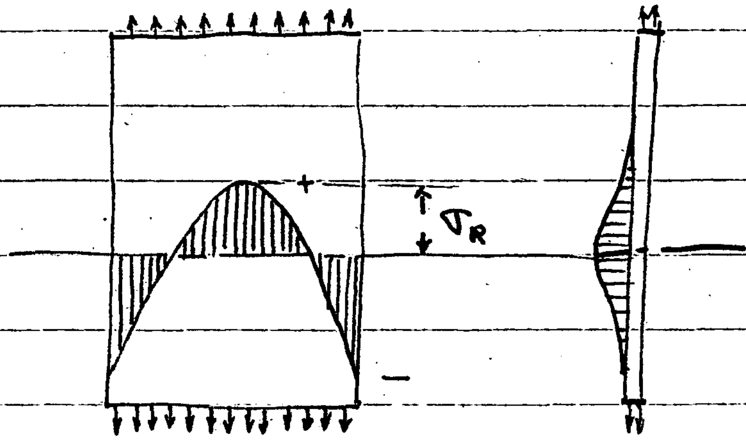
Effects of Residual Stress ~~to~~ on the Behaviors of Beams

"Residual stress makes an elastic problem plastic"

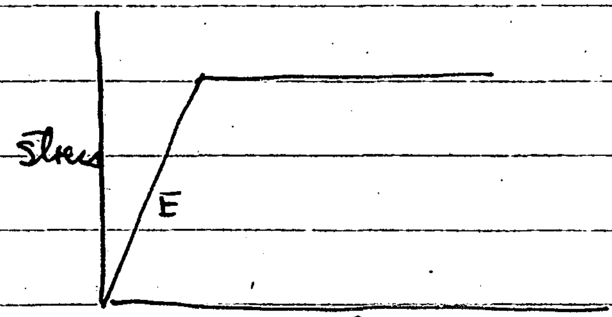
~~April~~
May 1, 1949

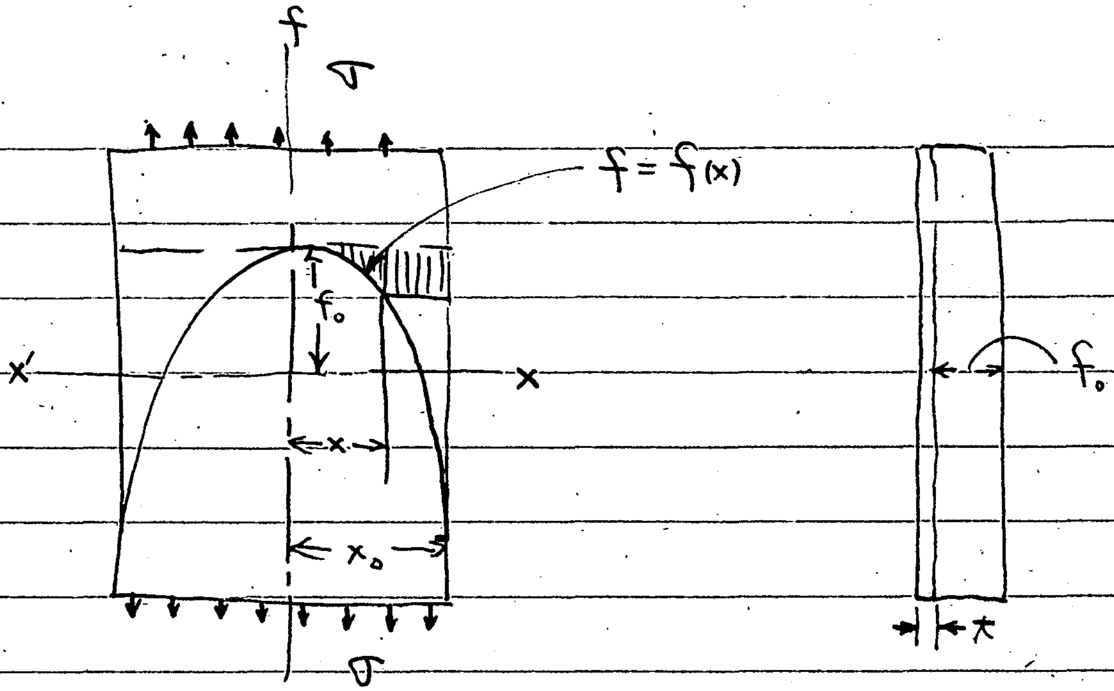
Residual stress under simple tension.



Assumption:

The stress and strain relation for the material follow the following diagram





A.

$A = X_o t$

A_E = Elastic area

$A_E = t(x_o - x)$

$$E_p = E\frac{A_E}{A} = \frac{X_o - X}{X_o} = E\left(1 - \frac{X}{X_o}\right) \quad \text{(A)}$$

Assume $f_o = f_{y.p}$

$$\sigma A = \int_0^x t(f_o - f)\,dx + t(X_o - x)f$$

$$\sigma x_o = \int_0^x (f_o - f)\,dx + (X_o - X)f$$

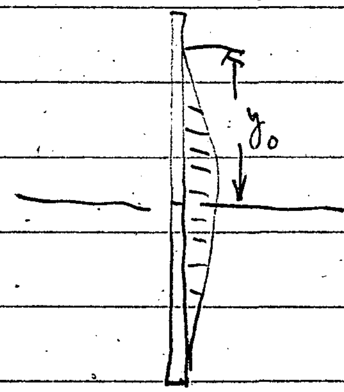
So we solve $\quad x = F(\sigma) \quad$ (B)

Substitute (B) to (A)

$$\therefore \quad E_p = E\left(1 - \frac{F(\sigma)}{X_o}\right)$$

$$E_p = \frac{d\sigma}{d\epsilon}$$

$$\therefore \epsilon = \int_0^{\sigma} \frac{d\sigma}{E_p}$$



$$\Delta L = L \times \epsilon$$

$$\Delta L = L \int_0^{\sigma} \frac{d\sigma}{E_p}$$

B. now if f_o vary with y axis

$f_o = g(y_o)$ but $y = 0$ $g(y_o) = f_{y.p.}$

$$\therefore \quad d(\Delta L) = \frac{1}{E}\int_0^{f_{y.p.} - g(y)} d\sigma \; dy$$

$$+ \int_{f_{y.p.} - g(y)}^{f_{y.p.}} \frac{d\sigma}{E_p} \, dy$$

$$\Delta L = \int_0^{y_o} \left[\frac{1}{E}\int_0^{f_{y.p.} - g(y)} d\sigma + \int_{f_{y.p.} - g(y)}^{f_{y.p.}} \frac{d\sigma}{E_p} \right] dy$$

C. If the aplied stress vary along the y axis.

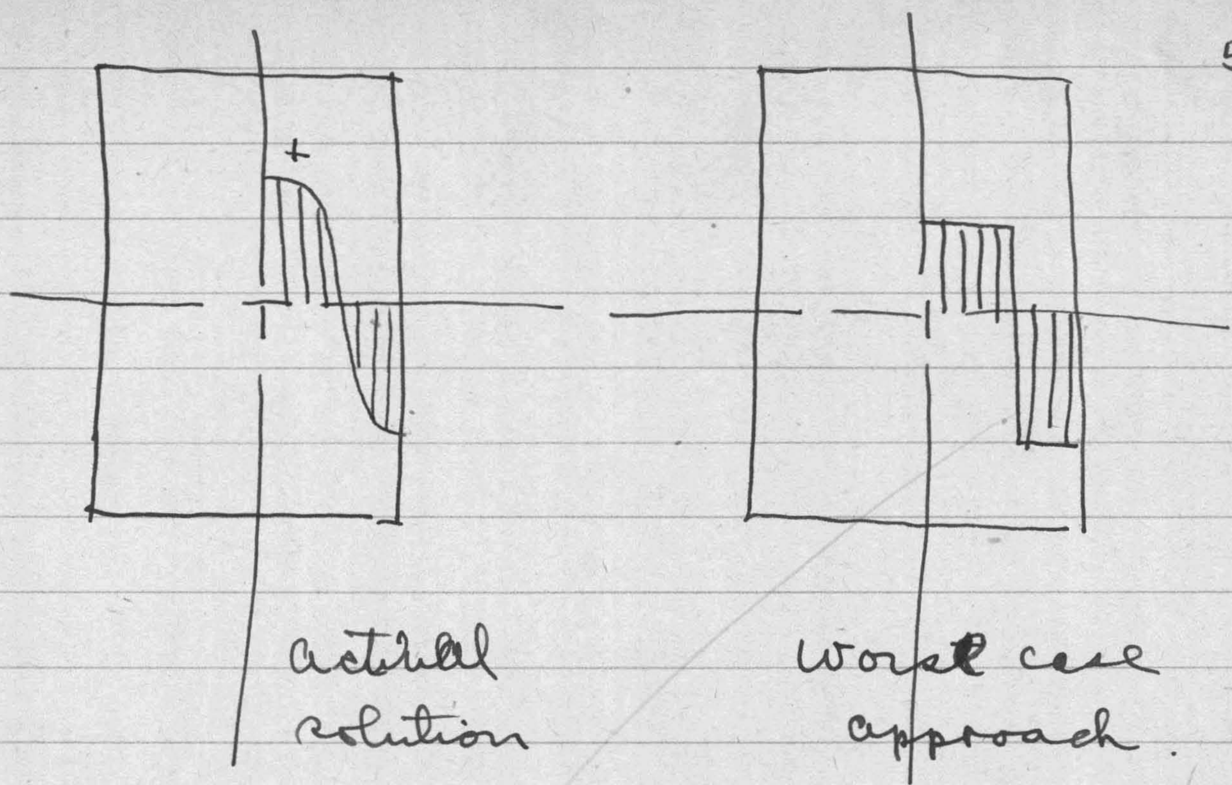
$$\Delta L = \int_0^{y_0} \left[\frac{1}{E} \int_0^{f_{y.p.} - g(y)} d\sigma + \int_{f_{y.p.} - g(y)}^{Q(y)} \frac{d\sigma}{E_p} \right] dy$$

where $Q_{(y)}$ = function of y

Represent the variation of stress along y axis.

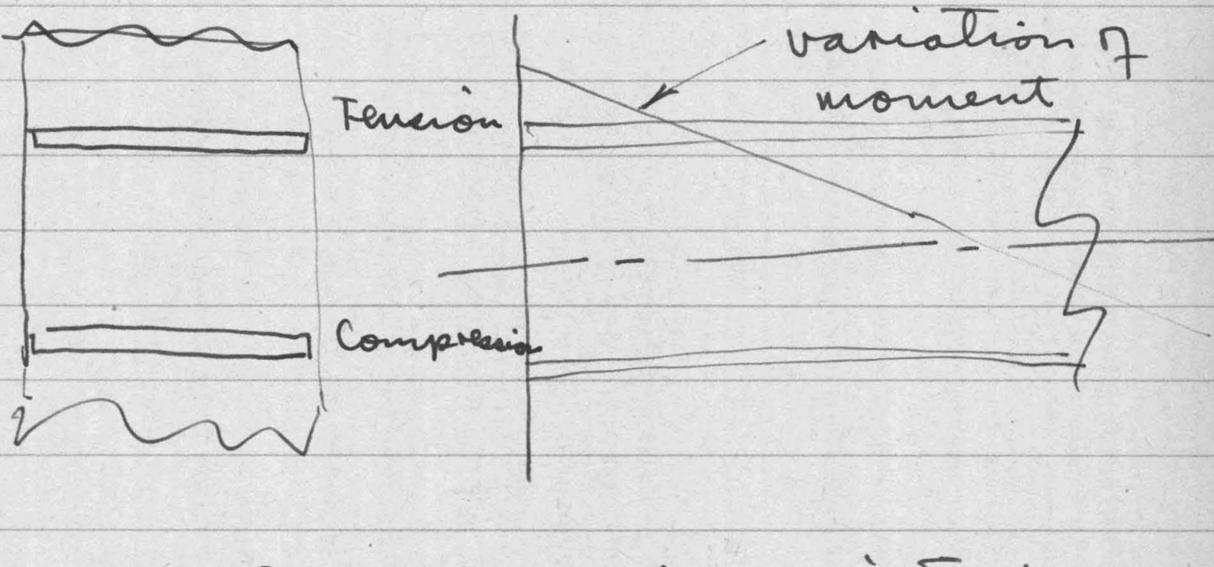
~~[illegible]~~

(as the fiber of a beam.)



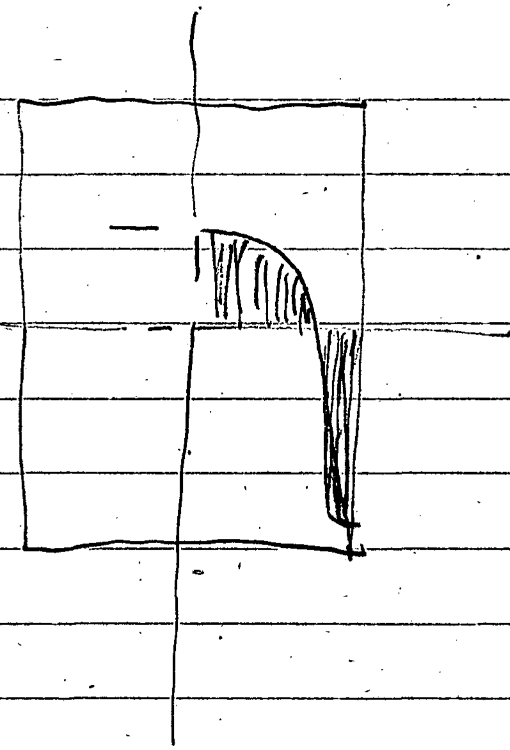
Average stress may be a closer (approach)

Take the joint as two welded plats.



Use the above method to find the relaxiation $\Delta L_1'$ & $\Delta L_2'$

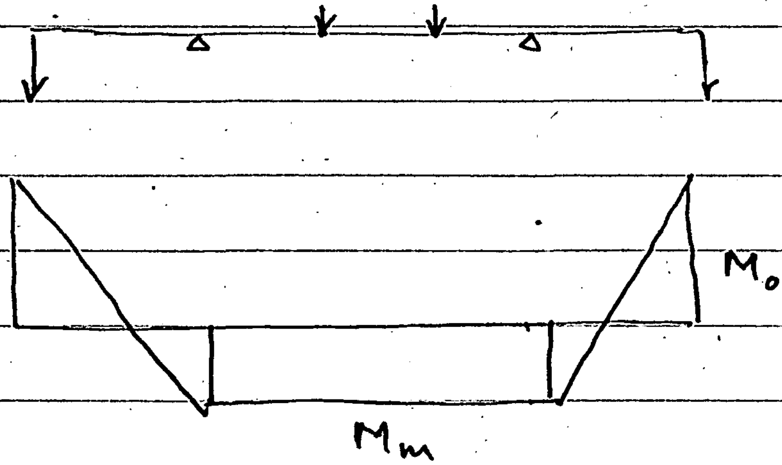
$$(\Delta L_1' + \Delta L_2')/h = \text{relaxiation angle.}$$

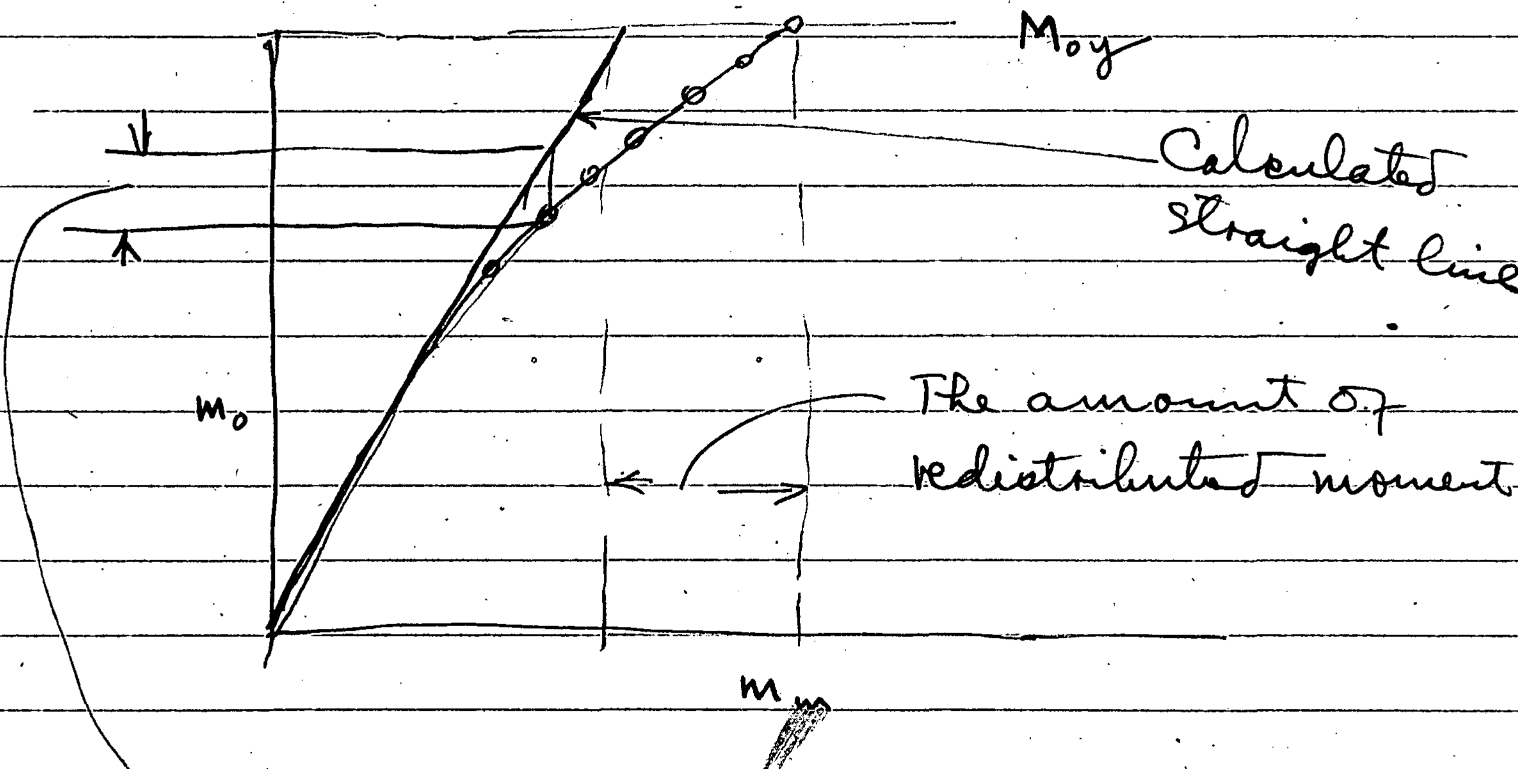


~~Test Results~~

Test B_2

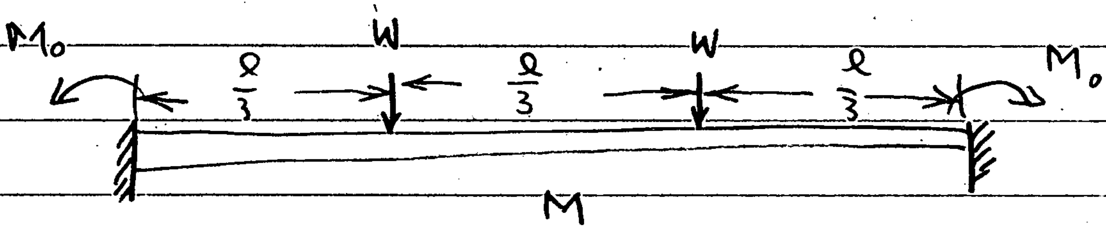
In the elastic range





The amount of moment reduced due to plastic relaxation.

Elastic behaviors of beams are effected by residual stresses as we have pointed out that the modulus of elasticity of the material is reduced by the presence of residual stresses



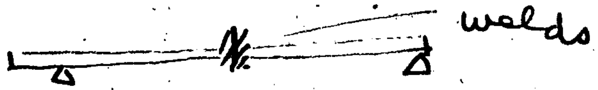
$$M_0 = f(W) = F(M) = \alpha M$$

In the case of a beam loaded as above we know

$$M_0 = 2M \qquad \alpha = 2$$

1. Suppose only residual stresses due to rolling are considered and they are assumed to be uniform. Then the const. α wouldn't be changed as load increase below the calculated yielding strength.

2. a. Residual stresses only in central part. It is clear α tends to increase.



b. Even if there is no residual stress, ~~[illegible]~~ as α smaller than one. Central part of the beam reaches the plastic range first so reduces the rigidity of the central part of the beam. That will make

α tends to increase.

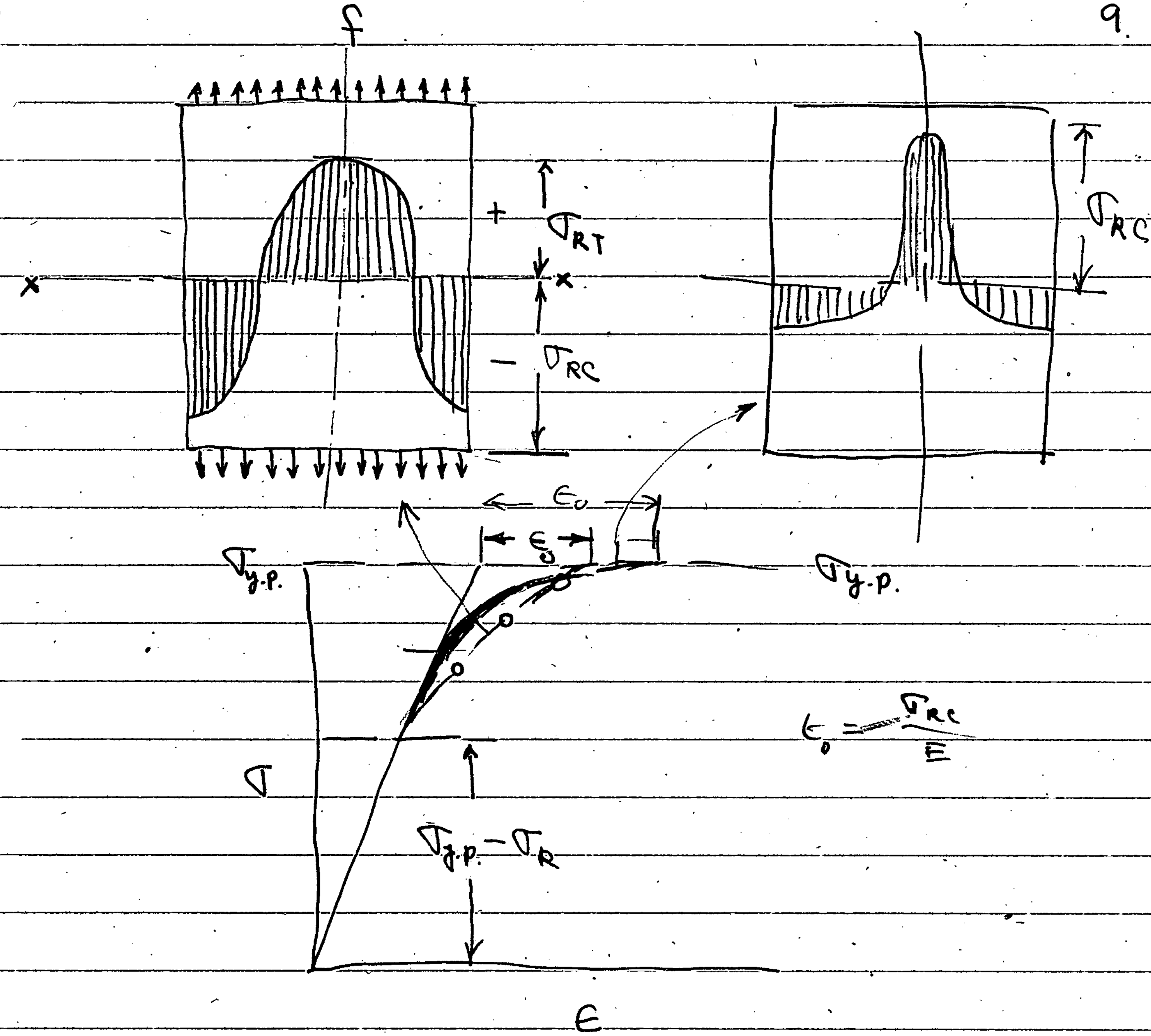
May be some tests should be done in case $\alpha < 1$

3. Residual stresses are located at the ends of the beam Then we will find

α tends to decrease.

see page 6.

Therefore during a test we can always plot M vs M_o to find the value of α and see how residual stresses effect the behavior of beams.

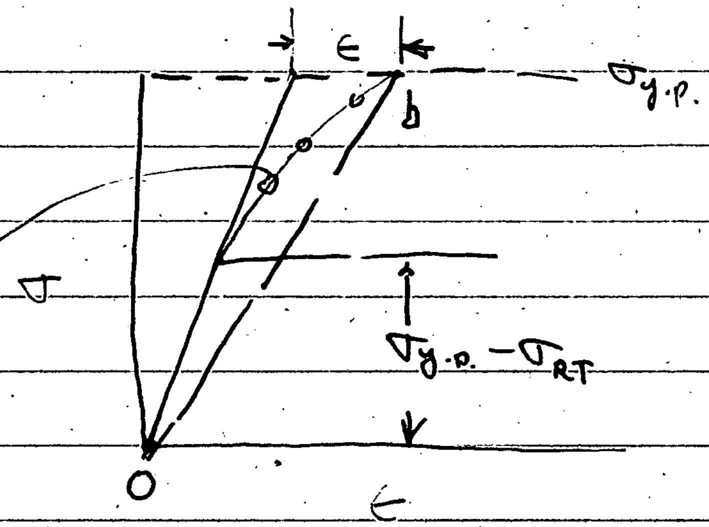


$$\epsilon_0 = \frac{\sigma_{RC}}{E}$$

$$\cancel{\epsilon = \frac{1}{E}(\sigma_{RT} - \sigma_{RC}) - \frac{\sigma_{y.p.}}{E} = \frac{1}{E}(\sigma_{RT} - \sigma_{RC} - \sigma_{y.p.})}$$

Where σ_{RT} = Mag of Tensile residual stress

σ_{RC} = Mag. of Comp. residual stress.



This new straight line could be recommended in design. The corresponding Residual stress diagram is on page 5.

By using of equations on page 4 we can find the amount of relaxd moment.

This curve is very important for tangent modulus load of columns