Axial Behavior of Damaged Tubular Columns

by

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Abstract

A simple "engineering" method was formulated for computing the axial load vs. axial shortening relationship of pin-ended tubular members damaged by a dent and/or out-of-straightness. The method predicts the pre- and post-ultimate load-shortening response, and can be used in analyzing the strength and behavior of offshore platform frames containing such damaged members. The method was developed from a parametric study and regression analysis of a database containing load-shortening data from published tests results on small-scale damaged specimens (31 load-deformation curves) and data generated from a finite element analysis (56 load-deformation curves).

The effects of geometric nonlinearity and elasto-plastic material property were included in the finite element analysis of the pre- and post-ultimate response of damaged tubular columns. Prior to generating data for the parametric study and regression analysis, the finite element model was verified by comparing calculated (finite element) load-shortening responses with small-scale test data.

In order to develop a regression model to be used as a basis for the simplified engineering method, a parametric study was conducted to determine the influence of each independent variable on the axial behavior of damaged members. The object of the study was to select the shortest suitable approximating (coordinate) function for each independent variable. The variables considered were: column slenderness, D/t ratio, dent depth to diameter ratio, out-of-straightness, yield stress, and axial shortening. The regression analysis of the load-shortening relationships in the database resulted in a set of 96 constants which is reduced to a four-term approximating function for the load-shortening response, once specific values are given for the member geometry, material and damage. The procedure is illustrated with some examples and a comparison with test results.

The method is valid for member geometries and material properties typically found in fixed offshore platforms with the limitation that relatively thick-walled members should have sustained significant damage. This limitation is based on the exclusion of members which exhibit a sharply peaked load-deformation relationship.
1. Introduction

The design of offshore structures must include considerations of strength, stability, and serviceability while providing for safe and reliable resistance to applied loads over the life of the structure. From monitoring and inspection of in-service platforms, it has become increasingly apparent that these basic requirements must be met for the structure in a deteriorated condition, i.e., the structure must be designed with some degree of damage tolerance. "Damage-tolerance may especially be crucial for deep-water fixed platforms, where inspection and maintenance of the deeply submerged parts of the structure may be difficult, if not impossible." [2] Minimum requirements are derived from consideration of the consequences of structural failure: loss of human life, of the structure, and/or environmental pollution. These dictate that operational and environmental loads must not result in collapse or progressive failure of the structure, particularly as a result of slight or undetected damage. Furthermore, the economy of operation/maintenance of a platform requires that a structure should have the capacity to withstand some minimal damage without the need for costly repairs.

Although some degree of damage tolerance is implicit in any redundant structure, quantification of the residual strength of a damaged member(s) and of the whole structure is needed for a rational approach to efficient, cost-effective design and maintenance. Consequently, the research described in this report was directed at the assessment of the effect of accidental damage on member behavior, whether anticipated as in the design process or real as a result of accidental overload on an in-service platform.

1.1 Problem Definition

Typically the effect of dents and/or out-of-straightness of a tubular member results in a reduction of the stiffness and/or capacity of the member and this may significantly affect the strength and/or serviceability of the structure. The remaining residual strength of the structure is dependent on the pre- and post-ultimate behavior of the damaged member since it is likely that service loads will lead to non-linear response of the damaged member resulting in a redistribution of forces in the structure. Therefore, the response of the damaged member must be known or estimated in order to assess the effect of damage on the structure.
In general, prediction of the pre- and post-ultimate load-shortening response of damaged tubular columns requires a shell analysis of the member including the effects of large deformations and material nonlinearity. However, analysis of this type is impractical even if possible with state-of-the-art finite element programs. Consequently, there is a need for a simplified yet reasonably accurate "engineering method" for predicting the behavior of damaged members.

1.2 Previous Research

In one of the first reported efforts to quantify the effects of damage in tubular members, a parametric study was made using finite element analysis of initially crooked tubular columns and experiments on small-scale specimens. [8] However, this was a beam-column analysis in which distortion of the cross section was not considered. In a later attempt to include the effect of dent damage, it was suggested that the effect of dents may be included in the analysis, not by modeling the geometry of the dent, but by retaining the circular cross section of the member and modifying the stress-strain relationship for fibers in the dent affected area. [10] A reduction or "knock-down" factor applied to the modulus of elasticity and yield stress was suggested based on empirical data from small-scale tests.

In a series of papers, Taby presented an analytical method which estimates the load deformation response of dented and/or initially crooked, simply supported columns. [11, 12, 14, 15] This approach is based on a simplified physical model for which the governing relations have been fit to empirical data from 109 small-scale tests. The computer program DENT A is based on this method. [16]

Ellinas quantified the effect of dent-damage with a lower bound prediction of the ultimate strength based on first yielding of a simplified physical model similar to Taby's. [1]

In their experimental work Taby and Smith carried out a number of tests on damaged (dented and crooked) small-scale specimens made from drawn tubing or cold-rolled plate. [8, 10, 11, 14, 15] From the tests of two large-scale members removed from an offshore platform retired from service and of two comparable small-scale specimens, Smith concluded that small-scale tests were adequate to predict the behavior of full-size damaged members. [9] However, the results of these tests showed that the small-scale specimens un-
derestimated the ultimate strength by as much as 15% and the post-ultimate strength by as much as 30%. Smith attributed the discrepancy in the post-ultimate range to the effect of different load vs. displacement control in the tests for the large- and small-scale specimens.

1.3 Need for Research

While it is possible to predict, with reasonable accuracy, the response of a damaged member by a finite element analysis using shell elements and including material non-linearity and large deformations, such an analysis is impractical in terms of cost, computer resources needed, and the time required to formulate a reliable model. Furthermore, analysis of an entire platform or even a sub-frame containing a damaged member with such a model would be even more impractical. Even if a non-linear finite element analysis could be performed efficiently, the results must be independently verified.

Of the approaches discussed above, only Taby's method (DENT A) includes the effect of cross-sectional distortion and estimates the load-shortening response in the pre- and post-ultimate ranges. However, DENT A is based on a simplified beam-column model that was "tuned" to experimental results from small-scale tests. The application of Smith's method may be limited to members with relatively low D/t ratios since it has been reported that the effect of cross-sectional deformation (amplification of dent-depth) is significant for D/t ratios as low as 40. [15] The research reported here represents a phenomenological approach based on experimental data and analytical data from a general shell finite element analysis.

1.4 Work Performed

The objective of this research was to produce a relatively simple yet reasonably accurate engineering method for predicting axial load as a function of axial shortening of damaged, pin-ended, tubular steel columns. The basic approach to the problem centers on the collection of experimental and generation of analytical column load-shortening responses followed by a parametric study and a regression analysis of the data, and formulation of a simplified method of analysis. This approach has been used successfully in the past to predict the pre- and post-ultimate response to in-plane loading of plates and stiffened plates and the load-indentation response of circular tubes. [3, 4, 5, 6]

The resulting simplified approximation of load-shortening behavior is a matrix of 96
coefficients which, for a given column with known geometry, damage and material, are reduced to a 4-term function for approximating the load-shortening behavior. The method can readily be implemented as a subroutine or a stand-alone program.

The work reported here includes the development of a database, the development of an analytical (finite element) model for generating additional load-shortening data, and the development of a regression model. Application of the resulting engineering method is also demonstrated.

The basic approach to the problem consisted of the following steps:
1. Collection and generation of experimental and analytical data on the load-shortening behavior of damaged columns.
   a. Literature search for published experimental data.
   b. Finite element analysis to generate additional data.
      i. Development of a model and verification with experimental results.
      ii. Generation of analytical data.
   a. Parametric study of the data and selection of suitable approximation functions for the regression model.
   b. Regression analysis and improvement of the model (an iterative procedure).

Details of these tasks are described in the following chapters.
2. Development of the Database

The development of an approximate method for predicting the load-shortening response of damaged tubular columns was based on a parametric study and a regression analysis of analytical and experimental load-shortening relationships contained in a database. Experimental data was collected from available literature, and analytical data was generated by a finite element analysis (See Chap. 3).

2.1 Database Management

A relational database was needed for managing the database in order to avoid storing all related data (geometry and material properties of the column) for each point of every load-shortening curve. An efficient system for storing relatively large amounts of data and the ability to program custom tailored manipulations of the stored data was also needed. The capability to produce graphical displays of data was desired to aid in the parametric study. After reviewing several mainframe and personal computer software packages, the SAS* software system was selected because of its capabilities to perform all these tasks. The SAS software was available on a Digital Equipment Corporation VAX 8530.

2.2 Included Data

For each case, the database included the basic data on column geometry (diameter, thickness, length, dent-depth, initial out-of-straightness), material yield strength, and pairs of load-shortening coordinates. At least ten points were included for each column, typically thirteen to eighteen depending on the length of the post-ultimate load-shortening curve. Related parameters (D/t, δ/L, λ, etc.) were readily calculated from the raw data for the parametric study and regression analysis through custom written SAS programs.

Published experimental load-shortening relationships were available from the empirical work done in this area by other researchers. Principally, published data was taken from Smith [8, 9, 10] and Taby [12, 13, 14, 15]. Although, in Taby's research, over 100 tubes

*SAS Institute, Cary, N.C.
had been tested with a variety of end conditions, only a representative sampling of load-
shortening curves were published, all of which (pin-ended tests) were included in the
database.

Analytical data generated by finite element analysis (See Chap. 3) were also incor-
porated into the database to expand it over a broader range of column geometry and damage. The number of load-shortening curves included in the database from each source and the
range of geometrical parameters, yield strength and damage are shown in Table 1.

Load-shortening relationships for columns with relatively minor damage and high D/t
ratios presented some difficulty in the simulation process and, at present, were excluded
from the regression analysis. The curves included and their source are listed in Table 2.
3. Generation of Analytical Data

In order to effectively study the behavior of damaged tubular columns, additional data were needed to supplement the limited number of published experimental load-shortening curves. Due to the complexity of the behavior of a damaged tubular member and the need to generate data on the pre- and post-ultimate response including large deformations and material nonlinearity, a finite element analysis was used. The 1984 version of the finite element software ADINA** (ADINA 84) was selected because of its capabilities for non-linear analysis and automatic load incrementation. The analysis was performed on a Control Data Corporation Cyber 850 Model 180 running the NOS/VE operating system.

3.1 Idealized Geometry of a Damaged Column

In the development of the finite element model for generating data on the load-shortening response, certain assumptions were made about the location and geometry of the dent damage and the initial out-of-straightness. The idealized geometry permitted description of the damage in terms of only two parameters: dent depth d, and the magnitude of the initial crookedness (out-of-straightness) δ.

A damaged tubular member with initial crookedness and a dent at mid-length has two planes of symmetry, one longitudinal and one at the dent perpendicular to the longitudinal axis, assuming that the dented cross section is symmetrical about the longitudinal plane of symmetry. (This is a reasonable assumption if the dent and out-of-straightness are caused by the same accident.) Although dent damage and initial crookedness may have much more general forms, the longitudinal location of the dent and variations in the shape of the dent and crookedness have been found by other researchers to have little effect on the behavior. For example, Smith concluded that “Comparison of test results ... indicates radical variations in the position of a dent and associated bending damage do not substantially change the damage effect” [10], and “Test results also support previous theoretical findings that loss of strength is insensitive to the shape and location of dents and to the shape of bending

**ADINA R & D Inc., Watertown, MA
deformation”. [10] Further verification comes from Taby, who concluded “The sensitivity to dent shape and location is, however, insignificant ...”. [13] Consequently, a single damage model (assumed dent geometry, location and shape of initial crookedness) was used in the analysis.

The longitudinal axis defining the initial crookedness of the damaged member was assumed to be of sinusoidal shape. Thus, for the origin at midlength, it is given by

\[ z = \delta \cos \frac{\pi x}{L} \]  

where \( x \) is the longitudinal distance from mid-length of the column, \( \delta \) is the magnitude of maximum initial lateral deflection, and \( z \) is the lateral deflection of the longitudinal axis of the tube as shown in Fig. 2. Thus, the initial crookedness is defined by a single parameter, \( \delta \).

The dent was assumed to be a sharp “vee” as if produced by a “knife-edge” loading perpendicular to the longitudinal axis. The geometry of the dent was defined in terms of the dent depth, \( d \), as shown in Fig. 3. The longitudinal profile of the dent, \( \zeta \), and the length of the dent, \( l_d \), were taken from an analytically derived relationship for a tube supported along its length with no end restraint and subjected to a “knife-edge” lateral loading. [17] The dent depth as a function of distance from its center at mid-length of the column ( \( \zeta = \zeta(x) \) ) is given by

\[ \zeta = d \left( 1 - \frac{2x}{l_d} \right)^2 \]  

where

\[ l_d = D \sqrt{\frac{\pi d}{4t}} \]  

with \( D = \) mean (mid-thickness) diameter of the tube. The expression for \( l_d \) (Eq. 3) is in good agreement with dent profiles published by Smith. [10]

The cross-sectional geometry of the dent is based on empirical observations and is composed of a flattened and a curved segment. The curved segment is defined by the radius which varies linearly as a function of the angle \( \phi \) as shown in Fig. 3. The radius increases from \( R \) (the radius of the undamaged tube) to \( R_d \) at the intersection of the curved and flattened segments. \( R_d \) is determined from the requirement that the circumferential length of the dented and circular cross sections must be equal.
\[ \pi R = \frac{R + R_d}{2} \phi + R_d \sin \phi \] (4)

This equation is readily transformed into

\[ \pi R = \frac{R + R_d}{2} \cos^{-1} \left( \frac{d - R}{R_d} \right) + \sqrt{R_d^2 - (R - d)^2} \] (5)

which gives \( R_d \), although not explicitly, in terms of \( R \) and \( d \). The discontinuous slope at the intersection of the flattened and curved segments, as shown in Fig. 3, corresponds to the plastic hinge formed during indentation.

### 3.2 Basic Concepts of the Finite Element Model

The basic concepts employed in the finite element modeling of the damaged column are illustrated in Fig. 1. Because of double symmetry of the problem (one longitudinal plane of symmetry and one plane of symmetry at mid-length), it was only necessary to model one-quarter of the tube. In addition to the symmetry of the problem, the model reflects the behavior of the column. For pin-ended boundary conditions, a portion of the column some distance away from the dent behaves as an elastic beam-column with no distortion of the cross section. The region near the dent is subjected to bending of the tube wall leading to distortion of the cross section and plastic deformations. These considerations are reflected in the model (See Fig. 1) where the region near and including the dent is modeled with shell elements while the remainder of the column with beam-column (line) elements.

The length of the portion of the column modeled with shell elements was taken to be one-half the length of the dent, \( l_d \), as determined from Eq. 3 plus twice the diameter. An elastic-plastic material model was used for the shell elements. The remainder of the column was modeled with beam (beam-column) elements with large displacement formulation and linearly elastic material. A rectangular cross section was used for the beam elements with the area and moment of inertia equal to those of the undamaged (circular) cross section.

### 3.3 Finite Element Discretization

The finite element model used to generate load-shortening data was one of three models developed. The validity of each model was assessed by analyzing test specimens for which published experimental data were available and comparing the calculated and empiri-
cal responses. The models differed primarily in the pattern of discretization and the type and number of shell elements used in modeling the portion of the column near the dent.

The first model, Model 1, was found to be inadequate for predicting the response of columns with D/t ratios greater than 60. Model 2 (the second in the series) was much more accurate but was extremely costly in terms of CPU time. Model 3 exhibited good correlation with experimental data and was much more economical than Model 2 and therefore was selected for generating data included in the database.

The discretization of the portion of the tube modeled with shell elements for Model 1 is shown in Fig. 4. Eighteen 16-node "quadrilateral" and four 9-node "triangular" isoparametric elements were used. Reasonable results were obtained with this model when compared to experimental data for tubes with a D/t ratio less than 60. For D/t greater than 60, the model significantly overestimated the strength with the error increasing approximately linearly with the D/t ratio. This was due to the inability of the model to simulate the amplification of the dent that coincides with attainment of the ultimate load for tubes with relatively large D/t.

In order to improve the predictions of Model 1, a finer discretization of the dented area was used for Model 2, and 16-node elements were incorporated as shown in Fig. 5. Model 2 resulted in much improved correlation with experimental data for tubes with larger D/t. However, the model was extremely expensive to use in terms of CPU time. This led to the development of Model 3.

The discretization of Model 3 is shown in Fig. 6 and it resulted in a much more economical computer usage than Model 2 while producing reasonable agreement with experimental data. Consequently, Model 3 was selected for generating data for the database. A comparison of the predicted responses from the three models, compared to experimental data, is shown in Fig. 7. The experimental data is from test specimen 1CDC with D/t=91, \( \lambda=0.65 \), D=0.141% and \( \delta/L=0.0024 \). (Ref. [14])

Model 3 had 32 shell elements, 4 beam elements, 268 nodes, and 1162 degrees of freedom. The shell elements were either 16-node elements or a variation of these needed for transition from the finer mesh at the dent to a coarser mesh outside the dent. Boundary
conditions were imposed on the nodes at the edges of the shell elements to reflect the symmetry of the model as indicated in Fig. 8. Nodes lying in a plane of symmetry are allowed to displace only within and have vectorial rotation perpendicular to the plane of symmetry. At the junction of the beam and shell elements, the displacements of the shell element nodes were constrained to the beam element node so that a section through the model remained plane (See Fig. 8). Lateral displacements of the shell element nodes and of the end node of the beam element at the juncture were constrained to be equal. Thus, due to symmetry, the beam elements were planar and had only lateral displacement and bending rotation.

The loading was applied to the model by imposing a displacement in the longitudinal direction to the end node of the beam element as shown in Fig. 8.

3.4 Verification of Calculated Response

The load-shortening curves calculated with Model 3 are shown compared to experimental data in Figs. 10 through 13 as well as in Fig. 7 which shows the calculated results from Models 1, 2 and 3 for Specimen 1CDC from Ref. [14]. The correlation with experimental data shown in the figures is representative for all others. In general, the finite element analysis (Model 3) accurately predicted the ultimate load of the test specimen. The calculated responses in the post-ultimate range generally underestimated the load compared to the experimental data with a few exceptions (See Fig. 7).
4. Approximation of Load-Shortening Behavior

A regression analysis was performed on the database to develop a simplified method for predicting the load-shortening behavior of damaged tubular columns. A brief overview of the regression method used and specifics with respect to the analysis of the load-shortening behavior are given in the following sections.

4.1 Regression Analysis

A multiple linear regression analysis was used to approximate the discrete data in the database. The known values of the dependent variable (for l data points) are arranged in a column matrix, F.

\[
F = \begin{bmatrix}
    f_1 \\
    f_2 \\
    \vdots \\
    f_i \\
    \vdots \\
    f_l
\end{bmatrix}
\]

For each value, \( f_i \), of the dependent variable there is a set, \( (x_{1i}, x_{2i}, \ldots, x_{ki}, \ldots, x_{ni}) \), of corresponding values for n independent variables. The approximation, \( \bar{f} \), to the data, F, is taken to be in the form of a linear series.

\[
\bar{f} = a_1 q_1 + a_2 q_2 + \ldots + a_j q_j + \ldots + a_m q_m
\]

where \( a_j \) are the regression coefficients and \( q_j \) are the regressors. Each regressor \( q_j \) is a function of the independent variable(s). In matrix notation, Eq. 7 can also be expressed by the product

\[
\bar{f} = QA
\]

where Q is a \( 1 \times m \) row matrix of regressors \( q_j \) and A is a \( m \times 1 \) column matrix of the coefficients, \( a_j \). Coefficients A are determined by solving the following set of linear equations which are derived from a least squares regression.

\[
B^T B A = B^T F
\]

Each \( i^{th} \) row of matrix B is the row matrix Q evaluated for a set of values of the independent
variables corresponding to the $i^{th}$ element of $F$. Thus, $B$ is a $l \times m$ matrix and the product $B^TB$ is a $m \times m$ matrix.

In multiple regression analysis it is convenient to study separately the effect of each independent variable on the dependent variable and select a suitable approximating (coordinate) function for it. The relationship between the dependent variable and an independent variable is observed while all other independent variables are kept constant, and an appropriate coordinate function is selected to approximate the relationship. In effect, the elements of the coordinate function are regressors for the particular variable and, analogously to Eq. 8, the approximation, $\bar{f}$, can be expressed in terms of a set of coefficients and functions of a single independent variable while all other independent variables are set to have constant values. Thus,

$$\bar{f} = H_k A_k$$  \hfill (10)

where $H_k$ is the coordinate function (a row matrix) and $A_k$ is the column of coefficients for the $k^{th}$ independent variable. After coordinate functions are selected for each independent variable, it is assumed that regressors $q_j$ which define the approximation $\bar{f}$ as a function of all the independent variables in Eq. 7 are formed as a product of the coordinate functions for the individual independent variables. In this study, the convention of a direct product was used to establish the regression model from the selected coordinate functions. For example, for a relationship with $n$ independent variables, the coordinate functions would be designated by the following row matrices:

$$H_1 = [h_{11} \ h_{12} \ ... \ h_{1m_1}]$$

$$H_2 = [h_{21} \ h_{22} \ ... \ h_{2m_2}]$$

$$H_n = [h_{n1} \ h_{n2} \ ... \ h_{nm_n}]$$

Regressors are then determined by the direct product defined as a prescribed sequence of multiplications of the elements of the $n$ coordinate functions $H_k$ as indicated below in the row matrix of Eq. 12.
\[ Q = \text{dir}(H_1, H_2, \ldots, H_n) = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1n} \\ h_{11} & h_{12} & \cdots & h_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ h_{11} & h_{2m} & \cdots & h_{nm} \end{bmatrix} \]

Note the sequence of the subscripts in each product where, the term \( h_{kj} \) represents the \( j^{\text{th}} \) element of the coordinate function for the \( k^{\text{th}} \) independent variable. The total number of elements in the *direct product* is \( m = \prod_{k=1}^{n} m_k \).

Standard least squares procedure is performed to determine the \( m \) coefficients \( A \) from Eq. 9. The approximation function for the given data is then \( \bar{f} = QA \) (Eq. 8). One advantage of this formulation by the *direct product* procedure is that, after the coefficients \( A \) are determined, the dependent variable expressed as a function of the \( m \) regressors and coefficients can be readily reduced to a function with fewer (or a single) independent variables after some of the variables are set to constant values and thus eliminated. The approximation is then the product of a row matrix of a reduced set of regressors, which are the *direct product* of the coordinate functions for the desired independent variables, and a corresponding column matrix of coefficients.

\[ \bar{f} = Q_r A_r \]  

(13)

For example, if all but one independent variable (the \( k^{\text{th}} \)) are set to be constant, \( Q_r \) is simply \( H_k \), and the approximation \( \bar{f} \) is reduced to Eq. 10 \( \bar{f} = H_k A_r \). Coefficients \( A_r \) are directly computed from the now known coefficients, \( A \), by

\[ A_r = C Q_e^T \]  

(14)

where \( C \) is formed by arranging the \( m \) elements of \( A \) into a rectangular \( (m_k \times \frac{m}{m_k}) \) matrix, and \( Q_e \) is the *direct product* (in sequential order) of coordinate functions to be eliminated, that is, all except for the \( k^{\text{th}} \) coordinate function \( H_k \). The arrangement of \( C \) is a direct outcome of the order in the *direct product*, \( Q_e \), since each \( j^{\text{th}} \) row of \( C \) contains the elements of \( A \) which are coefficients of the terms of the *direct product* matrix, \( Q \), which contain the \( j^{\text{th}} \) term of the coordinate function \( H_k \).

This procedure was used to determine the axial load-shortening relationship for constant values of all other variables as demonstrated in Sec. 4.4.
4.2 Parametric Study

The database of experimental and analytical load-shortening relationships was used to establish a set of variables which could define the behavior of damaged columns. The effect of each variable was then studied so that suitable coordinate functions could be selected. It was desirable to select a set of independent variables which result in a minimal scatter of the data and thus produce a better "fit". In this study, the non-dimensionalized parameters, $D/t$, $L/r$, $d/D$, $\delta/L$ and $\Delta/L$ were studied with respect to $P/P_y$. Note that for reasons discussed in Sec. 3.1, the dent shape and location were not included as variables in the study. It was found that the non-dimensionalized axial load $(P/P_y)$ could be represented as a function of the following parameters:

\begin{align}
1. \quad \lambda &= \frac{L}{\pi r} \sqrt{\frac{\varepsilon_y}{P}} \\
2. \quad D/t \\
3. \quad d/D \\
4. \quad \delta/L \\
5. \quad S &= \frac{\Delta}{L\varepsilon_y}
\end{align}

With these parameters, there was little scatter in the data for wide variations in the absolute size of the columns and yield stress of the material, and they were used as the independent variables in the regression analysis. In the notation used in Sec. 4.1 with respect to the order in the direct product, $H_1$ is the coordinate function for the variable $\lambda$, $H_2$ is the coordinate function for $D/t$, etc., as numbered above in Eq. 15.

In studying the load-shortening response, it was observed that for columns with a relatively small amount of damage (dent damage and initial crookedness) and large $D/t$ ratios, the behavior was characterized by an essentially linear response up to the ultimate load followed by very rapid decrease in load ("peaked" response). This peaked response was also observed to be a function of the slenderness ($\lambda$) of the column. For columns with greater damage, the load-shortening response was smoother, with a gradual approach to the ultimate load and a gradual reduction in load in the post-ultimate range.

Due to the difficulty in selecting coordinate functions to depict both types of behavior, the current study was limited to columns with a relatively smooth load-shortening response. For this purpose, it was convenient to combine both dent damage and initial out-of-straightness into one "damage factor" $u$. 
\[ u = 35 \frac{\delta}{L} + \frac{d}{D} \]  \hspace{1cm} (16)

Then, the domain of study was defined by a 3-dimensional space with coordinates of \( \frac{D}{t}, \lambda \) and the "damage factor" \( u \) over which the regression analysis was performed. This domain is bounded by three planes shown in Fig. 9 and can be defined by a maximum value of \( \frac{D}{t} \) ratio for which the coordinate functions for \( S \) are valid.

\[
\frac{D}{t} \leq 30 + 233.3 \ u \quad \text{for} \quad \lambda \leq 0.5 + u \\
\frac{D}{t} \leq 96.67 - 133.3 \lambda + 366.7 \ u \quad \text{for} \quad 0.5 + u < \lambda \leq 0.5 + 2.5 \ u
\]  \hspace{1cm} (17)

In practical terms, these limits state, for example, that if \( \frac{L}{r} = 80 \), \( \frac{d}{r} = 0.15 \), \( \frac{\delta}{L} = 0.002 \) and \( \sigma_y = 36 \text{ ksi} \), the value of \( \frac{D}{t} \) should not be greater than 59.

Due to the nature of the load-shortening relationships and the variation in relative strain, \( S = \frac{\Delta}{L \epsilon_y} \), at which the ultimate load is reached, the selection of satisfactory coordinate functions for \( S \) was very difficult. Consequently, an alternate formulation of the approximation function was developed in which the response (independent) variable was taken as the inverse of the non-dimensionalized axial load, that is, \( p = \frac{P}{F} \). The nature of \( p \) vs. \( S \) (\( \frac{P}{F} \) vs. \( \frac{\Delta}{L \epsilon_y} \)) relationship facilitated the selection of the coordinate function for relative strain. This is further discussed in Sec. 4.2.1.5.

With the response (dependent) variable established, a parametric study for each of the independent variables was conducted to determine an appropriate coordinate function.

### 4.2.1 Selection of Coordinate Functions

The objective in determining a coordinate function was to find a linear combination of as few terms as possible with the capability to approximate the given data with reasonable accuracy. Consequently, the relationship between the response (dependent) variable, \( p \), and the independent variable for which the coordinate function was sought was studied for constant values of all other variables. After coordinate functions for all the independent variables had been selected, the regression model was formed by the direct product. Then a trial and error approach was used to improve the "fit" of the model by "tweaking" the elements.
of the coordinate functions. Illustrative examples of the basis for the selection of the coordinate functions for each variable are given in the following sections. Note that some terms of the coordinate functions were multiplied by a factor of a power of ten so that the individual terms would all have approximately the same order of magnitude. This was done to preclude any numerical difficulties in the solution of Eq. 9 associated with a badly scaled solution vector A (large variation in the orders of magnitude).

4.2.1.1 Coordinate Function for $\lambda$

The relationship between $p$ and $\lambda$ for constant values of all other variables was studied by examining plots from the database. From this study it was determined that the relationship could be approximated with a parabolic function as can be seen in Figs. 14 and 15 which show $p$ vs. $\lambda$ for constant values of the other variables: $D/t=40$, $d/D=0.20$, $\delta/L=0.0$ and two values of $S$, $S=0.5$ and $S=1.1$. The following three-term coordinate function for $\lambda$ was selected:

$$H_1 = H_\lambda = [1 \quad \lambda \quad \lambda^3]$$  \hspace{1cm} (18)

4.2.1.2 Coordinate Function for $D/t$

Figure 16 shows the nature of the $p$ vs. $D/t$ relationship for $\lambda=0.8$, $\delta/L=0.0$, $d/D=0.20$ and $S=0.5$. Figure 17 is for the same constants except that $S=1.1$. Examination of these and other plots of the data resulted in the selection of the simple straight-line coordinate function

$$H_2 = H_{D/t} = [1 \quad D/t \times (10)^{-1}]$$  \hspace{1cm} (19)

4.2.1.3 Coordinate Function for $d/D$

The coordinate function for relative dent depth ($d/D$) is the following simple second order parabola

$$H_3 = H_{d/D} = [1 \quad (d/D)^2 \times (10)^2]$$  \hspace{1cm} (20)

Examples of $p$ vs. $d/D$ from the database are shown in Figs. 18 and 19 for $D/t=40$, $\lambda=0.8$, $\delta/L=0.0$, with $S=0.5$ and $S=1.1$.

4.2.1.4 Coordinate Function for $\delta/L$

The observed relationship between $p$ and $\delta/L$ appeared to be generally linear. This is shown in Fig. 20 for $D/t=25$, $\lambda=0.8$, $d/D=0.05$ and $S=0.5$, and in Fig. 21 for $S=1.1$. The coordinate function selected is a straight line.

$$H_4 = H_{\delta/L} = [1 \quad \delta/L \times (10)^3]$$  \hspace{1cm} (21)
4.2.1.5 Coordinate Function for $S$

Since, for the column geometries and damage considered, the maximum value of $P/P_y$ fell over a relatively wide range of values of $S$, it was difficult to find a linear combination of functions of $S$ to approximate the load-shortening relationships. The selection of $p=P_y/P$ (as opposed to $P/P_y$) as the response (dependent) variable was made in order to facilitate the selection of the coordinate function for $S$. This is illustrated by considering a fairly typical $p$ vs. $S$ relationship shown in Fig. 22. The nature of the inverse ($P_y/P$) relationship permitted the use of separate terms to approximate the descending and ascending portions of the curve. The following four-term coordinate function was selected for the two ranges of $S$.

$$H_S = H_S = \begin{cases} [S^{-1} \quad S^{-2} \quad \left(1 - \frac{1}{\cosh S}\right)^{1.5} \quad 0] & \text{for } 0 \leq S \leq 0.5 \\ [S^{-1} \quad S^{-2} \quad \left(1 - \frac{1}{\cosh S}\right)^{1.5} \quad (S - 0.5)^2] & \text{for } S > 0.5 \end{cases}$$ (22)

The individual terms of the coordinate functions for $S$ are plotted in Fig. 23 (note that the term $\left(1 - \frac{1}{\cosh S}\right)^{1.5}$ is multiplied by a factor of 5.0 for scaling purposes). From this figure it can be concluded that a linear combination of the terms can produce a minimum value of $p$ over a range of values of $S$. 
4.2.2 Formulation of the Regression Model

The coordinate functions are combined in an ordered procedure to form the regression model as described in Sec. 4.1. The order of the direct product was, as enumerated in Eq. 15, $\lambda$, $D/t$, $d/D$, $\delta/L$, and $S$. It resulted in a set of $3 \times 2 \times 2 \times 2 \times 4 = 96$ regressors. In Eq. 23, the computed coefficients ($A$) of the regressors are given in the form of the transpose of the rectangular $C$ matrix, $C^{T}$, (See Sec. 4.1) with $H_{5}$ ($H_{3}$) as the remaining free coordinate function for the relative axial shortening.

\[
\begin{bmatrix}
91.9227 & 3.60723 & -523.75 & 333.093 \\
-178.01 & -4.1406 & 1456.72 & -694.12 \\
100.402 & -0.65395 & -1267.9 & 442.201 \\
-23.015 & -0.86287 & 136.164 & -82.801 \\
45.1927 & 0.940674 & -377.91 & 172.317 \\
-25.81 & 0.258937 & 333.524 & -110.43 \\
-70.763 & -5.392 & -418.08 & -131.62 \\
132.794 & 8.44454 & 479.68 & 288.668 \\
-69.232 & -2.6308 & 77.1425 & -196.13 \\
17.8061 & 1.3343 & 101.51 & 33.0922 \\
-33.477 & -2.0771 & -110.61 & -72.826 \\
17.5542 & 0.627405 & -27.334 & 49.5474 \\
-37.029 & -0.28566 & 409.663 & -169.59 \\
63.0653 & 0.999531 & -665.13 & 262.062 \\
-26.079 & -0.98045 & 240.142 & -77.67401 \\
9.19786 & 0.243448 & -90.288 & 38.6192 \\
-15.416 & -0.60022 & 136.913 & -55.105 \\
6.00993 & 0.456124 & -38.061 & 10.5092 \\
23.8209 & 2.47795 & 199.192 & 24.385 \\
-41.639 & -4.4515 & -356.47 & -34.082 \\
18.4614 & 2.10251 & 166.031 & 5.36668 \\
-6.2383 & -0.67844 & -50.645 & -6.8102 \\
10.8385 & 1.23408 & 93.3905 & 8.523461 \\
-4.7286 & -0.59913 & -46.209 & 0.0
\end{bmatrix}
\]
Note that each $j$th column of $C^T$ contains the coefficients of the regressors which include the $j$th term of the coordinate function $H_s$. The zero in $C^T$ (the bottom right corner) is the consequence of collinearity which resulted from the database used, the selected coordinate functions, and the computational precision used by SAS. [7]

To determine $p$ as a function of $S$, the $C$ matrix is post-multiplied by $Q_e^T$ which is the direct product of all coordinate functions except $H_5 = H_s$ (See Sec. 4.1). The resultant row matrix $Q_e$ is

$$Q_e = \begin{bmatrix}
1 & \lambda & \lambda^3 & \frac{D}{t}10^{-1} & \frac{D}{t}10^{-1} & \lambda^3\frac{D}{t}10^{-1} & \left(\frac{d}{B}\right)^210^2 & \lambda\left(\frac{g}{B}\right)^210^2 & \lambda^3\left(\frac{d}{B}\right)^210^2 \\
\frac{D}{t}\left(\frac{d}{B}\right)^210 & \lambda\frac{D}{t}\left(\frac{d}{B}\right)^210 & \lambda^3\frac{D}{t}\left(\frac{d}{B}\right)^210 & \frac{\dot{g}}{t}10^3 & \lambda\frac{\dot{g}}{t}10^3 & \lambda^3\frac{\dot{g}}{t}10^3 \\
\frac{D}{t}\frac{\ddot{g}}{t}10^2 & \lambda\frac{D}{t}\frac{\ddot{g}}{t}10^2 & \lambda^3\frac{D}{t}\frac{\ddot{g}}{t}10^2 & \left(\frac{d}{B}\right)^2\frac{\ddot{g}}{t}10^5 & \lambda\left(\frac{d}{B}\right)^2\frac{\ddot{g}}{t}10^5 & \lambda^3\left(\frac{d}{B}\right)^2\frac{\ddot{g}}{t}10^5 \\
\frac{D}{t}\left(\frac{d}{B}\right)^2\frac{\ddot{g}}{t}10^4 & \lambda\frac{D}{t}\left(\frac{d}{B}\right)^2\frac{\ddot{g}}{t}10^4 & \lambda^3\frac{D}{t}\left(\frac{d}{B}\right)^2\frac{\ddot{g}}{t}10^4
\end{bmatrix} \quad (24)$$

Then $Q_e$ is evaluated for the particular values of $\lambda$, $D/t$, $d/D$, and $\delta/L$. As stated in Sec. 4.2.1 and shown in Eqs. 19, 20 and 21, some terms in $Q_e$ have multipliers of powers of 10 in order to bring these terms (and the values of $A$) to approximately the same order of magnitude.

Note that in this case, since

$$Q = \text{dir}(H_1, H_2, H_3, H_4, H_5) = \text{dir}(H_\lambda, H_{D/t}, H_{d/D}, H_{\delta/L}, H_5) \quad (25)$$

it follows that

$$Q = \text{dir}(Q_e, H_5) \quad (26)$$

After the calculation of the reduced set of coefficients, $A_r$, from Eq. 14, the $p$ vs. $S$ relationship is given by Eq. 13 or, specifically,

$$p = Q_r A_r = H_5 A_r \quad (27)$$

which defines $p$ as a four-term function of $S$. Finally, the load-shortening relationship is obtained from

$$\frac{P}{P_y} = \frac{1}{p} = p^{-1} \quad (28)$$
4.3 Fit to Existing Data

The agreement between the approximated load-shortening relationships and the data included in the database was generally excellent. As a measure of the goodness of fit to the data, standard deviation and the root-mean-square of the relative error were calculated for \( \frac{P}{P_y} \), as opposed to the dependent variable used in the regression analysis, \( p = \frac{P}{P} \). The standard deviation is defined by

\[
\sigma = \sqrt{\frac{\sum_{i=1}^{l} (f_i - \bar{f})^2}{l - m}}
\]  

where \( \bar{f} \) is the approximation of \( P/P_y \) and \( f_i \) is the given value, \( l \) is the number of data points and \( m \) is the number of regressors (96). The root-mean-square of the relative error is given by

\[
rms = \sqrt{\frac{\sum_{i=1}^{l} \left( \frac{f_i - \bar{f}}{f_i} \right)^2}{l}}
\]  

The standard deviation calculated from Eq. 29 was \( \sigma = 0.0364 \) and the root-mean-square of the relative error was \( rms = 0.0733 = 7.33\% \).

A representative sampling of plots of the approximate relationship and the data is shown in Figs. 24 to 29. The difference between the approximation and the data was quite acceptable.

4.4 Example Calculation

To illustrate the calculation of the approximate load-shortening curve, example computations for Specimen A3 (Ref. [8]) shown in Fig. 24 are given below. Specimen A3 has the following geometric/material variables: \( \lambda = 1.06 \), \( D/t = 29.1 \), \( d/D = 0.048 \), and \( \delta/L = 0.0055 \). The resultant \( Q_e \) calculated from Eq. 24 for these values, is the row matrix given in Eq. 31.
The coefficients, $A_r$, are calculated from Eq. 14 by post-multiplying matrix $C$ by $Q_c^T$. (Matrix $C^T$ is given in Eq. 23) The resultant four coefficients form a column matrix $A_r$ shown here as a transpose.

$$A_r^T = \begin{bmatrix} 0.932524 & 0.0134093 & 6.6042 & -0.66175 \end{bmatrix}$$

The $p$ vs. $S$ relationship is then given by Eq. 27.

$$p = \frac{P_y}{P} = 0.932524 h_{S1} + 0.0134093 h_{S2} + 6.6042 h_{S3} - 0.66175 h_{S4}$$

where the $h_{S1}$ to $h_{S4}$ are defined in Eq. 22. The approximate load-shortening relationship is then calculated from Eq. 33 for specific values of $S = \frac{\Delta}{\ell_y}$. To further illustrate the calculation of the load-shortening relationship, the load, $P/P_y$, is determined from Eq. 33 for $S=0.5$ and the calculated point is indicated in Fig. 24. For $S=0.5$, the coordinate function $H_S$ is

$$H_S = \begin{bmatrix} 2.0 & 4.0 & 0.038077 & 0.0 \end{bmatrix}$$

Then, $p$ is given by

$$p = \frac{P_y}{P} = 1.865 + 0.053637 + 0.25147 + 0.0 = 2.170$$

And finally, the nondimensionalized axial load is

$$\frac{P}{P_y} = \frac{1}{2.170} = 0.461$$
4.5 Range of Applicability

The approximation of load-shortening behavior is valid for ranges of column geometries generally found in fixed offshore platforms. The method is limited by the range of data in the database used in the regression analysis as indicated in Table 2.

- $D/t = 20 - 80$
- $\lambda = 0.4 - 1.06$
- $d/D = 0.05 - 0.30$
- $\delta/L = 0.0 - 0.01$

In addition, the applicability is limited by the relative damage constraints of Eq. 17.

\[
\frac{D}{t} \leq 30 + 233.3u \quad \text{for } \lambda \leq 0.5 + u
\]

\[
\frac{D}{t} \leq 96.67 -133.3\lambda + 366.7u \quad \text{for } 0.5 + u < \lambda \leq 0.5 + 2.5u
\]

The range of applicability of the method may be increased as more data become available.
5. Summary, Conclusions and Recommendations

5.1 Summary and Conclusions

A simplified engineering method was developed to predict the load-shortening response of damaged (dented and crooked), pin-ended, tubular columns. The method is based on an analytical model and a regression analysis of data from a finite element analysis and published experimental results.

5.1.1 Finite Element Computer Analysis

Since a dented member subjected to an axial load undergoes plastification and large deformations, it was necessary to use a finite element program which would be capable of taking these effects into account. The finite element program ADINA was selected since it has suitable shell elements for large-displacement analysis. After considerable experimentation, a discretization model that took advantage of the double symmetry of the problem was developed. The model had 32 shell elements, 4 beam elements and 268 nodal points (1162 degrees of freedom) and gave reasonably good correlation with the experimental curves in the database. The program was then used to generate load-shortening relationships to be included in the database to supplement the curves available from previous experimental research.

5.1.2 Database for Axial Behavior of Damaged Compression Members

All experimental data available in published literature on the axial behavior of dented and crooked tubular members under concentrically applied axial load was collected and put into a database (Total of 31 curves). Information for each specimen covers the following items: Source, identification, data on material, geometry, location and amount of damage, and the load vs. axial shortening relationship with at least 10 points and at least 4 points in the post-ultimate range. The data can be readily retrieved, manipulated and analyzed with the SAS software selected for this purpose. Computer generated results were used to fill in and supplement the experimental data mainly to cover sparsely populated ranges of parameters. (A total of 56 curves)
5.2 Selection of Parameters

The axial load capacity for damaged columns was defined as a function of five parameters; D/t, λ, d/D (relative dent depth), δ/L (out-of-straightness), and S (average axial strain divided by yield strain).

5.2.1 Selection of Coordinate Functions

The functional effect of each parameter on the axial behavior was studied by trial-and-error to find a simple yet accurate expression to approximate the relationship. Two, three or, at the most, four-term functions were tried, and the ones with better accuracy selected. Since the total number of terms in the final approximation function would be the product of the number of terms in all coordinate functions, as few terms as possible were used for the individual functions. The final selection gave a total of 96 terms. It was also necessary to divide the axial behavior curve into two ranges with different load-deformation coordinate functions in order to increase the degree of accuracy.

5.2.2 Development of Approximate Engineering Method

The engineering method developed here allows a rapid computation of the load-deformation relationship once the dimensions and material properties (yield stress) and the amount of damage are known or estimated. The resultant relationship which covers the elastic pre-ultimate, ultimate and post-ultimate ranges, can be used for practical application within the ranges of parameters specified. The method requires storage of a set of 96 constants and can be readily programmed and used as a subroutine in a larger program for structural analysis of offshore framed structures.

5.2.3 Limitations of the Method

At present, the engineering method, with the coordinate functions used, was found to be much more accurate for members with significant degree of damage and when D/t is relatively low, yet in the primary range of practical design (D/t<80). The final formulation of the method is applicable with confidence mainly in the following ranges of parameters:

- D/t = 20 - 80
- λ = 0.4 - 1.06
- d/D = 0.05 - 0.30
• \( \delta/L = 0.0 - 0.01 \)

with the additional constraints imposed by Eq. 17.

Comparison of the method with the data available showed good correlation as indicated by the standard deviation of 0.0364 and the root-mean-square of the relative error of 0.0733.

5.3 Recommendations for Future Work

Work that can be recommended on the basis of this study can be given in two parts: a direct extension and completion of the results obtained in this report, and the related topics which can be viewed as new areas.

5.3.1 Extension of Current Work

The following items can be viewed as a direct supplement and extension of the work completed and described in this report.

1. Tests on large-scale specimens are needed. Since the database contained only test results from small-scale tests and from computer program which had good correlation with the small-scale tests (thus, representing small-scale tests), there is an urgent need for tests on prototype-sized specimens which would incorporate residual stresses, imperfections, and other characteristics of tubular members as they are encountered in offshore structures. Note that the small-scale specimens were manufactured and stress relieved by annealing, rather than fabricated by cold-rolling and welding. These large-scale tests should include some specimens which, in terms of non-dimensional parameters, duplicate small-scale tests conducted in the past. They should also cover the whole range of parameters of practical interest. For example, \( D/t = 20 \) to 100.

2. Generation of more data, experimentally or by using a computer program, is needed in order to expand the database for a more thorough study of the behavior and of the influence of the parameters involved.

3. Extension of the approximate method into the range of "peaked" behavior is needed. This is considered to be an elaboration of the work completed in this study.

5.3.2 Further Research on Damaged Tubular Members

Three areas need attention: further work on dented and crooked members, corroded members, and members with fatigue cracks.
5.3.2.1 Dented and Crooked Members

This research should cover the following items:

1. The effect of end eccentricity on the behavior of dented members requires additional experimental and analytical work.
2. End restraints, elastic and inelastic, need consideration; experimental and analytical, possibly by using small-scale specimens.
3. Effect of lateral loading needs to be investigated since members in the splash zone are subjected to heavy wave action.

5.3.2.2 Corroded Members

The effect of corrosion on the behavior of tubular members needs to be investigated. This should include the effect of loss of member net section as well as the effect of holes resulting from severe corrosion. Research in this area should address the following items:

1. The establishment of parameters to quantify the amount and location of corrosion damage.
2. The investigation of the effect of corrosion, as defined by these parameters, on the pre-ultimate, ultimate, and post-ultimate response of the member.
3. The development of an engineering method (simplified procedure) for the behavior of corroded members, possibly, by modifying the method developed for dented members. Modified or new coordinate functions will be needed.

5.3.2.3 Members with Fatigue Cracks

There are two main areas of research related to fatigue:

1. Effect of fatigue cracks (size and location) on the short-term axial load-deformation behavior of tubular members. This study is closely related to the subject of this report and would follow a similar procedure.
2. Initiation and growth of fatigue cracks. Consideration of load spectra, structural details, presence of salt water and the wetting cycle would be the areas of work. Of particular interest is the effect of corrosion notching on fatigue life.
Acknowledgments

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Tables
Table 1: Data Included in Database

<table>
<thead>
<tr>
<th>Source</th>
<th>Quantity</th>
<th>$\lambda$</th>
<th>$D / l$</th>
<th>$d / D$</th>
<th>$\delta / L$</th>
<th>$\sigma_y$ (ksi)</th>
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<tr>
<td>Ref. [8]</td>
<td>8</td>
<td>0.66 - 1.06</td>
<td>29 - 86</td>
<td>0 - 8%</td>
<td>0.0003 - 0.0055</td>
<td>29 - 70</td>
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<td>Ref. [9]</td>
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<td>0.65 - 0.83</td>
<td>55 - 70</td>
<td>0 - 13%</td>
<td>0.0018 - 0.005</td>
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<td>Ref. [12]</td>
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<td>0.62 - 0.82</td>
<td>40 - 60</td>
<td>2 - 10%</td>
<td>0.0005 - 0.0018</td>
<td>30 - 70</td>
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<tr>
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<td>0.45 - 1.1</td>
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<td>0.0005 - 0.0037</td>
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<td>20 - 100</td>
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Table 2: Data Included in Regression Analysis

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<tr>
<th>Source</th>
<th>Quantity</th>
<th>λ</th>
<th>$\frac{D}{t}$</th>
<th>$\frac{d}{D}$</th>
<th>$\frac{\delta}{L}$</th>
<th>$\sigma_y$ (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ref. [8]</td>
<td>3</td>
<td>0.78 - 1.06</td>
<td>29 - 46</td>
<td>5 - 10%</td>
<td>0.005 - 0.0055</td>
<td>29 - 33</td>
</tr>
<tr>
<td>Ref. [9]</td>
<td>2</td>
<td>0.65 - 0.66</td>
<td>55 - 58</td>
<td>13%</td>
<td>0.0018 - 0.005</td>
<td>40 - 43</td>
</tr>
<tr>
<td>Ref. [12]</td>
<td>1</td>
<td>0.62</td>
<td>41</td>
<td>10%</td>
<td>0.0018</td>
<td>67</td>
</tr>
<tr>
<td>Ref. [10]</td>
<td>8</td>
<td>0.45 - 0.94</td>
<td>26 - 46</td>
<td>9 - 18%</td>
<td>0.0009 - 0.0037</td>
<td>48 - 70</td>
</tr>
<tr>
<td>ADINA</td>
<td>27</td>
<td>0.4 - 0.8</td>
<td>20 - 80</td>
<td>5 - 30%</td>
<td>0.0 - 0.006</td>
<td>36 - 72</td>
</tr>
</tbody>
</table>
Figures
Figure 1: Basic Concept of Finite Element Model
Figure 2: Damaged Column Geometry

Figure 3: Dent Geometry
Figure 4: Shell Element Discretization of Model 1
Figure 5: Shell Element Discretization of Model 2
Figure 6: Shell Element Discretization of Model 3
Axial Load vs. Shortening
1CDC

\[
\frac{D}{t} = 91
\]
\[
\lambda = 0.65
\]
\[
\frac{d}{D} = 0.14
\]
\[
\frac{\delta}{L} = 0.0024
\]

Figure 7: Comparison of Experimental Results with Models 1, 2 and 3
Figure 8: Boundary Conditions and Constraints
Figure 9: Domain Defining Smooth Load-Shortening Response
Axial Load vs. Shortening

\[ \frac{D}{t} = 41 \]
\[ \lambda = 0.62 \]
\[ \frac{d}{D} = 0.10 \]
\[ \frac{\delta}{L} = 0.0018 \]

Figure 10: Finite Element Model 3 and Experimental Load-Shortening for Specimen IIIICII (Ref. [9])
**Axial Load vs. Shortening**

A4

\[
\frac{D}{t} = 29 \\
\lambda = 1.04 \\
\frac{d}{D} = 0.0 \\
\frac{\delta}{L} = 0.005
\]

**Figure 11:** Finite Element Model 3 and Experimental Load-Shortening for Specimen A4 (Ref. [5])
Axial Load vs. Shortening
B3

\[ \frac{D}{t} = 45 \]
\[ \lambda = 0.78 \]
\[ \frac{d}{D} = 0.08 \]
\[ \frac{\delta}{L} = 0.005 \]

Figure 12: Finite Element Model 3 and Experimental Load-Shortening for Specimen B3 (Ref. [5])
Axial Load vs. Shortening

R1A

\[
\frac{D}{t} = 26 \\
\lambda = 0.92 \\
\frac{d}{D} = 0.15 \\
\frac{\delta}{L} = 0.0028
\]

Figure 13: Finite Element Model 3 and Experimental Load-Shortening for Specimen R1A (Ref. [7])
\[
\frac{D}{\dot{t}} = 40
\]
\[
\frac{d}{D} = 0.20
\]
\[
\frac{\delta}{L} = 0.0
\]
\[S = 0.5\]

**Figure 14:** Sample Data for Selection of Coordinate Function for \(\lambda\)
Figure 15: Sample Data for Selection of Coordinate Function for $\lambda$. 

- $\frac{D}{t} = 40$
- $\frac{d}{D} = 0.20$
- $\frac{\delta}{L} = 0.0$
- $S = 1.1$
\begin{align*}
\lambda &= 0.8 \\
\frac{d}{D} &= 0.20 \\
\frac{\delta}{L} &= 0.0 \\
S &= 0.5
\end{align*}

Figure 16: Sample Data for Selection of Coordinate Function for $D/t$
\[ \lambda = 0.8 \]
\[ \frac{d}{D} = 0.20 \]
\[ \frac{\delta}{L} = 0.0 \]
\[ S = 1.1 \]

Figure 17: Sample Data for Selection of Coordinate Function for D/t
\[ \frac{D}{t} = 40 \]
\[ \lambda = 0.80 \]
\[ \frac{s}{L} = 0.0 \]
\[ S = 0.5 \]

Figure 18: Sample Data for Selection of Coordinate Function for d/D
Figure 19: Sample Data for Selection of Coordinate Function for $d/D$
$\frac{D}{t} = 25$

$\lambda = 0.8$

$\frac{d}{D} = 0.20$

$S = 0.5$

Figure 20: Sample Data for Selection of Coordinate Function for $\delta/L$. 
Figure 21: Sample Data for Selection of Coordinate Function for $\delta/L$
Figure 22: Typical Inverse Load-Shortening Relationship
Figure 23: Coordinate Function for $S$

\[
\begin{align*}
0.0 & \quad \text{for } S \leq 0.5 \\
(S - 0.5)^2 & \quad \text{for } S > 0.5 \\
(1 - \frac{1}{\cosh S})^{1.5} \times 5
\end{align*}
\]
AXIAL LOAD VS. SHORTENING
A3

$\frac{D}{t} = 29$

$\lambda = 1.06$

$\frac{d}{D} = 0.05$

$\frac{\delta}{L} = 0.0055$

Figure 24: Approximation of Load-Shortening for Specimen A3 (Ref [5])
Figure 25: Approximation of Load-Shortening for Specimen B3 (Ref [5])

AXIAL LOAD VS. SHORTENING
B3

\[
\frac{D}{t} = 45
\]

\[
\lambda = 0.78
\]

\[
\frac{d}{D} = 0.08
\]

\[
\frac{\delta}{L} = 0.005
\]
AXIAL LOAD VS. SHORTENING
R1A

\[
\frac{D}{t} = 26 \\
\lambda = 0.92 \\
\frac{d}{D} = 0.15 \\
\frac{\delta}{L} = 0.0028
\]

Figure 26: Approximation of Load-Shortening for Specimen R1A (Ref [7])
AXIAL LOAD VS. SHORTENING

III.CII

\[ \frac{D}{t} = 41 \]
\[ \lambda = 0.62 \]
\[ \frac{d}{D} = 0.10 \]
\[ \frac{\delta}{L} = 0.0018 \]

Figure 27: Approximation of Load-Shortening for Specimen III.CII (Ref [9])
AXIAL LOAD VS. SHORTENING
STA3_17

Figure 28: Approximation of Load-Shortening for Analysis STA3_17 (ADINA)
AXIAL LOAD VS. SHORTENING
STA3_56

Figure 29: Approximation of Load-Shortening for Analysis STA3_56 (ADINA)
Appendix A

Nomenclature

A  Column Matrix of Regression Coefficients
A_i  Column Matrix of Regression Coefficients for the i^{th} Coordinate Function
A_r  Column Matrix of a Reduced Set of Regression Coefficients
B  Matrix of Regressors for Given Values of Independent Variables
C  Rectangular Matrix of Regression Coefficients
D  Diameter to Mid-thickness
d  Dent Depth
E  Modulus of Elasticity
H  Coordinate Function (Row Matrix)
h_{ij}  The j^{th} Term of the i^{th} Coordinate Function
i  Counter for Data Points
j  Counter for Regressors and Terms of Coordinate Functions
k  Counter for Independent Variables
L  Length of Column
l_d  Length of Dent
l  Number of Data Points
m  Number of Regressors
m_i  Number of Terms in the i^{th} Coordinate Function
n  Number of Independent Variables
P  Axial Load
P_y  Squash Load = \pi D t \sigma_y
p  Nondimensionalized Inverse Axial Load = \frac{P_y}{\bar{P}}
Q  Row Matrix of Regressors
Q_e  Row Matrix Formed by Direct Product of Coordinate Functions To Be Eliminated
Q_r  Row Matrix Formed by Direct Product of a Reduced Set of Coordinate Functions
q_i  The i^{th} Regressor
R  Radius to Mid-thickness
R_d  Radius to Dent (See Fig. 3)
\[ r \quad \text{Radius of Gyration} \]
\[ S \quad \text{Relative Shortening of Column, } S = \frac{\Delta}{L \varepsilon_y} \]
\[ t \quad \text{Thickness of Tube Wall} \]
\[ u \quad \text{Damage Factor} = \frac{d}{D} + 35 \frac{\delta}{L} \]
\[ x \quad \text{Longitudinal Coordinate Measured from Mid-length} \]
\[ z \quad \text{Lateral Deflection (Initial Crookedness) as a Function of } x \]
\[ \Delta \quad \text{Axial Shortening of Column} \]
\[ \delta \quad \text{Maximum Initial Crookedness} \]
\[ \varepsilon \quad \text{Strain} \]
\[ \varepsilon_y \quad \text{Yield Strain, } \varepsilon_y = \frac{\sigma_y}{E} \]
\[ \lambda \quad \text{Slenderness Parameter, } \lambda = \frac{L}{\pi r} \sqrt{\varepsilon_y} \]
\[ \sigma \quad \text{Stress} \]
\[ \sigma_y \quad \text{Yield Stress} \]
\[ \phi \quad \text{Angle as Defined in Fig. 3} \]
\[ \zeta \quad \text{Dent Depth as a Function of Its Length and Distance } x \]
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