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LOW-CONCENTRATION SAND-WATER MIXTURES

by Millard P. Robinson, Jr.¹, A.M. ASCE, and Walter H. Graf², M. ASCE

INTRODUCTION

This study deals with an important aspect of solid-liquid transport technology in pipelines: The critical deposit velocity, "V_c", which separates the "non-deposit" (deposit free) regime from the "deposit" regime. This velocity is sometimes also referred to as either the minimum transport velocity, the deposition velocity, or just the critical velocity.

The critical deposit velocity of low concentration mixtures (C ≤ 5%) is presently not well-defined, although it is needed for design application. Pressurized sewage collection lines, most often transporting low concentration loads, have been shown to be economically competitive with conventional means of sewage disposal but in need of additional design information. There exists an exhaustive list of Newtonian slurry transport applications which can be found in the literature. Condolios et al. (5,6, & 7), Shen et al. (27 & 28), Robinson et al. (24), Robinson (25), and Graf (17) report the most current state-of-the-art and economic significance of the critical deposit velocity determination.

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There exist generally two prerequisites in properly designing a solid-liquid transport system: (1) Consideration of criteria that will ensure operation in a region of stability, and thus, provide for safe, uninterrupted transport of solids, and (2) minimization of the power required to transport the solids, and optimization of system design parameters. The critical deposit velocity relates both of these requirements in designing a transport system which is both economic and safe to operate.

The present study continues the investigation of the critical deposit velocity problem through the use of a modified Froude number analysis. From a regression analysis of the Lehigh data, correlation of the tested parameters with different modified Froude numbers is evaluated, and equations quantifying the modified Froude number relationship are determined. The Lehigh data are subsequently compared with data reported in the literature, and the economic implications of applying the resulting Lehigh equations in systems design are discussed.

General Remarks on Solid-Liquid Mixture Flow. It is not within the scope of this paper to exhaustively present the general theory for flow of solid-liquid mixtures in pipelines. Shen et al. (27) and Graf (17) have presented comprehensive surveys on the current state-of-the-art of sediment transport in pipes, and the interested reader is referred to these texts. It should be noted, however, that transported solid-liquid mixtures may vary from suspensions in water of coal, sand, gravel, wood chips, chopped sugar cane, and ashes to slurries of sewage sludge, polymeric solutions, and concentrated suspensions.
Solids suspensions are transported either as "Non-Settling" (homogeneous) mixtures or as "Settling" (heterogeneous) mixtures. The distinction between these two classifications has been presented by Durand (9) and Govier et al. (15). The present study is concerned with a "Settling" mixture, which exhibits Newtonian flow characteristics and is analyzed as a two-phase flow phenomenon. The transport of "Settling" mixtures in pipes is qualitatively characterized by several different regimes of flow. Reference for an explanation of these different regimes is again made to Shen et al. (27) and Graf (17).

The variety of flow regimes and their associated solids concentration gradients is diagramatically presented in Fig. 1, which is a typical curve of mixture headloss versus mixture velocity. An important distinction is made between the "Deposit" transport regime and the "Non-Deposit" transport regime. Within the non-deposit regime, several modes of transport prevail: (1) Pseudohomogeneous flow, (2a) heterogeneous flow, and (2b) heterogeneous flow with saltation. Flow in the deposit regime, (4), is described by bed and dune form irregularities. Separating the deposit and the non-deposit flow regimes, (3), is the transition region identified by the critical deposit velocity, "$V_c$". One is reminded that these points of division between different flow regimes are somewhat arbitrary.

THE CRITICAL DEPOSIT VELOCITY, "$V_c$"

Definition and Significance. The transition between deposit and non-deposit flow regimes is identified by a "critical condition". In the present investigation, "critical condition" is taken as the velocity at which particles begin to settle from the flowing medium.
and form a stationary (non-moving) deposit along the invert of the pipe; this will be called the **critical deposit velocity**, \(V_C\).

At the "critical condition" a deposit-scour feedback mechanism transports solid particles in the form of a pulsating bed. Close to the pipe wall the solid particles are stationary. When this condition is observed, the critical deposit velocity is recorded. Above this layer of stationary particles the remainder of the bed is sliding. Other particles shove, roll, and saltate over the moving bed surface, and some will become completely suspended farther from the wall. The deposit of solids on the bottom of a pipe is a random phenomenon varying with local fluctuations of solid and liquid parameters. Within the same pump-pipe facility, duplication of results is not easily attainable.

The critical deposit velocity is sometimes referred to as the limit deposit velocity, by Durand (9) and Sinclair (29), the sediment limiting velocity, by Gibert (14), the minimum transport velocity, by Rose et al. (26), or the deposition velocity, by Wasp et al. (32). It is imperative that a clearly defined "critical condition" becomes a primary concern in every solid-liquid transport investigation.

When using data from other "critical condition" studies, one must be cautious of the following: (a) Some investigators, such as Blatch (2), Wilson (34), Bruce et al. (4), Thomas (31), Charles (8), and Shen et al. (28), define a minimum or economic velocity which corresponds to the minimum headloss required for transporting a certain concentration of solids. Use of this criterion is in accordance with how one wishes to define "critical condition". It
was found in the present and in other investigations that the critical deposit velocity is not in direct relationship with the minimum head-loss criterion. Implementation of the assumption that these two criteria are identical is good only for preliminary evaluation. (b) The critical deposit velocity, approached from the non-deposit regime, is most often different from the critical scour velocity. To scour a deposited bed requires usually a greater shear force, thus a higher flow velocity, than when the same bed is deposited. (c) Some studies define a transition velocity between saltating and sliding bedload transport; which is at times mistaken for the critical deposit velocity.

The critical deposit velocity is an important design criterion both for safe operation and for system economics, but it is often vaguely defined in reports of solid-liquid transport research. Due to a lack of good definition and reproduceability of results, it is suggested that a conservative critical deposit velocity be used [see also Bonnington (3)].

Previous Investigations. Interest in the "critical condition" of solid-liquid transport in pipes was initiated by Blatch (2) and continued by O'Brien et al. (21), Howard (18), and others. However, Wilson (34) developed the first relationship which quantitatively dealt with parameters related to the "critical condition", which is given in its final form as:

\[
V_c = \sqrt{\frac{3KCV_{ss}gD}{f}}
\]  

(1)
It should be noted that the flow velocity, $V_C$, at "critical condition" is defined here for minimum energy gradients. Nevertheless, the relationship given with Eq. (1) relates parameters which are of importance in the critical deposit velocity problem. These parameters are: $C$, the solids concentration; $v_{ss}$, the particle settling velocity; $D$, the pipe diameter; $f$, the friction factor indicating flow resistance; and a correlation parameter, $K$.

Durand (9) used as the lower limit of his heterogeneous flow relationship an equation defining the limit deposit velocity, $V_C$, of sand mixtures which separates the zones of the regimes with and without deposit on the pipe bottom, or:

$$V_C = F_L \sqrt{2gD (s_s - 1)}$$  \hspace{1cm} (2)

The parameter, $F_L$, known as a modified Froude number, varies with solids concentration, $C$, and particle diameter, $d$. This relationship was examined for the transport of uniformly graded material, and later, Durand et al. (10) report findings for non-uniform material.

Gibert (14) reported on and analyzed the extensive SOGREAH data to obtain best-fit curves for Froude number, $V_C/\sqrt{gD}$, plotted against solids concentration, $C$. Subsequent to the study of Gibert (14), Graf et al. (16) included the effect of relative density, given by $\sqrt{2 (s_s - 1)}$, as was similarly done by Durand (9) - and Gibert's best-fit curves were replotted and are given with Fig. 2. This figure shows the general trend of results to be remarkably invariant for sand and gravel of particle sizes $d \geq 0.37$ mm. The curve for this larger material can be thought of as being a maximum envelope of $F_L$-values. For finer materials,
in the range of \( d = 0.20 \text{ mm} \) and less, there are distinctive variations in the curves. Condolios et al. (6) report a figure similar to Fig. 2 but only include an envelope curve for graded and mixed sands of \( d \geq 0.44 \text{ mm} \). It is expected (1) that both Gibert (14) and Durand et al. (10) used the same set of SOGREAH data.

Gibert (14) also discussed a theoretical approach to the critical deposit velocity problem, considering the "critical conditions" of flow in a conduit irregardless of flow-through geometry, to be related through the Froude Law of similitude. A discussion of Gibert's analysis is found in Robinson et al. (24).

Sinclair (29) conducted tests on sand-water, iron-kerosene, and coal-water mixtures at concentrations up to 20\% flowing in 0.5-inch, 0.75-inch, and 1.00-inch pipe. Through a dimensional analysis of the variables expected to significantly influence the critical deposit velocity, Sinclair (29) arrives at an equation, such as:

\[
\frac{V_{\text{max}}}{\sqrt{g d_{s6} (s_s - 1)^{0.8}}} = f_s \left[ \frac{d_{s6}}{D} \right]^{0.8}
\]

where the modified Froude number is expressed with a solids particle diameter, \( d_{s6} \). He observed that the critical deposit velocity reaches a maximum between 5 and 20\% solids concentration, so that the effect of concentration could be eliminated by using \( V_{\text{max}} \) instead of \( V_C \). Sinclair (29) wrote Eq. (3), for \( d > 1.5 \text{ mm} \) (when \( C \) does not enter the problem), as:

\[
\frac{V_{\text{max}}}{\sqrt{2gD (s_s - 1)^{0.8}}} = 1.30
\]
This may be compared with Durand's results, similarly expressed by:

$$\frac{V_C}{\sqrt{2gD (s_s - 1)}} \approx 1.32$$  \hspace{1cm} (5)

For smaller particle sizes, Sinclair (29) examines the relevance of boundary layer theory to the problem, and suggests that particle diameter, $d_{ss}$, takes precedence over the pipe diameter, $D$, in their relative influence on the modified Froude number. It is within this smaller range of particle sizes that the present study is conducted.

Flow and particle Reynolds numbers have been investigated for their applicability as criterion in the critical deposit velocity problem. Spells (30), Charles (8), and studies by Cairns et al., as reported by Sinclair (29), correlate the Reynolds number with a modified Froude number relationship. Correlation in these studies, however, is related to the minimum energy gradient criterion.

A modified Froude number relationship apparently presents a rather good criterion for evaluation of solid-liquid mixture flow through pipes. Its relationship to other parameters significant in the critical deposit velocity problem will be re-examined in the present study, and experimental findings checked against the SOGREAH data.

A Modified Froude Number Analysis. When transporting a solid-liquid mixture through a closed conduit, one may expect the following variables to be of importance: (a) Flow Parameters - $V$, mixture flow velocity; $g$, gravitational acceleration; and $v_{ss}$, particle settling velocity. (b) Fluid Parameters - $\rho$, carrying fluid density; and $\nu$,}

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kinematic fluid viscosity. (c) Pipe Parameters - D, pipe diameter; ε, pipe roughness; and tan θ, pipe slope. (d) Sediment Parameters - ρₕ, solids particle density; d, mean particle diameter; ψₛ, particle shape factor (sphericity); (d₉₀/d₅₀), non-uniformity coefficient of grain distribution; and C, (moving) volumetric solids concentration.

Proper grouping of variables into dimensionless parameters was reported in Graf et al. (16) and is re-examined here:

\[ f \left[ \frac{V}{\sqrt{gD (s_s - 1)}}, \frac{VD}{\nu}, \frac{d}{D'}, \frac{\varepsilon}{D'}, \tan \theta, \frac{d_{90}}{d_{50}}, C \right] = 0 \]  

(6)

The relative density, \((s_s - 1)\), comes from \((\rho_s - \rho)/\rho\) where \(s_s = \rho_s/\rho\).

It is expected that the flow Reynolds number, \(VD/\nu\), does not play a significant role in this problem, and is thus omitted. Further, replacing the general flow velocity, \(V\), with the critical deposit velocity, \(V_C\), and considering the particle shape factor to be unity for natural quartz grains or already included in the adjustment of non-spherical particle sizes, Eq. (6) is rearranged and given by:

\[ f \left[ \frac{V_C}{\sqrt{2gD (s_s - 1)}}, \frac{d}{D'}, \frac{\varepsilon}{D'}, \tan \theta, \frac{d_{90}}{d_{50}}, C \right] = 0 \]  

(7)

Note that the flow Froude number, \(V/\sqrt{gD}\), and the relative density, \((s_s - 1)\), both given in Eq. (6), were combined in a densimetric or modified Froude number, \(V_C/\sqrt{2gD (s_s - 1)}\). Equation (7) is somewhat similar to relations proposed by Durand (9), Sinclair (29), and Barr et al. (1).

For a certain relative pipe material roughness, \(\varepsilon/D\), and solids grain size distribution, \(d_{90}/d_{50}\), the applicability of Eq. (7) will be tested in the form of:
The left side of Eq. (8) absorbs the \( \tan \theta \) argument, and the best trigonometric relationship was determined, after fitting data against several forms, to be:

\[
\frac{V_C}{\sqrt{2gD (s_s - 1)}} f_1 [\tan \theta] = f_2 \left[ \frac{d}{D}, C \right]
\]  

The left side of Eq. (8) is a modified Froude number. The form of this parameter, raising both \( D \) and \( (s_s - 1) \) to the \( 1/2 \) power, has been tested and shown to be a reliable criterion.

It is felt that without loss of generality, it may become frequently important to replace the relative particle to pipe diameter, \( d/D \), by the particle diameter, \( d \), itself. In this instance, the significance of \( D \) is considered to be wholly described in the Froude number.

In the subsequent discussion, data will be presented and compared in the way suggested with Fig. 2.

EXPERIMENTS

Facilities. A three-story, pressurized and self-contained solid-liquid transport system was constructed, modified from an open-tank recirculating system. The frequent use of Victaulic couplings hastened erection and provided flexibility throughout the pipe system.

The experimental facility consists of: (1) A vari-drive motor-pump assemblage, (2) an adequately flexible pipeline arrangement,
(3) a sediment feed and removal system, and (4) the necessary measuring and regulatory devices. Figure 3 schematically illustrates the general scale of the overall system; a detailed write-up is given in Robinson (25).

Vari-Drive Motor-Pump - The hydraulic horsepower was supplied from a vari-drive motor-pump assemblage, functioning as the heart of the system. The pump, furnished by Ellicott, is a single suction centrifugal type with cast bronze casing and impeller. The suction pipe is 5-1/2 inches I.D., discharge pipe is 4-1/2 inches I.D., and the impeller diameter is 13-5/8 inches O.D. During the operation of the pump, cooling water is added continuously to the seal on the motor side of the pump, also providing a lubricating interface. The drive unit is a Westinghouse (3 phase-60 cycle-125 Hp) "Magna Flow" motor and is regulated by a vari-drive control. The driving unit is of the integral type, is water cooled, and has an adjustable speed range from 100 to 2153 rpm. Along with the motor there is an operator's station, excitation unit, and a type 5L autostarter. The entire system operates on 208 volts AC.

Pumping efficiency and impeller capacity were not noticeably altered throughout the 18-month testing period.

Pipelines - From the pump, mixture flow is discharged through a 6-inch Foxboro Magnetic Flowmeter leading to a horizontal reach of 8-inch pipe. An 8-inch gate valve regulates pump discharge below flow-rates of 200 gpm. Often times the partially closed valve would cause difficulty in establishing stable flow conditions when critical flow-rates occurred in this lower flow range. The solid-liquid mixture is then lifted to the test-floor elevation in 6-inch pipe. Along the test length of approximately 40 ft, measurements are obtained, pipe slope is adjustable, and mixture flow phenomena are visually observed. A 4-inch
pipe was installed together with its Plexiglass observation section; subsequently, a 6-inch pipe and Plexiglass section were installed. A strobotac set at a high frequency response aided the observation of solids flowing through the Plexiglass section, such that an accurate description of flow regime was obtainable.

A "Loop System" follows which is employed as a device for simultaneously measuring mixture flowrate and solids concentration. Located atop the balcony-floor elevation between the 3-inch vertical pipe sections, commonly referred to as the "Riser" and Downcomer", is the main air-release for the system.

The flow, upon leaving the "Loop System", bypasses a closed 3-inch sediment flush valve and enters a 6-inch vertical pipe, where sediment is gravitationally fed when an increase in concentration is desired. Flow continues downward to where a 6-inch gate valve empties the system and a 2-inch pipeline connects the city water supply. The system pressure was maintained and water supply assured through use of a constant pressure control valve (A in Fig. 3) set at 20 psi on the 2-inch supply line. A 2-inch check valve (B in Fig. 3) prevented back-flow to the city supply under excessive system pressures. The circuit is completed with 5-1/2-inch pipe leading to the suction side of the pump.

Sediment Feed and Removal System - The sediment feeding apparatus underwent several adaptions until an adequate technique was successfully applied. Sand was supplied to a mixing chamber and gravitationally fed to the flowing medium. A sediment removal facility was employed as a time-saving technique for removing the solids or
undesirable foreign material from the system and preventing discharge of polluted water to the collection sump.

Measurement and Flow Regulation - The volumetric concentrations of solids and the mixture flowrates were determined from "Loop System" headloss readings. Arrows 1 and 2 on Fig. 3 indicate the respective locations of "Downcomer" and "Riser" pressure taps, both with 1.50 m (=59.1 in.) headloss lengths.

Loop readings were repeatedly checked against flow recordings from a Foxboro Magnetic Flowmeter by means of a Dynalog Receiver measuring accuracy to within 1 percent of full scale, throughout the scale (approximately ± 25 gpm). A Prandtl tube (C in Fig. 3) was employed to verify both the "Loop System" and flowmeter measurements of mixture velocities. A Pitot tube sediment-sampling device (D in Fig. 3) checked the "Loop System" indication of solids concentrations.

Two Venturimeters were investigated for their applicability as mixture flow measuring devices, the results of which are reported by Robinson et al. (24). A new 3 x 2 inch Venturimeter (E in Fig. 3) and an antiquated 4 x 2 inch device (F in Fig. 3) were tested and later used in checking flow conditions for this particular study.

The mixture headloss length for the test section was 3.60 m (=141.8 in.), as located at the arrows marked 3. At each pressure tap location, four holes, 3/32 inch in diameter, were drilled diagonally opposite about the circumference of the pipe. Brass fittings were assembled and connected with poly-flo tubing for transmitting the hydraulic pressure. Manometer fluids were selected according to the required range of readings. Most often air-water readings were adequate, however, a 2.95 fluid-water medium was needed at extreme flow
conditions. The 50.0 inch manometer scales were graduated in tenths of an inch, readings to a hundredth of an inch were estimated, and each reading was converted to feet of water column. Minor manometer fluctuations always existed, partly due to the uneven distribution of sediment concentration through the large system and also due to the effect that concentrated slugs of sediment had on the pump's capacity for maintaining a constant mixture flowrate.

Flowrates between 200 and 1000 gpm were regulated by a vari-drive rheostat control, located at the operator's station. The 8-inch discharge valve controlled lower range flowrates. Sediment feed rates were not rigorously monitored, except for an attempt to evenly distribute the sediment throughout the system.

Measuring Techniques. Clear-water calibration of the system was the initial course of action. The "Loop System" headloss readings were then evaluated and checked against flowmeter, Prandtl tube, and Pitot tube measurements.

Clear-Water Tests - Tests of clear-water flow were conducted to determine material roughness characteristics of the 3-inch "Loop System" pipes and the 4-inch and 6-inch diameter test lengths. Friction factors, $f$, were calculated from the Darcy-Weisbach equation, evaluating manometer headloss readings and Prandtl tube indication of velocities over the ranges of Reynolds number indicated in Table 1. Also summarized are the respective relative roughness values, $e/D$, and material values, $e$, determined from the Moody-Stanton Diagram of friction factors for commercial pipe. The friction factors for all three pipes fall in the transition regime. For further determination of friction factors at
any mixture flow Reynolds number, an explicit solution of the Colebrook-White equation was used. Evaluation of extensive "Loop System" data required this type of solution for $f$.

The Loop System - The "Loop System" developed by Einstein et al. (11) was used to simultaneously determine the mixture flowrate, $Q_m$, and the solid phase concentration, $C$. The device consists of two identical vertical pipe sections with opposite flow direction. Pressure head differences are obtained over these vertical pipe sections, namely, the "Riser" and the "Downcomer" section, and $Q_m$ and $C$ determined from relationships based on the sum and differences between the two readings.

To expedite the determination of $Q_m$ and $C$ from loop head loss readings obtained while testing, a program was developed and executed on the CDC 6400 Computer to print out data for plotting readoff charts.

**Description of Experiments.** A 4-inch and a 6-inch diameter pipe, each one having a different pipe roughness, as shown in Table 1, were evaluated. Each was tested separately at different slopes: Horizontal; a positive slope, $\tan \theta = +0.027$; and a negative slope, $\tan \theta = -0.060$ (geometrically speaking). Two types of solid particles, described in Table 2, were tested in various combinations with $D$ and $\tan \theta$ variables, as are listed in Table 3. The mean sand particle diameters and non-uniformity coefficients, $d_{50}$ and $d_{90}/d_{50}$, respectively, were determined from a standard sieving analysis and remained constant throughout the testing period. The settling velocities were found from a graph and equation presented after Budryck by Durand (9, p. 100). The specific weights of the solids, $s_p$, were provided by the material suppliers and are listed in Table 2.
Volumetric concentrations of $0.1% < C < 17\%$ were handled at flowrates ranging from $0.1 \text{ cfs (~50 gpm)} < Q_m < 1.8 \text{ cfs (~800 gpm)}$. The system temperature was recorded for each test run and sometimes varied from $60^\circ F < T^\circ < 100^\circ F$. The effect of temperature on the loop readings was accounted for.

For a particular test series, the solids are circulated in a nearly pseudohomogeneous flow condition which ensures uniform distribution of the particles throughout the system. Once conditions were stabilized, the flowrate, the moving solids concentration, and the test section headloss readings were recorded; these are compiled in Robinson (25). A qualitative description of the mixture flow, as observed through the Plexiglass section, is thereon commented. Flowrates are decreased to the heterogeneous flow regime, and eventually to a heavy bedload transport condition in which most particles are either rapidly sliding along the invert or saltating into the clear flow area of the pipe. Subsequent flowrate changes are more finely incremented. Lowering the flowrate to a velocity at which the bedload begins pulsating between deposit and non-deposit flow conditions, the sliding bed thickness builds and there exists no measurable transport of the bedload particles. In this study, this is the definition of the critical deposit velocity, $V_C$. The solids concentration corresponding to that particular $V_C$ is recorded just prior to the critical condition, when all particles are in transit.

EVALUATION OF EXPERIMENTAL DATA

Nine series of tests were conducted to determine the critical deposit velocities for varied concentrations of sand and plastic pellets
transported with water in a pipeline. Although a series of data was obtained for the transport of plastic pellets, pumping of the plastics created considerable problems. The system did not yield consistent results and these data were considered somewhat unreliable. Consequently, they are not reported here [see Robinson (25)]. Most data were recorded from sand-water tests in a horizontal pipe over a range of low solids concentration ($C < 7\%$). It is expected that within this lower range of solids concentration, both the particle diameter, $d$, and solids concentration, $C$, effect the critical deposit velocity value.

By testing various combinations of solids concentrations, $C$, particle diameter, $d$, specific weight of solids, $s_s$, pipe diameter, $D$, and pipe slope, $\tan \theta$, different critical deposit velocities were recorded and compared. All experimental data are first tabulated and then plotted as mixture headloss against mixture velocity in Robinson (25).

**Critical Deposit Velocities.** The critical deposit velocity data are summarized in Table 4 with indication of run numbers for each series of tests, the volumetric solids concentrations, the critical deposit velocities, and two modified Froude numbers. These two modified Froude numbers are defined in Table 4 and were computed for each critical deposit velocity. Froude number (I) is the modified form, after Durand (9), for critical deposit velocities in horizontal pipe flow; Froude number (II) is introduced to evaluate critical deposit velocities in sloping pipes as well.

From a preliminary study, plotting Froude numbers (I) and (II) against solids concentration, $C$, it was found that Froude
number (II) best correlates the data, including both horizontal and sloping flow values.

**Correlation of Data.** A regression analysis was made to correlate modified Froude number (II) with the following parameters; concentration, C; concentration, C, and particle diameter, d; and concentration, C, and relative particle size, d/D. The third correlation provided no additional information and is thus excluded from further discussion. The regression functions take two forms: (1) A least squares fit of modified Froude number, Fr, with concentration, C, written as:

\[
Fr = k_1 C^{k_2}
\]

where \( k_1 \) and \( k_2 \) are evaluated from logarithmic values of the data over three different particle size ranges, and (2) a least squares multiple regression, using Gaussian iteration to fit modified Froude number, Fr, to both concentration, C, and particle size, d, such as:

\[
Fr = k_3 C^{k_4} d^{k_5}
\]

The exponents, \( k_4 \) and \( k_5 \), and coefficient, \( k_3 \), are determined for the different sand particle ranges of data and also for the total range of sand-water data. An explanation of the multiple regression analysis and a statistical interpretation of the resulting equations are given in Robinson (25).

Two regression equations are found to fit the Lehigh data:

(1) Assuming solids concentration, C, to be the only important independent variable, the best-fit equation is given as:

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The coefficient of correlation is 0.870. (2) Including the influence of particle diameter, d, the following equation was developed:

\[
Fr = \frac{V_c}{\sqrt{2gd (s - 1)}} (1 - \tan \theta) = 0.928 C^{0.106} d^{0.056} 
\]

where the particle diameter, d, is in mm. The coefficient of correlation is 0.877. Note that the value of exponent \( k_2 = 0.016 \), given with Eq. (11), is very close to exponent \( k_4 = 0.105 \), given with Eq. (12). Further, coefficient \( k_3 = 0.928 \) in Eq. (12) differs only slightly from coefficient \( k_1 = 0.901 \) in Eq. (11). This similarity between the coefficients and exponents in Eqs. (11) and (12) is due to the almost negligible effect of particle diameter, d. Equations (11) and (12) are compared graphically in Fig. 4, where Eq. (12) is fit with the two different particle size data.

From the above discussion, it seems reasonable that the use of Eq. (11) be recommended.

Relative Influence of Tested Parameters - Needless to say, not all ranges of the parameters, D, d, s, C, tan θ, d_90/d_50, and ε/D, have been completely investigated and never will be. However, the resulting regression equations, Eqs. (9) and (10), offer insight to the relative influence of some of the tested parameters on the critical deposit velocity.

The influence of solids concentration, C, on the critical deposit velocity is found in this study to be of primary significance,
particularly within a low-concentration range of \( C < 7\% \). For concentrations above 5 to 10\%, both Sinclair (29) and Wilson (33) find that critical deposit velocities decrease with concentration. A similar observation was made in the present study when concentrations exceeded 5\%.

The particle diameter, \( d \), has no direct effect on the critical deposit velocity value within the range of particle diameters tested in the present study, \( 0.45 < d < 0.88 \text{ mm} \). However, with suspensions of fine particles in the range \( d < 20 \text{ mm} \), it is expected that solids settling is sufficiently delayed to decrease the critical deposit velocity. This is reported by Worster et al. (35) and Gibert (14).

Sloping of the pipe, \( \tan \theta \), noticeably altered the location of critical deposit. Upward sloped flow hastened settling, yielding a higher critical deposit velocity than would be expected with similar flow conditions in a horizontal pipe. The opposite was the case for downward sloped flow. This influence is explained by the effect that slope has on the tractive shear force developed in the sliding bed.

While the Lehigh data provide insufficient evidence that relative density, \( s_s^{-1} \), expressed as \( (s_s^{-1})^{0.5} \), is proportional to the critical deposit velocity, other studies have made this verification. Furthermore, the influence of both the grain size distribution, \( d_{90} /d_{50} \), and the relative material roughness, \( \varepsilon/D \), was not determined due to the limited range over which these parameters were tested.

**Comparison with Other Data.** Particularly important in the present study is the applicability of the modified Froude number relationship, given with Eq. (8), for low-concentration mixtures, \( C < 7\% \). The strength of the Lehigh data is in the range with \( 0.10 < C < 2.0\% \).
The low-concentration data are mainly responsible for the final form of the modified Froude number relationship, as given with Eqs. (11) and (12). In what follows we shall try to investigate as to how other experimental data compare with the present findings.

Many researchers have reported on sand-water mixture studies, but from all of these, only the studies by Gibert (14), Führbötter (13), Sinclair (29), and Durand, Smith, and Yotsurura, as reported by Wasp et al. (32), rendered useful data for the present investigation. The ranges of parameters investigated in these studies are listed in Table 5, and the data are plotted in Fig. 5 for comparison with the Lehigh sand-water data given with:

\[ Fr = 0.901 C^{0.106} \]  

Figure 5 together with the Lehigh sand-water data, represented with Eq. (11), suggest the following trends in the range where \( C < 5\% \):

1. The critical deposit velocity, \( V_C \), increases with solids concentration, \( C \); the increase becomes less evident as the concentration rises to 5%.
2. For particle sizes, \( d \geq 0.37 \text{ mm} \), the critical deposit velocity remains practically unchanged with increase in \( d \). The Lehigh data exhibit this trend showing particularly good agreement with the other data, and will give conservative design values.
3. For particle sizes smaller than \( d = 0.37 \text{ mm} \), the critical deposit velocity, \( V_C \), decreases with decreasing \( d \). It is expected that this decrease in \( V_C \) levels off for very fine particles, but the data reported give inconclusive verification of this.

Neither particle size distribution nor the pipe material roughness were considered to be of importance in this comparison.
CONCLUSIONS

The critical deposit velocity, $V_C$, tested in the form of a modified Froude number, is correlated with other parameters, which are significant in the solid-liquid transport problem, over the following ranges: $0.01 \leq C \leq 7.00\%$, $0.45 \leq d \leq 0.88$ mm, $4.00 \leq D \leq 6.00$ in., $-0.060 \leq \tan \theta \leq 0.027$, $1.07 \leq d_{90}/d_{50} \leq 1.21$, and $0.00009 \leq \epsilon/D \leq 0.00032$.

From a dimensional analysis of these parameters, a modified Froude number relationship is developed, as given with Eq. (8). The relationship is tested for sand-water transport; these data exhibit the following: (1) Agreement with the Gibert (14) curves for particle diameters, $d \geq 0.37$ mm; (2) the increase in critical deposit velocity, $V_C$, becomes less evident as solids concentration, $C$, rises to 5%; (3) for particle sizes, $d \geq 0.37$ mm, the critical deposit velocity remains practically unchanged with increase in $d$; and (4) the critical deposit velocity is higher than the velocity associated with the minimum headloss at low concentrations; however, the opposite is true for $C > 5\%$.

A regression analysis, made to correlate the Lehigh data, shows that the modified Froude number is highly dependent on concentration, $C$, affected by particle diameters, $d$, if $d \leq 0.37$ mm, and hardly influenced by relative particle size, $d/D$. The regression equation which best fits the data and is in reasonable agreement with data from other sand-water studies, is given with:

$$\frac{V_C}{\sqrt{2gD (s_s^{-1})}} [1 - \tan \theta] = 0.901 C^{0.108}$$

The Lehigh critical deposit velocity equations give conservative values and are presently the only relations available for
predicting critical deposit velocities for low-concentration sand-water mixtures. It is recommended that either Eq. (11) or Eq. (12) be used as a critical deposit velocity design criterion, certainly within the range of parameters tested in the present study, and cautiously in ranges of parameters extending outside of the tested bounds.

ACKNOWLEDGEMENTS

Sincerest thanks are extended to Oner Yuce1 for his unsullied partnership throughout the study. The research program was partially sponsored by the Federal Water Quality Office of the U.S.D.I. (WP-01478; 11020 EKD) and by Lehigh's Office of Research.

APPENDIX I - REFERENCES


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Pipe Specification | ε/D | ε (ft) | Reynolds Nos.
---|---|---|---
Loop System:  
3 in. commercial steel | 0.00004 | 0.00001 | 2.48 x 10^6 to 4.77 x 10^6
Test Length:  
4 in. galvanized | 0.00009 | 0.00003 | 1.97 x 10^5 to 3.58 x 10^5  
6 in. black steel | 0.00032 | 0.00016 | 1.39 x 10^6 to 3.76 x 10^5

Table 1: Relative Roughness and Material Roughness Values for the Three Pipe Sizes

<table>
<thead>
<tr>
<th>Solids Material</th>
<th>d_{so} (mm)</th>
<th>d_{go}/d_{so}</th>
<th>s_s</th>
<th>v_{ss} (ft/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quartz Sand:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#0</td>
<td>0.88</td>
<td>1.21</td>
<td>2.65</td>
<td>0.312</td>
</tr>
<tr>
<td>#00</td>
<td>0.45</td>
<td>1.07</td>
<td>2.65</td>
<td>0.189</td>
</tr>
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</table>

Table 2: Solid Particles Specification

| Pipe Diameter, D in.  
(Material Roughness, ε ft) | Mean Particle Diameter, d_{50} (Specific-Gravity, s_s) | Pipe Slope, tan θ | Symbol (Fig. 4) (Tab. 4) |
<table>
<thead>
<tr>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>4 (0.00003)</td>
<td>6 (0.00016)</td>
<td>0.88 (2.65)</td>
<td>0.45 (2.65)</td>
</tr>
</tbody>
</table>

Table 3: Tested Combinations of Pipe Diameter, Solid's Particle Diameter, and Slope
\[
\text{Fr (I)} = \frac{V_C}{\sqrt{2gD (s_n-1)}} \\
\text{Fr (II)} = \frac{V_C}{\sqrt{2gD (s_n-1)}} [1 - \tan \theta]
\]

<table>
<thead>
<tr>
<th>Run</th>
<th>Volumetric Solids Concentration (percent)</th>
<th>Critical Deposit Velocity (ft/sec)</th>
<th>Modified Froude Number</th>
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</thead>
<tbody>
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<td>(II)</td>
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Table 4: Critical Deposit Velocity Data

<table>
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<tr>
<th>Run</th>
<th>Volumetric Solids Deposit Concentration (percent)</th>
<th>Critical Deposit Velocity (ft/sec)</th>
<th>Modified Froude Number</th>
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</thead>
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Table 4: Continued
### Table 5: Range of Parameters of the Data Reported by Other Investigators for Sand/Water Mixtures; Data are Plotted in Fig. 5.

<table>
<thead>
<tr>
<th>Investigator</th>
<th>Mixture Veloc.</th>
<th>Mixture Head Loss</th>
<th>Deposit Regime</th>
<th>Non-Deposit Regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>Durand (1952)*</td>
<td>0.44, 2.04</td>
<td>5.90 in.</td>
<td>up to 15%</td>
<td>Extensive range of parameters tested</td>
</tr>
<tr>
<td>Smith (1955)*</td>
<td>0.18</td>
<td>3.00 in.</td>
<td>up to 26%</td>
<td>VC obtained from VC vs. C plot</td>
</tr>
<tr>
<td>Gibert (14)</td>
<td>0.27, 0.37</td>
<td>0.20</td>
<td>40.2 to 150.0 mm</td>
<td>up to 15%</td>
</tr>
<tr>
<td>Führbötter (13)</td>
<td>0.27, 0.53, 0.88</td>
<td>0.30 mm</td>
<td>up to 25%</td>
<td>2.64 sand/water</td>
</tr>
<tr>
<td>Yotsukura (1951)*</td>
<td>0.23, 0.59, 1.15</td>
<td>4.25 in.</td>
<td>up to 25%</td>
<td>2.65 sand/water</td>
</tr>
<tr>
<td>Sinclair (29)</td>
<td>0.35, 0.68</td>
<td>0.50, 0.75, 1.00 in.</td>
<td>up to 20%</td>
<td>2.61 sand/water</td>
</tr>
</tbody>
</table>

*Reported in Wasp et al. (32)

---

Fig. 1: Regimes of Flow

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Fig. 3: Solid-Liquid Transport Test System

\[ \frac{V_C}{\sqrt{2gD \left( \frac{d}{D} \right)^{1.5}}} \]

Adopted from Gibert (14)

- Sand of \( d \geq 0.37 \) mm
- Sand of \( d = 0.20 \) mm

Fig. 2: Modified Froude Number versus Concentration; Particle Diameter as Parameter

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Fig. 4: Best-Fit Equations for Lehigh's Sand-Water Data Only; Modified Froude Number versus Concentration, Particle Diameter as Parameter

\[
\frac{v_c}{\sqrt{2gD (s^-1)}} [1 - \tan \theta]
\]

\[Fr = 0.901 C\]  
\[d = 0.88 \text{ mm}\]  
\[d = 0.45 \text{ mm}\]

Fig. 5: Modified Froude Number versus Solids Concentration, Particle Diameter as Parameter (Data from Sand-Water Mixture Studies)