CUMULATIVE FATIGUE DAMAGE
UNDER RANDOMIZED LOADING

by
Thomas E. Gallagher

A Thesis
Presented to the Graduate Faculty
of Lehigh University
in Candidacy for the Degree of
Master of Science

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ABSTRACT

This thesis presents the results of an experimental investigation to determine the effects of a variable amplitude load program randomly applied to notched butt welded plate specimens. The load program simulated actual service conditions for steel beam highway bridges and the plate specimens duplicated the fabrication method used in joining these beams to form a continuous flange. The tests were performed using a high speed resonant fatigue testing machine which included a programming device for applying a random load. The results are presented and their application in the determination of service life is discussed.

Factorial experiments were designed to provide information regarding the effects of minimum stress, maximum stress, stress range, and mean stress on the fatigue behavior of the notched specimens. The geometry of the specimen was developed from the experimental program.

The test results were analyzed by a mathematical model to obtain the relative degree of correlation between the stress parameters selected. Analysis showed that the stress range produced the greatest correlation when plotted against the number of cycles to failure and that maximum stress, minimum stress, and mean stress had no significant effect on fatigue life. It was found that the slope of the regression line through the constant amplitude test data was statistically the same as the slope of the regression line through the variable amplitude test data.
This thesis is accepted and approved in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering.

May 22, 1968

Dr. John W. Fisher
Professor in Charge

Dr. David A. VanHorn
Chairman, Department of Civil Engineering
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This thesis presents the results of an experimental investigation to determine the effects of a variable amplitude load program randomly applied to notched butt welded plate specimens. The load program simulated actual service conditions for steel beam highway bridges and the plate specimens duplicated the fabrication method used in joining these beams to form a continuous flange. The tests were performed using a high speed resonant fatigue testing machine which included a programming device for applying a random load. The results are presented and their application in the determination of service life is discussed.

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1. INTRODUCTION

1.1 Background

The problem of structural fatigue has been recognized for more than one hundred years. It was first noted by railroad engineers who observed axle failures on railroad cars. The failures occurred even though the axles had not been overstressed during their lives. This phenomenon was studied by Rankine (1843), Wohler (1850), and others. Wohler conducted tests which showed that a material could fail under repetitive loading at a stress below the ultimate level and recognized that internal damage was occurring to the material. As the damage accumulated, the material was finally overstressed and failed. Failure in this manner became known as fatigue.

Wohler plotted his test results on a graph of load amplitude versus cycles to failure. This plot is known as an S-N diagram and is a means used in evaluating the fatigue properties of a material. An ideal S-N curve, as shown in Fig. 1, consists of two distinct parts: a sloping line and a horizontal line. The sloping line indicates the specimens which fail after continuous stress application while the horizontal line defines the so-called endurance limit. Specimens tested at stresses below the endurance limit are assumed to have an infinite life. The S-N curve is plotted with data derived from constant amplitude tests. The junction of the sloping line and the horizontal line is
called the knee of the curve. Test results in this area may be subject to a fairly large dispersion which is called scatter.

Failure by fatigue loading was recognized as a problem, but at the time most design was done by the allowable static stress method, so designers chose to handle the problem by using a greater factor of safety for structures subjected to cyclic loading. This procedure resulted in the use of excess weight and waste of materials in many instances, and still left the possibility that the structure might fail before reaching its expected service life.¹

The study of fatigue damage has received less attention from civil engineers than it has from mechanical, railroad, or aeronautical engineers since civil engineering structures are subjected to fewer cyclic loads than gears and engines, railway components, and aircraft. Nonetheless, fatigue studies are receiving increasing attention in the civil engineering field, particularly from bridge engineers who are required to design structures which are subjected to random variations of load where the peak stress values of succeeding cycles are different.

It is anticipated that future traffic patterns will reflect large increases in size, weight, and frequency of vehicles, with speeds up to the 100 mile per hour range. These patterns are a logical extension of the relationship between today's traffic and that of thirty years ago, and present a challenge to bridge engineers not only to design today for tomorrow's needs, but evaluate yesterday's structures considering the demands of today. It is of immediate urgency to be able to more reliably predict the remaining safe life of existing bridges. Although there are few recorded instances of fatigue failure of highway
bridges, this fortuitous circumstance is no doubt due to conservative
design practice and to the less severe traffic loadings of the past.

Today's construction codes are constantly revised to reflect
the use of more sophisticated methods of analysis and design. High
strength materials and welded fabrication methods are increasing in
use. All of these contribute to structures which are more susceptible
to failure by fatigue, and dictate that suitable methods be developed
for determining the service life of a structure. Service life is the
period during which the structure is able to perform its designed use.
This period may be expressed chronologically or as the amount of stress
applications that can be endured.

To design a structure or a structural component for fatigue
requires a prior knowledge of the load spectrum which the component
will undergo during its service life as well as the number of cycles
it must endure. A load spectrum is a representation of the loads and
their frequency of occurrence on a structure. It may be either recorded
from the structure or artificially generated. Once these are known, tests
can be conducted to provide guidelines for design. The aerospace
industry has made the most notable advances in fatigue. The industry
was the first to develop variable amplitude load tests. By incor-
porating them with load spectra representing the various stresses and
frequencies of stresses which a component must withstand during its
life, they were able to predict with some degree of accuracy the service
life of the particular component. Most data produced by this technique
have related to materials and spectra not typical of bridges. Its
applicability to civil engineering has not been fully explored.
A need exists to follow the lead of the aircraft industry and take advantage of the tools which have been developed for variable amplitude fatigue testing. Current structural systems, e.g., bridges and building components are designed on the basis of the estimated number of maximum load cycles. The selection of the design life has recognized indirectly the random nature of the loading and the inability to ascertain the anticipated load history. Constant cycle data has been used to develop design requirements.

More effort has been placed on the acquisition of data that will help define the load histories of structures. The Bureau of Public Roads is one agency that is sponsoring research in this area. Various state highway departments have been or are now collecting data regarding stress magnitudes and frequency of application which are placed on highway bridges, in order to determine actual load spectra which must be withstood.

As these histories are more fully defined, more information is needed on the behavior of components under these loadings if further refinements in design practice are to be made. No relationship has yet been found which accurately ties together the effects of traffic volume, loads, and bridge strains.

1.2 Cumulative Damage Theories

Grover suggests two approaches to the problem of treating the complexity of loading in order to predict the service life of a part. One is to seek a theory of cumulative damage which will permit the
estimation of life when constant cycle S-N curves for the material and a specified set of complex variable amplitude loadings are known; the second is to seek some spectral loading representative of service conditions and make a laboratory appraisal using this load spectrum. The first approach will be considered in this section.

Before the theories of cumulative damage can be considered it is needed to appreciate the term "fatigue damage". Fatigue damage is difficult to define since there is no simple measure of damage in the early stages of fatigue. Two measuring devices proposed are the loss of strength of the material and the size of the fatigue cracks present, but these both suffer from the problem that for a large part of their life, neither quantity is detectable. Despite difficulties in accurately detecting or understanding the damage, it is obvious that damage does take place and that eventually the specimen fails at a stress below its maximum static stress. Because of the difficulty in suitably defining the term "damage", and of evaluating the damage caused by complex stress variations, the cumulative damage theories proposed are based on assumptions regarding the occurrence of damage and methods for adding the damages produced by variable amplitude stress cycles. One indication of the difficulty in this definition is the plethora of cumulative damage theories which have been proposed, studied, and then rejected because they indicated, other than for a specific set of conditions, an incorrect theory. However, until a sound physical basis of behavioral evaluation is established, evaluation of cumulative fatigue damage must depend on theories. It is in order to briefly review some of these proposed theories and note the key assumptions which can be
identified, since, lacking a suitable alternative, these assumptions may indicate some factors involved in the design to resist cumulative damage.

Two characteristics of a cumulative damage theory are the consideration of stress independence or stress dependence and interaction or freedom from interaction. The magnitude of the stress level must also be included since the life of a specimen is determined by the number of cycles run at a particular stress amplitude, but a theory may be classified as stress independent if it assumes that equal amounts of damage are produced at equal fractions of life for all stress amplitudes. In such a theory both the life and the damage must be non-dimensionalized so that both equal 100% at failure. Otherwise it is not possible to relate cycles to failure or loss of static strength when considering different stress amplitudes. For a stress independent theory, a curve plotting damage versus cycle ratio would result in a relationship in which the effect of all stresses is represented by a single curve. This is shown in Fig. 2. For a stress dependent theory, any given value of stress would produce a unique curve. This relationship is shown in Fig. 3.

A theory may be classified as interaction free if the relationship between the damage and the number of cycles at a specified stress amplitude is assumed to be valid whether or not other stress amplitudes are applied. Conversely, a theory which holds that the damage at a particular amplitude may be altered depending on the prior load history is an interaction theory. If a specimen has been damaged to a certain level, and subsequently has another cycle of a given stress applied to it, the additional damage to the specimen must equal the increment of
cycle ratio n/N times the slope of the damage-cycle ratio curve or the theory is not an interaction theory. Both stress-dependent and stress-independent theories may be interaction free.

The following are among the cumulative damage theories which have been proposed:

1. **Miner Theory**\textsuperscript{7} - This theory, which was also proposed by Palmgren some twenty years earlier, and by Langer in a more general framework, is the most generally known of the cumulative damage theories. It is a stress-independent interaction free theory and has had more use than any other theory both in design and in theoretical development because of its simplicity and ease of application. Miner's theory states that when a mixture of stress amplitudes is applied to a specimen, failure will occur when the fractions of life expended at each stress level add up to unity. The fraction of life at a particular stress level is the ratio of the number of cycles\textsuperscript{7} applied at stress level to the number (N) which would cause fatigue failure at that amplitude as determined from a Wohler curve. It may be expressed as

\[ \sum_{i} \frac{n_i}{N_i} = 1 \]

2. **Valluri's Theory**\textsuperscript{8} - Valluri has developed a stress-independent, interaction-free theory which is therefore equivalent to Miner's if certain restrictions concerning
the definition of fatigue damage are observed. The
parameters which Valluri uses in his theory are much
more complex than that used by Miner, and the load
spectrum encountered must also be well mixed in order
for failure to occur at the instant when damage equals
unity. Because of the difficulties in applying Valluri's
theory without excessive effort, it is not popular.
It further suffers from not being able to define its
parameters for materials in which there is little loss
of strength at the appearance of the first visible crack.

3. Grover's Theory⁹ - Grover proposed a two stage damage
process with the first stage terminating after cracks
have been initiated and the second stage including the
propagation of these cracks to failure. In constant
level fatigue tests, the boundary between the two stages
is the function of "a", some fraction of the total stress
cycles. Thus crack initiation occurs during "a" and
crack propagation during (1-a). The value of "a" is a
constant for any stress level and is interaction free.
However the theory is stress-dependent since the damage
curves drawn using this theory differ for different
amplitudes.

Grover's theory may be expressed as

\[ \sum \frac{n_i}{a_i} N_i = 1 \]

\[ \sum \frac{n_i}{(1-a)N_i} = 1 \]
where $n_1$ is the number of cycles in the first stage, and $N_1$ the number of cycles in the second stage.

The theory is interesting in that it recognizes that fatigue damage occurs in two stages when viewed microscopically, the first stage being intragranular damage (within the grains) and the second being the intergranular damage resulting in visible cracks. However, there are insufficient data at present to determine the value of the parameter "a" or how it varies with stress amplitude.

Use of Grover's theory in design will always result in a heavier structure than that designed by Miner because its stress dependence would indicate an earlier failure for a well mixed spectrum.

4. Shanley's Theory$^{10}$ - Shanley proposed a mechanism for the generation of fatigue cracks caused by the unbonding of the atomic structure during application of stress cycles. At the point where the stresses applied have caused a crack length equal to the critical value $L_{cr}$, failure occurs. The crack length is a function of the atomic unbonding caused by the stress cycles applied. It may be expressed as

$$L_{cr} = L_{in} \exp \left( C \sigma_{a1}^x N_i \right)$$

where $L_{in} =$ initial crack depth produced by the first cycle, $C =$ constant, $\sigma_{a1} =$ stress at $i^{th}$ cycle, $x =$ constant, and $N_i =$ cycles to failure.
Kaechele\textsuperscript{6} points out that the equation of the S-N curve may be established by this approach where

\[ N = \frac{\text{constant/} \sigma}{\sigma_a} = \frac{K}{\sigma_a} \]

and then proposes that cumulative damage theories which contain a means for predicting the S-N curve as well as the life expectancy, be termed fundamental fatigue theories, while theories which must rely on previously established S-N curves in order to predict service life be termed cumulative-damage theories.

5. Freudenthal-Heller Theory\textsuperscript{11} - This interaction type theory was proposed on the premise that an S-N curve could be generated such that Miner's theory, coupled with this curve, would be valid for determining cumulative damage under spectrum loading. It may be expressed as

\[ N = \frac{k}{\sigma_a^x} \]

similar to the one established by Shanley. However, the generation of a value for \( k \) poses a problem, and experimental work on this theory has not yet attempted to handle other than smooth specimens. Until it is verified on specimens with stress raisers, or subjected to constant or variable mean stresses, it cannot be applied to design problems.

6. Corten-Dolan Theory\textsuperscript{12} - This interaction theory claims to determine interaction effects from constant amplitude fatigue tests at two levels. Interaction is introduced
by the assumption that the damage produced at the highest stress level affects damage growth at the other stress levels. Experimental work to check the theory has produced favorable results so far but more work must be done before this theory will find application in design.

7. French's Theory - This theory is similar to Miner's in that French defined a "damage line" below the generated S-N curve and claims that any specimen cycled below the line accumulates no damage, which is somewhat similar to Miner's hypothesis that a specimen cycled below the fatigue limit accumulates no damage.

8. Richart and Newmark's Theory - This theory is similar to Miner's with the exception that Miner assumed a linear relationship between damage and cycle ratio, and this theory assumes an exponential one. It may be expressed as

\[
\sum \left( \frac{n_i}{N_i} \right)^{P_i} = L
\]

Determination of \( P_i \) is generally not feasible for design purposes.

9. Marco and Starkey's Theory - This is almost identical to that of Richart and Newmark with the exception that the exponential value must always exceed unity. No method for determining this exponent is proposed.
10. Henry's Theory\textsuperscript{16} - This theory is limited to materials which exhibit an endurance limit, and is completely based on an S-N curve. Henry proposed to reduce the constants in the equation of the S-N curve as fatigue damage accumulates and determines failure as the point where the endurance limit reaches zero. It essentially entails the translation of the abscissa of the S-N curve at an amount determined by the damage accumulated from previous stress cycles in the test.

11. Manson, Freche, and Ensign's Theory\textsuperscript{17} - A double linear damage rule was proposed stating that two stages of damage occur under fatigue loading (crack initiation and crack propagation) and that each stage is linear.

While other theories have also been proposed, some more complicated than the preceding and some less complicated, they have in common the feature that they do not successfully predict fatigue life under the conditions that designers require. Some may work under given conditions, or on a given material, but a universally applicable theory has not yet been found. Until one is found, prediction of fatigue life within the safe limits required by designers can result only from full scale testing of the structural components.
1.3 Purpose of Test

The object of the test program was two-fold. It was intended:

1. To initiate a pilot fatigue study under variable loading conditions which simulated service conditions for steel beam highway bridges.

2. To develop data regarding the formation of fatigue cracks where welded plate girders have their flanges butt welded together and to attempt to relate the data from the pilot program to a concurrent test on welded beams.

In the test program it was attempted to use a load spectrum which was recorded by the Michigan Department of Highways from strain gages placed on interstate highway bridges.\(^{18}\) This spectrum was applied to specimens representing a butt welded flange on a plate girder in a region of transition from one width of the flange to another. Actual tests of welded beams under uniform cyclic loading are presently under way to test this same condition. It is intended to attempt a correlation of data generated by both tests in order to evaluate the accuracy of the load spectrum in simulating field conditions.
2. DEVELOPMENT OF RANDOM LOAD PATTERN

2.1 Determination of Testing Method Used

There are basically three types of testing methods available in the representation of service loading conditions by a load sequence, namely:

1. Repeated Load Test (or constant amplitude test)
2. Programmed Load Sequence Test
3. Random Load Sequence Test

Repeated Load Test

In this test a particular variable amplitude spectrum is replaced by a single representative load which should be the one causing maximum fatigue damage according to the linear cumulative damage theory. This method in effect assumes the shape of the S-N curve and the object of the test is to fix the curve by locating a single point. The advantages of this test are that it is quick and is no more subjected to scatter than programmed variable amplitude loading, and that it demonstrates the effects of design changes in a structure as well as indicating sections of the structure that may be critical in fatigue.

The disadvantages of the test are that the S-N curve may be assumed incorrectly since its shape varies considerably depending on the particular material being tested, and that it does not forecast the service life of a structure with any confidence.
**Programmed Load Sequence Test**

In this test, blocks of loads representing loads taken from the load spectrum which the structure would undergo are applied according to a prearranged sequence. Each block contains loads of the same amplitude and in the ratio of the frequency of occurrence of these loads in the spectrum. This type of test was first proposed by Gassner in 1946 and has been refined by him to the extent that he chooses a definite number of load levels and cycles to complete each program cycle. By experimenting with increasing numbers of load levels he found that no significant change in service strength occurred beyond eight levels. He also found that a negligible work-hardening effect occurs once the service life is above a certain high number of cycles, which he found to be one million cycles for aluminum alloy 2024T4. He claimed the number of cycles will vary for different materials but the eight load levels will not. Gassner also found that omission of low stress levels (between the fatigue limit and 5% of the maximum stress) may raise the fatigue life by two to three times and thus give a non-conservative indication of service life. Until he determined this, low stress levels were not accounted for since they were usually below the fatigue limit and contributed nothing to Miner's theory of linear cumulative damage.

Gassner's program test applies the load blocks in a particular pattern in order to obtain a mixing of high and low loadings, starting at an intermediate level as illustrated in Fig. 4c and alternately increasing and decreasing until failure occurs. Other modifications of this sequence are known as the high-low and the low-high application as shown in Figs 4a and 4b.
Unfortunately these three methods of load application do not generate results which are in agreement.

**Random Load Sequence Test**

There is disagreement among research workers as to the exact definition for this test. Since the natural load sequence would be random, and a high positive (or negative) load would not necessarily be followed by a high negative (or positive) load, laboratory representations of such conditions are extremely difficult. Work in this field has been done by Head and Hooke\(^2\) in the development of a random noise fatigue machine. A random noise generator gives a random output voltage which applies a fluctuating force to the fatigue specimen. The application of this process to a large specimen would be quite complicated.

Others feel that since there is difficulty in developing a fatigue machine capable of accepting a truly random load sequence, some modification of the programmed load sequence test should be made in which the load levels are applied randomly, as shown in Fig. 4d, rather than in a planned sequence. Freudenthal\(^2\) has done work in this field by taking several recorded load spectra and reducing them to six finite load increments which are applied to a test specimen in a random fashion. It should be noted that both Gassner and Freudenthal believe that the fatigue life of a specimen subjected to a constant mean stress and a programmed load block is twice as short as the actual life of a component which does not have a peak stress duplicated about the mean.
The testing method chosen in this study was the random load sequence test following Gassner's suggestion for the number of load levels, but with the sequence of application of these loads randomly chosen.

2.2 Histogram of Stress Range Selected

Since one purpose of the test was to simulate service conditions for steel beam highway bridges, it was necessary to locate data which was representative of a bridge presently in use on the highway system. A study undertaken by the Michigan Department of State Highways in 1966 contained the stress histories of certain bridges. From this study a typical histogram of stress range versus frequency of occurrence for an interstate highway bridge was selected and is shown in Fig. 5. The histogram was developed by placing a strain gage at the 1/4 point of the flange on the most highly stressed stringer in the bridge. It consisted of eleven stress ranges, a desirable number since it allowed a direct relationship between the histogram stress ranges and the programmed load levels, without introducing a possibility that service life indications might be affected by having too few load levels in the program.

The stress selector drum on the Amsler Vibrophore, the testing machine used on this program, is divisible into 100 increments. The histogram was already separated into eleven blocks by percentages, so it was a simple matter to round off decimal points and determine the number of drum increments each load level would require. After the stress level and relative frequency of the blocks had been determined,
the blocks were numbered sequentially and then related to numbers drawn from a random number table in order to choose a random load sequence as illustrated in Fig. 6. There were two lengths of program cycles used - one of five million cycles per drum revolution when the maximum stress range in the program was low enough so that more than five complete revolutions were expected before failure, and one of five hundred thousand cycles when it was felt failure would occur prior to the completion of twenty-five million cycles.

2.3 Factoring of Stress Ranges

The maximum stress range on the bridge chosen to represent the load spectrum was 6300 psi (see Fig. 5) superimposed on a minimum tensile stress of 6700 psi. An examination of the literature indicated that the entire load spectrum was below the endurance limit. In order to avoid excessive life cycles and to insure failure of the test specimens, the stress ranges were factored to higher values.

Justification for this factoring, in addition to expedience, is that the bridge was a modern highway structure, designed under federal interstate highway construction specifications in order to support extreme loads. Older bridges would not have so high a load capacity and under similar loads would have to withstand higher stresses. Additionally, the presence of stress raisers produce high local stresses in particular components, although not necessarily throughout the bridge. The factoring was begun at a value of 60,000/6300 = 9.5, with 60,000 psi the maximum stress the vibrophore could produce on the specimen. The load factor was decreased as appropriate from the pilot study results.
3. DEVELOPMENT OF SPECIMEN GEOMETRY

3.1 General Fabrication Details

The test specimens for this program were fabricated from a single plate of ASTM A36 steel. Strips 12 inches long were cut from the plate which was 48 inches wide and 1/2 inch thick. The 12 inch strips were cut in half and the cut edges shaped into vees. The halves were then butt-welded together with an automatic submerged arc welding machine. An E6012 electrode was used and excess weld reinforcement was removed. The welded plate was then saw cut into twenty four blanks, each approximately 2 inches by 8 inches. The test specimens were made from these blanks. The plate was cut so that the rolling direction and the direction of applied stress were the same. Figure 7 illustrates the fabrication process.

Similar weld details were used in the principal experiment on beams. The test specimens were intended to simulate the flange splices in the beams as shown in Fig. 8. A thinner plate was used in the pilot program than in the beam tests (1/2 inch versus 3/4 inch) because the capacity of the testing machine was limited.

Tensile coupons for coupon tests were taken transverse to the direction of rolling. They were tested on a Tinius Olsen mechanically operated universal testing machine having a 120 kip capacity, and equipped with an automatic load strain recording device. A typical load elongation
curve for the tensile coupon tests is shown in Fig. 9. The test results are presented in Table 1.

3.2 Arrival at Final Configuration

The shape initially tested was a reduced section coupon with parallel sides and approximately 0.50 square inches in cross-section as illustrated in Fig. 10a. Since the capacity of the vibrophore is 22,000 pounds, this meant that a maximum stress range of 44,000 psi could be applied to the weldment and adjacent sections. A single specimen was subjected to this maximum and 35,000,000 cycles were applied with no noticeable result.

In order to introduce a stress raiser, a weld bead was placed along the length of the specimen to simulate the flange to web connection. The extreme sensitivity of the vibrophore to eccentricity of the specimen prevented it from running. The bead was then ground smooth in the hope that the effect of heat input by the welding would introduce a metallurgical notch of sufficient severity to lower the life of the specimen. This specimen survived 57,000,000 cycles of maximum stress range. The weld bead was then reintroduced but in order to reduce the eccentric mass, the bead was tapered at the ends by grinding. Two specimens with this configuration were subjected to the maximum stress range and broke after two million cycles. Both breaks were initiated at the toe of the weld bead and in the radius, away from the butt weld test section.

At this point it was evident that in order to develop a basic S-N curve a higher range of stress was necessary. This meant altering
the specimen shape to that shown in Fig. 10b. Because the thickness of
the specimen was fixed at 1/2 inch in order to relate to the flanges of
the beams being tested in a concurrent project, it was decided to reduce
the width. A ratio of 1-1/2 to 1 in width to thickness was agreed
upon and since the specimens which failed had done so in the radius, it
was decided to increase the radius in order to cut down on stress con-
centrations. A constant radius of 6 inches was selected and a jig to
shape the blocks to this configuration was developed by Fritz Engineering
Laboratory machinists (see Fig. 11). It was hoped that since the minimum
section was in the weld area, failure would be induced there. Again the
shape proved unsuitable when it was tested. Subsequent modifications
such as a continuous weld bead, or intermittent weld beads down the side
occasionally resulted in failure, but the weld bead warped the specimen
after cooling and necessitated cold straightening of it. This changed
the residual stress pattern and introduced other variables into the
program. The fillet weldment was discarded to avoid this variable.

After considering the results derived from the preceding specimen
it was decided that there was no way to develop a crack in the area of
the butt weld at lower stress levels without introducing a notch to
accomplish it. In order to be sure that all data which the test would
produce could then be related to prior tests in this field, a previously
used notch was selected as the means of insuring failure of the specimen
and is shown in Fig. 10c. The configuration of the notched specimen
(actually a symmetric pair of notches opposing each other across the butt
weld) was made identical to the notched shape used by Payvar and
Vasarhelyi at the University of Washington in a study of the fatigue
behavior of welded structural steels. It consisted of a vee notch of 45° with a radius of 0.010 inch at the root and a depth of about 0.062 inch.

The reason for introducing a notch was to decrease the life and magnitude of applied stress. Results to this point had indicated an unacceptably high fatigue limit for unnotched specimens. Subsequent tests run using the notched shape generated results within acceptable stress levels and life cycles. All specimens tested in the constant amplitude and variable amplitude portions of the programs conformed to the geometry given in Fig. 10c.

3.3 Development of the Experiment Design

An inherent characteristic of the Amsler Vibrophore is that it generates stresses around a mean value. This meant that the minimum stress could not be held constant as was done in the constant amplitude tests of beams.

The experiment design of both the constant amplitude and the random load tests was based on two values of mean stress. A factorial design was developed for the constant amplitude test which allowed an S-N curve to be developed while testing thirty specimens, with fifteen assigned to each mean stress level. The experiment design is summarized in Fig. 12. Three specimens were included in each test sequence to provide for replication of data. Replication provided a means of evaluating the experimental error and determining the significance of the test variables. The stress range was chosen from 26 ksi to 50 ksi in increments of 6 ksi. It was felt that this range would correspond
to the sloping portion of the S-N curve. The experiment design provided an opportunity to evaluate the effects which maximum stress, minimum stress, stress range, and mean stress had on the cycle life.

The design of the random load test also provided for replication. Fourteen specimens were tested. The first two, one at each mean stress level, were run with the random load sequence drum set to the maximum load factor. The following four test sequences of three specimens each were then run at lower load factors. The results of the first two indicated the choice of load factors for the subsequent tests. A minimum of five complete program drum revolutions was desired prior to specimen failure.

The two mean values of stress used throughout the test program were 10 ksi and 16 ksi tension. While the selection of the actual numerical values was arbitrary, it was guided by data from the Michigan study which indicated minimum stresses of between 6 ksi and 9 ksi on the tension flanges of beams. These stresses were caused by dead load forces and are representative of the values usually encountered.

The development of the S-N curve by constant amplitude tests was done first. The curve was intended to serve as a basis for evaluating the random load test data. It was also hoped the curve could be correlated with data from the concurrent tests of welded steel beams having similar weld details at the flange splice.
4. TESTING MACHINE AND TEST PROCEDURE

4.1 Testing Machine

The fatigue tests were performed in an Amsler High Frequency Vibrophore Fatigue Testing machine shown in Fig. 13. It is a high speed resonance type testing machine which can be used for tension-tension, tension-compression, or compression-compression tests. The machine has a load capacity of 22,000 pounds and a maximum frequency of 18,000 cpm. The testing frequency is a function of the specimen, always coinciding with the natural frequency of the vibratory elements; namely, the driving mass, the specimen, the load measuring dynamometer, and the counter mass. The system is maintained at resonance by a driving magnet, controlled by a feedback system. The load amplitude is measured by the reflection of a beam of light from a mirror attached to the dynamometer onto a load scale. For static tests, when the load amplitude is zero, this light band reduces to a thin line on the load scale. The magnitude of the load is automatically maintained by means of a photocell controlling the optical-electrical feedback system. Fluctuations in line voltage have a negligible effect on the performance of the testing machine.

A small automatic control gear was especially developed by Amsler for programmed fatigue tests and is shown in Fig. 14. The device runs independently of the operation of the testing machine. A slowly rotating drum of non-conducting material contains slots which can hold
vertical strips of conducting material. A feeler attached to the load maintaining photocell makes contact with these strips as the drum rotates, activating an electric motor which moves the feeler towards the free end of the strip. As a consequence the height level of the feeler may be periodically changed which directly changes the amplitude of load. This mechanism allows a large variability in the choice of programs since the strips may be placed on the drum at any location and height. The rotation speed of the drum can be adjusted and generally is set so that one rotation corresponds to one period in a program test.

One feature of the vibrophore is the specimen fracture relay amplifier. With this apparatus it is possible to control the degree of fracture of the specimen. It may be adjusted to detect the initial crack formation and stop the machine at that time, or it may be set less sensitively and allow the crack to progress across the specimen. The fracture relay amplifier was set to correspond to the definition of failure for the particular program, either constant stress or variable stress.

4.2 Test Procedure

The method of gripping the specimens was that suggested by Amsler for flat bars. This was followed throughout the program despite changes in the geometry of the specimens. A set of standard vibrophore gripping heads were inserted into the machine and the specimen introduced. Each end of the specimen was then held between two wedges which were pre-loaded by strong screws so that both tensile and compressive stresses could be transferred to the specimen. Figure 15 shows a specimen installed
in the wedge grips. A torque wrench applied the pre-load in order to provide a uniform distribution of load. Double folded emery cloth was inserted around the specimen and between the wedges in order to increase the friction and reduce slip during the test.

The frequency used in the test program was about 185 cycles per second which conveniently equated to one million cycles every ninety minutes. The use of a high frequency reduces the danger of buckling the specimen when stress reversal occurs.

Alignment of the specimen in the machine was done with the aid of a scale, as suggested by the operating instructions. The vibrophore has the characteristic of showing misalignment by excessive lateral vibrations, the absence of the vibrations indicating satisfactory alignment. The alignment was checked by placing a metal pin sideways against the gripping head and observing any noticeable lifting of the object off the head.

4.3 Definition of Failure

After the specimen was installed in the machine, the loads corresponding to the desired stresses were calculated and the power needed to reach the load was applied. The automatic cycle counter, which adjusts to the operating frequency of the machine, was actuated and the specimen run until either failure or runout occurred. The value of runout was chosen as 100 million cycles. Failure for constant amplitude was defined as the point where the full stress amplitude could not be sustained by the specimen, and the specimen fracture relay amplifier was set accordingly.
A different criterion for failure was used for the variable amplitude tests. The specimen fracture relay amplifier was set to correspond to the least sensitive stress block in the program cycle. This was necessary to prevent the vibrophore from stopping during the program without any damage having occurred to the specimen. Because of the low sensitivity, the specimen was able to run after a sizeable crack was present even though the specimen could not sustain the full stress amplitude of the particular stress block. The crack continued to grow until the net area of the test section was reduced beyond that needed to carry the mean tensile load. At this point complete fracture occurred.
5. RESULTS AND DISCUSSION

5.1 Test Results

The constant amplitude test series included twenty-seven specimens, divided between mean stresses of 10 ksi and 16 ksi. The results of these tests are tabulated in Table 2 and plotted in Fig. 16 in the form of a S-N curve.

The variable amplitude test series included fourteen specimens, also divided between the two selected mean stress levels. The results of these tests are tabulated in Table 3 and plotted in Fig. 17 as a life function curve. This curve relates the number of cycles to failure against the maximum stress range in the program cycle.

Typical photographs of the fractured surfaces of some specimens are shown in Fig. 18. The patterns of failure are interesting but did not seem related in any consistent manner to the life, load factor, or mean stress of a given specimen. The patterns shown in Figs. 18a and 18b were most typical of failed specimens. The crack began as a small dark spot in a corner and emanated out as a circle. The failure surface became lighter as it progressed because the specimen changed its cross-section properties as the crack grew. This in turn changed the testing frequency. Figure 18c shows a specimen that cracked simultaneously along a notch and then grew uniformly across the specimen. Figure 18e shows the surface...
of the only specimen in the whole program which failed in fatigue at both sides of the specimen. Figure 18d and 18f are combinations of the first two failure patterns. This type failure began as a crack in a corner but instead of radiating out in a circular pattern, it tended to approximate the simultaneous edge crack in its propagation.

5.2 Discussion of Results

A statistical analysis of the data for the constant amplitude tests was made, and is summarized in Table 4. An analysis of variance was made to evaluate mean stress, maximum stress, minimum stress, and the stress range. A comparison of values generated indicated that the slope of the curves, or regression lines, was not significantly different regardless of the parameters chosen, but that the distance between the regression lines was significant for both maximum and minimum stresses for each mean stress. When the stress range was taken as the dependent variable there was no significant difference in variation due to the distance between the curves. This means that a single line could be used to represent all data. Hence, the mean stress level had no effect on the fatigue life of the specimen as the stress range accounted for all the variation.

The wide scatter of results was a source of concern. One factor contributing to the scatter was the welding effects on the specimen geometry. Very few of the specimens were perfectly flat before testing and most exhibited a slight bowing at the center in the butt weld region. As noted in Chapter 3, it was not considered desirable to cold-straighten any specimen since it would have changed the residual stress patterns.
This out-of-straightness contributed an eccentric bending moment which may have affected the fatigue life.

The analysis of the data from random loading was similar to that for the constant amplitude tests, except that only the mean stress levels and the maximum stress range within a particular load program were considered. There were three load programs used to develop this data, with load factors of 9.5, 8.0, and 7.0. There was no significant difference in either the slope of or the distance between regression lines, indicating that one regression line defined the life function curve and could be used to represent all data from the variable amplitude tests.

Finally, the relation between the life function curve and the S-N curve was analyzed. This is shown in Fig. 19. The analysis indicated that there was no statistical difference between the slope of the regression lines for each group of data. This result may be of significance, although an absolute judgment cannot be made with the limited data available. If it can be verified that the life function curve for any load spectrum has the same slope as the S-N curve for the same component under constant cycling and is merely translated from it, then it would be possible to establish the life function curve by a small number of tests. It would seem that a basis exists for future studies.

With the life function curve generated for a random spectrum of loading, the determination of the service life of a component is facilitated. The maximum stress range caused by the load spectrum is related directly to the curve. Thus, if the load spectrum is typical for the traffic pattern, and the maximum stress range within the spectrum
is known, the service life may be predicted. Some caution is required, however. The distinction between stress spectra and load spectra should be particularly noted since their relation will be dependent on the agreement between analytical and actual behavior of the structure, as well as the agreement between the anticipated (or recorded) and the actual load spectra.

Under uniform cyclic loading runout occurred at \( S_r = 32 \text{ ksi} \). Only one specimen, the first to be tested in the constant amplitude tests, was run at a level of \( S_r = 26 \text{ ksi} \). The experiment design required six to be tested at this level, but since runout was encountered at 32 ksi it was decided to test the remaining five specimens at another level. The level chosen was \( S_r = 35 \text{ ksi} \). This was between two levels where runout and failure were respectively occurring and was intended to locate more accurately the knee of the S-N curve.

Of the eleven load levels in each program cycle, more than half were at stresses below the endurance limit for the specimen geometry. For load factors of 9.5, 8.0, and 7.0, the program had six, seven, and eight load levels respectively below the endurance limit. The levels corresponded to 86%, 94%, and 96% respectively of the total number of stress cycles applied. By a linear cumulative damage rule, the specimen life should have been much longer than that produced by the tests, since stresses below the endurance limit are assumed to have no effect. However, the results show that there was a notable effect on the specimen life from these lower stresses. This is explained by considering the damage caused to the specimen by a high stress cycle. Although the damage is not sufficient to fail the specimen, it does weaken it. The
endurance limit is lowered and the specimen is more susceptible to the effect of the lower stress cycles. These contribute to the cumulative damage causing failure sooner than that predicted by a linear cumulative damage rule.
6. SUMMARY AND RECOMMENDATIONS

The results of the pilot test program to evaluate cumulative fatigue damage under random loading may be summarized as follows:

1. Data produced indicate that butt welded splices with the reinforcement removed should be no source of fatigue problems.

2. The fatigue life curve developed by the programmed loading had the same slope as the constant amplitude S-N curve for the specimen geometry tested, and was translated some distance from it.

3. The stress range was the most significant variable. It accounted for the variation in cycle life. Mean stress did not affect the cycle life.

The following recommendations concerning future tests are made:

1. Tests to determine the relationship between the slopes of the constant amplitude S-N curve and the fatigue life curves for other specimen geometries should be made. The object would be to prove (or disprove) conclusively whether the slopes of the curves are identical within significant levels.
2. The weld reinforcement should not be removed on some future group of specimens. This would produce a notch effect similar to actual conditions. The use of a machined notch presents problems in relating test results to actual conditions.
7. FIGURES AND TABLES
Fig. 1 Typical S-N Curve
Fig. 2 Stress Independent Damage - Cycle Ratio Relationship

Fig. 3 Stress Dependent Damage - Cycle Ratio Relationship
Fig. 4 Block (Programmed) Loading
Fig. 5 Histogram Used in Test Program

Fig. 6 Random Load Profile Used
Fig. 7 Fabrication Details
Fig. 8 Details of Beam Used in Main Tests

Fig. 9 Load-Elongation Curve (Typical)
Fig. 10 Specimen Geometry
Fig. 11 Photo of Shaping Jig
<table>
<thead>
<tr>
<th>$S_{\text{Mean}}$</th>
<th>10</th>
<th>A B C D E</th>
<th>16</th>
<th>F G H I J</th>
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<td>$S_{\text{Range}}$</td>
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<table>
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<td>$S_{\text{Max}}$</td>
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<td>+3 0 -3 -6 -9 -12 -15</td>
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Fig. 12 Experiment Design
Fig. 13 Photo of Vibrophore
Fig. 14  Photo of Selector Drum
Fig. 15  Photo of Gripped Specimen
Fig. 17 Variable Amplitude Data

Stress Range (ksi) vs. Number of Cycles to Failure.
Fig. 18 Photos of Several Fractured Specimens
Fig. 19 Composite Plot of All Data
### TABLE 1 MATERIALS TEST DATA

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Yield Strength (psi)</th>
<th>Tensile Strength (psi)</th>
<th>% Elongation in 8 inches (%)</th>
<th>% Reduction in Area (%)</th>
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<td>1</td>
<td>34,660</td>
<td>66,660</td>
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<td>36,240</td>
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Notes: 1. All specimens were cut longitudinally from the edge of the welded plate they represent, as shown in Fig. 7.

2. Specimens were tested to 1.5 percent strain at a strain rate of 0.025 inches per minute and thereafter at a strain rate of 0.200 inches per minute.
<table>
<thead>
<tr>
<th>Specimen</th>
<th>$S_{\text{mean}}$ (ksi)</th>
<th>$S_{\text{range}}$ (ksi)</th>
<th>N (kc)</th>
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<td>2-9</td>
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* No failure
### TABLE 3 SUMMARY OF DATA FOR VARIABLE AMPLITUDE TESTS

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### TABLE 4  RESULTS OF STATISTICAL ANALYSIS

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<th>Data Used</th>
<th>Coefficient of Correlation</th>
<th>Standard Error of Estimate</th>
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</thead>
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<tr>
<td>1. ( \log N = A + B \times S_r )</td>
<td>Constant amplitude, ( S_{\text{mean}} = 16 \text{ ksi} )</td>
<td>0.7030</td>
<td>0.4169</td>
</tr>
<tr>
<td>2. ( \log N = A + B \times S_r )</td>
<td>Constant amplitude, ( S_{\text{mean}} = 10 \text{ ksi} )</td>
<td>0.6851</td>
<td>0.2910</td>
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<tr>
<td>3. ( \log N = A + B \times S_r )</td>
<td>Constant amplitude, All data</td>
<td>0.6967</td>
<td>0.3310</td>
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<tr>
<td>4. ( \log N = A + B \times S_{\text{max}} )</td>
<td>Constant amplitude, All data</td>
<td>0.4752</td>
<td>0.4053</td>
</tr>
<tr>
<td>5. ( \log N = A + B \times S_{\text{min}} )</td>
<td>Constant amplitude, All data</td>
<td>0.4422</td>
<td>0.4131</td>
</tr>
<tr>
<td>6. ( \log N = A + B \times \log S_r )</td>
<td>Constant amplitude, All data</td>
<td>0.6558</td>
<td>0.3480</td>
</tr>
<tr>
<td>7. ( \log N = A + B \times S_r )</td>
<td>Variable amplitude, ( S_{\text{mean}} = 16 \text{ ksi} )</td>
<td>0.8120</td>
<td>0.3231</td>
</tr>
<tr>
<td>8. ( \log N = A + B \times S_r )</td>
<td>Variable amplitude, ( S_{\text{mean}} = 10 \text{ ksi} )</td>
<td>0.7228</td>
<td>0.3701</td>
</tr>
<tr>
<td>9. ( \log N = A + B \times S_r )</td>
<td>Variable amplitude, All data</td>
<td>0.7339</td>
<td>0.3534</td>
</tr>
</tbody>
</table>

Notes: Results considering the different mean stress levels are identical with those of Models 1 and 2.
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