Report No. 205.72

"PLASTIC DESIGN IN ACTION"

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Presented at AISC-USC Symposium

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The purpose of this article is to discuss some of the practical problems encountered in plastic design. Emphasis is placed on the assumptions of this design method and on how these assumptions are reflected in the design and fabrication of steel structures.

INTRODUCTION

In a recent publication[1] the basic concepts of plastic design were discussed in some detail. Reference 1 describes the characteristics of the plastic hinge and explains its function in redistribution of moment. The large reserve in strength beyond the elastic limit exhibited by continuous steel structures is due to the ductility of steel which results in moment redistribution.

The first conscious application of plastic design concepts was in Hungary in 1914. Reference 1 traces the progress of plastic design from this beginning and outlines its acceptance in codes and specifications in the United States and around the world. Since Reference 1 was published, numerous additional agencies have indicated their acceptance of the plastic design method.

Numbers indicate References.

In 1956, the ASCE Committee on Plasticity Related to Design joined with an existing Welding Research Council Committee for the purpose of preparing a "Commentary on Plastic Design in Steel." This important reference presents the theoretical considerations involved in the plastic theory and in certain secondary design problems. Experimental verification of theory is documented and approximations in the form of "design guides" are given. This commentary is now available as an ASCE Manual[2].

ASSUMPTIONS OF PLASTIC DESIGN

Any rational method of design involves a series of assumptions which are based on such factors as material properties, criteria of failure, and experience. An important phase of this work entails reviewing a tentative design to see that it satisfies these assumptions. Design specifications are frequently used as a guide in this review. The L/d and Ld/bt provisions of the AISC Specification for conventional (elastic) design are familiar examples of such guidance. In plastic design, such provisions are termed "secondary design considerations." The intelligent application of such provisions must rely on an understanding of their purpose and on the underlying assumptions.

Figure 1 outlines the assumptions and secondary design considerations germane to plastic design. Some of the most interesting questions with respect to plastic design in action have to do with these secondary design considerations, most of which are related to the assumptions.

The remarks which follow will consider questions apropos to these assumptions and secondary design considerations. The first question concerns a topic which appears twice in Figure 1 - Connections.

Q. <u>How should one proportion high strength bolts in a moment connection?</u>

To provide a basis for answering this question, consider the behavior of a bolt in tension, shown in Figure 2[3]. This figure shows how a bolt stretches under applied tensile load. Of particular significance is the flat portion of the curve which shows that the bolt is ductile. This important property justifies assigning to each bolt a constant "yield" value in tension - no matter how far from the neutral axis of the joint the bolt is placed. At ultimate load, some bolts will have elongated more than others in the joint but the loads carried by all tension bolts will be approximately equal as a result of this ductile behavior.

Figure 2 indicates that the ultimate (maximum) tensile load is about 1.4 times the proof load for high strength bolts. Therefore, the proof load is a conservative estimate of the "yield" value. For design purposes, each bolt on the tension side of a moment connection is assumed to carry a load equal to the proof load at failure of the connection.

The design concept for high strength field bolted moment connections is illustrated in Figure 3. The joint at the upper right in this figure is similar to one that was used at the ridge of the Heelan Catholic High School, Sioux City, Iowa. The steps in the design of this joint are:

1. Find the maximum moment, thrust, and shear at the joint. Figure 3 considers the design for moment and thrust.

2. Make a tentative layout of the joint.

3. Estimate the lever arm "a" from the compression flange to the center of gravity of the tension bolts.

4. Compute $T = C = M/a$. The forces T and C form the moment-resisting couple. Find the number of bolts (acting at proof load) and the compression area A_c (acting at yield stress) to resist T and C.

5. Find the compression Area A_H (acting at yield stress) to resist the thrust H. This is usually a small value in rigid frames which carry loads primarily by flexure.

6. Check the resisting moment of the tension bolt forces. Choose a moment center at the CG of the compression area A_c (frequently close to the CG of the flange). Adjust bolt spacing as required.

7. Check the joint for shear. A H. T. bolted joint resists shear by (a) friction developed by initial tension (proof load) in the bolts or (b) by bearing. Since this joint is at mid-span, shear is no problem.

8. Find the thickness of the end plate. Note that small bolt

spacing in the vertical direction (less than about 4 in.) may result in large plate thicknesses.

Q. <u>Is it necessary to grind the edges or corners of plates in the area of plastic hinges?</u>

This question involves measures to preserve ductility, some of which are illustrated in Figure 4. These measures are aimed at avoiding a premature brittle fracture originating at the edge of a plate or hole, triggered by severe cold working.

1. Remove cold worked material around holes by a sub-punch and reaming operation, or use drilled holes. This applies to all holes in regions where stress may approach the yield stress level in tension, regardless of the type of fastener. For instance, holes in the tension splice plate and tension flange of beam should be drilled full size or sub-punched and reamed. If the girder develops a plastic hinge in the vicinity of this connection, holes in the girder web (but not in the beam clip angles) should also be drilled or sub-punched and reamed.

2. The edges of all plates in a tension region must be free of cold worked material. Use UM plates (universal mill plates with rolled edges) in width increments of 1/2 inch or use flame-cut edges. Avoid sheared edges on tension plates. Generally, UM plates are more economical since they avoid cutting extras. If sheared plates must be used, grind or plane sheared edges smooth to remove cold worked material. No special treatment is required for UM or flame-cut plates.

Other than the treatment of tension holes and sheared edges, no special provisions are required for this joint. Other details should follow accepted practices for conventional design.

Q. <u>What is the transition in a beam from plastic to elastic behavior? How does one determine the spacing of lateral bracing?</u>

The transition from plastic to elastic behavior is indicated at the left in Figure 5. At the left end of the beam segment, the moment $M_1 = M_p$ causes complete plastification of the section while at the right end the moment $M_2 \not< M_p$ is resisted elastically. If residuals are neglected the yield zone extends from the left end of the beam to the point where $M = M_y$, shown in the moment diagram below the beam. The extent of the yield zone obviously depends on the end moment ratio, $P = M_2/M_1 = M_2/M_p$. Thus, the end moment ratio is a controlling factor in determining the spacing of lateral bracing at a plastic hinge which must rotate at constant moment.

The purpose of lateral bracing in the vicinity of a plastic hinge is to assure that rotation capacity (plastic hinge action) will not be limited by lateral buckling. The lateral bracing provisions of the AISC Rules for Plastic Design are shown at the right in Figure 5. The coordinates in this chart are the end moment ratio M_2/M_p and the slenderness ratio L/r_y. Notice that steep moment gradients permit larger unbraced lengths.

It is important to distinguish between lateral bracing provisions which are intended to preserve rotation capacity (shown in Figure 5) and those which are concerned only with lateral stability (the Ld/bt rules). The former apply to all hinges which must rotate in order to reach ultimate load, that is to all but the last formed plastic hinge. The Ld/bt rules apply to all parts of the structure which remain elastic and to the last formed plastic hinge, since no rotation capacity is required at this hinge. Thus, if the beam in Figure 5 were braced at mid-span by the purlin shown dotted, the maximum distance from this purlin to the hinge at the left end would be determined from Figure 5 while the Ld/bt rules would apply to the right part of the beam.

To see how the lateral bracing rules are applied in practice, consider the rectangular frame in Figure 6. Lateral support must be provided at the three hinge locations - at mid-span and two corners. Assume purlin and girt spacing and draw the moment diagram at ultimate load. The unsupported spans next to each plastic hinge must be checked using the plastic design rules shown in Figure 5. The critical span with the smallest allowable unsupported length will be that span with the largest moment ratio (unless the last hinge forms at this span). Thus the critical spans are AB for the columns and CD for the beam.

The critical length of span AB is determined by one of the equations shown in Figure 6, depending on the value of M_B/M_p. This will show whether the assumed girt spacing is satisfactory. If architectural or other considerations prohibit girts or other bracing for the columns, the designer may use a larger column to force the hinge at A in the beam.

To determine the critical length of span CD, one should note that the hinge at C is the last to form. Therefore, no rotation capacity is required at this hinge and the critical length of span CD can be estimated using the same Ld/bt rule which applies to elastic sections of the frame. This procedure is safe when one considers the influence of end restraint of adjacent spans on span CD.

In the event that the assumed spacing of purlins or girts is larger than that permitted by the plastic design rules (Figure 5) the designer has several alternatives:

1. Use the more refined analysis described in Chapter 6 of the "Commentary."[2]

2. Add secondary bracing members.

3. Select another member size with larger r_y.

4. Use side plates welded across the flange tips to form a "box section" in the critical span.

Q. Is a contilever designed plastically?

The cantilever is a determinate structure with moments at any section which are controlled by statics. Since failure occurs after the formation of one plastic hinge, redistribution of moment is not necessary, nor does it occur in a cantilever. Member sizes of cantilevers or any other statically determinate structural members would be the same whether elastically or plastically designed.

Differences would arise in the spacing of lateral supports according to the two different procedures. Since redistribution of moment does not occur, rotation capacity is not required. Therefore, the "elastic" bracing rules would apply.

Q. What constitutes adequate lateral support?

A convenient rule of thumb is that the lateral support should transmit two per cent of the force in the flange of the member being braced. In the vicinity of plastic hinges, it is important to provide lateral support for the compression flange. Thus in Figure 6, the lateral bracing at the corners and at the purlin and girt adjacent to the corners should support the compression (inside) flange of the frame.

Not only must the compression flange be braced but torsional motion must be restrained -- and this leads to the next question.

Q. <u>When beams are continuous over columns, could the stiffener be on the column center line?</u>

Four possible joints used where a beam is continuous over a column are shown in Figure 7. These joints vary in the use of stiffeners which serve two purposes. One is to transmit the column flange thrust into the web of the beam. The second is to provide torsional restraint needed to brace the beam.

The first detail in Figure 7 is used when the column flange thrust is too large to be resisted in bearing by the beam web. If the bearing stress at the root of the fillet in the beam web is not a controlling factor, only one pair of stiffeners is required to brace the beam as in the next three details in Figure 7.

In the second detail, the beam supports a purlin which is located over the column flange. The obvious place for the stiffener is over the same column flange. A single pair of stiffeners is adequate in the third and fourth details if the cap plate is thick enough to transmit a force in the stiffener to the column flanges.

The lower portion of Figure 7 illustrates what may happen if the beam is not torsionally restrained at the column. The tensile forces in the top flange result in vertical compression forces in the web. A small eccentricity in the connection will tend to cause tension flange buckling. Stiffeners are effective in preventing this type of failure.

Q. <u>If deflections in continuous beams must be checked and must be determined by elastic methods, why not design elastically and use 24 ksi?</u>

The first part of this question implies that the deflection of continuous beams must be checked in all designs. Such checks rarely indicate that deflections are a controlling factor in conventional (elastic) designs except possibly for long spans. The same conclusions are valid in plastic design practice.

To corroborate this statement, consider the following designs for a beam spanning 60 feet and carrying a live load of 1.25 kips/ft. This is a relatively long span for which deflections might be a controlling factor.

Comparative Designs

L = 60 Ft.   LL = 1.25 k/f

	End conditions and Design Method	Section Required	Working Load Defn./Span Ratio
A.	Simple Beam Elastic Design	33 WF 130	1/360
B.	Continuous Beam Elastic (24 ksi Rule)	24 WF 94	1/720
C.	Continuous Beam Plastic Design	24 WF 76	1/560

The simple beam design A would satisfy the deflection requirements of the AISC Specification (Sect. 17). The deflections of the two continuous beam designs B and C are well within the deflection requirement. Thus, it is evident that the deflection of continuous beams is not a critical factor in design, regardless of the design method used.

A convenient rule of thumb for deflection control of plastically designed continuous beams may be stated as follows:

> To limit live load deflections to less than L/300, use a beam depth greater than L/23.

This rule is based on conservative estimates of end restraint and live load to dead load ratios. Smaller depths may sometimes be justified as is evident from design C above where d = L/30.

Some notion of the relative economy of continuous beam designs using plastic and elastic methods is evident from the above comparative designs. There is a weight saving of 24% for the plastic design as compared with the elastic design (using the 24 ksi rule) of the continuous beam, and a 76% saving as compared with the simple beam design.

To gain further insight into the implications of the "24 ksi rule" of the AISC Specification (Sect. 15a3) consider the chart in Figure 8.* This figure shows the required section modulus (divided by WL) as a function of the side to center span ratio for the continuous beam indicated in the inset. The horizontal line shows the required size for a plastic design. Notice that the load factor (or safety factor) is the same, regardless of the span geometry.

The curved lines in Figure 8 indicate the size required for an elastic design using the 24 ksi rule. Two observations are evident from this figure.

1. For a limited a/L range, the load factor for an elastic design using the 24 ksi rule is less than that for a simple beam (1.85).

2. For most of this range, a larger section is required using elastic design.

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* From unpublished memorandum by R. L. Ketter

Although this is a somewhat idealized example, it serves to illustrate that the dual requirements of safety and economy are best met by plastic design.

Q. <u>May plastic design be applied to multi-story frames?</u>

The answer to this question is a qualified "yes." Present AISC specifications permit plastic design for the floor framing of multi-story structures properly braced against lateral forces, if the columns are proportioned according to conventional "elastic design" specifications.

Figure 9 [4] gives an indication of the results of currently available design methods for multi-story frames. The example considered is a 10 story, 5 bay frame with diagonal bracing, designed by four different methods. The bar graphs in the lower portion of Figure 9 tabulate the steel tonnage for each method. The shaded part of the bars indicates the weight of beams, and the remaining portion the weight of columns. The four design methods are:
1) elastic, simple beam; 2) elastic continuous beams; 3) plastic continuous beam, elastic (AISC) column; and 4) plastic continuous beam, ultimate strength column.

Figure 9 indicates a 25% saving in weight of steel for the plastic design (3) as compared with the elastic simple beam design and a 7% saving in comparison with the elastic continuous beam design. In addition, the elastic continuous beam design involved more design time. The increased weight of column steel required by the ultimate strength column design (4) is due to the fact that the lower story column sizes had to be increased to keep the P/Py ratio within the 0.6 limit set by the AISC Plastic Design Rules. There is good reason to expect that further study of the column problem will justify raising this limit and thus reveal even greater economy in plastic design.

Several multi-story buildings have been built using plastic design. One of these is the 18 story Tower Building in Little Rock, Arkansas.[5] The floor framing in this building was designed plastically and field welded for continuity. A tubular K bracing system was used to resist wind forces. Conventional design specifications were followed in proportioning the columns. Separate bids were taken for both a plastically designed steel frame and a reinforced concrete frame. The steel frame proved lower in cost and required two months less construction time, resulting in earlier rental income.

An 8 story apartment building in Canada utilized plastic design for floor beams spanning 39 feet[6]. This resulted in large column free areas and flexible architectural treatment. The designers estimated a saving of $10,500 in favor of the plastically designed steel frame, which cost 7-1/2% less than a reinforced concrete frame.

SUMMARY

These remarks have discussed some of the practical problems met in plastic design practice, such as connections, edge preparation, lateral bracing, stiffeners, and deflections. Emphasis is placed on the assumptions and secondary design considerations of the plastic method which form the basis for solving many of the practical problems.

Comparisons of the economy and rigidity resulting from the plastic and conventional design methods are included. They indicate that the dual requirements of safety and economy are best realized by plastic design.

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PLASTIC DESIGN

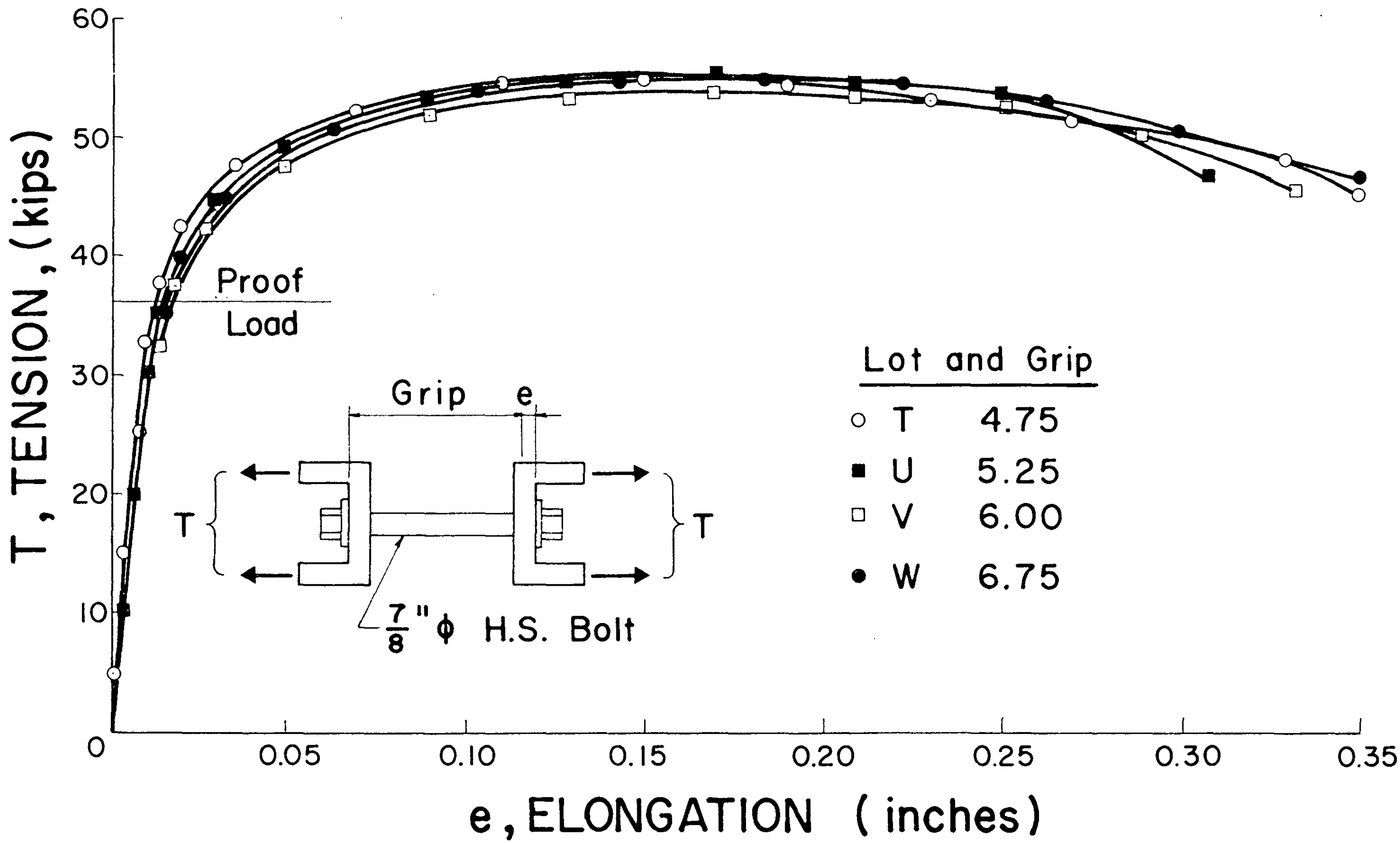
Assumptions

1. DUCTILITY
2. PLASTIC MOMENT
3. ROTATION CAPACITY
4. CONTINUITY AT CONNECTIONS
5. REDISTRIBUTION OF MOMENT
6. MECHANISM AT ULTIMATE LOAD

Secondary Design Considerations

1. AXIAL FORCE AND SHEAR FORCE
2. INSTABILITY: LOCAL, LATERAL, AND COLUMN BUCKLING; FRAME STABILITY
3. BRITTLE FRACTURE
4. DEFLECTION STABILITY
5. DEFLECTIONS
6. CONNECTIONS

FIG. 1



T, TENSION, (kips)
60
50
40
30
20
10
0
0.05
0.10
0.15
0.20
0.25
0.30
0.35
e, ELONGATION (inches)
Proof Load
Grip
e
T
T
$\frac{7}{8}$" φ H.S. Bolt
Lot and Grip
○ T 4.75
■ U 5.25
□ V 6.00
● W 6.75

FIG. 2

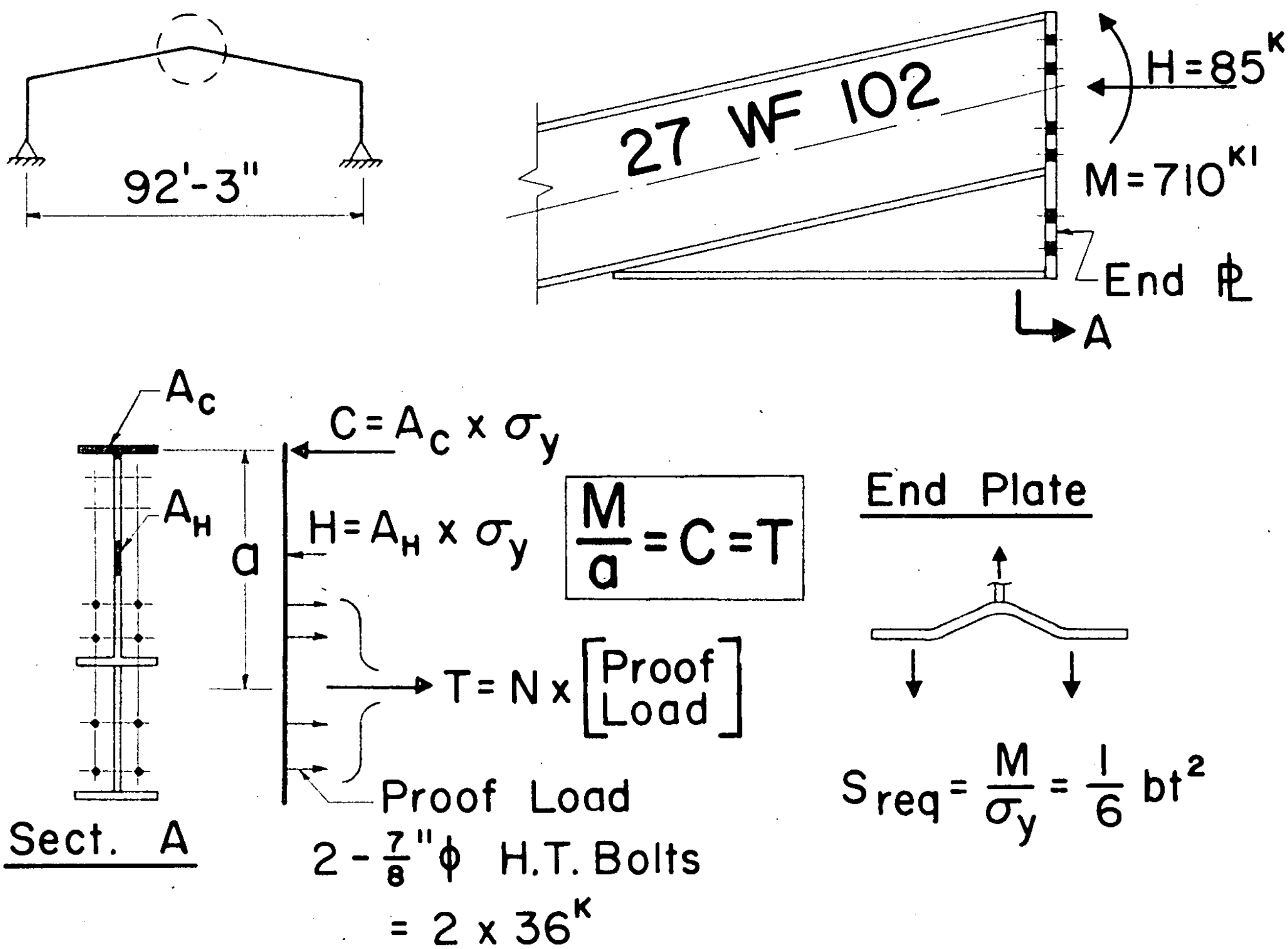

92'-3"
27 WF 102
H=85^K
M=710^K'
End PL
A
A_C
A_H
a
Sect. A
C = A_C x σ_y
H = A_H x σ_y
M/a = C = T
T = N x [Proof Load]
Proof Load
2 - 7/8" φ H.T. Bolts
= 2 x 36^K
End Plate
S_req = M/σ_y = 1/6 bt^2

FIG. 3

205.316

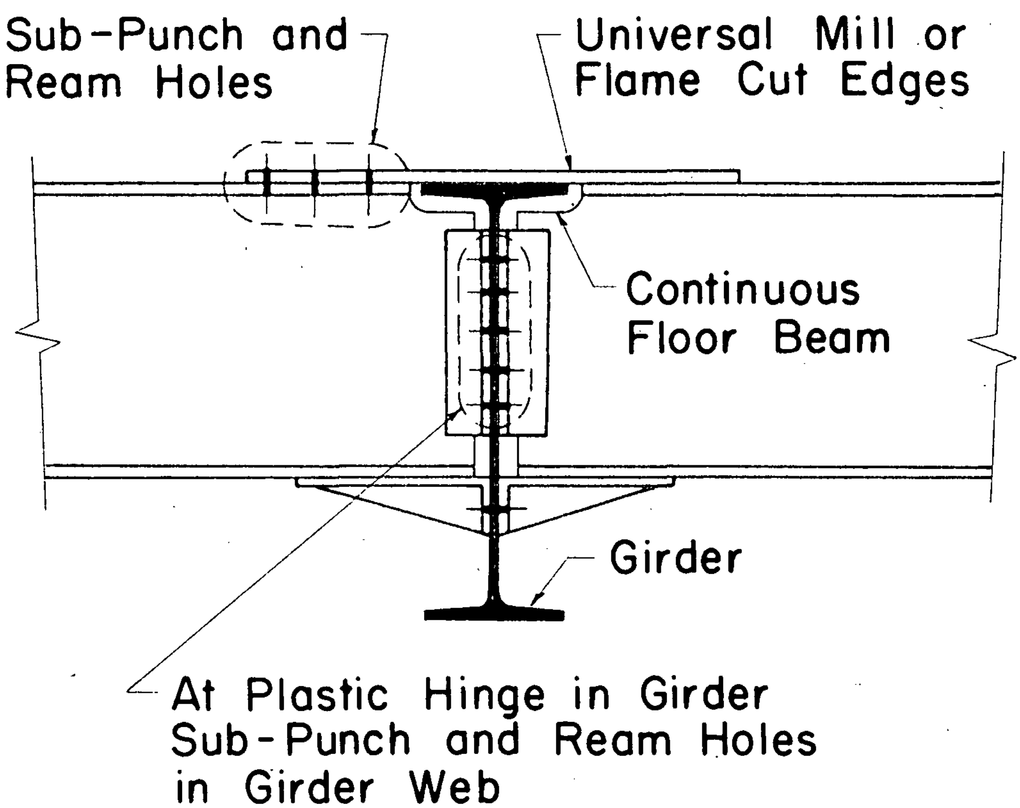

Sub-Punch and Ream Holes
Universal Mill or Flame Cut Edges
Continuous Floor Beam
Girder
At Plastic Hinge in Girder Sub-Punch and Ream Holes in Girder Web

FIG. 4

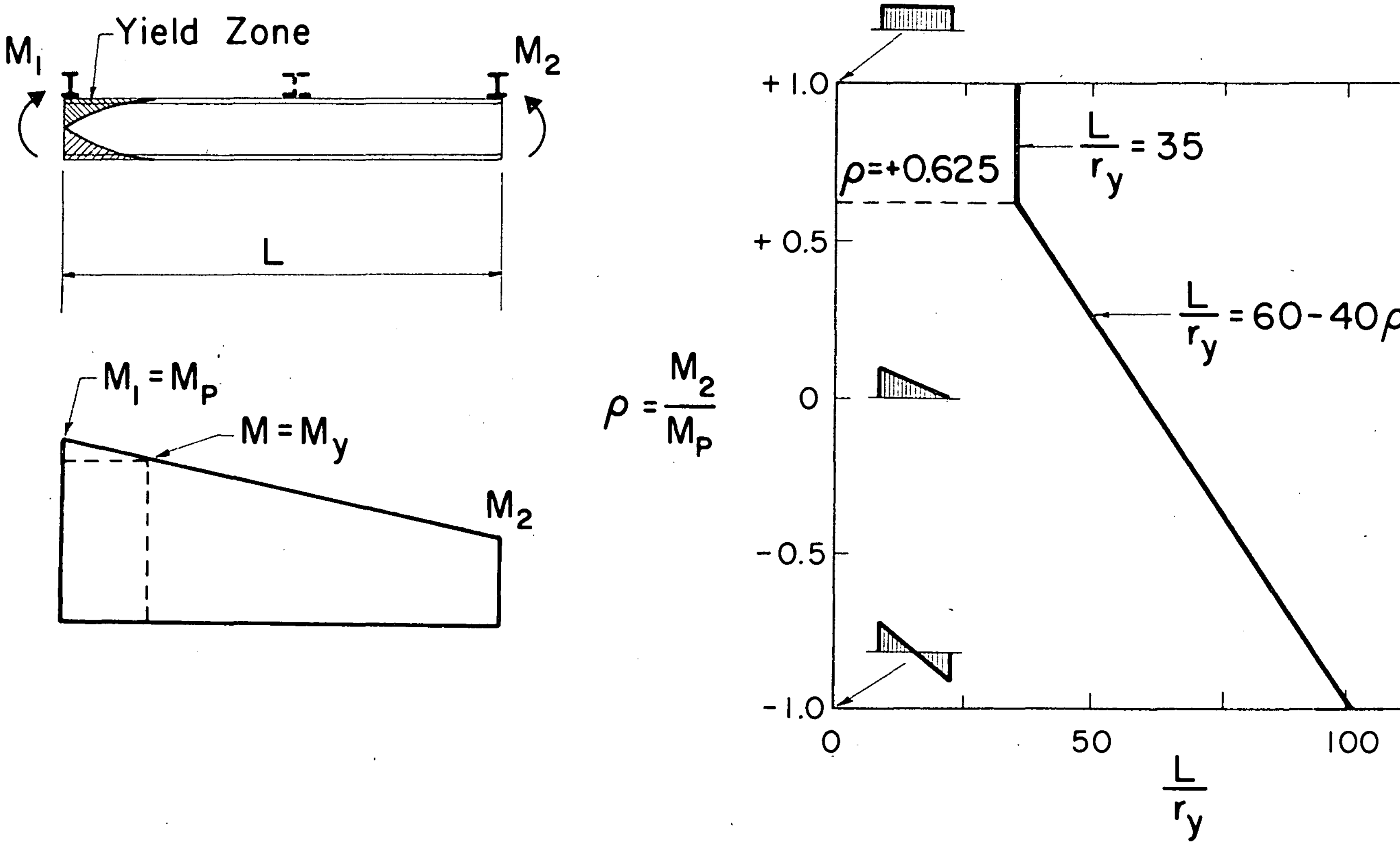
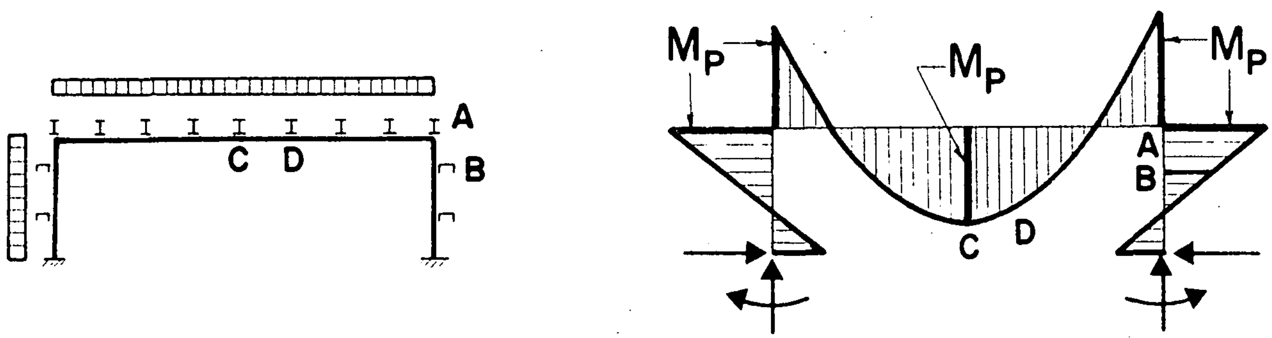

Yield Zone
M_1
M_2
L
$M_1 = M_P$
$M = M_y$
M_2
$\rho = \frac{M_2}{M_P}$
$\rho = +0.625$
$\frac{L}{r_y} = 35$
$\frac{L}{r_y} = 60 - 40\rho$
+1.0
+0.5
0
-0.5
-1.0
0
50
100
$\frac{L}{r_y}$

FIG. 5



Lateral Bracing for Columns - Span AB

$$\frac{M_B}{M_P} < +0.625\,,\quad \left(L_{AB}\right)_{cr} = \left(60 - 40\,\frac{M_B}{M_P}\right) r_y$$

$$\frac{M_B}{M_P} > +0.625\,,\quad \left(L_{AB}\right)_{cr} = 35\, r_y$$

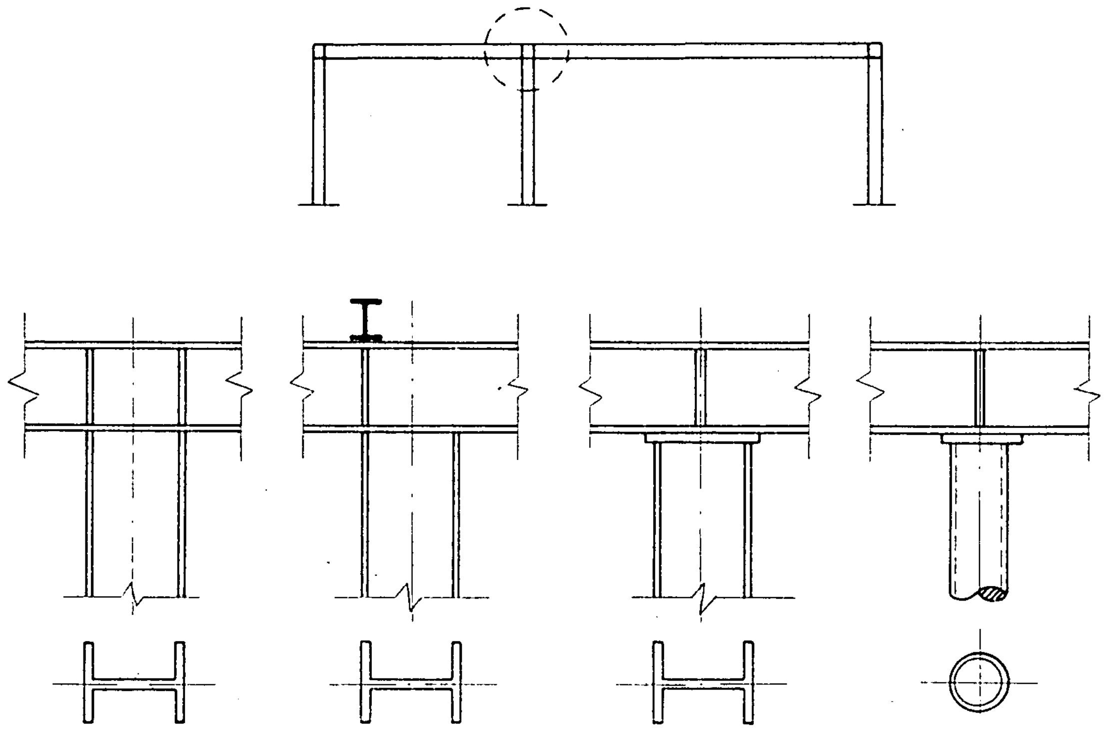
Lateral Bracing for Beams - Span CD

Last Hinge at C - No Rotation Reqd.

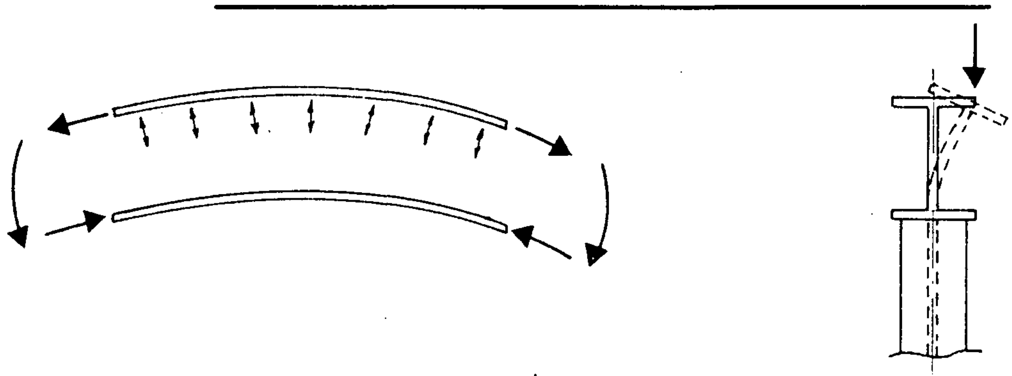
$$\left(L_{CD}\right) \times \frac{d}{bt} < 600$$

FIG. 6

205.320



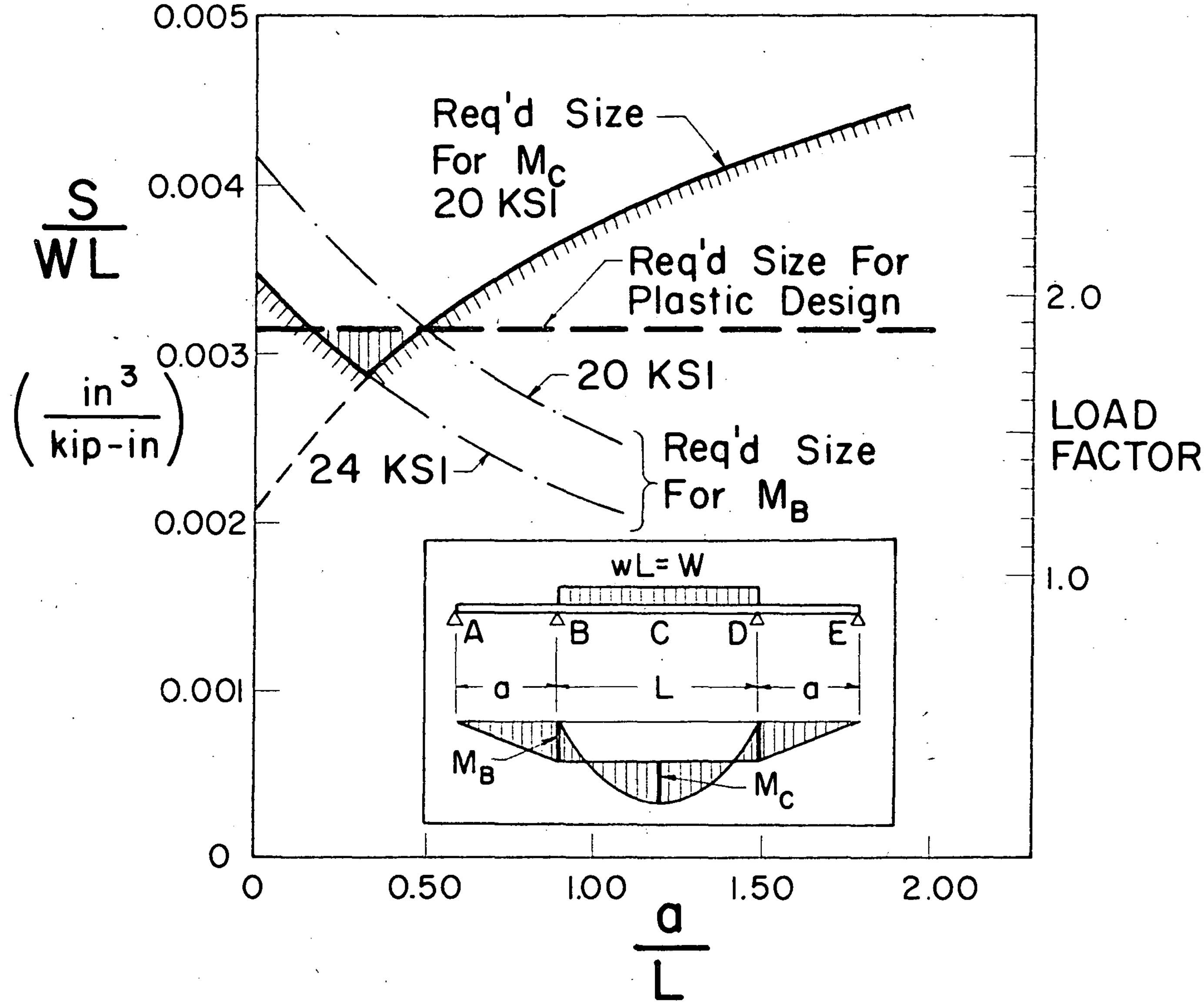
BRACING STIFFENER DETAILS



TENSION FLANGE BUCKLING

FIG. 7

205.321



0.005
0.004
0.003
0.002
0.001
0
S/WL
(in³/kip-in)
Req'd Size For M_C 20 KSI
Req'd Size For Plastic Design
20 KSI
24 KSI
Req'd Size For M_B
LOAD FACTOR
2.0
1.0
wL= W
A
B
C
D
E
a
L
a
M_B
M_C
0
0.50
1.00
1.50
2.00
a/L

FIG. 8

205.323

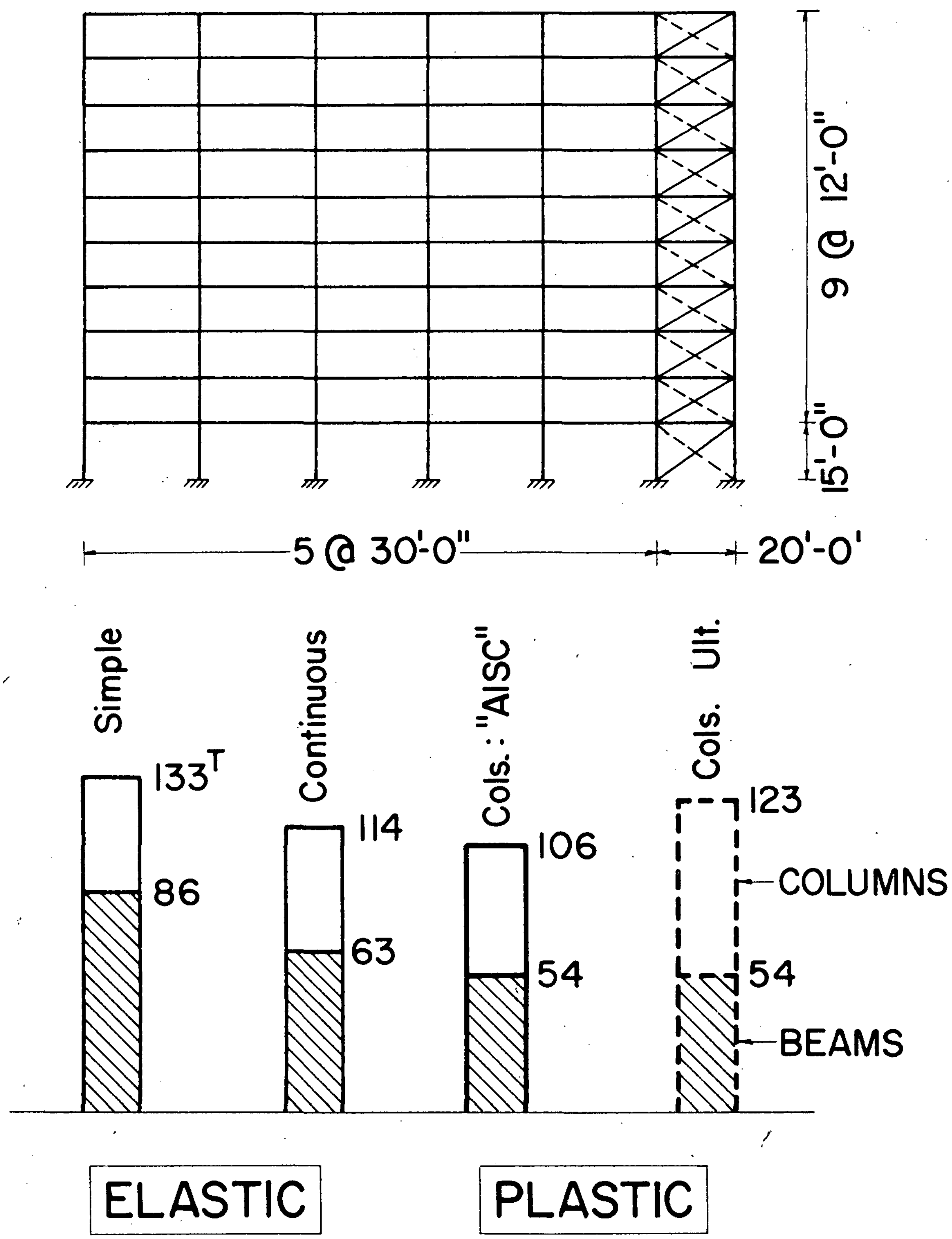

9 @ 12'-0"
15'-0"
5 @ 30'-0"
20'-0'
Simple
Continuous
Cols.: "AISC"
Cols. Ult.
133^T
86
114
63
106
54
123
54
COLUMNS
BEAMS
ELASTIC
PLASTIC

FIG. 9

DA OU