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PLASTIC DESIGN IN STEEL

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It is the purpose of this paper to define and describe plastic design in structural steel. A brief discussion of its advantage is aimed at showing why the time of the engineer and architect is warranted in studying the method. Next some examples will be given to show how plastic design is applied to continuous beams and to rigid frames.

A considerable number of structures have already been built both in Europe and in the United States according to designs based upon the plastic method. Some examples will be shown, and possible future applications will be discussed.

WHAT IS PLASTIC DESIGN?

Plastic design is a design based upon the maximum load the structure will carry. This maximum load is determined from an analysis of the strength of steel in the "plastic" range -- which explains the origin of the term.

Conventional elastic design assumes that the limit of usefulness of a structure is the first attainment of yield-point stress. However, the final criterion of the usefulness of a frame is its ability to carry load. If its strength is not limited by brittle fracture, fatigue, instability, or excessive deflection, then the only logical remaining design criterion is the maximum load it will support. The mere fact that computed yield-point stress has been reached at some point is of no importance in itself; in fact, many present design assumptions rely upon ductility of the material to provide a safe structure. Plastic design goes one step further and makes "conscious" use of this same ductility upon which the engineer has become accustomed to rely.

To what kind of a structure would this method be applied? From the present state of knowledge, plastic design may be applied to statically-loaded structural steel frames with rigid joints, and to continuous or restrained beams. It is suitable for statically indeterminate structures which are stressed primarily (although not exclusively) in bending. It is not intended for simple beams, because present procedures for such members afford sufficient ease of design and at the same time are not wasteful of material.

The concept of a safety factor, is, of course, retained. Whereas in elastic design a section is selected on the basis of a maximum allowable stress at working load, in plastic design one computes first the ultimate load and selects a member that will fail at that load. The ultimate load is determined, in fact, by multiplying the expected or working load by a factor which assures that there will be the same margin of safety in rigid frames as is presently afforded in the conventional design of simple beams.

A steel rigid frame attains its maximum strength through the formation of so-called "plastic hinges". These hinges, in turn, form under overload because of the ability of structural steel to deform plastically after the yield point is reached. This ductility of steel is characterized by a flat "plateau" in the stress-strain diagram as shown by the typical curves in Fig. 1. They were obtained from tension coupons cut from a rolled WF shape. Notice that after the elastic limit is reached, elongations of about 15 times the elastic limit value take place without any decrease in load. Afterwards some increase in strength is exhibited as the material strain-hardens.

Fig. 2 shows that these strains are really quite small. At ultimate load the maximum strain will not have exceeded about 1.5% elongation; and for ordinary structural steel, final failure by rupture occurs only after a specimen has stretched about 20 times this maximum strain that is encountered in plastic design.

Based upon the idealized curve of Fig. 2, there is shown in Fig. 3 the action of a beam under bending moment. If it is assumed that all of the material in a WF shape is concentrated in the flanges then (when the elastic limit is exceeded) the compression flange shortens at constant load and the tension flange lengthens at constant load. The resulting moment is therefore constant; the member acts just like a hinge except that deformation occurs under constant moment. The magnitude of this maximum moment is called the plastic moment, M_p. Since the member is entirely plastic, M_p is equal to the yield point stress multiplied by the combined statical moment of the areas above and below the neutral axis.

It should now be evident why the attainment of yield-point stress does not correspond to failure. At the point of peak moment a zone of yielding develops, a hinge forms, and the structure can then call upon its less-heavily-stressed portions to carry further increase in load. Eventually, when enough plastic hinges form, the structure will reach its ultimate load and fail by continued deflection at that load.

A Hungarian, Gabor Kazinszy, first applied these concepts to the design of some apartment-type buildings in 1914. Early tests in Germany were made by Maier-Leibnitz. Van den Broeck, Baker, Roderick, Horne, Heyman, Prager, Symonds, Neal, and Johnston have all made important contributions to the plastic theory of structures. The work at Lehigh University has featured the verification of the plastic method through appropriate tests on large structures, the systematizing of design operations, and the theoretical and experimental study of secondary design requirements that must be met. Plastic design is already a part of certain specifications and engineers are now making use of it.

JUSTIFICATION FOR PLASTIC DESIGN

Quite properly one might ask, "What can plastic design do for me?" Plastic design gives promise of economy in the use of steel and of saving of time in the design office by virtue of its simplicity. Further it will provide building frames more logically designed for greater over-all strength.

The plastic method is certainly not the solution to all problems in structural steel design. However, for the type of structure noted above, it is a powerful tool to assist the designer.

Since more effective use is made of all of the material in a structure, then for the same working load, a plastically-designed frame will be lighter than its elastically-designed counterpart. Attention is focused upon the entire structure in plastic design rather than upon individual members. The saving in weight will depend on a number of factors; while the "average" saving might run from 15 to 20%, higher values than this have been reported.

The design technique is simpler because the essence of the method is that it reduces an indeterminate structure to a determinate one. This, in fact, is the role of the "plastic hinges"; they provide for rotation at constant moment and thus "determine" the magnitude of the moment at critical sections. Since it is much easier to design a determinate structure than an indeterminate one, a simpler design procedure may properly be expected when the plastic method is used. The advantage of continuity is retained without the difficulty of elastic rigid frame analysis.

ILLUSTRATIVE EXAMPLES

The first example is a plastic analysis for ultimate load rather than a design procedure. The problem of the rigid cross-bar supported by three rods shown in Fig. 4 is admittedly impractical, but it should serve to illustrate some important principles. The problem is to determine the ultimate load this assembly will support.

According to elastic concepts the maximum allowable load would be the yield load, P_y. However, the equilibrium equation ($\sum V = 0$) does not solve the problem, because the assembly is statically indeterminate. We only know that $P = 2T_1 + T_2$. To compute the yield load it would be necessary to consider, in addition the relative elastic deformation of the three bars (the "continuity" condition).

Notice how much simpler it is to determine the ultimate load according to the principles of plastic analysis. After the center bar reaches the yield stress, the partially plastic assembly deforms as if it were a two-bar system except that a constant force equal to yield-point stress times the area is supplied by Bar 2 (the member is in the plastic range). This situation continues until the load reaches the yield value in the two outer bars, after which the assembly would continue to deform at the load P_u. The ultimate load is simply equal to the sum of the yield forces in each rod, or,

$$P_u = 3f_y A$$

Quite evidently, yielding of the center bar reduced the indeterminate assembly to one which could be analysed by statics, and this simplified the solution to a procedure that took one step!

Design of a Restrained Beam:

Suppose it is desired to support a total load of 200 kips on a beam in a building with several spans of 40-feet. One of the interior spans is shown in Fig. 5. The first step in the "Statical" method of plastic design would be to multiply the working load by the load factor. This gives an ultimate load of 370 kips, and the problem is to select a section that will just fail at this factored load.

The next step would be to remove the restraining end moments and draw the moment diagram considering the beam as simply-supported. The maximum moment ordinate at the center, from statics, is $W_uL/8$.

Next, the redundant end moments M_A and M_B are applied to the beam giving the redundant moment diagram as shown.

Finally these two moment diagrams are combined (the redundant moments are subtracted from the determinate moments) in such a way that plastic hinges are formed at the center and at the two ends. If less than these three hinges were formed, then part of the beam would not be used to maximum effectiveness.

From the graphical construction of the composite moment diagram, the center determinate moment ($W_uL/8$) is equated to M_p at the end plus M_p at the center, or

$$\frac{W_uL}{8} = 2M_p$$

from which

$$M_p = \frac{W_uL}{16} = 925^{k-ft}$$

As noted above, every beam has its own plastic moment of resistance (M_p = yield stress times statical moment of the area above and below the neutral axis)*. It is found that a 30 WF 108 will be adequate, furnishing a plastic moment of 949 k-ft.

How does one know when a structure is being used to maximum effectiveness? The answer: when it will carry no additional load. At the ultimate load the plastic hinges continue to rotate and therefore the structure simply deflects at constant load. In this respect the action is similar to a linkage or "mechanism" (there is no change in load with deformation). So this same term ("Mechanism") will be used to describe the <u>further</u> deflection of a beam or frame after it reaches the ultimate load. Such a mechanism is shown at the bottom of Fig. 5, and thus what was actually done in drawing the composite moment diagram was to draw

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*A table of these M_p values for rolled WF and I shapes used as beams has been prepared by the AISC and it is included at the end of this paper.

it in such a way that a mechanism was formed.

Design of a Rigid Frame

As a third example, a single-span rigid frame will be designed with haunched connections. Although the structure shown in Fig. 6 is quite different from that of the previous example, the steps in the design are exactly the same as before.

Step 1: The ultimate load is determined by multiplying the expected working load (40-kips) by the load factor. Thus the ultimate load is 74 kips.

Step 2: The redundant is selected as the force H at the column bases. This amounts to assuming that the right-hand column is resting on a roller support.

Step 3: Since the structure is now determinate, the moment diagram can be drawn as shown by the heavy solid line. (The separate construction of the determinate and the redundant moment diagrams has been eliminated).

Step 4: The next step is to draw the redundant moment diagram in such a way that a "mechanism" is formed (in this way the material is used most efficiently).

The problem will first be solved neglecting the fact that there are haunches. The redundant moment diagram (line 1-a-b-c-7) is drawn in such a way that the horizontal line a-b-c bisects the center ordinate of 1110 k-ft. The required plastic moment is therefore 1110/2 = 555 k-ft. From the "Plastic Moment Table" it is found that a 24 I 79.9 will be adequate (M_p = 558 k-ft.)

Considering the frame with a haunch makes the problem no more difficult. The redundant moment diagram is drawn in such a way that plastic hinges form in the beam at the ends of the haunches and at the center (sections 3, 4 and 5).

Step 5: The magnitude of the required plastic moment can either be computed from the geometry of the moment diagram or scaled from the sketch. It is found to be 407 k-ft. The lightest shape that is suitable is a 21 WF 68 (M_p = 439 k-ft.)

At section 2 the moment is 491 k-ft or greater than the plastic moment value of 407 k-ft. A larger shape is needed there, and turning to the table of M_p-values a 24 WF 76 (M_p = 550 k-ft) will be specified.

A final step in all designs is to examine details such as connections, bracing, column buckling, etc., to make sure that some secondary design factor does not limit the usefulness of the structure. Such design guides will be available in Ref. 10 and Ref. 11 which will give the background of theory and experiment upon which they are based.

EXAMPLES OF STRUCTURES DESIGNED BY THE PLASTIC METHOD

About two-hundred industrial (single-story) frames have been designed in England by the plastic method -- also a school building and a five-story office building. Ref. 7 contains in its bibliography seven papers describing actual plastic designs. Figs. 7 and 8 are two of these.

On this continent the first building to be designed plastically was in Canada (Ref. 1). It was a two-story frame with beams continuous over 6 spans.

At least four plastically-designed structures have been built in this country. Figs. 9 and 10 show one of these during erection and after the structure was completed. The span of the frames of this warehouse in Souix Falls, South Dakota is 88-ft. (Ref. 2). They are spaced at 18'-6" centers and the structure is designed for a dead load of 12 psf and a snow load of 30 psf. It required a 24 WF 94 shape, uniform throughout. A comparison with the 30 WF 108 required by conventional design shows a saving of about 13% in structural steel.

FUTURE TRENDS

Plastic design is suitable for the design of continuous beams, single-story industrial building frames and multistory buildings in which horizontal forces are resisted by wall supports (such as by cross-bracing). The method should be applied to those structures that have been sufficiently studied and tested to give confidence that the calculated maximum strength will be realized.

Undoubtedly the scope of application will extend to other types of construction when such studies have been completed. Even now the philosophy of plastic analysis is being used to proportion certain elements of naval vessels, and current research is aimed at application in a more general way in the design of main ship frames and the stiffened plates that make up decks, etc. Studies have also been made of the application of plastic design to arches, Vierendeel girders, and to stiffening frames of cylindrical shells.

What will plastic design mean? To the "sidewalk superintendent" it may mean nothing. The structure will look just the same as a conventionally-designed rigid frame. To the engineer it will mean a more rapid method of design. To the owner it will mean economy because plastic design requires less steel. For the building authority it should mean more efficient operations because designs could be checked faster. For all of us it means that better use has been made of the natural resources with which Almighty God has so richly blessed us.

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PLASTIC MOMENT TABLE

Based on $f_y = 33$ ksi

If $f_y \neq 33$ ksi, multiply M_p by $f_y/33$

(courtesy, American Institute of Steel Construction)

M_p ft. kips	SHAPE
3450	36 WF 300
3210	36 WF 280
2960	36 WF 260
2770	36 WF 245
2590	36 WF 230
2520	33 WF 240
2300	33 WF 220
2110	36 WF 194
2080	33 WF 200
2020	30 WF 210
1972	36 WF 182
1833	36 WF 170
1814	30 WF 190
1714	36 WF 160
1631	30 WF 172
1594	36 WF 150
1535	33 WF 152
1532	27 WF 177
1411	33 WF 141
1387	27 WF 160
1282	33 WF 130
1275	24 WF 160
1243	27 WF 145
1201	30 WF 132
1144	24 WF 145
1120	30 WF 124
1038	30 WF 116
1015	24 WF 130
982	21 WF 142
950	30 WF 108
943	27 WF 114
926	24 WF 120
874	21 WF 127
857	12 WF 190
846	24 WF 110
837	27 WF 102
820	24 I 120
765	24 WF 100
765	21 WF 112
764	27 WF 94
751	24 I 105.9
713	12 WF 161
696	24 WF 94
682	18 WF 114
667	14 WF 136
657	24 I 100
623	18 WF 105
622	21 WF 96
621	14 WF 127
616	24 WF 84
606	24 I 90
578	14 WF 119
577	12 WF 133
567	18 WF 96
558	24 I 79.9
550	24 WF 76
539	14 WF 111
528	20 I 95
527	21 WF 82
513	12 WF 120
513	16 WF 96
488	18 WF 85
488	20 I 85
473	21 WF 73
465	16 WF 88
449	12 WF 106
441	18 WF 77
439	21 WF 68
417	12 WF 99
417	20 I 75
406	10 WF 112
400	16 WF 78
400	14 WF 84
398	18 WF 70
396	21 WF 62
386	12 WF 92
378	20 I 65.4
369	14 WF 78
362	18 WF 64
362	16 WF 71
358	10 WF 100
355	12 WF 85
345	14 WF 74
340	18 I 70
337	18 WF 60
328	12 WF 79
324	16 WF 64
316	14 WF 68
315	10 WF 89

(continued)

PLASTIC MOMENT TABLE

(continued)

M_p ft. kips	SHAPE
307	18 WF 55
297	12 WF 72
292	16 WF 58
285	18 I 54.7
282	14 WF 61
277	18 WF 50
269	10 WF 77
255	16 WF 50
249	10 WF 72
240	14 WF 53
238	12 WF 58
228	10 WF 66
226	16 WF 45
216	14 WF 48
215	12 WF 53
210	15 I 50
207	10 WF 60
200	16 WF 40
200	12 WF 50
192.8	8 WF 67
191.5	14 WF 43
188.6	15 I 42.9
184.2	10 WF 54
178.4	12 WF 45
175.7	16 WF 36
169.1	14 WF 38
166.8	12 I 50
165.8	10 WF 49
164.7	8 WF 58
158.4	12 WF 40
151.1	10 WF 45

M_p ft. kips	SHAPE
149.9	14 WF 34
144.2	12 I 40.8
141.4	12 WF 36
134.8	8 WF 48
129.5	14 WF 30
129.1	10 WF 39
122.0	12 I 35
120.9	12 WF 31
114.3	12 I 31.8
109.7	8 WF 40
104.4	12 WF 27
96.7	10 I 35
95.4	10 WF 29
95.4	8 WF 35
81.1	10 WF 25
80.7	12 B 22
77.1	10 I 25.4
74.5	8 WF 28
68.1	12 B 19
67.1	14 B 17.2
66.3	10 WF 21
64.4	8 M 24
63.5	8 WF 24
59.3	10 B 19
56.7	12 B 16.5
52.7	8 I 23
52.5	8 WF 20
52.3	6 WF 25
51.2	10 B 17
49.1	6 M 25
48.0	8 M 20

M_p ft. kips	SHAPE
47.8	12 B 14
45.0	8 I 18.4
43.9	10 B 15
43.4	8 WE 17
41.4	6 WE 20
40.0	6 M 20
39.5	7 I 20
39.1	12 JR 11.8
37.4	8 B 15
32.8	7 I 15.3
31.9	6 B 16
31.2	8 B 13
31.2	5 WF 18.5
30.5	5 M 18.9
28.8	6 I 17.25
26.4	5 WF 16
25.4	10 JR 9
23.0	6 I 12.5
22.7	6 B 12
20.3	5 I 14.75
17.3	4 WF 13
16.8	4 M 13
15.3	5 I 10
15.0	8 JR 6.5
11.1	7 JR 5.5
11.1	4 I 9.5
9.5	4 I 7.7
7.8	6 JR 4.4
6.4	3 I 7.5
5.3	3 I 5.7

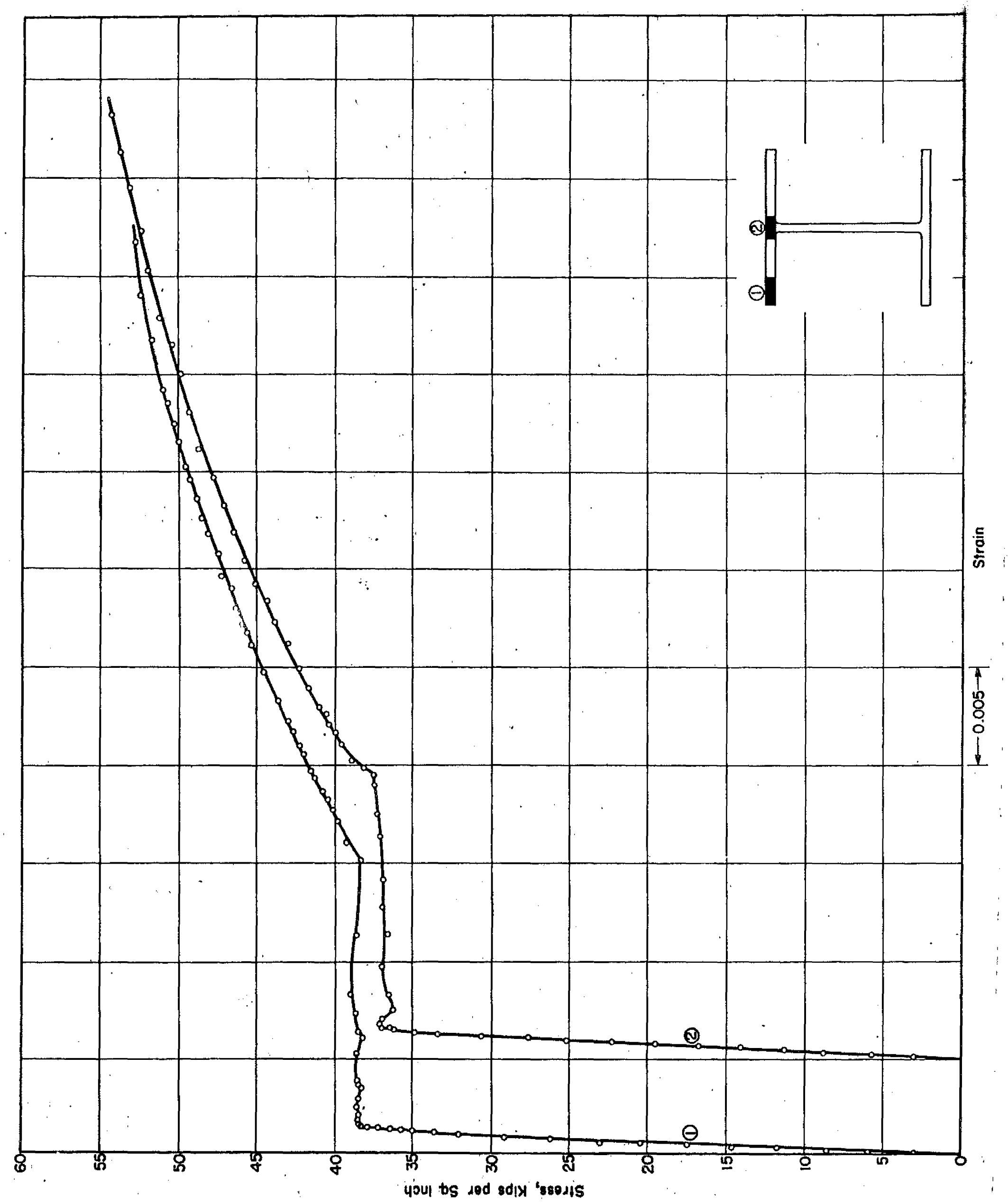
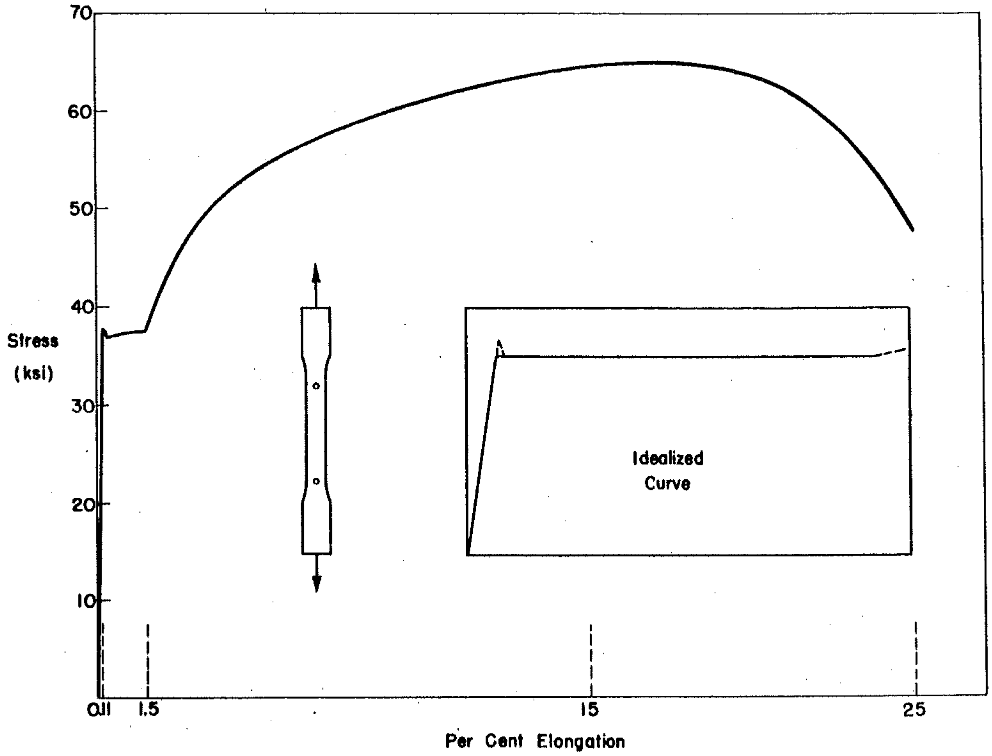
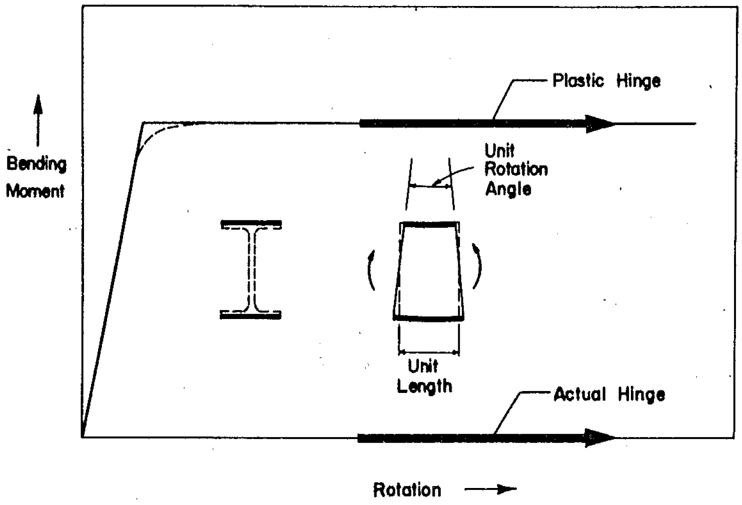

Stress, Kips per Sq. Inch
0
5
10
15
20
25
30
35
40
45
50
55
60
Strain
0.005
①
②
①
②

Fig. 1



70
60
50
40
30
20
10
Stress
(ksi)
Idealized
Curve
0.11
1.5
15
25
Per Cent Elongation

Fig. 2



Plastic Hinge
Bending
Moment
Unit
Rotation
Angle
Unit
Length
Actual Hinge
Rotation

Fig. 3

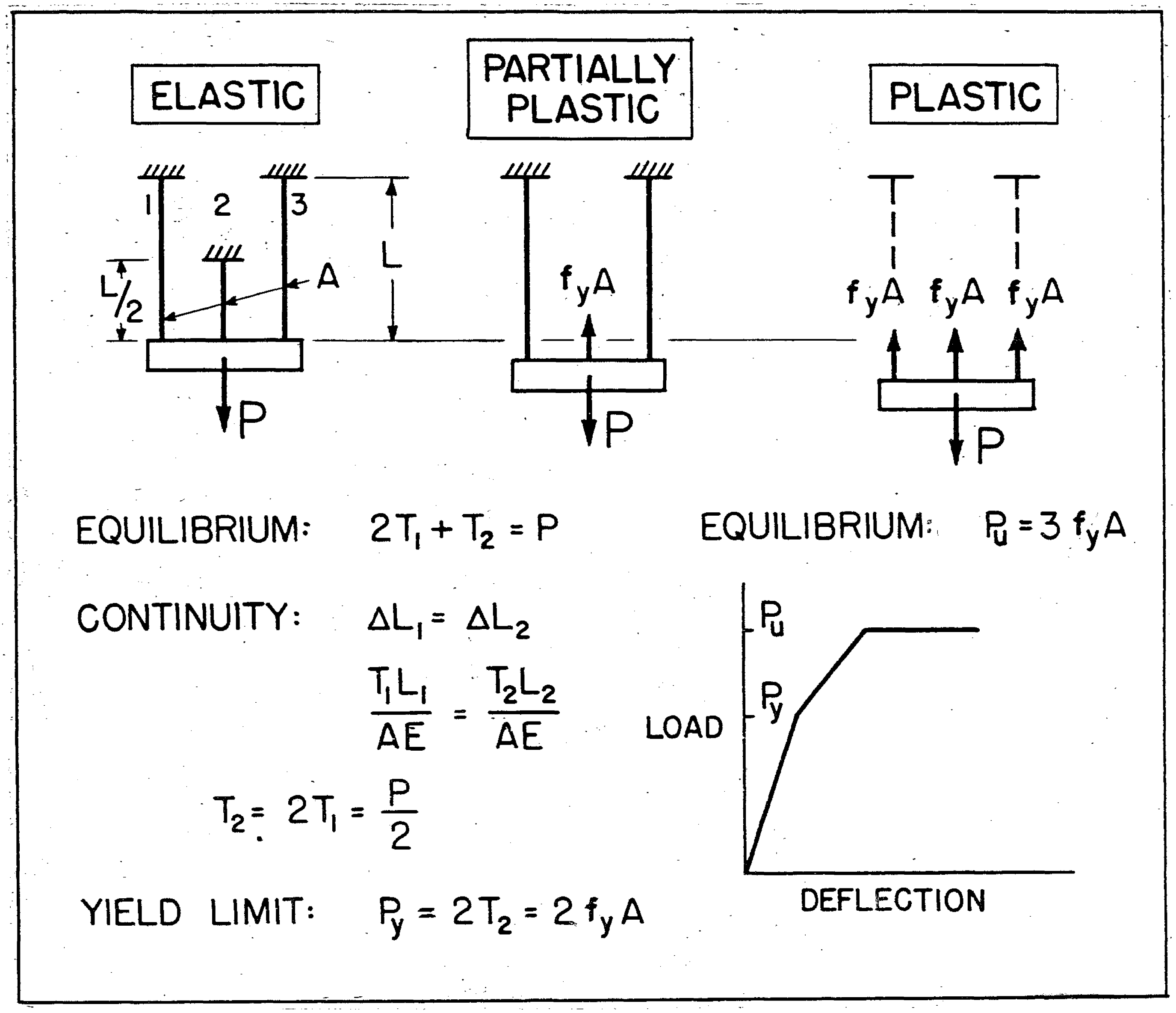
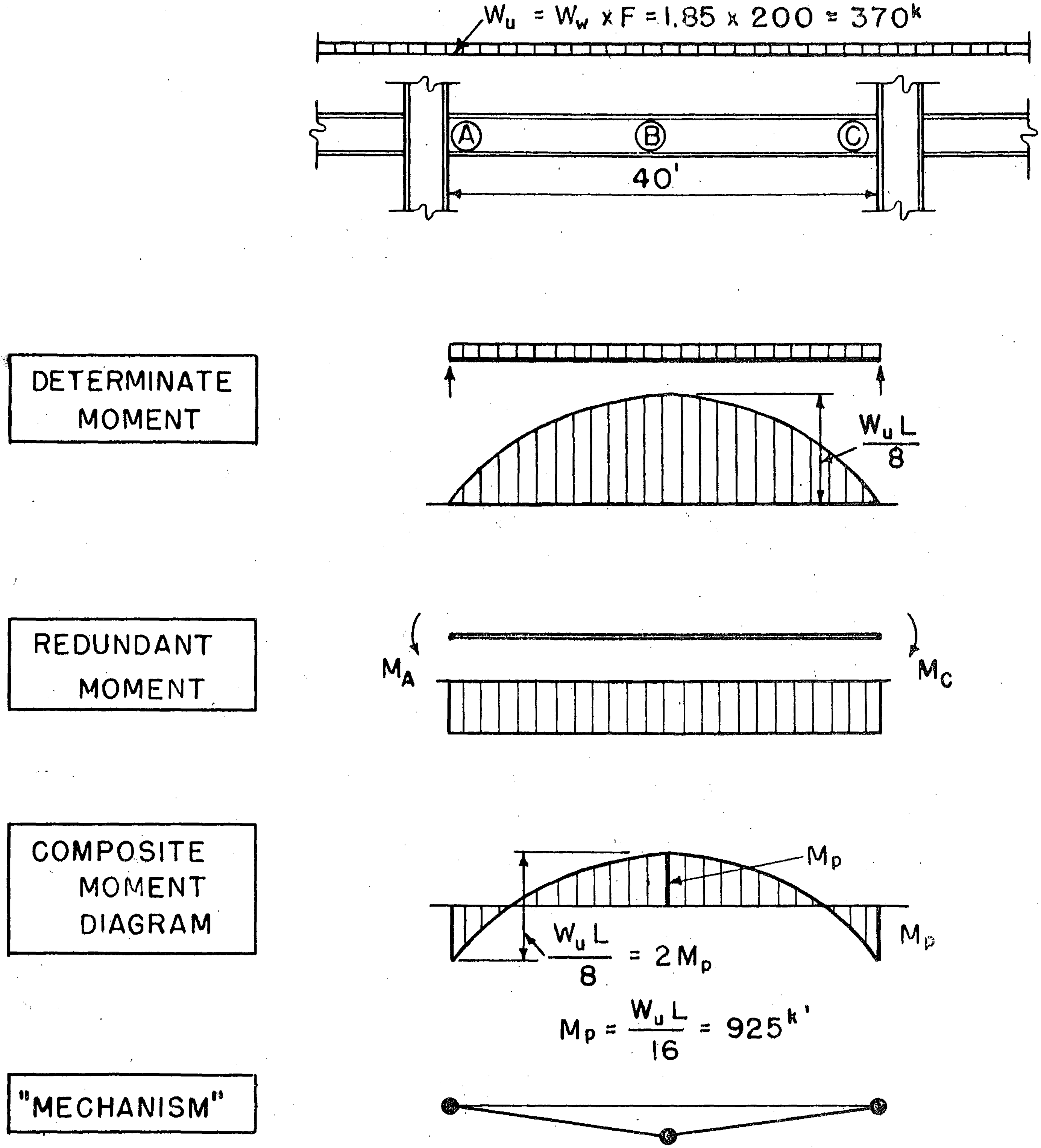

ELASTIC
PARTIALLY PLASTIC
PLASTIC
1
2
3
$L/2$
A
L
f_yA
f_yA f_yA f_yA
P
P
P
EQUILIBRIUM: $2T_1 + T_2 = P$
CONTINUITY: $\Delta L_1 = \Delta L_2$
$\frac{T_1L_1}{AE} = \frac{T_2L_2}{AE}$
$T_2 = 2T_1 = \frac{P}{2}$
YIELD LIMIT: $P_y = 2T_2 = 2f_yA$
EQUILIBRIUM: $P_u = 3f_yA$
P_u
P_y
LOAD
DEFLECTION

Fig. 4



Wu = Ww x F = 1.85 x 200 = 370k
A
B
C
40'
DETERMINATE MOMENT
WuL/8
REDUNDANT MOMENT
MA
MC
COMPOSITE MOMENT DIAGRAM
Mp
Mp
WuL/8 = 2Mp
Mp = WuL/16 = 925k'
"MECHANISM"

Fig. 5

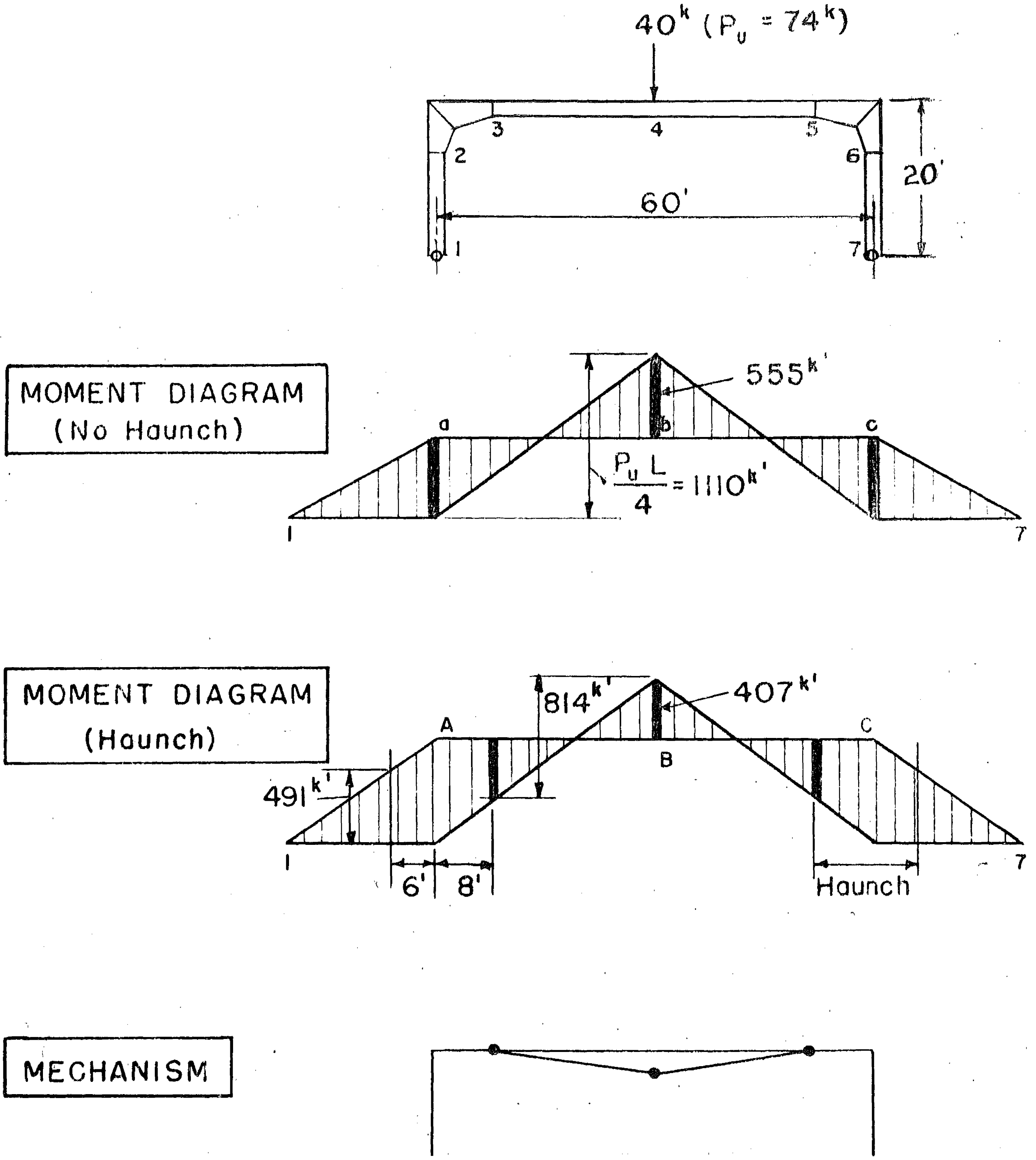

40^k (P_u = 74^k)
1
2
3
4
5
6
7
60'
20'
MOMENT DIAGRAM (No Haunch)
a
b
c
555^k'
P_u L / 4 = 1110^k'
1
7
MOMENT DIAGRAM (Haunch)
A
B
C
814^k'
407^k'
491^k'
6'
8'
Haunch
1
7
MECHANISM

Fig. 6