

WHY PLASTIC DESIGN

by

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## 1. I N T R O D U C T I O N

Steel possesses ductility--A unique property that no other structural material exhibits in quite the same way. Through ductility structural steel is able to absorb large deformations beyond the elastic limit without the danger of fracture.

Although there are a few instances where conscious use has been made of this property, by and large the engineer has not been able to fully exploit this feature of ductility in structural steel. As a result of these limitations it turns out that we have been making a considerable sacrifice of economy.

Engineers have known of this ductility for years, and since the 1920's have been attempting to see if some conscious use couldn't be made of this property in design. Plastic design is the realization of that goal. We are now equipped to apply this new design concept to statically loaded frames of structural steel for continuous beam and single story building frames, continuous over one or more spans. This category accounts for a very substantial portion of the total tonnage of fabricated structural steel produced annually in the United States. Not only are we equipped, these techniques have already been applied in Europe, and during this past summer a structure was designed according to the plastic methods and actually erected in Canada.

We have only been able to achieve that goal--namely the producing of a practical design method--because two important conditions were satisfied. First, the theory concerning the plastic behavior of continuous steel frames has been systematized and reduced to simple design procedures. Secondly, every conceivable factor that might tend to limit the load-carrying capacity to something less than that predicted by the simple plastic theory has been investigated and rules have been formulated to safe-guard against such factors.

It is our objective in this conference to present to you the available methods of plastic analysis, the design procedures that have been worked out using these methods, the experimental verification of these procedures by actual test results, and to cover in the time that permits the secondary design considerations that so often are a stumbling block to any new design technique. In short, we hope to stimulate your further study of the topic. If we do not succeed in demonstrating the simplicity of the plastic method, please remember that you didn't master all you know about present design methods in a few one-hour lectures.

It is the purpose of this particular talk to describe the fundamental concepts involved in plastic design, to justify its application to structural steel frames, and to demonstrate that some of the concepts are actually a part of our present design procedures.

To some of you these concepts may be new. Others of you have had some experience in solving problems. It won't hurt any of us, however, to have definitions clearly in mind and these are included in an appendix.

## 2. STRUCTURAL STRENGTH

(Functions of Structures)

### 1. Limits of Usefulness

The design of any engineering structure, be it a bridge or building, is satisfactory if it can be built with the needed economy and if throughout its useful life it carries its intended loads and otherwise performs its intended function. As already mentioned, in the process of selecting suitable members for such a structure, it is necessary to make a general analysis of structural strength and secondly to examine certain details to assure that local failure does not occur.

The ability to carry the load may be termed "structural strength". Broadly speaking, the structural strength or design load of a steel frame may be determined or controlled by a number of factors, factors that have been called "limits of structural usefulness". These are: first attainment of yield point stress (conventional design), brittle fracture, fatigue, instability, deflections, and finally the attainment of maximum plastic strength.

### 2. Plastic Design As An Aspect of Limit Design

Strictly speaking, a design based on any one of the above-mentioned six factors could be referred to as a "limit design", although the term usually has been applied to the determination of ultimate load as limited by buckling or maximum strength. <sup>(1)</sup> "PLASTIC DESIGN" as an aspect of limit design and as applied to continuous beams and frames embraces, then, the last of the limits --the attainment of maximum plastic strength.

Plastic design, then, is first a design on the basis of the maximum load the structure will carry as determined from an analysis of strength in the plastic range (that is, a plastic analysis). Secondly it consists of a

consideration by rules or formulas of certain factors that might otherwise tend to prevent the structure from attaining the computed maximum load. Some of these factors may be present in conventional design. Others are associated only with the plastic behavior of the structure. But the unique feature of plastic design is that the ultimate load rather than the yield stress is regarded as the design criterion.

It has long been known that whenever members are rigidly connected, the structure has a much greater load-carrying capacity than indicated by the elastic stress concept. Continuous or "rigid" frames are able to carry increased loads above "first yield" because structural steel has the capacity to yield in a ductile manner with no loss in strength; indeed, with frequent increase in resistance. Although the phenomenon will be described in complete detail later, in general terms what happens is this: As load is applied to the structure, the cross-section with the greatest bending moment will eventually reach the yield moment. Elsewhere the structure is elastic and the "peak" moment values are less than yield. As load is added a zone of yielding develops at the first critical section; but due to the ductility of steel, the moment at that section remains about constant. The structure therefore calls upon its less-heavily stressed portions to carry the increase in load. Eventually zones of yielding are formed at other sections until the moment capacity has been used up at all necessary critical sections. After reaching the maximum load value, the structure would simply deform at constant load.

### 3. Elastic Versus Plastic Design

We can't repeat too often the distinction between elastic design and plastic design. In conventional elastic design, a member is selected such that the maximum allowable stress is equal to 20,000 pounds per square inch at the working load. Probably the most common equation we now use is  $S = \frac{M}{20}$ . As shown in Figure 1 such a beam has a reserve of strength of 1.65 if the yield point

stress is 33,000 pounds per square inch. Due to the ductility of steel there is a bit more reserve (14% for a wide flange shape). So the total inherent overload factor of safety is equal to 1.88.

In plastic design, on the other hand, we start with the ultimate load. For if we analyze an indeterminate structure we will find that we can compute the ultimate load much easier than we can compute the yield load. So we multiply the working load,  $P_w$ , by the same load factor (1.88) and then select a member that will reach this factored load.

If we took the trouble to draw the load -vs- deflection curve for the restrained beam we would get the curve shown in Figure 1. It has the same ultimate load as the conventional design of the simple beam and the member is elastic at working load. The important thing to note is that the factor of safety is the same in the plastic design of the indeterminate structure as it is in the conventional design of the simple beam.

While there are other features here, the important thing to get in mind at this stage is that in conventional procedures we find the maximum moment under the working load and select a member such that the maximum stress is not greater than 20 ksi (the factor of safety inherent in this procedure is 1.88 in the case of a simple beam); on the other hand in plastic design we multiply the working load by  $F = 1.88$  and select a member which will just support the ultimate load.

Already we have used two new terms: limit design and plastic design. Let's include them in our list of definitions (see appendix).

### 3. HISTORICAL DEVELOPMENT

The concept of design based on ultimate load as the criterion is more than 40 years old! The application of plastic analysis to structural design appears to have been initiated by Dr. Gabor Kazinczy, a Hungarian, who published results of his tests of clamped girders as early as 1914.<sup>(2)</sup> He also suggested analytical procedures similiar to those now current, and designs of apartment-type buildings were actually carried out.

In his *Strength of Materials*<sup>(3)</sup>, Timoshenko refers to early suggestions to utilize ultimate load capacity in the plastic range and states

"such a procedure appears logical in the case of steel structures submitted to the action of stationary loads, since in such cases a failure owing to the fatigue of metal is excluded and only failure due to the yielding of metals has to be considered."

Early tests in Germany were made by Maier-Leibnitz<sup>(4)</sup> who showed that the ultimate capacity was not affected by settlement of supports of continuous beams. In so doing he corroborated the procedures previously developed by others for the calculation of maximum load capacity. The efforts of Van den Broek<sup>(1)</sup> in this country and J. F. Baker<sup>(5)</sup> and his associates in Great Britain to actually utilize the plastic reserve strength as a design criterion are well know. (Prof. Van den Broek was teaching about ductility in 1918). Progress in theory of plastic structural analysis (particularly that at Brown University) has been summarized by Symonds and Neal<sup>(6)</sup>. A survey of design trends, by Winter<sup>(7)</sup>, discusses briefly many of the factors germain to plastic design.

For more than ten years the American Institute of Steel Construction, the Welding Research Council, the Navy Department, and the American Iron and Steel Institute have sponsored studies at Lehigh University. These studies have featured not only the verification of this method of analysis through appropriate

tests on large structures, but have given particular attention to the conditions that must be met to satisfy important secondary design requirements<sup>(8)</sup>. Much of this will be discussed later in the conference.

#### 4. FUNDAMENTAL CONCEPTS

With this background of the functions of a structure, let us start with the fundamentals of the simple plastic theory.

##### 1. Mechanical Properties

An outstanding property of steel, which (as already mentioned) sets it apart from other structural materials, is the amazing ductility which it possesses. This is characterized by Figure 2. In Figure 3 are shown partial tensile stress-strain curves for a number of different steels. Note that when the elastic limit is reached, elongations from 8 to 15 times the elastic limit take place without any decrease in load. Afterwards some increase in strength is exhibited as the material strain hardens.

Although the first application of plastic design is to structures fabricated of structural grade steel, it is no less applicable to steels of higher strength as long as they possess the necessary ductility. Figure 3 attests to the ability of a wide range of steels to deform plastically with characteristics similar to A-7 steel.

It is important to bear in mind that the strains shown in this figure are really very small. As shown in Figure 4, for ordinary structural steel, final failure by rupture occurs only after a specimen has stretched some 15 to 25 times the maximum strain that is encountered in plastic design. Even in plastic analysis, at ultimate load the critical strains will not have exceeded about 1.5% elongation. Thus the use of ultimate strength as the design criterion still leaves available a major portion of the reserve ductility of steel which can be used as an added margin of safety. Bear in mind that this maximum strain of 1.5% is a strain at ultimate load in the structure--not at working load. In most cases under working load the strains will still be below

the elastic limit. We must distinguish also between the term "ultimate load" as we use it here to mean the maximum load a structure will carry, as distinct from the "ultimate strength" exhibited by an acceptance test coupon. So, we will add that definition to our list.

## 2. Maximum Strength of Some Elements

On the basis of the ductility of steel (characterized by Figure 4) we can now quickly calculate the maximum carrying capacity of certain elementary structures.

As our first example take a tension member such as an eye bar (Figure 5). If we compute the stress we find that

$$\sigma = \frac{P}{A}$$

If we draw the load-versus-deflection relationship it will be elastic until the yield point is reached. As shown in Figure 5 the deflection at the elastic limit is given by

$$\delta_y = \frac{P_u L}{AE}$$

Since the stress distribution is uniform across the section, unrestricted plastic flow will set in when the load reaches the value given by

$$P_u = \sigma_y A$$

This is, therefore, the ultimate load. It is the maximum load the structure will carry without the onset of unrestricted plastic flow.

As a second example we will consider the three-bar structure shown in Figure 6. It is selected because it is an indeterminate structure since the state of stress cannot be determined by statics. Consider first the elastic state: we will first take equilibrium and obtain:

$$2T_1 + T_2 = P$$

Next we would consider continuity and use the condition that (with a rigid cross bar) the total displacement of bar 1 will be equal to that of bar 2.

Therefore

$$\frac{T_1 L_1}{AE} = \frac{T_2 L_2}{AE}$$

$$T_1 = \frac{T_2}{2}$$

With this relationship between  $T_1$  and  $T_2$  obtained by the continuity condition, we then find from the equilibrium condition that

$$T_2 = \frac{P}{2}$$

The load at which the structure will first yield may then be determined by substituting in this expression the maximum load which  $T_2$  can reach, namely  $\sigma_y A$ .

Thus  $P_y = 2T_2 = 2\sigma_y A$

If we were interested in the displacement at the yield load it would be determined from

$$\delta_y = \epsilon_y L_2 = \frac{\sigma_y L}{2E}$$

Now, when the member is partially plastic it deforms as if it were a two-bar structure except a constant force equal to  $\sigma_y A$  is supplied by Bar 2 (the member is in the plastic range). This situation continues until the load reaches the yield value in the two outer bars. Notice how easily we can compute the ultimate load:

$$P_u = 3\sigma_y A$$

The continuity condition need not be considered when the ultimate load in the plastic range is being computed.

The load deflection relationship for the structure shown in Figure 6 is indicated at the bottom. Not until the load reaches that value computed by a plastic analysis did the deflections commence to increase rapidly. The deflection at ultimate load can be computed from:

$$\delta_u = \epsilon_y L_1 = \frac{\sigma_y L}{E}$$

The three essential features to this simple plastic analysis are as follows:

1. each portion of the structure (each bar) reached a plastic yield condition
2. the equilibrium condition was satisfied at ultimate load and
3. there was unrestricted plastic flow at the ultimate load

These same features are all that are required to complete the plastic analysis of an indeterminate beam or frame, and in fact, this simple example illustrates all of the essential features of a plastic analysis.

### 3. Plastic Bending

Let us now trace the stages of yield stress penetration in a rectangular beam subjected to a progressive increase in bending moment. At the top of Figure 7, is replotted the stress-strain relationship. We retain the assumption that bending strains are proportional to the distance from the neutral axis.

At Stage 1, as shown in the next line of Figure 7 the strains have reached the yield strain. When more moment is applied (say to Stage 2), the extreme fiber strains are twice the elastic limit value. The situation is similar for Stage 3 ( $\epsilon_{\max.} = 4\epsilon_y$ ). Finally at Stage 4 the extreme fiber has strained to  $\epsilon_{st}$ .

What are the stress distributions that correspond to these strain diagrams? These are shown in the next line of Figure 7. As long as the strain is greater than the yield value, notice from the stress-strain curve that the stress remains at  $\sigma_y$ . The stress distributions, therefore follow directly from the assumed strain distributions.

As a limit we obtain the "stress block"-- a rectangular pattern which is very close to the stress distribution at Stage 4.

One term contained in Figure 7 and not as yet defined is the curvature,  $\phi$ . This is the relative rotation of two sections a unit distance apart. Just as it is basic to the fundamentals of elastic analysis, the moment-curvature ( $M-\phi$ ) relationship is a basic concept in plastic analysis.

#### 4. Moment-Curvature Relationship

In the upper portion of Figure 8 is shown, then, the moment-curvature relationship for the rectangular beam just considered in Figure 7. The numbers in the circles correspond to the "stages" of Figure 7. Notice that Stage 4 approaches a limit and represents the complete plastic yield of the cross section. There is a 50% increase in strength above the computed elastic limit (Stage 1) due to plastification of the cross-section. This represents one of the sources of reserve strength beyond the elastic limit of rigid frame.

(We've used another new term that we can now add to our list of definitions. "Plastification" is the development of full plastic yield of the cross-section. It is the process that takes place as we move from the elastic limit to full yield.)

The moment-curvature relationship for a typical wide flange beam is shown in the lower portion in Figure 8 (the stress distributions correspond to the lettered points on the  $M-\phi$  curve) It should be noted that the reserve strength beyond the elastic limit is smaller than for the rectangle. The average value for all WF beams is 1.14. Correspondingly there is a more rapid approach to  $M_p$  when compared with the rectangle. As a matter of fact, when the curvature is twice the elastic limit value (Stage c) the moment has reached within 2% of the full plastic moment.

#### 5. Plastic Modulus ( $Z$ ), Shape Factor ( $f$ ) and Plastic Hinge

When the yield point is reached in bending, the total resistance provided by the cross-section, as shown in Figure 9, is given by (Stage 1):

$$M_y = \sigma_y S$$

where  $S$  is the section modulus. When the section becomes completely plastic the resisting moment approaches a maximum value called the plastic moment,  $M_p$ . The stress block for this case is also shown in Figure 9. The magnitude of this moment may be computed directly from this stress distribution. It is equal to the couple created by the tensile and compressive forces  $(\sigma_y \frac{A}{2})$ . Thus,

$$M_p = (\sigma_y \frac{A}{2})(\bar{y})(2)$$

The quantity  $A\bar{y}$  is called "plastic modulus" and is denoted by  $Z$ ; therefore

$$M_p = \sigma_y Z$$

The plastic modulus is equal to the combined statical moments of the cross-section areas above and below the neutral axis, since the stress at every point on these areas is the same. (We have two additional terms to add to our dictionary: "the plastic moment" and "the plastic modulus").

The ratio of maximum moment of resistance ( $M_p$ ) to the elastic moment of resistance ( $M_y$ ) is dependent only upon the form of the cross-section. It is therefore called the shape factor,  $f$ , and may be computed from

$$\frac{M_p}{M_y} = \frac{Z}{S} = f$$

For wide flange shapes the average value of  $f$  is 1.14, varying from a low 1.09 to a high of 1.22. In Figure 10 are shown a number of shapes with the corresponding value of  $f$  being tabulated.

You will remember from Figure 8 that when the moment approached the limiting value then rotation increased without limit. In other words the member was free to rotate through many times the angle that the same length of beam could bend elastically. Actually the section under maximum moment is acting just like a hinge except that it is restrained by a constant moment,  $M_p$ . We

call this phenomenon a plastic hinge and in our later calculations we will assume that the hinge forms at discrete points of zero length. Thus the member under bending behaves elastically up to the full plastic moment,  $M_p$ , and then is free to rotate at that constant moment.

## 6. Redistribution of Moment

Now, if the shape factor were all that were involved in plastic design we wouldn't be discussing the topic at all. A second factor contributing to the reserve of strength (and usually to a greater extent than the shape factor) is called "redistribution of moment". It is due to the action of plastic hinges. As load is added to a structure, eventually the plastic moment is reached at a critical section. That critical section is the one that is most highly stressed in the elastic range. As further load is added, this plastic moment value is maintained while the section rotates. Other less-highly-stressed sections maintain equilibrium with the increased load by a proportionate increase. This process of redistribution of moment due to successive formation of plastic hinges continues until the ultimate load is reached.

This is exactly the process that took place in the case of the three-bar truss except that forces instead of moments were involved. When the force in Bar 2 reached the yield condition it remained constant there while the forces continued to increase in Bars 1 and 3. The ultimate load was reached when all critical bars became plastic.

Let's illustrate the phenomenon now with the case shown in Figure 11, a fixed ended beam with a concentrated load off-center. As the load  $P$  is increased the beam reaches its elastic limit at the left end (Stage 1). The moments at sections B and C are less than the maximum moment. Note that in this example we will consider the idealized  $M-\theta$  relationship as shown in the lower left hand portion. (The dotted curve shows the more "precise" behavior).

As the load is further increased, a plastic hinge eventually forms at Section B. The formation of the plastic hinge at A will permit the beam to rotate there without absorbing any more moment. Referring to the load-deflection curve immediately below the moment diagrams the deflection is increasing at a greater rate.

Eventually, at Stage 3, when the load is increased sufficiently to form a plastic hinge at section C, all of the available moment capacity of the beam will have been exhausted and the ultimate load reached.

It is evident from the load-deflection curve shown in the lower part of the figure that the formation of each plastic hinge acts to remove one of the indeterminates in the problem and the subsequent load-deflection relationship will be that of a new (and simpler) structure. In the elastic range, the deflection under load can be determined for the completely elastic beam. Starting from point 1 the segment 1-2 represents the load-deflection curve of the beam in sketch b loaded within the elastic range. Likewise the load-deflection curve of the beam in sketch c looks similar to the portion 2-3.

Beyond Stage 3 the beam will continue to deform without an increase in load just like a link mechanism if the plastic hinges were replaced by real hinges. (We add the word "mechanism" to our list of terms because we will be referring to it frequently.)

The fundamental concepts involved in the simple plastic theory may be summarized as follows (and indeed are demonstrated by Figure 11):

1. Plastic hinges will form in structural members at points of maximum moment.
2. A plastic hinge is characterized by large rotation at the plastic moment value ( $M_p = \sigma_y Z$ )
3. The formation of plastic hinges allows a subsequent redistribution of moment until  $M_p$  is reached at each critical ("maximum") section.

4. The maximum load will be reached when a sufficient number of plastic hinges have formed to create a mechanism.

### 6. Introduction to Methods of Analysis

On the basis of the principles we have just discussed perhaps you have already visualized how to compute the ultimate load: Simply sketch a moment diagram such that plastic hinges are formed at a sufficient number of section to allow mechanism motion. Thus in Figure 12, the bending moment diagram for the uniformly-loaded, fixed-ended beam would be drawn such that  $M_p$  was reached at the two ends and the center. In this way a mechanism is formed.

By equilibrium,

$$\frac{W_u L}{8} = 2M_p$$

$$W_u = \frac{16M_p}{L}$$

How does this compare with the load at first yield? At the elastic limit (see dotted moment-diagram in Figure 12) we know from a consideration of continuity that the center moment is one-half the end moment. Thus,

$$\frac{W_y L}{8} = M_y + \frac{M_y}{2} = \frac{3}{2} M_y$$

$$W_y = \frac{12M_y}{L}$$

Therefore the reserve strength due to redistribution of moment is

$$\frac{W_u}{W_y} = \frac{16 M_p/L}{12 M_y/L} = \frac{4 M_p}{3 M_y}$$

Considering the average shape factor of WF beams, the total reserve strength due to redistribution and shape factor (plastification) is

$$\frac{W_u}{W} = \frac{4}{3}(1.14) = 1.52$$

For this particular problem, then, the ultimate load was 52% greater than the load at first yield, representing a considerable margin that is disregarded in

conventional design.

There are other methods for analyzing a structure for its ultimate load, in particular the "Mechanism Method" (to be described later) which starts out with an assumed mechanism instead of an assumed moment diagram. But in every method, there are always these two important features: (1) the formation of plastic hinges and (2) the development of a mechanism.

5. JUSTIFICATION OF  
PLASTIC DESIGN

The reasons now should be evident as to why the application of plastic analysis to structural design is justified: they are

- (1) Economy
- (2) Simplicity
- (3) Rationality

A word, now, about each of these:

Since there is considerable reserve of strength beyond the elastic limit and since the corresponding ultimate load may be computed quite accurately, then structural members of smaller size will adequately support the working loads when design is based on maximum strength.

An analysis based upon ultimate load possesses an inherent simplicity because the elastic condition of continuity need no longer be considered. This was evident from our consideration of the three-bar truss (Figure 6) and the fixed-ended beam (Figure 12).

Finally the concept is more rational. By plastic analysis the engineer can determine with an accuracy that far exceeds his presently available techniques the real maximum strength of a structure. Thereby the factor of safety has more real meaning than at present. It is not unusual for the factor of safety to vary from 1.65 up to 3 or more for structures designed according to conventional elastic methods.

6. INADEQUACY OF STRESS  
AS THE DESIGN CRITERION

The question immediately arises, can't one simply change the allowable stress and retain the present stress concept? While theoretically possible, the practical answer is "no". It would mean a different working stress for every type of structure and would vary for different loading conditions.

It is true that in simple structures the concept of the hypothetical yield point as a limit of usefulness is rational. This is because the ultimate load capacity of a simple beam is but 10 to 15% greater than the hypothetical yield point, and deflections start increasing rather rapidly at such a load. While it would seem logical to extend elastic stress analysis to indeterminate structures, such procedures have tended to over-emphasize the importance of stress rather than strength as a guide in engineering design and have introduced a complexity that now seems unnecessary for large number of structures.

Actually the idea of design on the basis of ultimate load rather than allowable stress is a return to the realistic point of view that had to be adopted by our forefathers in a very crude way because they did not possess knowledge of mathematics and statics that would allow them to compute stresses.

As a matter of fact, to a greater extent than we may realize, the maximum strength of a structure has always been the dominant design criteria. When the permissive working stress of 20 ksi has led to designs that were consistently too conservative, then that stress has been changed. Thus the benefits of plasticity have been used consciously or unconsciously in design. It is also patently evident to most engineers that present design procedures completely disregard local over-stressing at points of stress-concentration, etc. Long experience with similar structures so designed shows that this is a safe proce-

dure. Thus, the stresses we calculate for design purposes are not true maximum stresses at all, they simply provide an index for structural design.

In the remainder of these remarks a number of examples will be given in which the ductility of steel has been counted upon (knowingly or not) in present design. Plastic analysis has not generally been used as a basis for determining these particular design rules and as a result so-called elastic stress formulas have been devised in a rather haphazard fashion. A rational basis for the design of a complete steel frame (as well as its details) can only be attained when the maximum strength in the plastic range is selected as the design criterion.

Some examples in which we rely upon the ductility of steel for satisfactory performance are the following and are listed in two categories:

#### 1. Factors that are Neglected

- (1) Neglect of residual stresses in the case of flexure.
- (2) Cambering of beams and the neglect of the resulting residual stresses.
- (3) Neglect of erection stresses.
- (4) Neglect of foundation settlements.
- (5) Neglect of over-stress at points of stress-concentration (holes, etc).
- (6) Neglect of bending in angles connected in tension by one leg only.
- (7) Neglect of over stress at points of bearing.
- (8) Design of connections on the assumption of a uniform distribution of stresses among the rivets, bolts, or welds.
- (9) Neglect of the difference in stress-distribution arising from the "cantilever" as compared with the "portal" method of wind stress analysis.

#### II. Revisions in working stress due to reserve plastic strength

- (10) Bending stress of 30 ksi in round pins.
- (11) Bearing stress of 40 ksi on pins in double shear.
- (12) Bending stress of 24 ksi in framed structures at points of interior supports.

Figures 13- 18 are illustrative of items 1, 3, 8, 10, and 12, above. In Figure 13 it is demonstrated, for example, that cooling residual stresses (whose in-

fluence we neglect and yet which are present in all rolled beams) cause yielding in the flange tips even at the working load!

Structural members experience yield while being straightened in the mill, fabricated in a shop or forced into position during erection. Actually it is during these three operations that ductility of steel beyond the yield point is called upon to the greatest degree. Having permitted such yielding in the mill, shop and field, there is no valid basis to prohibit it thereafter, provided such yield has no adverse effect upon the structure. As an example, Figure 14 shows how erection forces will introduce bending moment into a structure prior to the application of external load. Although the yield-point load is reduced as a result of these "erection moments", there is no effect whatever on the maximum strength.

Consider, next, the design of a riveted or bolted joint. The common assumption is made that each fastener carries the same shear force. This is true only for the case of two fasteners. When more are added (Figure 15), then as long as the joint remains elastic, the outer fasteners must carry the greater proportion of the load. For the example with four rivets, if each rivet transmitted the same load, then, between rivets C and D one plate would carry perhaps three times the force in the other. Therefore it would stretch three times as much and would necessarily force the outer rivet (D) to carry more load. The actual forces would look something like those shown under the heading "Elastic". What eventually happens is that the outer rivets yield, redistributing forces to the inner rivets. Therefore the basis for design of a riveted joint is really its ultimate load and not the attainment of first yield. Figure 16 shows a line of bolts demonstrating the differential plastic action in the various fasteners.

Figure 17 is a further example and is concerned with the design of a round pin. In a simple beam of WF shape, when the maximum stress due to bending reaches the yield point, most of the usable strength has been exhausted. However, for some cross-sectional shapes, much additional load may be carried without excessive deflections. The relation between bending moment and curvature for WF and round beams is shown in Figure 17. The upper curve is for the pin, the lower for a typical WF beam, the non-dimensional plot being such that the two curves coincide in the elastic range. The maximum bending strength of the wide flange beam is  $1.14 M_y$ , whereas that of the pin is  $1.70 M_y$ . The permissible design stresses according to AISC are 20 ksi for the WF and 30 ksi for the pin. Expressing these stresses as ratios of yield point stress,

$$\text{WF: } \frac{\sigma_w}{\sigma_y} = \frac{20}{33} = 0.61$$

$$\text{Pin: } \frac{\sigma_w}{\sigma_y} = \frac{30}{33} = 0.91$$

For a simply-supported beam the stresses, moments, and loads all bear a linear relationship to one another in the elastic range and thus

$$\frac{P}{P_y} = \frac{\sigma}{\sigma_y} = \frac{M}{M_y}$$

Therefore, the moment at allowable working stress ( $M_w$ ) in the WF beam is  $.61 M_y$ ; for the pin, on the other hand,  $M_w = .91 M_y$ . What is the true load factor of safety for each case?

$$\text{WF: } F = \frac{P_{\max}}{P_w} = \frac{M_{\max}}{M_w} = \frac{1.14 M_y}{.61 M_y} = 1.87$$

$$\text{Pin: } F = \frac{P_{\max}}{P_w} = \frac{M_{\max}}{M_w} = \frac{1.70 M_y}{.91 M_y} = 1.87$$

The exact agreement between the true factors of safety with respect to ultimate load in the two cases, while somewhat of a coincidence, is indicative of the influence of long years of experience on the part of engineers which has resulted

in different permitted working stresses for various conditions resulting in practice. Probably no such analysis as the foregoing influenced the choice of different unit stresses that give identical factors of safety with various sections, nevertheless, the choice of such stresses is fully justified on this basis. When years of experience and common sense have led to certain empirical practices these practices can usually be justified on a scientific basis.

The final example (Figure 18) demonstrates the use of 24 ksi at points of interior support in continuous beams and shows how this is a safe procedure according to plastic analysis.

## 7. CONCLUDING REMARKS

As a background for the talks that are to follow, it has been my purpose to introduce to you the simple plastic theory, to define some terms, and to demonstrate that our dependence upon the ductility of steel has always been very real. Perhaps we can do no better than to refer to Figure 1 again, as a basis for reviewing what we have learned thus far:

- (1) Whereas in elastic design a section is selected on the basis of a maximum allowable stress at working load (we do not deal at all with the overload factor of safety!) in plastic design we compute first the ultimate load ( $P_w \times F$ ) and select a member that will fail at that load.
- (2) Since every beam has a limiting (plastic) moment of resistance ( $M_p$ ) the ultimate load may be computed on the basis that a sufficient number of plastic hinges will have formed to create a mechanism.
- (3) Use of maximum strength as the design criterion provides at least the same margin of reserve strength as is presently afforded in the conventional design of simple beams.
- (4) At working load the structure is still in the so called elastic range.
- (5) In most cases, a structure designed by the plastic method will deflect no more at working load than will a simply-supported beam designed by conventional methods to support the same load.
- (6) Plastic design gives promise of economy in the use of steel, of savings in the design office by virtue of its simplicity, and of building frames more logically designed for greater over-all strength.

Plastic design has come of age. Considerable literature is available in the form of lecture notes<sup>(8)</sup>, reference books<sup>(9, 10)</sup>, and various technical proceedings<sup>(11)</sup>. Mr. Higgins and Mr. Estes of the American Institute of Steel Construction are nearing completion of a manual on plastic design which will afford the designer with even more specific examples and techniques. Thus engineers in this country will be able to join with those in England and in Canada who have already applied plastic analysis to their design problems.

Plastic Design In SteelSOME DEFINITIONS

- LIMIT DESIGN A design based on any chosen limit of structural usefulness.
- PLASTIC DESIGN A design based upon the ultimate load-carrying capacity (maximum strength) of the structure. The term "plastic" is derived from the fact that the ultimate load is computed from a knowledge of the strength of steel in the plastic range.
- ULTIMATE LOAD ( $P_u$ )  
or  
MAXIMUM STRENGTH The highest load a structure will carry. (It is not to be confused with the term as applied to the ultimate load carried by an ordinary tensile coupon.)
- PLASTIFICATION The development of full plastic yield of the cross-section.
- PLASTIC MOMENT ( $M_p$ ) Maximum moment of resistance of a fully-yielded cross-section.
- PLASTIC MODULUS ( $Z$ ) Combined statical moments of the cross-sectional areas above and below the neutral axis.
- PLASTIC HINGE A yielded section of a beam which acts as if it were hinged, except with a constant restraining moment.
- SHAPE FACTOR ( $f$ ) The ratio of the maximum resisting moment of a cross-section ( $M_p$ ) to the yield moment ( $M_y$ ).
- MECHANISM A "hinge system" -- A system of members than can move without an increase in load.
- REDISTRIBUTION OF MOMENT A process which results in the successive formation of plastic hinges until the ultimate load is reached. By it, the less-highly stressed portions of a structure also may reach the ( $M_p$ )-value.

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