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REPORT
NOTES ON SHANLEY'S THEORY OF
INELASTIC COLUMNS

TO

BY

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Discussion on Shanley's Theory of Inelastic Column

Assume the column reach at a load $P_r = \frac{E_c A}{L}$ without bending, and then start to bend at that load when load increases. We'll see if we can get an equilibrium system.

$$\Sigma M = 0$$

$$\therefore P = \frac{AE_c}{L} \left[\frac{e_1 + e_2 k}{e_1 + e_2} \right] \quad \text{--- (A)}$$

$$\Sigma F = 0$$

$$(1) \quad \therefore P = P_r + \Delta P = \frac{AE_c}{L} + \frac{(e_1 - e_2 k) AE_c}{2}$$

$$\Delta P = \frac{e_1 AE_c}{2} - \frac{e_2 AE_c}{2}$$

$$= \frac{AE_c}{2} (e_1 - e_2)$$

$$E_2 = k E_c$$

$$\therefore \Delta P = \frac{AE_c}{2} (e_1 - k e_2)$$

$$\therefore P = \frac{AE_c}{L} \left[1 + \frac{(e_1 - e_2 k)}{2} \right] \quad \text{--- (B)}$$

$$\therefore 1 + \frac{L(e_1 - e_2 k)}{2} = \frac{e_1 + e_2 k}{e_1 + e_2}$$

$$\frac{e_1 + e_2 - e_1 - e_2 k}{e_1 + e_2} = \frac{L(e_1 - e_2 k)}{2}$$

is the condition of equilibrium

$$e_2(1-k) + \frac{L}{2}(e_1 - e_2 k)(e_1 + e_2) = 0 \quad \text{--- (C)}$$

Substitute (C) to (A) we find the relation to maintain an equilibrium condition after $P = \frac{E_c A}{L}$, between P & e_1 or e_2

We find

$$P = \frac{AE_0}{L} \left(1 + \frac{2\delta(k-1)}{k-1 + 2\delta(1+k)} \right)$$

Suppose the column has been loaded up to a point between tangent modulus load and reduced modulus load with lateral support. Then the lateral support moves away and load increase.

Let the load at the column = P_0 .

$$\sum M = 0 \quad P = \frac{AE_0}{L} \left(\frac{e_1 + e_2 k}{e_1 + e_2} \right)$$

$$\text{where } k = \frac{E_1}{E_0}$$

$$\sum \bar{F} = 0 \quad P = P_0 + \Delta P = P_0 + \left(\frac{e_1 - e_2 k}{2} \right) \frac{AE_0}{L}$$

Condition of Equilibrium

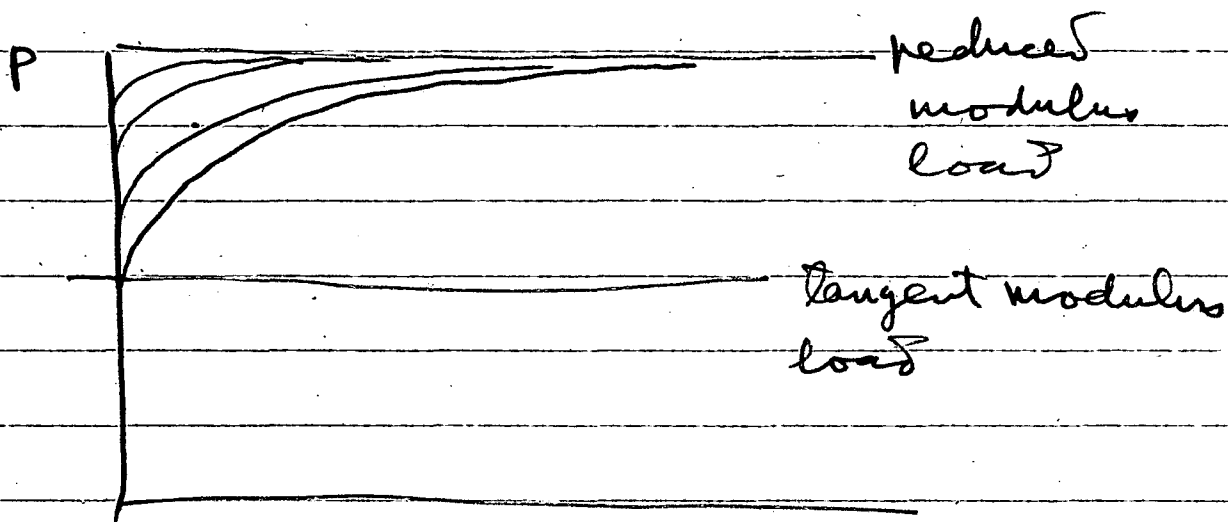
$$P_0 + \frac{AE_0}{L} \left[\frac{(e_1 - e_2 k)L}{2} - \frac{e_1 + e_2 k}{e_1 + e_2} \right] = 0$$

$$\text{Let } P_0 = K' \frac{AE_0}{L} \quad K' > 1$$

we have

$$\left[K' + \frac{e_1 - e_2}{\alpha} \right] = \frac{e_1 + e_2 K}{e_1 + e_2}$$

$$P = \frac{AE_0}{L} [F(K', K, \delta)]$$



Millions of paths ^{the} δ column can be deviated from straight between these two limits

Stability of the system

From reduced modulus theorem we know after the column part its largest modulus load but before its reduced modulus load the system is stable if the load has been kept const. while the infinitesimal displacement is given.

But if "p" varies while the displacement is given we can investigate the stability of the system in the following way

$$P = \frac{AE\alpha}{L} \left(1 + \frac{2\delta(k-1)}{k-1 + 2\delta(1+k)} \right)$$

$$\frac{dP}{d\delta} = \frac{AE\alpha}{L} \left(\frac{2(1-k)^2}{(k-1 + 2\delta(1+k))^2} \right)$$

$$\frac{dP}{d\delta} = \text{Positive whatever the value } \delta \text{ is}$$

$$\lim_{\delta \rightarrow \infty} \frac{dP}{d\delta} = 0 \quad \text{Neutral stability}$$

$$\begin{aligned} \lim_{\delta \rightarrow \infty} \frac{2\delta(k-1)}{k-1 + 2\delta(1+k)} &= \lim_{\delta \rightarrow \infty} \frac{2(k-1)}{\frac{(k-1)}{\delta} + 2(1+k)} \\ &= \frac{k-1}{k+1} \end{aligned}$$

$$\therefore P = \frac{AE\alpha}{L} \left(1 + \frac{k-1}{k+1} \right)$$

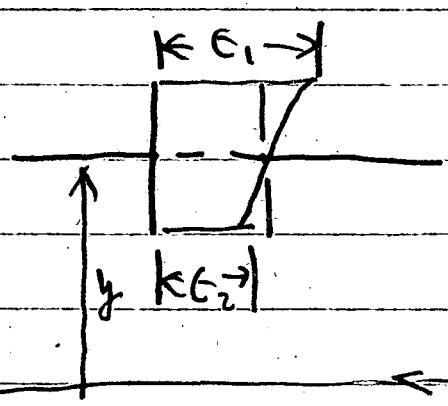
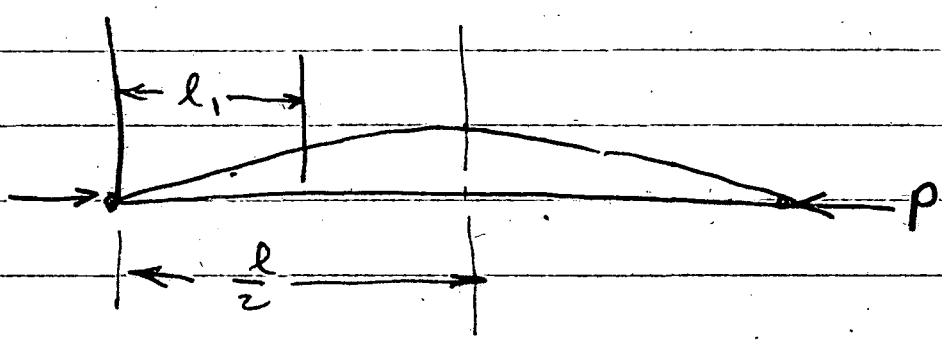
reduced modulus
load

This proves the system is stable all through to the end till P reaches reduced modulus load the system become neutral stable.

Generalized Formulation (Rectangular Cross section)

①

From $0 \rightarrow l_1$



$$P = \frac{E_x I \pi^2}{l^2} + \Delta P$$

$$\Delta P = \frac{E_1 + E_2}{2} \times A \times E_x$$

$$E_x I \frac{d^2 y_1}{dx^2} + P y_1 = 0$$

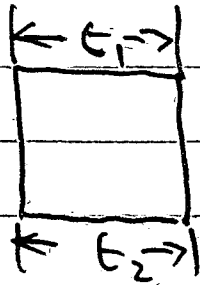
$$y_1 = C_1 \cos \sqrt{\frac{P}{E_x I}} x + C_2 \sin \sqrt{\frac{P}{E_x I}} x$$

1. $x = 0 \quad y_1 = 0 \quad \therefore C_1 = 0$

2. $x = l_1$ a. $\left| \frac{dy_1}{dx} \right| = \left| \frac{dy_2}{dx} \right|$ determine C_2

b. $y_1 = y_0$ determine C_1

at $x=0$

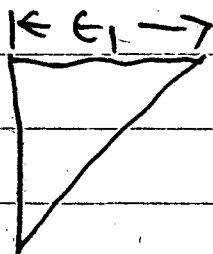


$$\epsilon_1 = \epsilon_2$$

$$\Delta P = \epsilon_1 E_x A$$

at $x=l_1$

$$y_1 = y_0$$



$$\epsilon_2 = 0$$

$$\Delta P = \frac{\epsilon_1}{2} E_x A$$

$$\text{Moment (internal)}_1 = E_x I \frac{d^2 y_1}{dx^2}$$

$$\frac{d^2 y_1}{dx^2} = \frac{1}{R} = \frac{\epsilon_1}{R} = \frac{2 \Delta P}{E_x A R}$$

A = cross section area

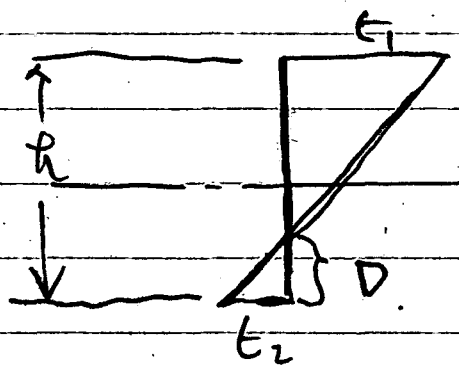
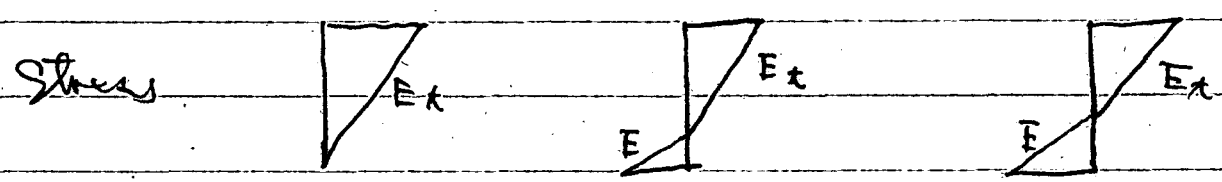
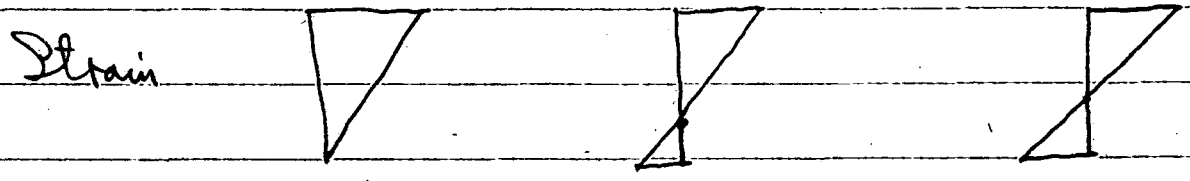
h = depth of the beam

$$M_{(\text{external})} = P y_0 = \left(\frac{E_x I \pi^2}{l^2} + \Delta P \right) y_0$$

$$\therefore y_0 = \frac{2 \Delta P I l^2}{A R (E_x I \pi^2 + \Delta P)}$$

From l_1 to $\frac{l}{2}$

$x = l_1$ $l_1 < x < \frac{l}{2}$ $x = \frac{l}{2}$



① $\frac{1}{R} = \frac{\epsilon_1 + \epsilon_2}{h} = \frac{d^2 y}{dx^2}$

② $D = \frac{t_2}{\epsilon_1 + \epsilon_2}$ $h - D = \frac{\epsilon_1}{\epsilon_1 + \epsilon_2}$

③ $E_t h t \left[\epsilon_1 \times \frac{t_1}{\epsilon_1 + \epsilon_2} - \epsilon_2 \times \frac{k t_2}{\epsilon_1 + \epsilon_2} \right] = 2AP$

Where $t =$ thickness of section
 $h =$ depth
 $k = \frac{E}{E_t}$

$$\therefore E_x R t \left[\frac{\epsilon_1^2}{\epsilon_1 + \epsilon_2} - \frac{k \epsilon_2^2}{\epsilon_1 + \epsilon_2} \right] = 2 \Delta P$$

From ①, ②, ③, we have

$$\text{Solve now } \left\{ \begin{array}{l} \frac{R}{h} = \epsilon_1 + \epsilon_2 \\ \frac{R}{h} E_x A t \left[\epsilon_1^2 - k \epsilon_2^2 \right] = \Delta P \\ \frac{R \epsilon_2}{h} = D \end{array} \right.$$

$$\cancel{R = f(D)} \quad D = f\left(\frac{d^2 y}{dx^2}\right)$$

$$\text{then } P y_2 + E_x I \frac{d^2 y}{dx^2} + F(D, E, E_x) = 0$$

$$P y_2 + E_x I \frac{d^2 y}{dx^2} + F\left(\frac{d^2 y}{dx^2}, E, E_x\right) = 0$$

$$y_2 = \psi(\Delta P, E_x, E, k, h, x)$$

$$\text{① } x = l_1 \quad y_2 = y_0$$

$$x = \frac{l}{2} \quad \frac{dy_2}{dx} = 0$$

$$x = l_1$$

$$\left(\frac{dy_2}{dx}\right) = \left(\frac{dy_1}{dx}\right)$$

$$\frac{P}{R} = \epsilon_1 + \epsilon_2 \quad \dots \quad (1)$$

$$\frac{R}{h} E_x A t [\epsilon_1^2 - K \epsilon_2^2] = \Delta P \quad \dots \quad (2)$$

$$\frac{R \epsilon_2}{h} = D \quad \dots \quad (3)$$

Take (2) $K=1$

$$\epsilon_1 = \frac{P}{R} - \epsilon_2$$

$$\therefore \frac{R}{h} E_x A t \left[\frac{P^2}{R^2} - 2 \frac{P}{R} \epsilon_2 \right] = \Delta P$$

$$\frac{R}{2h} \left(E_x A t \frac{P}{R} - \Delta P \right) = \epsilon_2$$

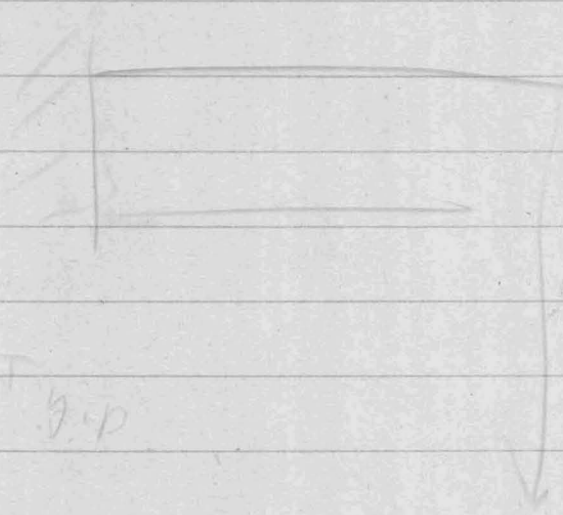
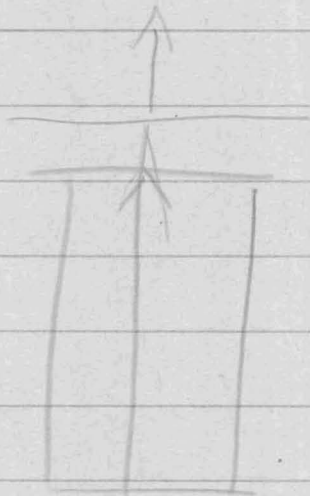
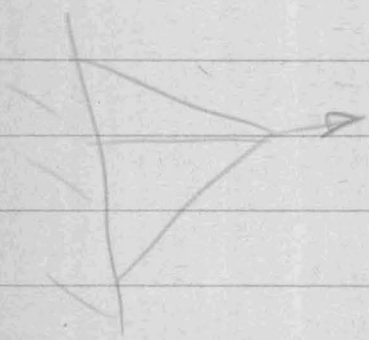
$$\therefore \epsilon_2 = \frac{E_x A t}{2} \cdot \frac{R \Delta P}{2h}$$

$$\therefore D = \frac{R}{h} \left(\frac{E_x A t}{2} - \frac{R \Delta P}{2h} \right)$$

$$\therefore D = \frac{d^2 y}{h dx^2} \left(\frac{E_x A t}{2} - \frac{\frac{d^2 y}{dx^2} \Delta P}{2h} \right)$$

Non linear equation.

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