ABSTRACTS OF

EUROPEAN REPORTS ON REINFORCED COLUMN INVESTIGATIONS.

Prepared by
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<table>
<thead>
<tr>
<th>Author</th>
<th>Year</th>
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<th>Page</th>
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<td>Otto Graf</td>
<td>1921</td>
<td>27</td>
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Total 1078 Columns
COMPRESSION STRENGTH OF LONGITUDINALLY AND LATERALLY
REINFORCED CONCRETE

"RESISTANCE A LA COMPRESSION DU BETON ARME ET DU BETON PRETTE"

by Armand Considere

In Beton und Eisen, 1902-1903 (Genie Civil at the same time)

Considere found that a given amount of steel used as lateral reinforcement gave a larger effect on the strength of a column than used as longitudinal reinforcement. This is due to the internal friction in the material and experiment with sand gave a lateral friction ratio of 2.04.

Considere started to investigate if this holds true also for concrete. At first a few tests were made at Quimper on 1:4 mortar cylinders of 40 mm. (1.6 in.) dia. reinforced with wire.

The results were as follows:

Age at test: - - - - - 8 14 22 23 100 days
Standard strength - - - 40 50 60 60 170 kg./cm²
Percentage of spiral - - 2 3 4 2 3.4%
Strength obtained - - 342 440 518 347 740 kg./cm².
Increase in strength - - 302 390 458 287 570 kg./cm².

A comparison between the strength of a spiral column and a steel column of same weight showed that both had approximately the same strength, that is, the strength was proportional to the specific gravity of the specimens.
Later tests were made in Paris on 38 columns of 15 cm. (6 in.) diameter and various lengths. Some of the columns had a length of 90 cm. (36 in.) while others were 130 cm. (52 in.) long.

These tests showed that the spirally reinforced columns were greatly superior to the plain or longitudinally reinforced columns.

The results did not give any definite relation between the amount of spiral reinforcement and the strength of the columns, but showed the effect of the pitch of the spiral. The columns having 2.03% spirals and 15 cm. (6 in.) pitch were considerably higher in strength than those having 2.47% spirals and 30 cm. (12 in.) pitch.

They also showed that the longitudinal reinforcement raised the strength of spirally reinforced columns considerably. A decrease of the pitch below one-fifth of the diameter of the column did not seem to effect the strength materially when no longitudinal reinforcement was used. When longitudinal reinforcement was used, the proper pitch seemed to lie between one-seventh and one-tenth of the column diameter.

In order to study the plasticity of such spiral cylinders of 1:3 mortar a specimen was kept under a load of 557 kg./cm². Under this load it deformed into an S-shape, which in certain places had a radius of curvature of 30 cm., with a total shortening of 17%. The spiral was then removed from the
column and tension, compression, and bending tests showed that practically no change had taken place in the wire, indicating that the spiral wire had not been stretched beyond its yield point.

Modulus of elasticity tests of spiral columns gave very erratic results. It was found, however, that the modulus changed with repeated loadings and that the columns suffered a permanent set after being loaded. The change in modulus of elasticity increased with the percentage of the ultimate load to which the repetition of load was carried. It was also found that the change in modulus of elasticity was largest with the columns having low initial modulus.

The lateral expansion was so small that it could not be measured for a stress in the columns of 94.5 kg. per cm². This indicated that the spirals were taking practically no load until the concrete had exceeded its ultimate strength. Beyond this point the spirals had to take a large amount of the load. It seemed as though the spirals were laying loose in the concrete at the beginning of a test of dry cured columns, while for wet cured columns the spirals would be put in action at an earlier stage. This fact is attributed to the shrinkage of concrete during dry and expansion during wet curing.
Considere recommended two different allowable stresses for columns made under rigid control. One for columns made in laboratories and on certain important jobs, and one for columns made under ordinary field conditions. For concrete having a cement content of 600 kg. per cubic meter of aggregates, he recommended 160-150 kg. per cm². for the first class columns and 77-66 kg. per cm². for the second class. These columns will have an ultimate shortening of 20 to 30 times that due to the designed loads.

During the publication of these articles, further experiments were carried out on 11 by 11 by 50 cm. prisms reinforced with spiral and longitudinal bars. The results obtained from these tests checked the first series of tests.
COMPRESSION TESTS ON REINFORCED CONCRETE SPECIMENS

"DURCKVERSUCHE MIT EISENBERTONKORPERN"

by C. Bach

In Mitteilungen ueber Forschungsarbeiten auf dem Gebiete des Ingenieurwesens. Heft 29, 1905

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SERIES A: Longitudinally Reinforced Concrete Columns Having Lateral Ties

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This series of tests included 18 columns 1.0 m. long and 25 by 25 cm. in cross section. One group of columns had no reinforcement, while five other groups had four longitudinal bars which were tied together by 7 mm. ties spaced differently. Groups 2, 3, and 4 had 15 mm. round bars, while group 5 had 20 mm., and group 6, 30 mm. bars. The amount of longitudinal reinforcement varied from 0 to 1.13%. The distance between the ties was 250 mm. (= d) for group 2, 125 mm. (1/2 d) for group 5, 62.5 mm. (1/4 d) for group 4, and 250 mm. for groups 5 and 6.

The strength of the concrete was obtained on 30-cm. cubes.

All specimens were made of a concrete mix of 1:3:2 (cement, sand, and gravel respectively) by volume, and the water content was 15%.
The age at test was about three months.

The compression tests of the cubes gave an average strength of 150 kg. per cm² for tamped specimens. All longitudinally reinforced columns indicated end failure. The steel bars extended to a point 5.0 cm. (2.0 in.) from the ends of the columns.

During the testing of the column the elastic and permanent deformations were observed on at least two specimens of each group, and the maximum load was obtained on all specimens. The loads at low stresses were repeated until the total deformation did not change any more. For higher stresses the constant (elastic and permanent) total deformation was not reached before a large number of repeated loadings had been applied. The deformation obtained after ten repetitions was therefore used in the tabulation of the results.

The average maximum strengths of the six different groups were as follows:

<table>
<thead>
<tr>
<th>Group</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strength in kg. per cm².</td>
<td>141</td>
<td>168</td>
<td>177</td>
<td>205</td>
<td>170</td>
<td>190</td>
</tr>
</tbody>
</table>

These results show clearly the increase in strength with decrease in spacing of ties. They also show the increase in strength with increase in longitudinal steel when the spacing remains constant. The increase in strength per sq.in. of column area, however, was less for high percentage of steel than for low.
It was found that the increase in strength due to the lateral reinforcement was much greater than the increase caused by corresponding increase in percentage of longitudinal reinforcement.

**SERIES B: Spiral Columns**

These tests included 61 columns tested for the contracting firm, Wayss and Freytag A-G.

The cross section of the columns was octagonal with a minor diameter of 27.5 cm. The height was 1.0 meter and the spirals as well as the longitudinal bars extended to 2.5 cm. from each end of the column. The spirals had a protective shell of about 1.5 cm. thickness. The test program was as shown in the table. The concrete mix was 1:4 (sand : gravel) by volume, and the water content was 14%. Three columns were made for each group except group 1, which had four columns.

The age at the test was from 4 to 4-1/2 months.

The deformation measurements were taken for three repetitions of load.
<table>
<thead>
<tr>
<th>Group No.</th>
<th>Wire Dia. m/m</th>
<th>Pitch m/m</th>
<th>Number of bars</th>
<th>Size, Dia. m/m</th>
<th>Maximum Load kg/sq.cm²</th>
<th>lb/lbf/sq.in²</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>133</td>
<td>1890</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>38</td>
<td>4</td>
<td>7</td>
<td>159</td>
<td>2260</td>
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<td>3</td>
<td>7</td>
<td>37</td>
<td>4</td>
<td>7</td>
<td>178</td>
<td>2530</td>
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<td>10</td>
<td>42</td>
<td>4</td>
<td>7</td>
<td>240</td>
<td>3410</td>
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<td>11</td>
<td>226</td>
<td>2610</td>
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<tr>
<td>6</td>
<td>7</td>
<td>37</td>
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<td>11</td>
<td>230</td>
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<td>7</td>
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<tr>
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<tr>
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<td>80</td>
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<td>7</td>
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<td>2630</td>
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<tr>
<td>16</td>
<td>10</td>
<td>80</td>
<td>8</td>
<td>10</td>
<td>181</td>
<td>2670</td>
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<tr>
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<td>80</td>
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<tr>
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<td>8</td>
<td>10</td>
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</tr>
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<td>120</td>
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<td>14</td>
<td>207</td>
<td>2940</td>
</tr>
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</table>

Both the load at first cracking and at maximum was recorded. It was found that the actual pitch of the spirals, as well as the placing of the bars varied considerably from the designed values.

The report was submitted to Considere for criticism and his reply was as follows:

"The concrete, ordinarily tamped, was porous and had the low strength of 153 kg. per cm²."

"The pitch of the spirals as shown in the table was very irregular."

"The longitudinal bars were unevenly spaced. For one of the columns the distance from center to center of bars varied between 35 and 125 m/m."
"The spirals were placed neither centrical nor parallel to the end surfaces of the columns. One column showed a displacement of the spiral of 3 cm.

"In spite of these errors the specimens in groups 5, 6, and 7 give strength results which approached those obtained from the formula. In this formula the following notations are used:

\[ A_c = \text{Core area of concrete in column} \]
\[ f_c = \text{Strength of plain concrete, in these tests 135 kg. per cm}^2. \]
\[ A_s = \text{Area of longitudinal bars} \]
\[ A'_s = \text{Imaginary area of spirals} \]
\[ f_s = \text{Yield point strength of steel} \]

The maximum load is then determined by:

\[ P = 1.5 f_c A_c + f_s (A_s + 2.4 A'_s) \]

"In spite of the irregularity in the columns the obtained maximum loads agree approximately with the computed strengths, and even exceed those for the columns with low percentage of reinforcement.

The other groups showed following valuable information:
PITCH OF SPIRALS

The groups which had pitch of 80 and 120 m/m gave rather low results. Even though the strengths were increased they are below that given by the formula. This fact seems to be caused by a wrong relation between the diameter of the spirals and that of the longitudinal bars, and this was not corrected for by any decrease in the pitch of the spiral.

RELATION BETWEEN SPIRALS AND LONGITUDINAL BARS

In many of the groups the percentage of longitudinal reinforcement was very low, and the strength results were also correspondingly lower as the size of the spirals increased.

As a whole, the tests indicate that the pitch of the spirals must be decreased and the size or number of longitudinal bars increased as the size of the spirals increases in order to make the concrete to resist a large compression load, which tends to produce side pressure and force the concrete to flow out between the longitudinal bars."
TESTS ON REINFORCED CONCRETE COLUMNS
AND ON EMBEDDED STEEL COLUMNS

"VERSUCHE MIT SAULEN AUS EISENBETON UND MIT EINBETONIERTEN
EISENSAULEN, IN STUTTGART UND IN WIEN"

by Fritz V. Emporerger

In Forscherarbeiten auf dem Gebiete des Eisenbetons
Heft 8, 1908

In the introduction Emporger points out that the maximum strength of reinforced concrete columns (structural steel reinforcement) is determined by the sum of the strength of the steel and the strength of the concrete. The concrete will prevent the steel from buckling and the steel will give lateral restraint to the concrete so that column actions are eliminated. Talbot's finding that the lateral ties did not increase the strength of the column is attributed to the fact that the length of overlap of the ties was insufficient. Talbot's conclusion that the concrete in the reinforced columns was weaker than in the plain columns, was due to the fact that he used the total column area as effective concrete area and that the lateral ties were not properly anchored.

Another fault with Talbot's experiment was the deficient arrangement for the end caps. The steel was not carried close enough to the ends, and the result was that the concrete had to carry the total load alone in this portion of the column.
In 1901 experiments on 23 columns were made of a 1:3 mix, of which one was lost, 8 of a 1:4 mix, and 7 of a 1:5 mix. The water-cement ratios by weight was 0.48, 0.40, and 0.50 respectively.

The reinforcement consisted either of ordinary round iron or of structural angle iron. The percentage of reinforcement varied from 2.7 to 29.2%. The columns were of 18 in. square cross section and the lengths were 3.6, 2.4, and 1.0 m.

The columns were made on the job. At the age of 1 to 1-1/2 months they were shipped to the laboratory (in Vienna) and stored for about four years. They were then shipped to Stuttgart for testing.

The plain columns showed satisfactory uniformity in strength, but the tests of the reinforced columns did not seem conclusive.

A number of columns with structural shape reinforcement showed strengths equal to the sum of the strength of the steel and the strength of the concrete as measured on 20 cm. (8-in.) cubes.
TESTS OF REINFORCED CONCRETE COLUMNS LOADED ECCENTRICALLY

"VERSUCHE MIT ECCENTRISCH BELASTETEN BETONEBERNEM BAULEN"

by Maximilian Ritter von Thulie

In Forscherarbeiten auf dem Gebiete des Eisenbetons

Heft 10, 1909

The length of the testing machine limited the length of the columns to 1.5 m. (59 in.). The cross-section of the columns was 8 cm. (3-5/16 in.) square. The slenderness ratio was $\frac{150}{8} = 18.75$. The ends had capitals of 16 cm. (6-5/8 in.) square cross section. All columns were made at the laboratory in Lemberg and tested at the age of 45 days. The percentages of longitudinal reinforcement were 0, 1.25, 1.57, 2.44, and 2.75. The lateral reinforcement consisted of ties with different spacings and of spirals. The eccentricity varied from 0.05 cm. (0.02 in.) to 4 cm. (1.57 in.). A total of 495 columns were tested. Five of these were defective so that a total of 490 useful columns were tested.

Hinged bearing blocks were used at both ends for the testing of columns loaded eccentrically. The concentric loaded columns had ball bearings.

Most of the columns had extra reinforcement at the ends so as to prevent the head from breaking off. In some of the columns this reinforcement consisted of longitudinal
rods, in others it consisted of spirals.

The reinforcement had a maximum strength of about 4500 kg. per cm$^2$. (64,000 lb. per sq.in.).

A detailed description is given of the failure of each individual column. The tables, together with the photographs, fill most of the bulletin. The effect of the spacing of the ties (3 and 4 cm.) was found to be negligible for concentric loaded columns. For eccentrically loaded columns the strength was found to decrease with an increase in the spacing of the ties. The effect of the pitch of the spiral was also found to be negligible for concentric loading. Eccentric loading showed a slightly higher strength for a decrease in pitch. Concentric tests of columns 1.0 and 1.5 m. long showed that for plain concrete columns the strength of the short columns was considerably above the strength of the long columns. The reinforced columns, however, showed but little effect of the length of the column.

For eccentric loaded columns no definite effect of the length was found except for the spiral columns.

The concrete strength as measured on the cubes was 212.9 kg. per cm$^2$. (3030 lb. per sq.in.).

The test results showed that for a slenderness ratio up to 18.9, the buckling effect of eccentrically loaded columns was negligible.
SERIES 1

The tests reported in this paper were made for the purpose of obtaining information regarding the most proper lateral reinforcement for reinforced concrete columns. All columns were made from a 1:4 concrete mix, and tested at the age of 45 days. Three specimens were made of each set of columns. All columns were 2 m. (6.5 ft.) long. The lateral reinforcement was spaced 20 cm. (8 in.) apart. Thirteen different types of lateral reinforcement were used. The size of wire was 7 mm. in diameter (9/32 in.). The longitudinal reinforcement consisted of four bars of 16 mm. diameter (5/8 in.). In addition, Groups 1, 2, and 12 were repeated with no longitudinal reinforcement, and the same groups were also repeated with longitudinal reinforcement consisting of four bars of 20 cm. (8 in.) lengths with the joints half between each set of lateral reinforcement. Five plain columns were included in the series, bringing the total number of columns up to 62.
The columns in group 1-11 had square cross section of 30 cm. (12 in.) side length. The columns in groups 12 and 13 had octagonal cross section of 15.65 cm. (5-5/8 in.) side length. The cross sectional area was equal for all columns.

Tension tests of the 16-mm. reinforcing bars gave the following results:

- Proportionate stress - 1510 kg. per cm².
- Yield-point stress - 2360 kg. per cm².
- Ultimate strength - 4230 kg. per cm².
- Modulus of elasticity - 2,035,000 kg. per cm².
- Elongation in 52 cm. - 25.6%

The corresponding values for the 7-mm. bars were as follows:

- Proportionate stress - 2770 kg. per cm.
- Yield-point stress - 3750 kg. per cm.
- Ultimate strength - 4830 kg. per cm.
- Modulus of elasticity - 2,025,000 kg. per cm.
- Elongation in 16 cm. - 21.2%

The concrete mix had a water content corresponding to 9.5% of the total amount of dry materials (cement, sand, and gravel). The water carried by the sand and gravel was taken into account when determining the amount of mixing.
water. The amount of materials for each batch was determined by weighing. The concrete was mixed 1/2 minute dry and 2-1/2 minutes wet, in a machine mixer. Wooden forms were used. The concrete was placed in 10 cm. (4-in.) thick layers and tamped thoroughly. The columns were left in the molds for 3 days and remained in the air of the laboratory for 10 to 12 days more. They were kept moist during this time by spraying. The columns were then placed in moist sand and cured until 10 days before testing, at which time they were capped with neat cement paste. The ends of the longitudinal bars had equally thick caps. These caps proved to be very good and did not break even above the rods.

The control specimens consisted of 30 cm. (12-in.) cubes. Three cubes were made on each of five different days. The average strength results on the cubes at the age of 45 days were as follows:

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>185</td>
<td>180</td>
<td>190</td>
<td>192</td>
<td>192</td>
<td>189 kg. per cm².</td>
</tr>
</tbody>
</table>

The cubes were cured the same way as the columns and capped by means of a 1:1 mortar. The large effect of the water content on the strength of the column should be kept in mind when studying the results.
The columns were tested in a 600-ton machine. The loads were applied in six intervals and ten repetitions of loadings and unloadings were made for each interval. A complete set of observations was made for each repetition. The longitudinal deformations were measured to the nearest 0.00001 cm. by means of a Martens optical gage.

During the testing of the columns it was noted that the lateral reinforcement had been distorted by the tamping of the concrete. The lateral ties had always moved downward. The failure of the columns always occurred in the top part of the column, regardless of whether the top was up or down, in the testing machine. As the largest dislocations did not always occur at the point of failure, the failure of the columns did not seem to be affected by the dislocation of the lateral ties. The strength results indicated that the columns made on days when the moisture content in the air was high, were weaker than columns made on dry days.

The deformation of the columns during the testing was not affected by the repetition of the load as long as low loads were used. For higher loads, however, a considerable increase in deformation was noted with the increase in number of repetitions and a constant value had not been reached at the end of ten repetitions.
The distribution of the load between the concrete and the steel changed with the amount of the applied load as well as with the repetition of the load, in such a way that the steel would have to carry an increasing percentage of the total. A proof of this indication was found in the fact that columns with no longitudinal reinforcement deformed more under the same load and number of repetitions than did longitudinally reinforced columns. The lateral reinforcement did not seem to have any effect on the longitudinal deformation. It was found that the relation \( m \) between the lateral and longitudinal deformation is smaller at low loads (0-6450 kg.) than at higher loads.

The maximum loads showed large variations for individual columns of a given reinforcement. It was noted that columns made in moist, rainy, weather showed lower strengths than did columns of similar groups made in dry weather. This effect of the air moisture at time of making, makes it very difficult to judge the effect of the different amounts of the reinforcement on the strength of columns.

Weight determinations on samples cut from top, center and bottom elevation of the columns showed that the unit weight was largest at the bottom, smaller at the center, and least at the top. It was found that the longitudinal reinforcement did not increase the strength of the columns materially. A marked increase in the modulus of elasticity of the
column was found for an increase in longitudinal reinforcement.

The strength of the plain columns was 156 kg./cm², or 82.5% of the strength of the control cubes. The circumferential ties were found more effective than diagonal ties.

It is stated in the summary that the effect of the different types of lateral reinforcement was considerably less than variations caused by workmanship, and the tests made were not sufficient to establish any relation between types of lateral reinforcement and strength of the columns. The ratio of longitudinal to lateral deformation varied between 4.8 and 7.9 and gave an average value of 6.5.

SERIES 2

At the committee meeting on Oct. 9, 1908, it was decided to extend the work of Series 1. The columns to be used should be 30x30 cm. (12x12 in.) cross section and 90 cm. (36 in.) long. The distance between lateral ties was set at 15 cm. (6 in.) and the ties should be tied to the longitudinal bars in order to secure proper spacing. Five columns of a kind should be made in a closed room so as to avoid the effect of moisture in the air. The materials should be stored inside several days before the making and the sand and gravel separated and recombined before use. All columns should be tested at an age of 45 days.
On Jan. 15, 1909, it was decided to extend the Series so as to include one group octagonal columns of spiral and one of circular ring enforcement. The total number of columns was 35.

The cement, sand and gravel were of same type as in Series 1. The sand and gravel were dried in a room before being used. The concrete consisted of a 1:4 mix by volume. The water content amounted to 8.2% of the dry materials, or considerably less than the 9.5% used in Series 1. The square columns had 4 round bars of 1.6 cm. dia. while the octagonal columns had 8 bars of 1.1 cm. dia. The spiral wire was 0.6 cm. dia. and the pitch was 3.5 cm. The circular ties were of same size and number as the size and turns of the spirals. The forms were removed at the age of 2 days. The columns remained 10 days in the laboratory and were wetted regularly during this time. They were then covered in moist sand until 8-10 days before the test, at which time they were brought to the testing room and left in the air until tested. The control cubes were tested at the age of 28 days. Additional cubes were made with concrete of a water content of 9.5%. The strength tests gave an average strength of 292 kg./cm². for the concrete of 8.2% water and 242 kg./cm². for the concrete of 9.5%.

The lateral deformation was found to be greater in the center of the columns than at the ends. The failure of most of the columns occurred at the top portion. The maximum strength was found to be highest for the columns with spirals and lowest for the plain columns.
The average strength of the plain columns was 172 kg./cm². The steel used for spirals was of very high strength (Y.P. = 6100 kg./cm²). This high strength of the spirals may not have affected the occurrence of cracks, but it increased the maximum load on the column. The circular rings did not give quite as high ultimate load as did the spirals.

Tests on plain columns showed that the water content increased from bottom to top of column. The cement content also increased towards the top of the column. The unit weight decreased from bottom towards the top.
INVESTIGATION OF THE EFFECT OF CAPITALS ON DEFORMATIONS
AND STRENGTH OF REINFORCED CONCRETE COLUMNS

"UNTERRSUCHUNGEN UEBER DEN EINFLUSS DER KOPFTE AUF DIE FORMANDERUNGEN
UND FESTIGKEIT VON EISENBETON-SAULEN"

by Prof. M. Rudolff

In Deutscher Ausschuss fuer Eisenbeton, Heft 21,1912

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SERIES A

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The fact that the columns in the previous Series (Heft 5) failed near the top made it desirable to give the columns such a shape as to prevent the occurrence of end failure. The columns designed for this Series had capitals as this was considered the easiest way of eliminating the end effect.

The columns had square cross section of 30 cm. (12 in.) side length and the reinforcement consisted of 4 longitudinal bars of 3 cm. (1.18 in.) dia. The lateral reinforcement consisted of ties having a wire dia. of 7 mm. (.27 in.) with the ends hooped around one of the longitudinal bars. It was seen to that the ties lay against the bars. The vertical distance between the ties was 5 cm. (2 in.)

Series A contained 9 plain columns, namely 3 with no capital and 2 m. length, 3 with capitals and 2 m. length, and 3 without capital and of a length corresponding to the shaft length of the reinforced columns, that is 1.5 m. and 6 reinforced columns
of 2 m. overall length. Three of the reinforced columns had capitals and 3 had plain ends. The concrete mix was 1:4 by volume and the water content used was 10% of the weight of the dry materials. The temperature of the mixing water was maintained at 18°C. The columns remained 2 days in the forms and were then placed in a closed room. Up to the 8th day the columns were wetted every day. At the age of 28 days the columns were tested to failure. Ten days before test the ends of the columns were planed off as parallel as possible and planed steel plates were placed in both ends by means of a 1:1 mortar.

The control specimens consisted of 30 cm. (12 in.) cubes. The 28-day strength of the cubes varied from 199 kg./cm² to 250 kg./cm². (2830 to 3550 lb./sq.in.) with an average of 237 kg./cm². (3370 lb./sq.in.)

Both longitudinal and lateral deformations were measured. The deformations were found to be larger for the plain columns than for the reinforced columns both in longitudinal and lateral direction. The lateral deformation was larger for the columns without capital than for the columns with capitals regardless of amount of reinforcement.

It was found that Poisson's ratio was smaller near the ends than near the center of the columns.

The strength results showed difference between individual columns. It was found that these differences corresponded to the
humidity of the air at time of making. High strength corresponded to low humidity and low strength to high humidity. No similar difference in the strength of the control cubes was present. This difference in behavior is probably due to the fact that steel molds were used for the cubes while wooden forms served for the columns. The wooden forms would dry up more in dry weather and absorb more of the mixing water, thereby increasing the strength of the columns.

The plain columns of 2 m. length having capitals showed a lower strength than columns of same length and plain ends. Most columns failed in the upper end indicating that the strength of the concrete was less at the top than at the bottom.

A reduction in the length of the column shaft by the use of capitals did not increase the strength.

For the reinforced columns, however, the capitals increased the strength of the columns by about 10%.

**SERIES B**

The fact that most of the columns in Series A failed near the top or just below the capital led to an extension of the investigation. It seemed as if the concrete was crushed at the end before the steel had come properly into action. The additional columns therefore were made according to the following plan:

(a) 3 reinforced columns of 2 m. length with plain ends and steel flush with the end.
(b) 3 control cubes of 30 cm. (12 in.) side length having a 3 cm. (1.18 in.) dia. steel bar in the center. The bar extended to within 10 cm. (4 in.) from each end.

(c) 3 tests on the unit of reinforcement without the concrete filling.

(d) 3 reinforced columns of 2 m. length with plain ends and ties 5 cm. (2 in.) from the ends.

(e) 3 reinforced columns of 2 m. length with capitals of the type proposed by Moersch.

(f) 3 plain columns of length equal to the shaft length of Moersch’s columns or 1 m. (39 in.)

(g) 3 reinforced columns of 1.5 m. length with plain ends and steel flush with the ends.

A total of 18 columns was therefore included in Series B. The columns with plain ends and steel flush with ends did not fail near the ends. The strength of the plain columns showed practically no decrease with the increase in length of the column.

The extension of the steel until it became flush with the ends increased the strength of the columns considerably so that the strength became equal to the strength of the columns with capitals.

SERIES C

As the columns with plain ends and steel flush with ends showed the highest strength in Series B it was decided to make experiments for the purpose of studying how close the ends of the
steel have to be to the ends of the columns in order to give full strength to the column. Three groups of columns were made, or a total of 9 columns. The distance from the end of the steel to the end of the column was 1, 2 and 5 mm, respectively.

The results showed that in order to obtain maximum strength of the columns the steel should extend to within 10 mm. (.4 in.) from the end of the column.

The author maintains that the total strength of the column should be the sum of the yield-point strength of the steel and the ultimate strength of the plain concrete.

\[ P = P_{\text{steel}} + P_{\text{concrete}}, \]

where \( P_{\text{concrete}} \) means the strength of the plain column of same size as the reinforced. If the strength is higher it is due to the lateral reinforcement.

The results have been summarized as follows:

1. Both lateral and longitudinal deformations were reduced by the reinforcement.

2. The lateral deformations were reduced by the use of capitals.

3. The wooden form may effect the strength by its absorptive capacity.

4. The strength of the plain columns was unaffected by the use of capitals while the strength of the reinforced columns increased for columns with capitals.

5. The strength of the plain columns of 1 to 2 m. length was 75 to 80% of the cube strength.

6. No increase in strength was accompanying a decrease in length from 2 m. to 1.3 and 1 m.
7. A welded hoop around the reinforcement rods did not increase the strength of the column having ends of steel 10 mm. (.4 in.) from ends of column.

8. The failure outside of the end area is due to accidental lower strength at the place of failure.

9. The strength of the reinforced column increased with the decrease in distance between end of steel and end of column until a distance of 2 to 5 mm., where a maximum strength was reached. The manufacture of columns with plain ends was found to be considerably cheaper and easier than columns with capitals.

10. The resistance to deformation of a reinforced column is less than the sum of the resistance of the steel and the concrete.

11. The strength of the reinforced columns was less than the sum of the strength of the concrete and the steel when the rods stopped more than 10 mm. from the end of the column, but more than this sum when the distance from the ends was 1, 2 and 5 mm.

On bases of these results the sub-committee decided in its meeting of August 6, 1912 that:

A) Steel molds should be used for further experiments.

B) For further tests the distance from the end of the steel to the end of the column should be 2 to 3 mm. and that plain column ends should be used.
This bulletin describes some of the most recent European investigations on spirally reinforced concrete columns. Considere's formula for ultimate loads on spiral columns is given as:

\[ P = 1.5 fc Ac + 2400 (As + 2.4 A's) \]

where \( P \) = ultimate load, \( fc \) = strength of concrete, \( Ac \) = concrete core area, \( As \) and \( A's \) are areas of longitudinal and spiral reinforcement respectively. It is pointed out that this relation does not hold in actual tests as the ultimate loads are found to be considerably below those obtained from the formula. The series of tests described in the bulletin were made under the author's supervision at the Technical Institute at Dresden on 24 columns. The cross section of the columns was octagonal with 33 cm. (13 in.) small diameter and the length was 1.0 m. (39 in.). The percentage of longitudinal reinforcement varied from 0 to 2.0 and the spiral from 0 to 3.6. The pitch of the spirals varied from 2.5 cm. (1.0 in.) to 4.5 cm. (1.8 in.). The columns were tested at an age of 45 days. In one set of columns the concrete consisted of 1 part cement - 3 sand - 2 crushed rock by volume and the water content amounted to 8.5% by weight of the dry ingredients. Another set of columns had a 1:2-1/2:1-1/2 mix by volume and the same water content as in the first set. Three equal specimens were made for each group.
The columns were cured in moist sand until a short time before testing.

Both longitudinal and lateral deformation measurements were taken on most columns. Longitudinal on two opposite faces and lateral at 3 elevations.

The author also gives a brief review of tests by Bach and by Rudeloff and shows that Bach's tests gave ultimate strength varying from 50 to 97% of the strength computed by Considere's formula, while Rudeloff's tests gave 84 to 93% and Kleinlogel's 90 to 95%. French tests reported in 1907 on 20 columns showed that 8 columns had a strength from 1 to 15% above the computed strength, 5 gave results between 90 and 100%, 6 lay between 80 and 90%, and one gave only 75% of computed strength. As the value 1.5 in Considere's formula was obtained on one single specimen which had the spiral removed, this value should be changed to 1.0 so that the formula would read:

$$ P = f_c A_c + 2400(A_s + 2.4 A') $$

Spirals should not have a pitch larger than 1/7 or rather 1/9 of the diameter of the spiral.

The average Poisson's ratio for Kleinlogel's experiment was found to be about 1/6, varying from 1/4 to 1/9 for the different loading intervals. The ratio was depending upon the modulus of elasticity of concrete so that a large ratio was obtained for concrete having a low modulus of elasticity.
The lateral reinforcement in form of spiral seemed to have very little effect on the deformations for loads up to about 35 to 40 kg./cm², but it was detectable. At higher loads, but within working limits, the effect of the spirals was very marked. This is contrary to previous findings which seem to indicate that the spirals become effective only near ultimate load.
BUCKLING TESTS ON REINFORCED CONCRETE COLUMNS

"KNICKUNGSVERSUCHE MIT EISENBETONSAULEN"

By C. Bach

In Zeitschrift des Vereines Deutscher Ingenieure, Page 1969, 1913

Very little reliable data on the buckling effect or relation between length and strength of columns existed at the time of this investigation. The tests by Menager reported in the Annales des Ponts et Chaussées 1909, 2 vol., page 106, on three specimens 8 cm. thick, 40 cm. wide and 320, 400 and 480 cm. high respectively cannot be classified as buckling tests. The thickness of 8 cm. was too small for reliable information on the behavior of concrete.

The tests reported by Spitzer in HEFT 3, MITTEILUNGEN UEBER VERSUCHE, EISENBETONAUSCHUSS DES OSTERREICHISCHEN INGENIEUR UND ARCHITEKTENVEREINS are not very suitable for comparison. The length of the columns varied from 300 to 700 cm., but the age at test varied. Some columns were tested at 91 days while others were tested at 65 to 70 days. Columns for real comparison should be made simultaneously of the same mix.

This situation stirred the author to discuss the question with Dr. Moersch who promised to have his firm (Wayss & Freytag) donate the materials and the forms for the making of 900 cm. and 120 cm. high columns. It was agreed to have the specimens made at the Stuttgart laboratory. The number of columns to be tested was 27.
The program which was accepted by Dr. Moersch was as follows:

(1) a. 3 columns 900 cm. high having 320 cm. square cross section and 4 longitudinal bars of 30 mm. dia. corresponding to 2.0% reinforcement. The spirals were of 5 mm. wire, had a pitch of 45 mm. in the center portion and 50 mm. at the ends. The concrete mix was 1 cement to 4 parts sand and gravel by volume, and contained 9.3% water by weight of the dry materials. The mix was therefore rather plastic.

(1) b. 3 columns of 120 cm. length made from the same mix, and with the same percentage of reinforcement as group (1) a.

(1) c. 3 plain columns of 120 cm. length made from the same mix.

(1) d. Nine 30 cm. cubes.

(2) a. 3 columns of 900 cm. length and as (1)a, except that only 1.2% longitudinal reinforcement was used (4 bars 20 mm.)

(2) b. 3 columns as above but of 120 cm. length.

(2) c. 3 plain columns 120 cm. long.

(2) d. Nine 30 cm. cubes.

(3) a. 3 columns 900 cm. long. Reinforced as in (1) but having a different concrete mix. The concrete mix was 1 cement - 2 sand - 2 gravel by volumes, and 11.3% water by weight.

(3) b. 3 columns 120 cm. long but otherwise as in (3) a.

(3) c. 3 plain columns of same concrete mix as in (3) a, and (3) b, and 120 cm. long.

(3) d. Nine 30 cm. cubes.
The tests of the steel gave following results:

<table>
<thead>
<tr>
<th>Yield-point stress</th>
<th>Size of round bars mm.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5</td>
</tr>
<tr>
<td>Group 1</td>
<td>3384</td>
</tr>
<tr>
<td>Group 2</td>
<td>2899</td>
</tr>
<tr>
<td>Group 3</td>
<td>3027</td>
</tr>
</tbody>
</table>

Cement Tests:

**Tensile Strength at 28 days**

A) 1 day moist air, 27 days water: 36.7 kg./cm²

B) 1 day air 6 days water 21 days air: 43.6 kg./cm²

**Compressive strength at the age of 28 days**

A) 1 day moist air 27 days under water: 438 kg./cm²

B) 1 air 6 water 21 air curing: 492 kg./cm²

**Cube strength of the concrete specimens at an age of 45 days**

Average for group 1 = 360 kg./cm².
" " " 2 = 376 "
" " " 3 = 283 "

The tests were carried out in a vertical testing machine with spherical bearing block at both ends of the column. The bearing blocks were well oiled so as to eliminate friction as far as possible.

The following observations were made:

A) On the 900 cm. long columns the total, permanent and elastic deflection at midheight and the maximum load at which the buckling occurred. The buckling usually appeared at the center of the columns.
B) On the 120 cm. long columns the total, the permanent and the elastic deformations as well as the maximum load at failure.

C) On the cubes the compressive strength.

The obtained maximum loads on the 900 cm. long columns in group (1) were as follows:

\[(302,000 + 506,000 + 261,000) + 3 = 289,667 \text{ kg.}\]

For the 120 cm. long columns of same group:

\[(399,000 + 594,000 + 374,000) + 3 = 385,667 \text{ kg.}\]

The ratio therefore becomes:

\[289,667 : 385,667 = .75\]

For the 900 cm. columns in group (2) the maximum loads were:

\[(290,000 + 270,000 + 260,000) + 3 = 270,000 \text{ kg.}\]

For the 120 cm. long columns:

\[(390,000 + 390,000 + 350,000) + 3 = 370,000 \text{ kg.}\]

The ratio is:

\[270,000 : 370,000 = .75\]

For the 900 cm. long columns in group (3) the maximum loads were:

\[(230,500 + 228,500 + 259,500) + 3 = 232,835 \text{ kg.}\]

For the 120 cm. long columns:

\[(329,000 + 324,000 + 279,000) + 3 = 310,667 \text{ kg.}\]

The ratio is:

\[232,835 : 310,667 = .75\]

The percentage strength of the 900 cm. long columns as compared with the 120 cm. long columns was for group (1) = 75%, for group (2) = 75%, and for group (3) = 75%.
This shows that approximately the same ratio holds for all three groups, indicating that the effect of amount of reinforcement and the quality of concrete is very small.

The maximum loads on the 120 cm. long plain columns were as follows:

For group (1):
\[
(316,000 \times 306,000 \times 306,000) \div 3 = 310,667 \text{ kg.}
\]
or an average of 305 kg./cm².

For group (2):
\[
(332,000 \times 331,000 \times 356,000) \div 3 = 339,667 \text{ kg.}
\]
or an average of 330 kg./cm².

For group (3):
\[
(225,000 \times 245,000 \times 229,500) \div 3 = 233,167 \text{ kg.}
\]
or an average of 226 kg./cm².

The ratio between the strength of the 120 cm. long plain columns and the cube strength was as follows: Group 1 = 305/360 = .85; Group 2 = 330/375 = .84; Group 3 = 226/285 = .80.

The permissible load \( P \) on the columns subjected to buckling is ordinarily computed from either:

1) Navier's buckling formula:
\[
P = \frac{f}{1 + \frac{k}{E}} \frac{I^n}{I}
\]  
(1), or

2) Euler's formula:
\[
P = \frac{\pi^2 E I}{L^2}
\]  
(2), where:
f = Cross section of column in cm$^2$.
k = Allowable compressive stress in the material in kg./cm$^2$.
x = Buckling coefficient
l = Length of column in cm.
I = Moment of inertia of column in cm$^4$.
S = Factor of safety
E = Modulus of elasticity.

As the 900 cm. long columns had the same cross section and amount of reinforcement as the 120 cm. columns, Navier's formula is suitable for comparison and the buckling coefficient can be found:

$$V = \frac{1 \times \frac{f \cdot 120^2}{I}}{1 \times \frac{f \cdot 900^2}{I}}$$

(3)

where $V$ is the ratio between the strength of the columns of various lengths.

$V$ is known from the experiment and the equation is solved for $x$:

$$x = \frac{1 - V}{\frac{f}{I} (V \cdot 900^2 - 120^2)}$$

If the cross section of the steel is introduced as 15 times the equal area of concrete the following results are obtained:

a) For the columns in group (1) having cube strength of 360 kg./cm$^2$ and 2.8% reinforcement: $x = \frac{1}{6000460}$

b) For columns in group (2), cube strength of 376 kg./cm$^2$ and 1.2% reinforcement: $x = \frac{1}{6000459}$

c) For group (3) cube strength of 283 kg./cm$^2$ and 2.8% reinforcement: $x = \frac{1}{6000448}$

The values of $x$ is practically constant and the average is 0.0000448.
It is common to use a value of $x$ of .0001. These experiments show that it is sufficient to use half this value, that is $x = .00005$. When formula (1) is used it should be kept in mind that the compressive strength of a concrete prism with length 3 to 4 times the lateral dimension is considerably lower than the cube strength. The value of $k$ to use will then be $4/5 = .8$ of the results here given. If a factor of safety of 6 is required for a cube strength of 270 kg./cm$^2$, the value to use for $k$ is

$$k = \frac{270}{6} \cdot 4/5 = 36.$$  

For the ultimate strength according to formula (1) and the values of $x = .00005$, $k = 4/5$ of the cube strength, $n = 15$ for the computation of $f$ and $I$, the following results are obtained:

<table>
<thead>
<tr>
<th>Group No.</th>
<th>Computed kg.</th>
<th>from Test kg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>296291</td>
<td>299667</td>
</tr>
<tr>
<td>2</td>
<td>258666</td>
<td>270000</td>
</tr>
<tr>
<td>3</td>
<td>235916</td>
<td>232833</td>
</tr>
</tbody>
</table>

It should be kept in mind that the value of $n$ depends upon the yield-point strength of the steel and the strength of the concrete. For a yield-point strength of 3000 kg./cm$^2$ and a concrete strength of 300 kg./cm$^2$, the value of $n$ becomes $3000/300 = 10$.

For comparison of the results with Euler's formula the value of $E$ immediately before the buckling must be known. For the total deformation of the concrete measured immediately before the buckling the following results were obtained:

a) For groups 1 and 2: $E = 193300$ kg./cm$^2$.
b) For group 3: $E = 131400$ kg./cm$^2$.  


If the value of \( n \) to be used in the computation of the moment of inertia is taken as the ratio: \( n = \frac{\text{Yield-point of steel}}{\text{Prism strength of concrete}} \) which gives \( n = 9 \) for group 1 and 2 and \( n = 12 \) for group 3, the following results are found:

<table>
<thead>
<tr>
<th>ULTIMATE LOAD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group No.</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

The value of the computed and observed strength of the columns is found to be very nearly the same if the values of the modulus of elasticity \( E \) and the ratio \( n \) are taken from the observed data and used in Euler's formula.

If the values specified by the Prussian Government are used, that is \( E = 140,000 \) and \( n = 15 \), the results from Euler's formula are:

<table>
<thead>
<tr>
<th>Group No.</th>
<th>Computed</th>
<th>Tested</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>253752 kg.</td>
<td>289967 kg.</td>
</tr>
<tr>
<td>2</td>
<td>202817 kg.</td>
<td>270000 kg.</td>
</tr>
<tr>
<td>3</td>
<td>261418 kg.</td>
<td>232835 kg.</td>
</tr>
</tbody>
</table>

This shows that the test results deviate considerably from the computed results.

**SUMMARY**

The tests showed that for the given conditions the value of \( x \) to use in formula (1) is \( 0.000045 \) or approximately \( x = 0.00005 \) or half the ordinary assumed value \( x = 0.001 \).
For practical purposes the values to be used with formula (1) in computing short and long reinforcement concrete columns are:

\[ x = 0.0005, \text{ and } k = \frac{\text{prism strength of concrete}}{\text{factor of safety}} = \frac{\text{s cube strength}}{\text{factor of safety}} \]

Formulas (1) should be preferred for formula (2) when the specified values of \( E \) and \( n \) are used in the computations.
TEST RESULTS ON COMPRESSIVE STRENGTH OF PLAIN CONCRETE
COLUMNS OF VARIOUS Heights

"DIE ERGEBNISSE VON VERSUCHEN ZUR ERMITTELUNG DER DRUCKFESTIGKEIT
VON UNEINBEHÄRTEN BETONSAULEN BEI VERSCHIEDENER HÖHE DERSELLEN"

by G. Bach

In Deutsche Bauzeitung, Mitteilungen über Zement, Beton- und
Eisenbetonbau. No. 5, page 55, 1914

--------------------------------------------------------------------------------

Up to the time of making these tests the relation between
the strength and the height of a concrete specimen had not been
studied very fully and very little reliable data had been given.
The investigation was made for the purpose of clearing up some of
the doubts regarding this relation without going to so large
heights that buckling would be introduced.

The cross-section of the specimens in these tests was
square of 32 cm. side length (a = 32 cm.). The ranges covered
were as follows:

(1) 3 specimens, a = 32 cm. h = 16 cm. h/a = .5
(2) 3 specimens, a = 32 cm. h = 32 cm. h/a = 1.0
(3) 3 specimens, a = 32 cm. h = 64 cm. h/a = 2.0
(4) 6 specimens, a = 32 cm. h = 120 cm. h/a = 3.7
(5) 3 specimens, a = 32 cm. h = 256 cm. h/a = 8.0
(6) 3 specimens, a = 32 cm. h = 384 cm. h/a = 12.0

The concrete mix was 1:4 with 9.3% water by weight of the
dry materials. The specimens were tested at an age of 45 days.
Besides the prisms 30 cm. cubes were made, 3 for group (1) and
one for each of the other groups. The prisms were made in wooden
molds while iron forms were used for the cubes. The results obtained were as follows:

<table>
<thead>
<tr>
<th>h/a</th>
<th>Av. Strength, Kg/cm²</th>
<th></th>
<th></th>
<th>Strength ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Prisms</td>
<td>Cubes</td>
<td>Prisms / Cubes</td>
<td></td>
</tr>
<tr>
<td>.5</td>
<td>427</td>
<td>303</td>
<td>1.41</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>310</td>
<td>303</td>
<td>1.02</td>
<td></td>
</tr>
<tr>
<td>2.0</td>
<td>288</td>
<td>303</td>
<td>.95</td>
<td></td>
</tr>
<tr>
<td>3.7</td>
<td>316</td>
<td>365</td>
<td>.87</td>
<td></td>
</tr>
<tr>
<td>8.0</td>
<td>270</td>
<td>313</td>
<td>.96</td>
<td></td>
</tr>
<tr>
<td>12.0</td>
<td>263</td>
<td>313</td>
<td>.94</td>
<td></td>
</tr>
</tbody>
</table>

The results show that the ratio between the strength of prisms and that of cubes approached a constant value of about .8, that is 4/5 of the cube strength, when the height of the prisms is increased without going so far as to introduce buckling. The conclusion reached is therefore: **the column strength can for practical purposes be taken as 4/5 = 80% of the cube strength.**

A similar relation has been found both for cast-iron and wrought iron prisms giving approximately same values as found for concrete.

The decrease in strength with increased height is a result of the method of applying the load rather than of the property of the material.
TESTS ON REINFORCED AND PLAIN CONCRETE SPECIMENS SUBJECT TO
CENTRIC AND ECCENTRIC LOAD

"VERSUCHE MIT BEWEHRTEM UND UEBERWEHRTEM BETONKÖRPERN, DIE DURCH
ZENTRISCHEN UND EXCENTRISCHEN DRUCK BELASTET WERDEN"

by C. Bach

In Deutsche Bauzeitung, Mitteilungen ueber Zement, Beton- und
Eisenbetonbau, No. 6, page 45, 1914

These tests were made for the purpose of studying the re-
liability by which the stresses in concrete specimens can be com-
puted for plain and reinforced sections loaded centric as well as
eccentric. The tests were sponsored by Dr. Moersch and carried
out at the Stuttgart laboratory.

The following specimens were made:

A. Reinforced specimens:

a. 12 specimens reinforced on one side with 4 longitudinal
rods of 16 mm. diameter corresponding to 1/2% reinforcement. 3
specimens were loaded centric while 9 had eccentric loading. The
amount of eccentricity was 200, 300 and 500 m. respectively.

b. 15 specimens reinforced on both sides with 4 longitudi-
dinal bars, a total of 8 bars of 16 mm. diameter corresponding to
1.0% reinforcement. The rods were encircled by a spiral and con-
nected in pairs by means of hoops. 3 specimens were straight with
centric loading, 9 with enlarged ends and loaded at an eccentricity
of 100, 200, 300 and 500 mm. respectively, and 3 with hoops instead
of spirals and an eccentricity of 200 mm. These latter columns were made on a special request by Dr. Kosen.

e. 15 columns reinforced with 8 longitudinal rods of 22 mm. diameter corresponding to 2% reinforcement. The same system of overall spiral and individual hoops as in (2) was used. 5 specimens had centric loading and 10 had eccentric loading with an eccentricity of 100, 200 and 500 mm, respectively.

B. Plain concrete specimens:

15 specimens had no reinforcement. They had all enlarged ends and 3 were loaded at the center while the other 10 had eccentricities of 100, 150 and 200 mm, respectively.

The concrete mix consisted of 1:2:3 parts by volume with a water content of 9.2% by weight of the dry materials. The water content was such as to secure concrete within the range used in ordinary reinforced concrete constructions. The average compressive strength of 50 cm. cubes from this mix obtained from 55 specimens at the age of 45 days was 225 kg./cm².

The columns were tested in a vertical machine at an age of 45 days. The following observations were made:

1. The load at the observation of the first crack with record of the opening of the crack during the increased loading.

2. The ultimate load, that is, the load at failure of the specimen.

3. The total, the permanent and the elastic deformations as well as the shortening on both sides of the specimen on a gage line of 100 cm.

The following comments are made on the tests of the 55 columns and 55 cubes:
"Of special interest is the consecutive occurrence of cracks in the specimen during the loading. In one example the first crack was noted at a load of 56,000 kg. This specimen had an eccentricity of 200 mm. The crack was short and fine. At a load of 40,000 kg. three new cracks were noted, one of which extended across the whole back of the column and also was noticeable at the sides. At a load of 44,000 kg. a total of 14 cracks was noticeable on the back while 5 showed on the sides. The number and size of cracks increased with the load. At a load of 95,000 kg. a gradual failure of the concrete occurred in the tension section. When this load had been applied for 9 minutes the specimen failed."

The relation between the ultimate load and the amount of eccentricity is shown in a graph which gives different curves for each percentage of reinforcement. The tabular results are given below:

**ULTIMATE LOAD IN Kg. (2,205 lb.)**

<table>
<thead>
<tr>
<th>Amount of Eccentricity</th>
<th>0%</th>
<th>1/2%</th>
<th>1%</th>
<th>2%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>276167</td>
<td>260333</td>
<td>336333</td>
<td>404667</td>
</tr>
<tr>
<td>100 mm.</td>
<td>136000</td>
<td>202500</td>
<td>225000</td>
<td></td>
</tr>
<tr>
<td>150 mm.</td>
<td>91853</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>200 mm.</td>
<td>23967</td>
<td>92000</td>
<td>124000</td>
<td>157500</td>
</tr>
<tr>
<td>300 mm.</td>
<td>60333</td>
<td>69600</td>
<td>105000</td>
<td></td>
</tr>
<tr>
<td>500 mm.</td>
<td>29967</td>
<td>32350</td>
<td>53500</td>
<td></td>
</tr>
</tbody>
</table>
The gross cross section was 40 by 40 cm. The overall lengths were 200 and 250 cm. The centric loaded columns were straight while the eccentric loaded ones had enlarged ends. The straight section between the enlargements was 122 cm. The cross section of the head was 40 by 90 cm, except for the plain columns which had a cross section of 40 by 60 cm.

The results showed that the plain concrete column was more affected by the amount of eccentricity than the reinforced one. The results have also shown that the resistance to combined axial and bending stresses for these columns very nearly followed the same laws as developed for homogen materials.

A more detailed report on the same investigation is given in Heft 166-169, FORSCHUNGSARBEITEN AUF DEM GEBIETE DES INGENIEURWESENS, JULIUS SPRINGER, 1914.
Eighty-eight columns were included in this investigation. The tests reported in Heft 21 of Deutscher Ausschuss fur Eisenbeton showed that:

1. In order to obtain the highest efficiency of the reinforcement in concrete columns, the columns should be provided with strong end capitals, and that the longitudinal reinforcement should be carried to within 2 or 3 mm. of the ends of the column, and

2. That the use of wooden forms may affect the result by the fact that the wooden surfaces absorb part of the mixing water.

Iron forms were therefore used for the columns included in the Series reported in Heft 22. The longitudinal bars extended to about 2.5 mm. from the ends of the columns.

The various lateral reinforcements used were:

a) Welded rings or ties
b) Rings or ties looped together

Columns of square cross-section
c) Diagonal ties
d) Spirals
e) Welded rings or ties
f) Rings or ties looped together

Octagonal columns
g) Spirals
Each group had 6 columns, 3 with lateral reinforcement of 5 cm. wire and 3 of .7 cm. wire.

The square columns had a cross section of 30 by 30 cm. with broken corners, and an area of \( 900 \text{ cm}^2 \). The octagonal columns had a cross section of \( 200 \text{ cm}^2 \). The square columns had \( 12.56 \text{ cm}^2 \) of longitudinal reinforcement and the octagonal columns had \( 12.32 \text{ cm}^2 \). The length of the columns was 1.50 m.

The following specimens were used as control specimens:

- 3 plain square columns
- 3 plain octagonal columns
- 36 concrete cubes.

The concrete in this series consisted of a 1:4 mix.

Furthermore tests were made on additional 20 columns and 13 cubes:

- 6 columns were made for the purpose of studying the effect of the richness of the concrete mix, 3 were plain concrete and 3 were reinforced with rings looped together. The rings had a wire dia. of .7 cm. and were spaced 5 cm. on centers. The concrete mix was 1:2-1/2 for these columns.
- 7 columns were made for the study of difference in effect of lateral reinforcement in hollow and in solid columns.
- 4 columns were made for the study of the effect of the concrete shell outside the lateral reinforcement and 3 columns were made with a large amount spiral reinforcement. The spirals had a wire dia. of 1.2 cm. and a pitch of 3 cm.

Preliminary tests on concrete specimens having coarse sand and 8% water gave a strength of 351 kg./\text{cm}^2. (5000 lb./\text{sq.in.})
while the specimens having fine sand and 8.75% water failed at 305 kg./cm² (4350 lb./in²). Samples of concrete made with fine sand and 8% water (same mix in all cases) gave strength of 360 kg./cm² (5120 lb./in²).

Two concrete mixes were used in this series, 1:4 and 1:2-1/2. The columns of the 1:4 concrete mix had a water content equal to 10% of the materials while the concrete of 1:2-1/2 mix had 11% water.

The columns were made in steel molds. They were left in molds for two days, then placed in a closed room and wetted down once a day for the first 6 days. Ten days before the testing, the columns were capped on both ends with a 1:1 mortar.

The concrete control specimens consisted of one 30 cm. cube for each column. The cube was made, cured and tested in a similar manner as the column.

The columns were tested in a 600-ton (1,320,000 lb.) machine.

Longitudinal deformations were measured on all columns and lateral deformations on 18 square columns. The deformation observations were made by means of Marten's mirror apparatus to the nearest 1/100,000 cm. (1/254,000 in.)

No distinct difference in the lateral deformations was found for the different kinds of lateral reinforcements.

The different types of lateral reinforcement seemed to have no effect upon the longitudinal deformations.
The concrete in reinforced columns deformed more for a
given load than did the concrete in plain columns.

The ratio between the strengths of the plain square
columns and the strengths of the control cubes was found to be
0.94. The corresponding ratio for octagonal columns was 0.81.

The deformation of the columns for a given load increased
with the increase in number of repetition of loading.

It was found that the rings had the same effect upon the
strength of the column as a spiral of same size and pitch equal
to the distance between the rings. The diagonal reinforcement
did not add to the strength of the columns.

A majority of the columns failed near mid height. The
concrete of a 1:2-1/2 mix gave a cube strength of 326 kg./cm²
(4650 lb./in².), whereas the 1:4 mix gave a cube strength of 234
kg./cm². (3320 lb./in².). The ratio of column strength to cube
strength was 0.81 for the 1:2-1/2 concrete. It was found that the
effect of lateral reinforcement on the strength of the columns
was less for the rich mix than for the lean mix.

Tests on hollow and solid columns showed that the solid
columns utilized the material more advantageously than did the
hollow columns both for plain and reinforced specimens.

The stress-strain curve for columns having protective
shell was nearly in accordance with value computed from the column
with no shell plus the value for the area of the shell. This re-
lation held to near the ultimate strength of the column at which
the shell partly fell off so that practically no strength was added by the shell.

Columns having different grades of steel for lateral reinforcement showed an increase in strength with increase in yield-point stress of the steel.

The tests showed that for high percentage of lateral reinforcement the strength added by the rings was equal to the strength added by an equal amount of spiral reinforcement of same grade of steel.

As a conclusion of the report Rudolff states that:
"The present tests conform completely the conclusion reached in my first report on earlier tests, (Heft 5, page 71) that the division of compressive stresses on the concrete and the steel areas of the column changes continually for increasing loads as well as for repetition of loads, and that the stress-distribution is different at different heights in the column.

Furthermore they confirm the contention that the effects of the reinforcement on the strength of the columns depends upon the composition of the concrete.

Under these conditions I have at present refrained from computing the strength of the columns from the compressive strength of the concrete and from the areas of concrete and reinforcement. I want to reserve this question until next Series of column tests on the effect of different percentages of reinforcement has been completed."
"A cylinder subjected to axial load undergoes a shortening in its longitudinal direction and an expansion in its lateral direction. If the lateral expansion is prevented, the strength of the cylinder will increase. This old knowledge has promoted, especially for concrete columns, the use of lateral steel ties or spirals. The following notations are used:

\[ h = \text{pitch of spiral or distance between lateral ties}. \]
\[ d = \text{diameter of wire used for ties or spirals}. \]
\[ D = \text{average diameter of ties or spirals}. \]

With these notations the effect of the lateral restraint may be visualized as resulting from a hollow cylinder of a height \( h \) and a volume \( \pi D \cdot d^2 \), or of a cross section of \( \frac{2}{4} \cdot \frac{D \cdot d^2}{h} \), as \( D \) is the average diameter of the hollow cylinder. The wall thickness of the cylinder would then be:

\[ s = \frac{1}{4} \cdot \frac{d^2}{h} \]

A theoretical study of the increase in strength due to lateral restraint by means of the elastic theory will give much smaller effect of the restraint than actually obtained on columns tested to failure. The increase in strength is strange insofar that with decreasing \( h \) it first increases slowly but later increases very much faster than expected. Tests made in 1904 showed
this very clearly. The fact that the use of the ordinary elastic theory only gives approximate correct values for certain load conditions led to the following explanation:

The effect of the ties or spirals on the strength of the column is larger than expected from a lateral reinforcement. This is found especially for the case \( h = d \), that is, for no space between the rings. The rings touch each other and in addition to the lateral restraint comes the increased resistance to longitudinal deformation. If the rings or spirals do not touch each other, but have small openings the material between the rings will add considerably to the longitudinal restraint on account of not being free to slip towards the interior of the column. The effect of this stiffening must decrease very rapidly with the size of the free opening between the rings or between spiral turns.

In order to investigate these relations 30 columns were made. The purposes of the tests were:

(a) to observe the load at appearance of the first crack in the test columns, and

(b) to observe the maximum load on the columns.

The columns were octagonal in cross section with a small diameter of 330 mm. The overall height of the columns was 510 mm, with 5 mm caps at each end. Spirals and longitudinal bars were carried through to the caps.

Group 1 had a concrete mix of 1-2-3 by volume and the water content by weight of the dry materials was 9.2%.
This group included 13 columns, 2 plain and 2 of each of reinforced columns having values of \( h = 100, 75, 50, 40, 32, 24, 16, 8 \) mm. Group 2 included 12 columns, 2 plain and 2 of each having values of \( h = 75, 32, 24, 16, 8 \) mm. The concrete mix in group 2 consisted of 1 cement-5 sand by volume. The water content was 15.4% by weight.

The spiral wire was 8 mm. in dia. and the diameter of the spiral was 27 cm. for both groups. The longitudinal reinforcement consisted of 8 bars of 1.02 cm. dia. The yield-point strength of the longitudinal reinforcement was 3486 kg./cm², for the spirals in Group 1 it was 2775 kg./cm², and for the spirals in Group 2, 2583 kg./cm². The strength of the plain columns in Group 1 was \( \frac{137+121}{2} = 129 \) kg./cm², and in Group 2 \( \frac{62+61}{2} = 61 \) kg./cm². The following table gives the average results obtained in both groups of tests:

<table>
<thead>
<tr>
<th>Spiral Reinforcement</th>
<th>Pitch h mm.</th>
<th>Group 1</th>
<th>Group 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Load at crack Kg. Max.</td>
<td>Load at crack Kg. Max.</td>
</tr>
<tr>
<td>0 plain</td>
<td>0</td>
<td>117500</td>
<td>117500</td>
</tr>
<tr>
<td>.77</td>
<td>100</td>
<td>127500</td>
<td>156500</td>
</tr>
<tr>
<td>1.15</td>
<td>75</td>
<td>127500</td>
<td>163750</td>
</tr>
<tr>
<td>1.54</td>
<td>50</td>
<td>115000</td>
<td>187500</td>
</tr>
<tr>
<td>1.96</td>
<td>40</td>
<td>157500</td>
<td>205000</td>
</tr>
<tr>
<td>2.41</td>
<td>32</td>
<td>145000</td>
<td>235000</td>
</tr>
<tr>
<td>3.25</td>
<td>24</td>
<td>142500</td>
<td>274500</td>
</tr>
<tr>
<td>4.75</td>
<td>16</td>
<td>160000</td>
<td>348500</td>
</tr>
<tr>
<td>9.79</td>
<td>8</td>
<td>157500</td>
<td>550000</td>
</tr>
</tbody>
</table>
It is noted from these results that the strength increases with the decrease in the value of \( h \). The load at appearance of first crack increases only slightly with the decrease in \( h \), nearly in conformity with the values obtained from the elastic theory.

The maximum loads on the columns increases first slowly but as \( h \) becomes small increases very fast with the decrease in \( h \). This must be due to the increased stiffening effect mentioned before, as the pitch becomes small.

An equation for the strength of a column at maximum load should therefore include terms for the pitch \( h \) in such a manner that the effect becomes very large as \( h \) approaches small values.

The increase in maximum load found here for spirally reinforced columns holds also for other kinds of lateral reinforcement. It is found for hooped columns where the ends of the hoops are wound around the longitudinal bars. Each case has to be considered careful however before a general conclusion should be drawn.

In the case of square columns with longitudinal bars hooped together it must be remembered that the lateral restraint is not so large as by circular sections but the stiffening effect from decreasing value of \( h \) will also here play an important role. The results from 1904 showed this very plainly. The effect of the decrease in distance between lateral reinforcement can therefore be increased very much as the distance between the layers of reinforcement becomes small."
Fig. 1 shows the relation between pitch of spiral and strength of column, and Fig. 2 the relation between the amount of spiral reinforcement and the strength. The load at observation of first crack is shown by broken lines. The results indicate that the spirals added strength to the column equal to about 1.3 times an equivalent amount of longitudinal reinforcement of same yield-point stress.
Fig. 2. Relation between Percentage of Spiral Reinforcement and Strength of Column. Same data as used in Fig. 1.
Fig 1: Relation between Pitch of Spiral and Strength of Column.
Diameter of Spiral 270 mm (10.6 in), Wire Diameter 8 mm (9/32 in)