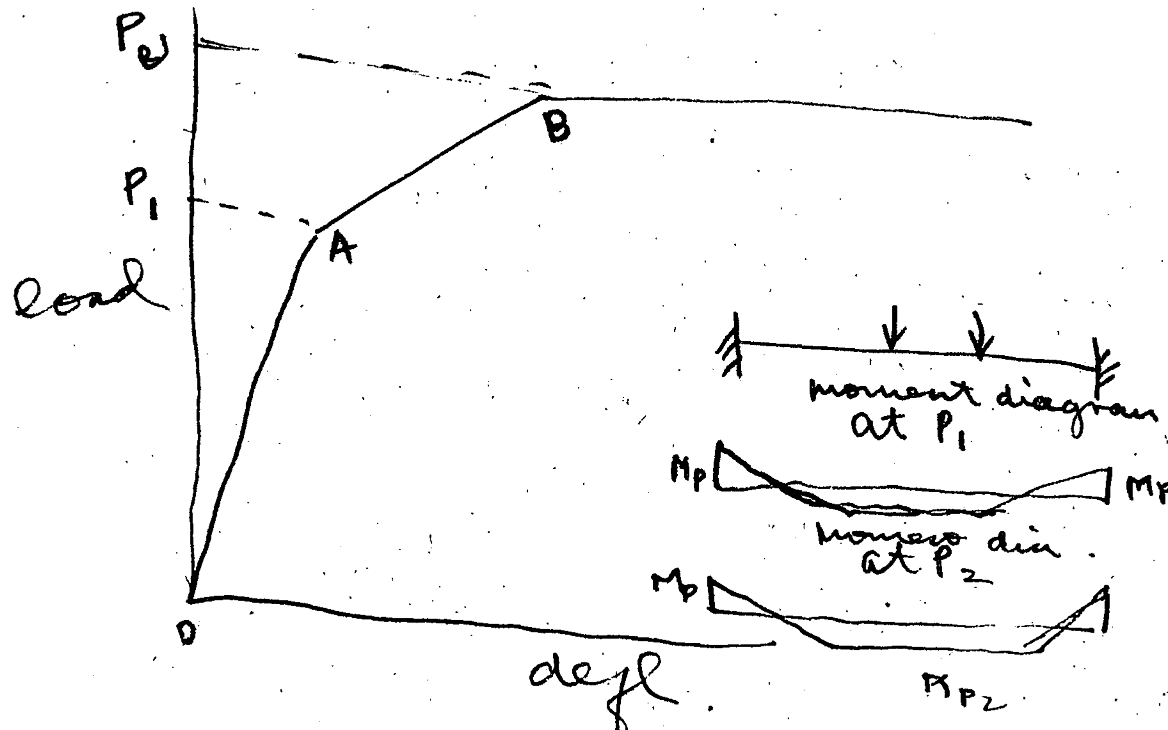

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Fig 7

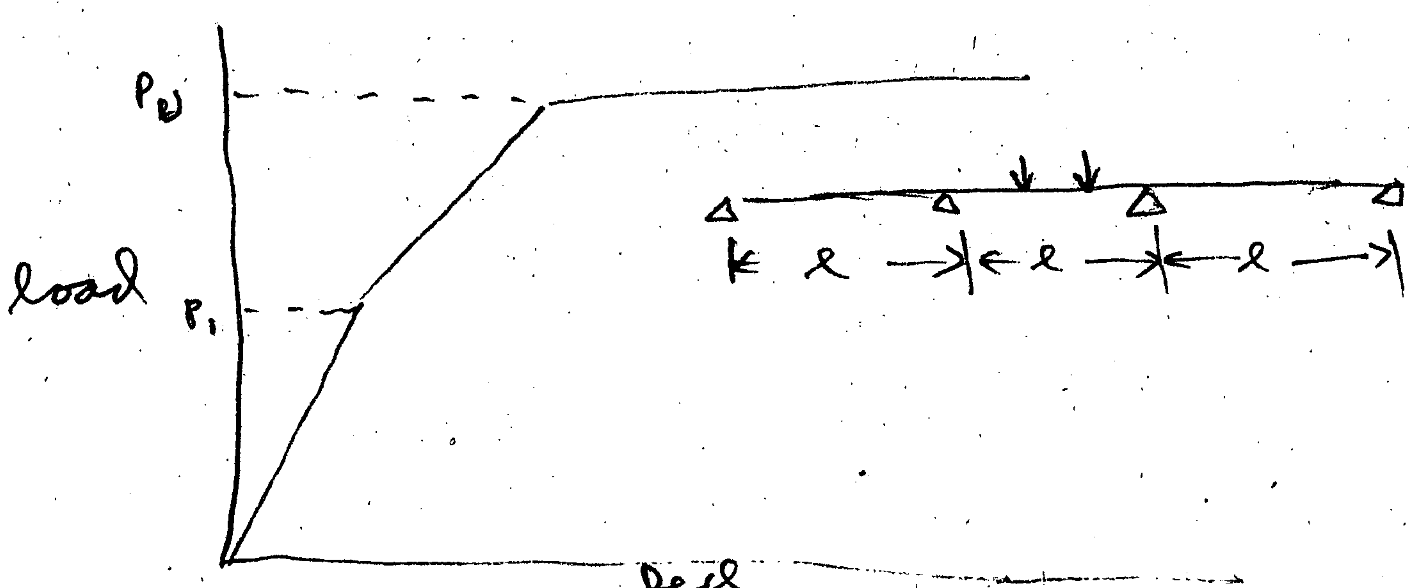
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Fig 8

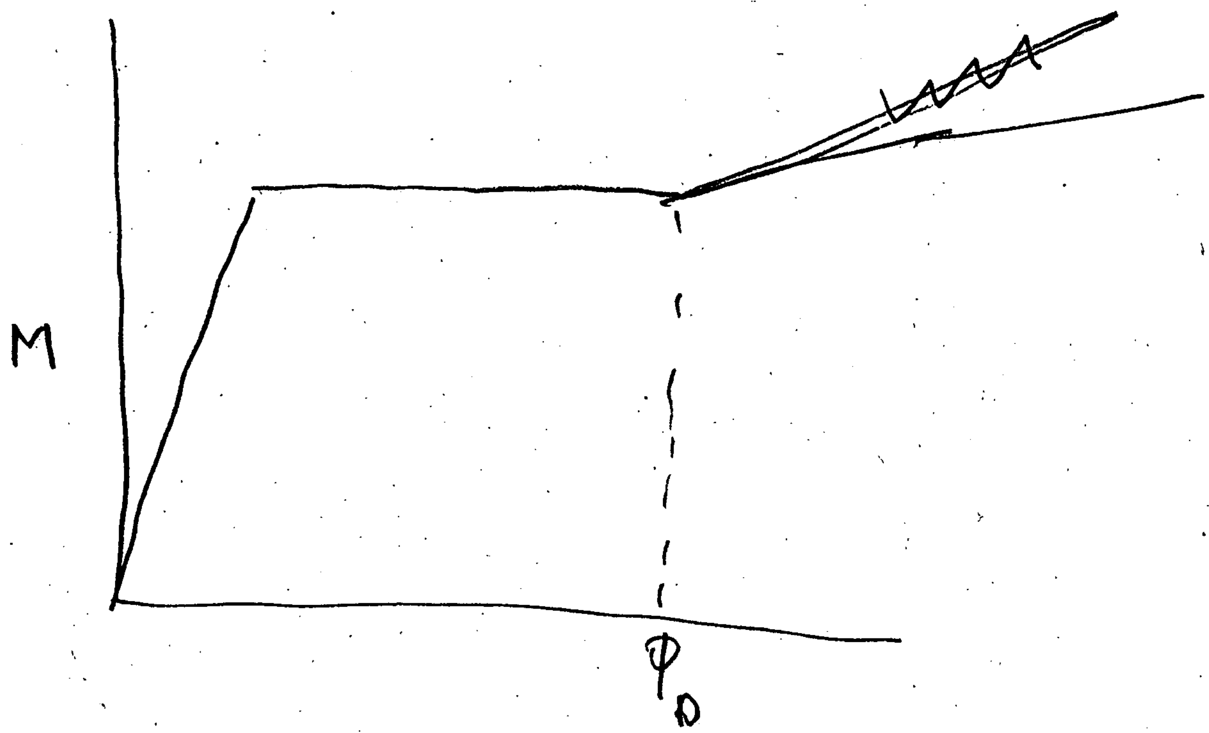

Fig. 9.