PLASTIC ANALYSIS AND DESIGN AT THE UNDERGRADUATE LEVEL

by
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Since the philosophy of basing design on the maximum carrying capacity of a structure is rational, since methods for predicting ultimate strengths are available, since these methods are as simple (if not simpler) than conventional elastic procedures, and since numerous full scale and model tests have confirmed the adequacy of the theory, it is inevitable that such methods will eventually be taught at the undergraduate level. As an added incentive, the teaching of the principles will make it possible to certify the safety of structures designed by certain approximate procedures currently being used. For example, if a given tier building is analyzed by both the Portal and the Cantilever methods variations in caluclated stresses of as much as 100% may be observed (and some of these may even be of the opposite sign). It would therefore seem that one or both of the methods will result in an unsafe structure. But to be able to justify either by elastic reasoning there would be required a comparison with the "exact" solution. From a "plasticity point of view" however, it is noted that both systems represent possible equilibrium states and are therefore lower bounds to the true carrying capacity of the structure. A design based on either of these procedures is therefore safe.

The justification of the methods of designing certain details (e.g. riveted or welded connections) according to the current AISC specification presents another troublesome problem when approached elastically. When considered plastically however, they are easilty explained. The influence of foundation movements, residual stresses, etc., are similar problems the answers to which can be found from plastic analysis.

At this point it should be emphasized that plastic analysis is not a "cure-all" and <u>will not</u> replace elastic design. Actually, the types of structures which fall within the domain of this so-called "new" design procedure are rather limited in number. On the other hand, for steel structures they represent a major portion of the tonnage of rolled shapes. It should also be pointed out that if deflection rather than strength controls the design, plastic analysis <u>will not</u> apply.

Having considered the question "Why teach plastic analysis?", the next question for discussion is "What are the limitations of plastic analysis?" or "What types of structures can be analyzed by plastic methods?".

First of all, to be able to use these methods of solution the material and cross-section must have moment-unit rotation characteristics such that, as the full plastic moment is approached, large increases in rotation are associated with relatively small increases in applied moment. This necessary condition rules out many engineering materials. Mild structural steel, however, is ideally suited. Secondly, plastic analysis does not apply where fatigue may be a problem (a factor that is fortunately not presend in ordinary building construction). Local and/or lateral buckling also must be guarded against. However, by placing certain restrictions on the geometry of the cross-section and by specifing the unsupported length (or a function of it) between lateral bracing, these conditions will not influence the design of the main members. An additional condition which at first caused much concern was the problem of variable repeated loading. It was known that variations in loads below those predicted on the basis of simple plastic theory could result in the failure of the structure. Fortunately, recent analytical and experimental work has pointed out that for the majority of practical cases encountered, this problem is not important. One problem that is currently receiving attention and for which plastic analysis is not at present recommended is the design of members subjected to high axial thrust in addition to bending moments (e.g. the columns in the lower stories of a tier building).

At present then, the types of structures best suited for plastic analysis are the single and multiple span, one and two storied, rigid, steel frames.

The next question is, "How can plastic analysis be included in an already overcrowded undergraduate program of study?". Here it should be pointed out that it is not at the present time considered necessary to add a separate course on this subject at the undergraduate level. The concepts of and the limitation to plastic analysis as well as the necessary conditions for and the methods for obtaining a plastic analysis solution can be covered in existing courses.

Consider the first course in mechanics, i.e. "Statics". In this course linkages (i.e. mechanisms) as well as other types of problems could also be solved for static equilibrium by virtual displacement methods. This would give the student a good foundation in "work principles" on the most elementary structures and would prepare him for more advanced consideration of the same principle that would be covered in later courses. Strength of Materials and Indeterminate Analysis could profitably draw upon this knowledge for both elastic and plastic analysis solutions.

In a course on "Strength of Materials" the inelastic as well as the elastic bending behavior of members could be discussed. Concepts such as the plastification of the cross-section, the redistribution of bending moments (for indeterminate structures) as well as the necessary conditions for a plastic analysis solution could be covered. In addition several continuous beam problems that had been analyzed by elastic methods could be solved plastically using the "Equilibrium" method of solution. The emphasis however should not be on the idea that the same problem is being solved by two different methods but rather that two different problems are being considered; one that describes the behavior of the structure in the elastic range and the other which gives the maximum carrying capacity of the structure. The philosophical question of which is the more desirable criterion of design for a particular problem should not be considered at this stage.

The laboratory course in Strength of Materials could include the testing of an indeterminate beam of mild structural steel to illustrate the phenomenon of redistribution of bending moments. If time were available, a similar test could be carried out for a brittle material to point up the basic difference in their post yield bending behavior.

If an "Advanced Strength of Materials" course is offered, this could include a more detailed discussion of those concepts introduced in the first course. There could also be considered the influence of cross-sectional shape and axial thrust on altering the full plastic moment value of the cross-section. The various methods for obtaining a plastic analysis solution could also be introduced.

"Engineering Materials" could include a discussion of the properties of materials beyond the elastic limit. The advantages of a ductile type of bending behavior could be discussed and conditions that limit ductility could be considered.

Since the full advantage of plastic analysis is realized in the indeterminate structure, it is in such a course that plastic analysis as such should be taught. If two semesters of this subject are offered at the undergraduate level this can easily be accomplished. However, if only one is available it may be difficult to do justice to both the elastic and the plastic methods. If a choice between the two is necessary, it is the opinion of the author that the elastic methods should receive first preference. One who knows elastic analysis can easily pick up plastic analysis, but the converse is not necessarily true. One or two lectures on plastic analysis would have to suffice for the time being.

The senior steel design course is another place where plastic analysis could logically be included. For example, a mill type building that had been designed elastically could be redesigned plastically in a relatively short amount of time.

The supervised special problem course or thesis at the undergraduate level is still another place where the student could introduced to plastic methods. Students of advanced standing should be encouraged to enroll in such courses.

It is realized that the above outlined coverage of plastic analysis at the undergraduate level is ambitious to say the least. Furthermore, it is a program that must develop gradually and be altered and adjusted to meet the individual situation. However, because of the recent interest in this subject as demonstrated by the increasing number of articles, short courses, etc., it is considered desirable to acquaint the undergraduate student with the concepts and limitations of this method of analysis as soon as possible. This could be accomplished by a series of 3 or 4 lectures and, possibly, a laboratory session in the senior steel design course. It has been observed that with this much time, the average student can grasp the basic idea behind plastic analysis and can solve several simple problems.

We have considered several aspects of the "Why" and "How" of plastic analysis at the undergraduate level. Plastic analysis <u>will</u> be taught. The only question remaining is WHEN.