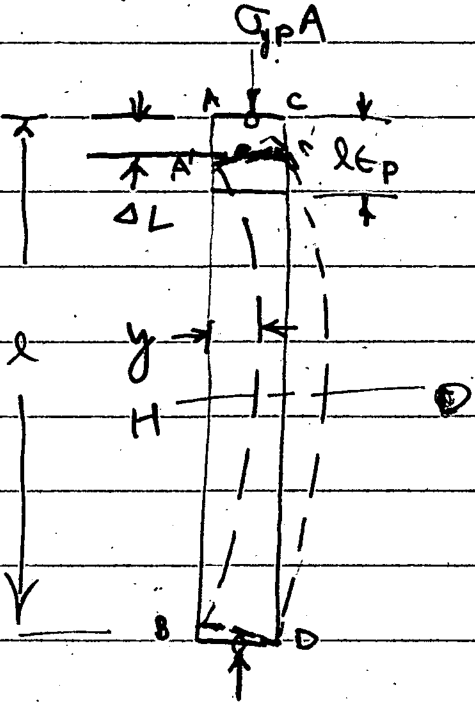
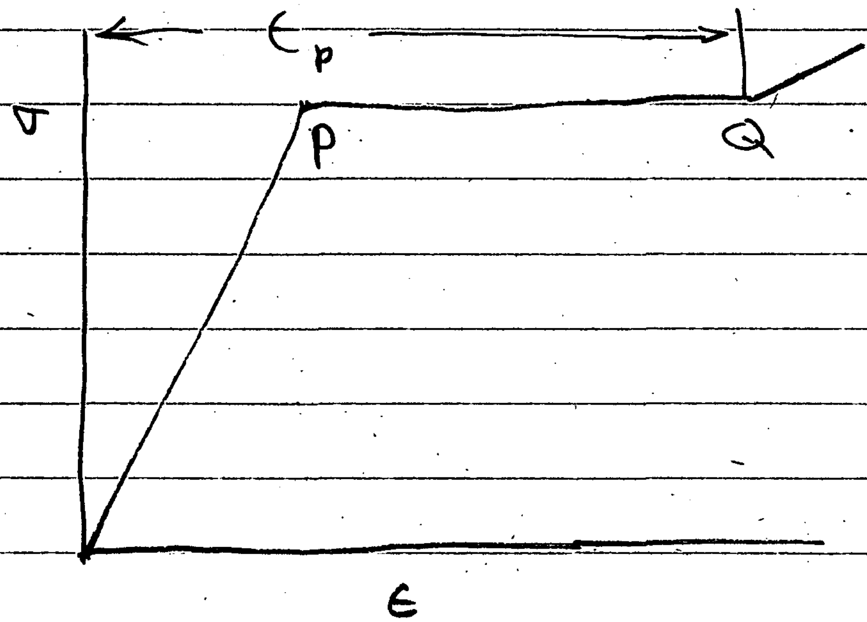
<u>Inelastic Buckling</u>
<u>of Structural Steel</u>

May 18, 1949

a. Plastic Column under dead load at σ_p.



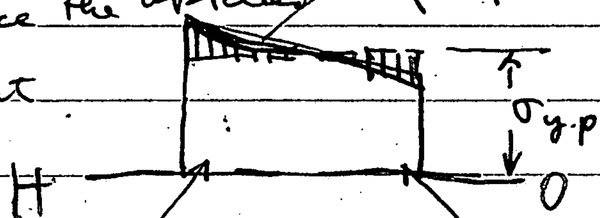


A column is loaded up to its yielding point, say, corresponding to a point as "p" in the diagram. Then keep the load const.
Suppose if the material is homogeneous the column may shorten a length $L\epsilon_p$ and remain straight.
But suppose the column material is not homogeneous. CD side may of (very slightly) higher yielding strength.
Then the column may tend to bend.

The column may be bent to the curvature as shown. CD straight line changed to C'D curve with ~~CD~~ the length will ~~may~~ be ~~unchanged~~ than CD ~~stretched~~ AB, changes to A'B with plastic straining

Take a section HO. The moment acts at the section equals $\sigma_{yp} \times A \times y = M_{HO}$.

The stress distribution will ~~may~~ change to ~~as~~ follows to balance the applied moment (is not ~~may not be~~ straight line)



Higher stress due to plastic flow strain rate

The drop of stress decrease in comp. strain. It follows the elastic straight line independent of strain rate.

If we know the relation

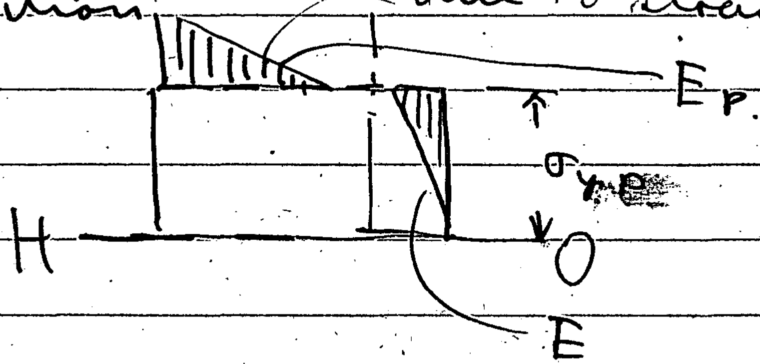
$$\sigma = f(\dot{\epsilon})$$

in plastic we can solve the problem

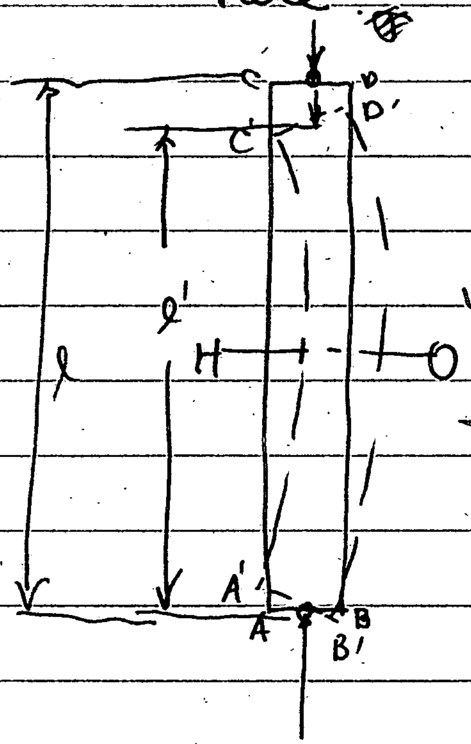
in dynamics

It is clear that the column will ~~deflecting~~ be deflecting in an accelerated speed
(Moment increases as deflection increases)

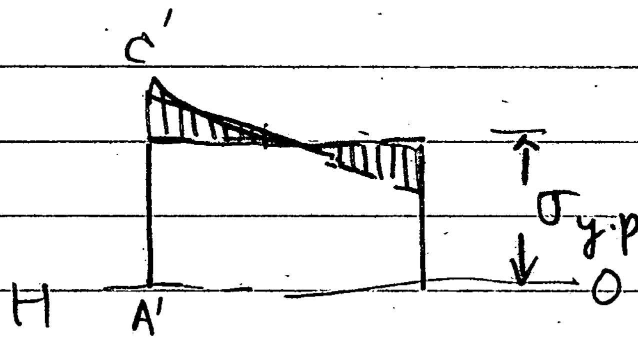
As soon as the ~~water~~ strain at AB side reaches point "Q" in the stress strain diagram ~~The~~ It starts to be strain hardened, Then when the following equalibrium condition obtained the column get into ~~that equilibrium condition~~ a stable condition due to strain hardening



b. Plastic column under change loading rate



Assume the column is under plastic deformation as described in previous article. Suddenly we stop loading and keep l' const.



Then the strain rate at A'C' will be ~~gradually~~ stoped and the A'C' side stress will drop to $\sigma_{y.p.}$ as strain rate reduces to zero. The load "p" will vary (reduce) till another new equalibrium condition is achieved

General case $y_1 = f(x)$

$y_2 = \psi(x)$

$$\epsilon_1 = z \frac{d^2 f}{dx^2}$$

$$\epsilon_2 = z \frac{d^2 \psi}{dx^2}$$

$$\epsilon_3 = \frac{\Delta L}{L} = \frac{1}{2L}\left[\int_0^l \left(\frac{d\psi}{dx}\right)^2 dx + \int_0^l \left(\frac{df}{dx}\right)^2 dx\right]$$

$$\Delta\epsilon = \epsilon_2 + \epsilon_3 - \epsilon_1 \qquad z_0 \text{ at } \Delta\epsilon = 0 \quad \text{---- (3)}$$

$$\Delta\sigma = E\Delta\epsilon$$

$$\int_{-z_0}^{\frac{h}{2}} (z+z_0)\Delta\sigma \, dy\, dz = P\psi \quad \text{------ (1)}$$

$$\int_{-\frac{h}{2}}^{-z_0} \sigma_{y.p.}\, dy\, dz + \int_{-z_0}^{\frac{h}{2}} (\sigma_{y.p} - \sigma)\, dy\, dz = P \quad \text{--- (2)}$$

~~Solve (1), (2) & (3) (impossible)~~

~~Either use Fourier's series~~

or

Special case

$\epsilon_3 + \epsilon_2 - \epsilon_1 > 0$ in the range of the beam

~~ϵ_3 = a const~~

$\therefore$

$$\int_{-\frac{h}{2}}^{\frac{h}{2}} E\,z\left(z\frac{d^2\psi}{dx^2} - z\frac{d^2 f}{dx^2} + \epsilon_3\right) dy\,dz = P\psi$$

$$EI\left(\frac{d^2\psi}{dx^2} - \frac{d^2 f}{dx^2}\right) + EQ\epsilon_3 = P\psi \quad \text{---} (1)$$

$$\int_{-\frac{h}{2}}^{\frac{h}{2}} \left(\sigma_{y.p.} - E\Delta\epsilon\right) dy\,dz = P$$

$$A\sigma_{y.p.} - E\left(Q\frac{d^2\psi}{dx^2} - Q\frac{d^2 f}{dx^2} + \epsilon_3 A\right) = P \quad \text{--} (2)$$

$$\epsilon_3 = \frac{1}{2l}\left[\int_0^l \left(\frac{d\psi}{dx}\right)^2 dx + \int_0^l \left(\frac{df}{dx}\right)^2 dx\right] \quad \text{---} (3)$$

assume "sine" functions

~~assume $\psi = A_n \sin\frac{n\pi x}{L} + B_n \cos\frac{m\pi x}{L}$~~

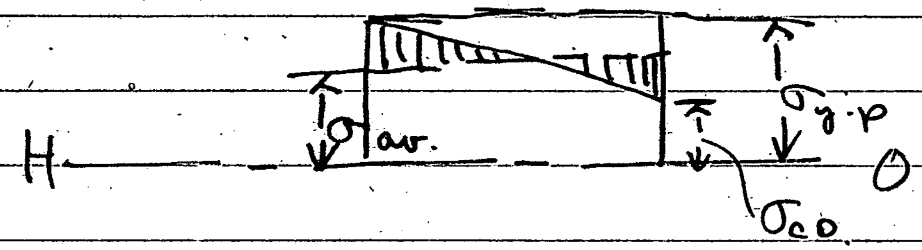
Eleminate ϵ_3 in (1) & (2)

~~Solve (1), then (3), substitute in (2) we get value of P.~~

Approximate estimation may be ~~B~~ 7.

~~The equalibrium condition will have a stress distribution as follows~~

made as follows.

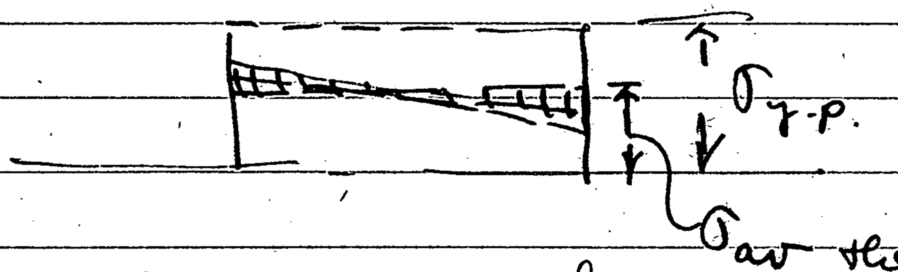


$$P = \sigma_{av} \times A$$

$P \times y_0$ = Moment represent by shaded area.

$$\therefore \frac{1}{2}(\sigma_{co} + \sigma_{y.p})A = P.$$

Where y_0 = max. deflection
HO is at the max. deflection section
in other sections:



where y = defl. at ~~other~~ the sections

Therefore if we have the value

y_0 & $\sigma_{y.p.}$ we can determine axial load p.

External moment

$$p \times y_0 = M$$

Internal moment

$$M = \frac{\frac{1}{2}(\sigma_{y.p.} - \sigma_{CD}) I}{\frac{h}{2}}$$

$$M = \frac{(\sigma_{y.p.} - \sigma_{CD}) I}{h}$$

where h = width of column

and $$p = \left(\frac{\sigma_{y.p.} + \sigma_{CD}}{2}\right) A \quad \ldots (1)$$

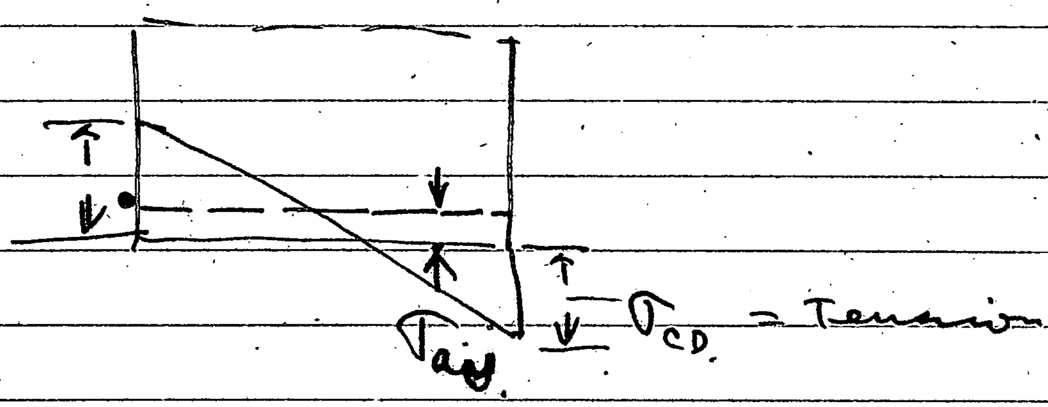
$$p \times y_0 = \frac{(\sigma_{y.p.} - \sigma_{CD}) I}{h} \quad \ldots (2)$$

Solve (1) & (2) we have

$$P = f(\sigma_{y.p.}\, y_0)$$

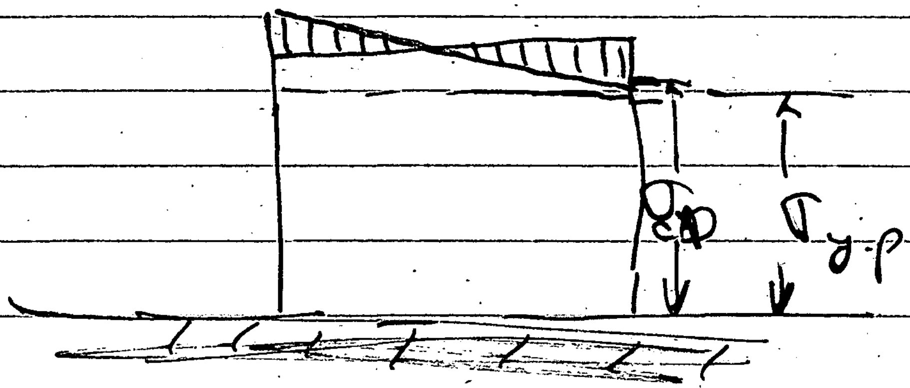
It is clear that P is smaller than $\sigma_{y.p}A$ anyway.

It is very interesting to see that the case on page 4. When the initial defl. is considerable larger than the loading suddenly stop and it hold const. the whole column may streching to increase the curvature. P will be reduced to obtain a new eqhailibrium condition and the stress dist. at the max. defl. may as follows.



C. It is clear when the column is ideal no eccentric loading it would not bent before reaching the strain hardening point
But if it starts to bent at "p" then the AD side will never get a chance to plastic range.

But when the stress applied is higher than $\sigma_{y.p.}$ and there is a little initial curvature in the column Then the column may have a stress distribution as follows ~~as~~ shortening and sidewise motion occurs at the same time ~~with a stress and strain diagram as follows~~



As deflection increases moment increases after stress $\sigma_{CD} < \sigma_{y.p.}$ then CD side starts to stretch instead of compressing

The actual case happens very close to discussed above even with the stress over the column only about $\sigma_{y.p.}$ because the material is not a perfect plastic body

Case "a" discussed only apply to perfect plastic body.

Case "b" discussed may apply to the actual case.

Both go to the strain hardening range and get a static equilibrium

But the actual case may get plastic comp. strain on both sides of different amount, of course, while the other one only gets plastic strain on one side.

Therefore the final state of the column is a function of stress strain relation of the material, the geometric shape, the plastic flow speed, the $\sigma = f(\dot{\epsilon})$ relation, the applied load, and the initial eccentricity of the member.

The curvature of the Beam flange under compression is of course a function Inelastic local buckling