RESIDUAL STRESS DUE TO COLD BENDING

Residual Str.

The unloading and reloading beam behavior after it has been loaded pass the yielding strength.

April 22, 49

1. The beam deflection under the same load during repeated loading increases.

2. The amount of deflection increment from initial load to max. load applied in the repeated ~~loading~~ test approaches a constant.

$$\Delta\delta = 0.808''$$

Calculation

$$\Delta\delta = \frac{23 l^3}{27 \times 24 \times EI} \times 20$$

$$E = 29.5 \times 10^3$$

$$I = 143.2$$

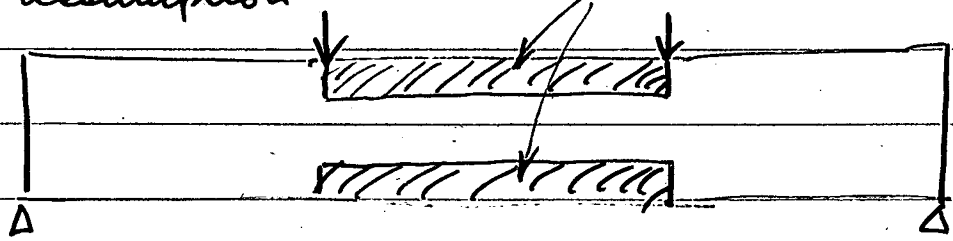
$$\Delta\delta = 0.801''$$

3. It shows that the strain has been hardened the strain and stress relation ~~flows~~ follows the Elastic modulus of Elasticity very close.

4. The reason that the deflection is not stable is due to some new local spots yielded during repeat loading

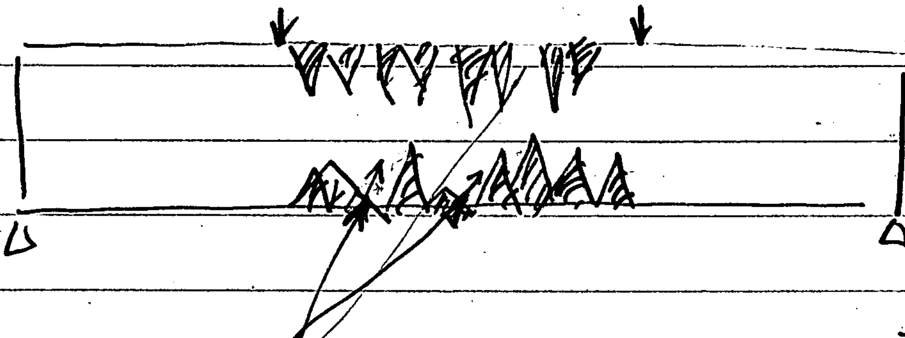
a. Theoretical assumption

Yielded



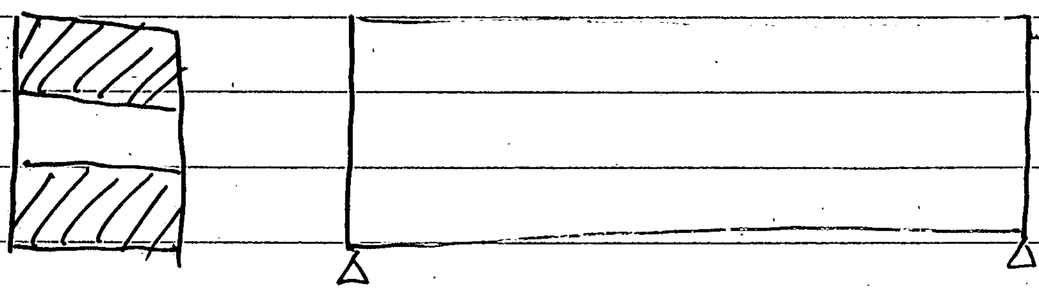
Actual case

b/.



New spots start to yield.

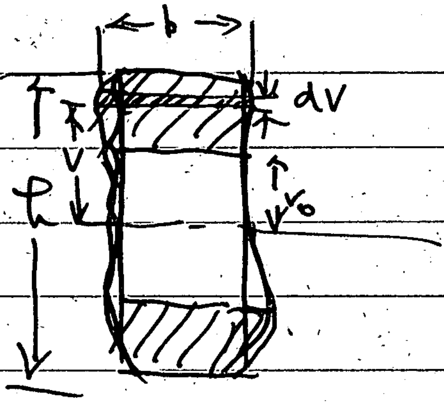
5.

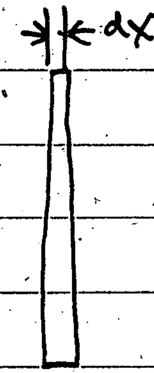


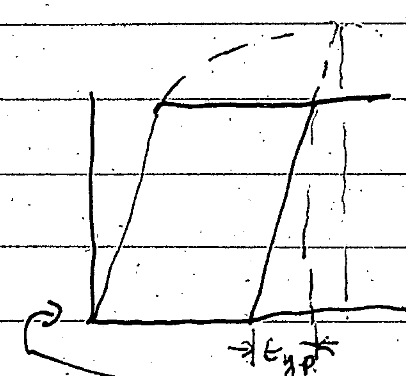
The deflection of the beam should follow the beam formula after strain hardened. This can be proved

The Problem:

A beam is loaded over yielding strength. What will be the behavior of the beam for other types of loading (The moment at each section by the new loading system must be smaller than the previous one in those yielded regions)







In general case it's a function of V.

$$\frac{\epsilon_{y.p.}}{V_0} = \frac{\epsilon_V}{V} \qquad \therefore \epsilon_V = \frac{V}{V_0}\,\epsilon_{y.p.}$$

$\epsilon_{y.p.}$ = Elastic strain

$\therefore \epsilon'$ = permanent strain $= \left(\frac{V}{V_0} - 1\right)\epsilon_{y.p.}$

ϵ' in Elastic range $= 0$

Assume curvature of the beam at this section = R

Assume the new loading system produce a moment M1 at the section

$\epsilon = VR$

$\sigma_1 = \text{stress} = \left[VR - \left(\frac{V - V_0}{V_0}\right)\epsilon_{y.p.}\right]E$

$\sigma_2 = VR$ in elastic range

$\Sigma \sigma V dA = M$

$$\therefore \; 2\int_0^{V_0} V\,(RV)\,E\,b\,dV + 2\int_{V_0}^{\frac{h}{2}} V\sigma_1\, b\,dV = M$$

$$\therefore \; 2\int_0^{V_0} V^2 R E b\,dV + 2\int_{V_0}^{\frac{h}{2}} bE\left[\frac{V}{\cancel{V_0}} R - \left(\frac{V - V_0}{V_0}\right)\epsilon_{y.p.}\right] V dV = M$$

$$\therefore \; \int_{-\frac{h}{2}}^{\frac{h}{2}} V^2 R\,dV = \frac{M}{E} + 2\int_{V_0}^{\frac{h}{2}} \frac{bV^2}{V_0}\epsilon_{y.p}\,dV - 2\int_{V_0}^{\frac{h}{2}} V\epsilon_{y.p.}\,b\,dV$$

$$IR = \frac{M}{E} + \frac{I_2\,\epsilon_{y.p}}{V_0} - Z_1\,\epsilon_{y.p.}$$

$$R = \frac{M}{IE} + \left(\frac{I_2}{IV_0} - \frac{Z_1}{I}\right)\epsilon_{y.p.}$$

I_2 = Moment of inertia of plastic region about N.A.

Z_1 = moment of plastic region about N.A.

When $M=0$

$$K = R_0 = \left(\frac{I_2}{IV_0} - \frac{Z_1}{I}\right)\epsilon_{y.p.}$$

$$= \frac{\epsilon_{y.p.}}{I}\left(\frac{I_2}{V_0} - Z_1\right)$$

R_0 = ~~radius of~~ curvature when moment all released.

$$R = R_0 + \frac{M}{IE}$$

Therefore in reference with R_0 the beam behaves as perfect elastic.

$$R = \frac{M}{EI} + \frac{\epsilon_{y.p.}}{I}\left(\frac{I_2}{V_0} - Z_1\right)$$

~~Now~~ ~~R_0~~ I_2, Z_1 and M may be represented by function of X a

$$-\frac{d^2y}{dx^2} = R$$

Thus solve the general case of loading

$$\text{Stress} = \sigma = E\epsilon'' = \left[VR - \left(\frac{V-V_0}{V_0}\right)\epsilon_{y.p}\right]E$$

Residual stress #1 (general expression $M=0$)

$$\sigma_0 = \left[VR_0 - \left(\frac{V-V_0}{V_0}\right)\epsilon_{y.p}\right]E$$

$$= \left[\left(\frac{I_2}{IV_0} - \frac{Z}{I}\right)\epsilon_{y.p}V - \left(\frac{V-V_0}{V_0}\right)\epsilon_{y.p}\right]E.$$

$$= E\epsilon_{y.p}\left[V\left(\frac{I_2}{IV_0} - \frac{Z}{I}\right) - \left(\frac{V-V_0}{V_0}\right)\right]$$

Suppose $V = V_0$

$$\sigma_0 = E\epsilon_{y.p}\left[\frac{I_2}{I} - \frac{ZV_0}{I}\right] = \frac{E\epsilon_{y.p}}{I}\left[I_2 - ZV_0\right]$$

Suppose $V = \frac{h}{2}$

$$\cancel{\sigma_0 = E\epsilon_{y.p}\left[\left(\frac{I_2}{I} - \frac{ZV_0}{I}\right)\right]}$$

$$\sigma_0 = E\epsilon_{y.p}\left[\frac{h}{2I}\left(\frac{I_2}{V_0} - Z\right) - \left(\frac{h-2V_0}{2V_0}\right)\right]$$

$$= \sigma_{y.p}\left[\frac{h}{2I}\left(\frac{I_2}{V_0} - Z\right) - \left(\frac{h-2V_0}{2V_0}\right)\right]$$

$$M = M_0.$$

$$R = \frac{\epsilon_{y.p.}}{V_0}.$$

$$\therefore \frac{\epsilon_{y.p.}}{V_0} = \frac{M_0}{IE} + \left(\frac{I_2}{IV_0} - \frac{2}{I}\right)\epsilon_{y.p.}$$

$$\therefore M_0 = \epsilon_{y.p.}E\left(\frac{I}{V_0} - \frac{I_2}{V_0} + 2\right)$$

$$M_0 = \frac{\epsilon_{y.p.}E}{V_0}\left(I - I_2 + 2V_0\right) \#$$