COMPRESSIVE STRENGTH OF CONCRETE IN FLEXURE AS DETERMINED FROM TESTS OF REINFORCED BEAMS

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### ERRATA

Page 34. The values in Columns 6 and 9 of Table 9 are incorrect, and the following values should be used.

<table>
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<th>Group No.</th>
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<th>Column 9</th>
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The errors are not of sufficient magnitude to affect any conclusion in the paper. Even the curves in Figures 21 and 22 are not affected appreciably.
INTRODUCTION

1. General Statement. The practice in designing reinforced concrete beams in the United States has been shaped and controlled largely by the recommendations of a Joint Committee on Concrete and Reinforced Concrete appointed in 1903, which made its final report in 1916. This Committee recommended the straight line formula for use in computation of working stresses in reinforced concrete beams, that is, a formula which assumes a constant modulus of elasticity. The working stress so computed was fixed at a maximum of 32.5 per cent of the strength of the concrete as determined from compression tests of 8 by 16-in. cylinders 28 days old. The recommended working stress in the reinforcement was 16,000 lb. per sq. in. This combination of stresses requires about 0.75 per cent reinforcement.

This recommendation is found in the 1909 and 1912 progress reports and is repeated in the final report of 1916. It appears to provide a factor of safety of approximately 3 for compressive stresses and 2.5 for tensile stresses. Test beams proportioned to meet these conditions have always failed in tension in the reinforcement with a factor of safety between 2.5 and 3.0. However, tests on beams with sufficient reinforcement to develop compression failure have shown factors of safety ranging approximately from 4 to 8, but with the majority of cases above 5 when judged on the basis of working stress in the concrete of 32.5 per cent of the cylinder strength.

Such conservative stresses were proper for the design of earlier days and in many cases may still be proper. However, two
important results have followed the adherence to these working stresses computed from the straight line formula. Many designers appear to have lost sight of the facts that (a) the compressive stresses in reinforced concrete beams computed by the straight line formula for loads near the maximum generally are considerably greater than the compressive strength of the control cylinders, and (b) that it is sometimes desirable and permissible to use a larger percentage of reinforcement than that which will give a computed stress in the concrete of 32.5 per cent of the cylinder strength, simultaneously with 16,000 lb. per sq. in. in the steel.

With the improvement gained in recent years in our knowledge of the properties of concrete and control of its quality, it is desirable to make modification in the relation between the computed compressive stresses and the computed tensile stresses, which will result in utilizing both materials (concrete and steel) economically. The fact that among the thousands of beams tested in laboratory investigations only a very small percentage have failed in compression indicates that in general more steel is required to utilize the strength of the concrete sufficiently.

In the 1921 and the 1924 reports of the Joint Committee on Standard Specifications for Concrete and Reinforced Concrete the maximum working stress for concrete in flexure was raised to 40 percent of the cylinder strength. This working stress, however, has not been universally accepted.

This investigation was planned to determine the relation between the compressive strength of 6 by 12-in. control cylinders and the strength of the same concrete computed from beam tests in which the concrete failed in compression. The investigation included two series of tests, one in which with beams of equal dimensions the cylinder strength of the concrete ranged from about 1400 to 5800 lb. per sq. in. and another series in which for two different strengths of concrete (2800 and 4000 lb. per sq. in.) the total depth of the beams varied from 6 to 17 inches. A large enough percentage of reinforcement was used in all cases to insure that failure would not occur in tension of the longitudinal reinforcement. Web reinforcement was used to guard against tension failure of the web. Thus it was intended to eliminate all causes of failure except compression.
This investigation was carried out in the Fritz Engineering Laboratory of Lehigh University, Bethlehem, Pennsylvania, from January to March 1930.

2. Acknowledgment. The cement for the investigation was furnished by the Lehigh Portland Cement Company of Allentown, Pennsylvania, the reinforcement steel by the Rail Steel Bar Association of Chicago, Illinois and the aggregates by the Warner Company of Philadelphia. All of these materials were furnished without charge. All other expenses were borne by the Portland Cement Association of Chicago and Lehigh University.

The success of this investigation has been largely due to the interested cooperation of the men connected with the work. Especial acknowledgment is made to C. C. Keyser, Laboratory Assistant and C. L. Kreidler and R. E. Gohl, Senior students in the Department of Civil Engineering.

3. Earlier Tests. The earliest study the writers have found comparing the strength developed in the beam with that developed in the cylinder was reported by Dr. Fritz Emperger in “Beton und Eisen,” Heft I, 1903, page 23. This study was based upon tests carried out at different places, some by Ransome in Boston and some by Sanders in Amsterdam, Holland. Values of the ratio of strength of concrete computed from the beams tested by Sanders to the strength shown by the control cubes are shown in Fig. 1. The strengths for the beam tests were computed by a straight line formula. As a result of this study the “German Committee for Reinforced Concrete” initiated a more extensive investigation of the same subject. The tests by the German Committee were carried out at the Technischen Hochschule at Stuttgart, Germany, under the direction of C. Bach and O. Graf. The results of these tests were originally reported in Heft 19, “Deutscher Ausschuss für Eisenbeton” and have been abstracted from Morsch’s book “Der Eisenbetonbau,” Fifth Edition, page 330, and plotted in Fig. 1. The control specimens for these beam tests were 30-centimeter (11.8-in.) cubes. In order to compare the results on the basis of ordinary 6 by 12-in. cylinders the corresponding ratios of beam strength to cylinder strength are shown at the right-hand side of the diagram. For convenience this ratio is termed the beam-cylinder strength ratio. In making
the conversion the strength per unit of area of the cube has been taken as 13 per cent* greater than that for the 6 by 12-in. cylinder.

A similar study of results from different sources in the United States was published in a paper by Slater and Zipprodt in the

![Diagram of beam-cube strength ratio vs. compressive strength of cube, lb per sq in.]

**FIG. 1—FOREIGN TESTS OF HEAVILY REINFORCED CONCRETE BEAMS**

*Proceedings* of the American Concrete Institute, 1920, page 120. The essential results from these tests are shown in Fig. 2.

Figures 1 and 2 show that for all these tests the strength of the concrete computed by the straight line formula was considerably greater than the strength of the concrete as shown by tests of cubes or cylinders, that is, the beam-cylinder strength ratio was always greater than one. The tests also indicate that the beam-cylinder strength ratio increased as the strength of the concrete decreased. The variation of this ratio with the strength of the concrete was less marked for the tests reported by Bach and Graf than for the others reported.

4. Program. Since the study was mainly on the compression

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*H. F. Gonnerman: "Effect of Size and Shape of Test Specimen on Compressive Strength of Concrete," *Bulletin 16*, Structural Materials Research Laboratory, 1925.
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resistance of the beam, the most important consideration in the layout of the program was to design the beams in such a way that failure would occur due to crushing of the concrete in the beam. In the calculations the assumptions made were that the yield-point stress would be 40,000 lb. per sq. in. for the reinforcement, and that a large enough amount of reinforcement would be

![Graph showing beam-cylinder strength ratio against compressive strength of cylinder, lb. per sq. in.]

**Fig. 2—Tests of Heavily Reinforced Concrete Beams Made in United States**

used to avoid danger of failure by tension in the steel. Additional security against tension failure resulted from the fact that the rail steel reinforcement supplied for the tests had a yield-point of approximately 65,000 instead of 40,000 lb. per sq. in.

For the series in which the compressive strength of the concrete varied the total depth of the beams was 13 in. The depth from the compression surface of the concrete to the center of gravity of the steel, here termed the effective depth, was 10 in. The designed cylinder strengths of the concrete for this series varied from 1000 to 5000 lb. per sq. in. and are shown in Table 1.
TABLE I—DESIGN OF REINFORCED CONCRETE BEAMS

<table>
<thead>
<tr>
<th>Group No.</th>
<th>Designed Concrete Strength, Lb. per sq. in.</th>
<th>Ratio of Reinforcement</th>
<th>Reinforcement Bars</th>
<th>Dimensions</th>
<th>Computed Shearing Stress, Lb. Per Sq. In.</th>
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Beams in Group 1 to 7, inclusive, have $\frac{3}{8}$-in. U stirrups spaced 3 in. center to center except No. 5 and 6A which have $\frac{1}{2}$-in. double U stirrups 3 in. on center. Beams 8 to 10A, inclusive, have $\frac{5}{8}$-in. bars at an inclination of 20 degrees with longitudinal reinforcement and spaced 9 in. center to center.

In order to study the effect of variation in depth of the beams on the beam-cylinder strength ratio a series of tests was planned in which the effective depth of the beams varied from 4 to 14 in. as shown in Table 1. Two strengths of concrete (designed as 2000 and 3000 lb. per sq. in. cylinder strength) were used with each of these variations in depths of beams.

The computed shearing stress for the loads calculated to cause compression failure in all beams are also given in Table 1. Obviously most of them are so high as to require web reinforcement to prevent diagonal tension failure. For beams having a depth to center of gravity of steel of 10 inches or more, vertical stirrups were used. For shallower beams the depth was too small for proper anchorage of vertical stirrups and inclined web reinforcing bars were used. As a criterion for the minimum amount of web reinforcement required the formula

\[ v = (0.005 + r) \cdot f_v \]

was used. In this formula \( v \) is equal to the shearing stress, \( r \) is equal to the ratio of web reinforcement and \( f_v \) is equal to tensile stress in reinforcement here taken as 40,000 lb. per sq. in. This formula was derived from tests carried out for the United States.
Shipping Board during the war.* However, the minimum size of the stirrups was fixed at $\frac{3}{8}$ in. diameter and the maximum spacing at 3 in. for the vertical stirrups. This generally resulted in considerably more web reinforcement than that called for in the formula. Fig. 3 shows the arrangement of the web reinforcement for all the beams.

All the beams were 8 in. wide and 11 ft. long.

Three beams of each kind were made for each group. The control specimens generally consisted of 6 by 12-in. cylinders and three cylinders were made with each beam. Prisms 8 by 8 by 12 in. were made with two of the beams for testing under eccentric load. At a later date three prisms and three cylinders were made in order to secure further information on the properties of the concrete.

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5. Cement and Aggregates. The cement and aggregates used in this investigation came from the same lots as those furnished for a series of column tests which is being carried out at Fritz Laboratory for the American Concrete Institute. The cement had a fineness of about 89 per cent passing the No. 200 sieve and gave a high 28-day strength. The aggregates came from near the Delaware River at Morrisville, Pennsylvania. The sand had an average fineness modulus of about 2.82, had 4 per cent passing the No. 48 sieve, and about 50 per cent between the No. 48 and No. 28 sieves. The sand content used was 40 per cent by weight of the total aggregates. This combination gave very workable concrete for all mixes included in the series. The coarse aggregate consisted of gravel of a maximum size of $\frac{3}{4}$ in. At the laboratory it was screened and recombined so as to contain 40 per cent between the No. 4 and the $\frac{3}{8}$-in. sieves, and 60 per cent between the $\frac{3}{8}$-in. and the $\frac{1}{2}$-in. sieves. The aggregates had been stored indoors some time before the making of the concrete. At the time of making specimens, the gravel contained about 0.5 per
cent water as moisture and the sand about 1 per cent. It was assumed that this was close to the percentage which would be absorbed and no correction for absorption was made.

At the time of designing the mix only the 7-day strength of concrete of one water content was available. This strength was converted into estimated 28-day strength by use of the formula

$$f_{28} = f_7 + 30\sqrt{f_7},$$

in which $f_7$ and $f_{28}$ are the 7-day and the 28-day strengths. An estimated water-cement-strength curve was drawn so that it passed a little above this estimated strength (see curve 1, Fig. 4). Fig. 4 also shows the strengths obtained from the tests and indicates that for the cement used the strength increase between the ages of 7 and 28 days was considerably greater than the increase given by the formula. After the water content per sack of cement for each strength of concrete had been taken from curve 1, a few trial batches were made with one of these water contents to determine the mix which would give suitable workability. The mix found to give the proper workability contained 40 gal. of water per cubic yard of concrete and this mix was used as the basis for designing the other mixes. It was assumed that the water content per cubic yard of concrete remained practically constant for mixes of a given consistency. All mixes were therefore designed so as to contain 40 gal. of water per cubic yard for the different water-cement ratios used. The mixes arrived at in this manner are given in Table 2 which contains all data of the concrete used in the series. In Fig. 5 the slumps have been plotted as ordinates of two independent graphs.
In one of them the water content, shown at the top, and in the other the cement content, shown at the bottom, represent the abscissas. For the resulting concrete 75 per cent of the slumps lay between 4 and 6 in. The extreme range was from 3 to 7½ in. As large a variation as this may be expected on account of lack of uniformity in the moisture content of the aggregates.

**Fig. 5—Relation between Water Content, Cement Content, and Slump of Concrete**

If the assumptions made were entirely correct and if the operation had been under perfect control, the curve representing the relation between slump and cement content should be a straight horizontal line. Due to variation in slump already pointed out, this condition was only partially realized as is shown in Fig. 5. However, the conformity was good enough to warrant the characterization of this method as a satisfactory working basis.

6. **Cement-Strength Relation.** In Fig. 6 28-day cylinder strengths of the concrete have been plotted as ordinates of two independent curves with the water content and cement content per cubic yard of concrete as abscissas. In this series, however, the water content was held constant and the relation between strength and water content was necessarily a vertical straight
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It will be seen from this figure that the cement strength relation is approximately a straight line, that is, the increase in strength was proportional to the increase in cement content. It should be kept in mind that the same aggregate was used throughout the series and that the proportion of sand to coarse aggregate was constant, 40 per cent fine and 60 per cent coarse, and that

![Graph showing relation of strength to cement content and water content for concrete of constant consistency](image)

Fig. 6—Relation of strength to cement content and water content for concrete of constant consistency

the consistency of the concrete was fairly constant as measured by the slump. The increase in strength was approximately 1100 lb. per sq. in. per sack of cement per cu. yd. of concrete.

It should not be expected that with variations in types of aggregate or with variation of proportion of fine to coarse aggregate in different mixes, a constant water content should give a constant consistency or that proportionality between increase in cement content and increase in strength should result.

7. Uniformity of Concrete. Even though each cylinder represented a different batch of concrete in a beam, the variation in strength between individual cylinders for a given beam was less than the variation between the averages for beams of the
same group. Therefore, in studying the uniformity of concrete the average strength of the three cylinders from each beam was considered as a single strength. The maximum variation of these strengths from the average of a group was computed for each group of three beams. The largest of these maximum variations for any group in the entire series was 12.9 per cent and the average of the maximum variations for all groups was 6.4 per cent. Table 3 gives the average cylinder strength for each beam and also contains the average strength for each group and the maximum variations from the average.

8. Reinforcement. The reinforcement bars were of rail steel which gave high yield-point stress and high ultimate strength. Table 4 shows the test results for coupons from the reinforcement bars. The average yield-point stress for all specimens was found to be 64,800 lb. per sq. in., the ultimate strength 106,400 lb. per sq. in., and the modulus of elasticity 29,500,000 lb. per sq. in. This modulus of elasticity was used in computing the ratio \( n \), that is, the ratio of the modulus of elasticity of steel to that of concrete.

MAKING, CURING AND TESTING OF SPECIMENS

9. Fabrication of Reinforcement. The longitudinal reinforcement was in one layer in some and in two layers in others of the beams. The clear distance between the two layers of bars was 3/4 in. In order to hold the reinforcement in its proper position with respect to the bottom of the beam, supporting legs of the necessary depth were welded to the lower layer of the longitudinal bars.

The proper horizontal spacing was maintained by means of short cross bars welded to each longitudinal bar of the layer.

The web reinforcement was looped around and attached to the longitudinal reinforcement by electric arc welding. Generally, where two layers of longitudinal bars were used the stirrups were looped around the bars of the lower layer and welded to both layers of the longitudinal reinforcement, thus insuring the proper spacing and position of stirrups and longitudinal bars.

For simplification in making up stirrups the same depth of stirrups was used for beams of various depths, as far as possible. During the making up of the beams it appeared that the stirrups
were too short to be adequate as web-reinforcement for beams with an effective depth of 14 inches. To correct for this deficiency in some measure these stirrups were looped around and welded to the bars of the upper layers in two out of three of the 14-in. beams instead of to bars of the lower layer. The upper and lower layers of bars in these two beams were tied together by means of spacers welded to both bars. For beams whose depth was too small to provide adequate anchorage for vertical stirrups inclined web bars were used. These also were welded to the longitudinal reinforcement, which in these cases included only one layer of bars.

This method of fabricating all the reinforcement for each beam into a rigid unit appeared to be very effective in maintaining the correct position of the reinforcement during the placing of concrete in the beam. It also reduced the danger of slipping of the reinforcement during the testing of the beam. Fig. 7 shows the fabricated reinforcement for beams of different depths.

10. Forms. For the beams having an effective depth of 8 in. or less, wooden forms were used. For those having a depth of 10 in. or more, steel forms were used. In order to prevent leakage from the forms, all joints were made with care, and as a further precaution, all joints (with both wooden and steel forms) were puttied before placing the concrete. In this way the amount of leakage was made very small. Before placing the concrete all forms were oiled as a precaution against sticking of the concrete to the forms. In the case of the wooden forms oiling was a precaution also against absorption of water and warping of the forms.

11. Mixing and Placing. The concrete was mixed in an inclined kettle-shaped mixer of approximately 2.5 cu. ft. capacity. The dry materials were dumped into the mixer and mixed for one minute; water was then added and the mixing was continued for three minutes more. The concrete was dumped into the metal container shown in Fig. 8 and shoveled from it into the forms. The rounded ends of this container facilitated the complete emptying of it. Each beam required more than one batch. The concrete of each batch was placed in the form while the succeeding
batch was being mixed. Each batch was spread to an approximately uniform depth over the full length of the beam. The concrete was rodded into place by means of a 5/8-in. steel bar, and the forms were hammered lightly in order to consolidate the mass. Generally one control cylinder was taken from each batch so as to represent the different layers of the concrete in the beam. Fig. 8 shows the arrangement during the making of the beams in which wooden forms were used.

12. Curing. The control cylinders, as well as the beams, were generally left in the forms for 48 hr. after making. The forms were then removed and the specimens placed in a moist room of 100 per cent humidity and a temperature of approximately 70°F, until a 28-day age was reached. A typical chart of the temperature in the moist room covering one week of the curing, February 24 to March 3, is shown in Fig. 9. The temperature range throughout the curing period was from 68°F to 72°F.

13. Testing. At the age of 28 days the specimens were removed from the moist room and tested to failure. The testing machine used, both for the control cylinders and the beams, was an Olsen screw-power machine equipped with supporting wings
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Fig. 8—Making the Beams

Fig. 9—Record of Temperature in Moist Room for Week Feb. 24—March 3, 1930
designed for the testing of beams. The 6 by 12-in. cylinders were tested in the ordinary manner, with an idling speed of the head of the machine of .05 in. per minute. The beams rested on a roller at one support and on a spherical bearing block at the other support. The center of each support was 9 in. from the end of the beam, making a distance of 9 ft. 6 in. between centers of supports. The load was transferred from the head of the machine to the concrete beam through a heavy structural steel loading beam having a reinforced web. This loading beam transferred the load through a roller at one end and a spherical bearing block at the other end to the test beam at the load points, which were placed 21 in. on either side of the center of the beam. This arrangement of loads produced a constant moment in the central 42-in. portion of the concrete beam. The spherical bearing blocks at one support of the test beam and at one load-point were introduced for the purpose of eliminating torsional moments and eccentric loading in the testing of the beams. Fig. 10 shows the arrangement during the testing of one of the beams.

During the testing of the beams close examination was made for the appearance of cracks.

14. Deformation Measurements.* Deformation measurements were taken on at least one control cylinder from each beam. Two metal collars surrounding the cylinders at sections 10 in. apart were attached to the cylinder by means of a pair of screws in each collar 180 degrees apart. All four screws lay in the same diametral plane. A steel rod used as a distance piece held the collars a fixed distance apart at a point on each collar 90 degrees from the axis of the screws. This arrangement allowed each collar to rotate about a horizontal axis through the screws as the cylinder shortened under the load. Opposite this distance piece a .0001-in. Ames gage was attached to the upper collar, and another distance piece, attached to the lower collar bore at its upper end upon the plunger of the Ames gage. The readings on the dial, therefore, indicated twice the amount of the shortening of the cylinder during the testing. The deformation results secured in this manner were used for the determination of the modulus of elasticity of the concrete.

*In this report: (1) the word "deformation" denotes total change of length in a given gage length. (2) The word "strain" denotes the change per unit of length which is caused by stress. (3) The word "stress" denotes the intensity per unit area of the internal force.
Table 3 gives the modulus of elasticity for the concrete in each group of beams. This modulus is the tangent modulus at a stress of 500 lb. per sq. in. This table also gives the corresponding values of $n$ for each group.

The deformations of the concrete in the beams were measured on each side of the beam by means of Ames gages reading generally 0.001 in. One gage line was placed near the top of the beam, one near the elevation of the center of gravity of the reinforcement, and in some cases, one was placed near the neutral axis of the beam. The gage length was 35 in. for the gages near the top and near the center of gravity of the reinforcement, and 30 in. for the gages near the neutral axis. The Ames gages used for the observation near the neutral axis read to 0.0001 in. The deformation apparatus may be seen in Fig. 10.

15. Observed Deflections. For measuring the center deflection a fine wire attached to the side of the beam near the neutral axis
at each support spanned the length of the beam. A mirror bearing a scale graduated to fiftieths of an inch was attached to the side of the beam at the center of the span at the elevation of the neutral axis. The elevation of the wire on the scale was read without parallax by lining up the wire with its image in the mirror at the time of reading the scale. The deflections thus observed were read by estimation to the nearest .01 in. Fig. 10 shows the arrangement for the deflection measurements during the testing.

16. Duration of Test. The time required for the testing of one beam, that is, the time between initial load and the maximum load, depended upon the strength of the beam and the number of deformation and deflection measurements made. In general, the testing was completed in less than one-half hour. Beams carrying a maximum load of more than 20,000 lb. were loaded in 5000-lb. increments between successive sets of readings, while for the beams of lower carrying capacity smaller increments between successive sets of observations were used.

Results

17. Occurrence of Cracks. During the testing of the beams close inspection was made for the appearance of cracks. Usually the first cracks appeared at the bottom of the beam just below the loading point or near the center of the span. Usually no cracks appeared in the portion of the beam between the supporting points and the loading points. However, a few beams
showed diagonal tension cracks between the supports and the loading points. These were the beams of groups 6, 6A and 7, having an effective depth of 12 and 14 in. which showed diagonal tension cracks prior to failure. These cracks occurred between the supports and the loading points at both ends of the beam. Generally, several cracks occurred parallel to each other and only a few inches apart. In these cases the cracks ordinarily extended from the neutral axis to the bottom of the beam. Most of these cracks were first discovered near the bottom, but some appeared first at approximately the level of the neutral axis. The load at which the first crack was observed is given in Table 5 which also gives the percentages of the maximum load at which the first crack was observed. The average load at which the first crack was observed was about 48 per cent of the maximum load for all the beams.

18. Forms of Failure. All beams except two failed in compression of the concrete between the loading points of the beam. Generally the failure was very close to the center. Beams of low strength concrete gave a gradual failure, while beams of higher strength concrete broke suddenly without previous warning.

The other two beams failed in diagonal tension. These were from the series in which the average strength of the concrete was 2800 lb. per sq. in., and in which the effective depth varied from 4 to 14 in. The effective depth was 14 in. for one of these beams, and 8 in. for the other. The 14-in. beam which failed in this manner was the one which had stirrups of 11-in. depth attached to the lower layer of the longitudinal steel reinforcement. The stirrups in this beam extended only partially into the compression area of the concrete. In the other two beams of the same group the stirrups were attached to the longitudinal bars of the upper layer. These beams failed in compression, even though diagonal tension cracks appeared before failure. The maximum loads, shown in Table 5, indicate that the compressive strength of the concrete was almost reached before the diagonal tension failure occurred. Fig. 11 is a photograph of the 14-in. beam which failed in diagonal tension.

Beam 8B, 8 in. deep, had inclined bars and failed in diagonal tension without previous warning. The two companion beams
### TABLE 4—SUMMARY OF TESTS ON REINFORCING BARS USED IN BEAMS

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Notes:  
*Computed from the weight and length of bar.  
**Yield point detected by the drop of the beam in only half of the tests made.  
†Reduced area computed from an average reduced diameter.  
‡Strains were measured in only half of tests made.

### TABLE 5—RESULTS OF BEAM TESTS

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<tr>
<th>Group No.</th>
<th>Depth d in.</th>
<th>Width b in.</th>
<th>Ratio of Reinforcement, p</th>
<th>Maximum Load on Beams, Lb.</th>
<th>Max. Var. From Ave.</th>
<th>Load at First Crack Per Cent**</th>
<th>Max. Shearing Stress Lb. Per Sq. In.†</th>
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*Failed in diagonal tension, omitted from average.  
**Percentage of maximum load.  
†\( W = \frac{V}{2gbd} \)

of the same group, 8A and 8C, failed in compression with no diagonal cracks or other indication of being near failure in diagonal tension. It is to be noted also, that the maximum load was slightly higher for beam 8B, which failed in diagonal tension, than the average load for 8A and 8C, which failed in compression. In order to investigate whether any slipping of the inclined bars took place, thus causing the diagonal tension failure, the concrete in beam 8B was removed so that the inclined bar could be in-
Compressive Strength of Concrete in Flexure

Fig. 11—Diagonal failure of beam 6A. (a) Left end, (b) Right end

Specified throughout its length. No indication of slipping of the bar or stressing the steel beyond the yield point could be detected. The welded connection of the inclined to the longitudinal bar was still intact and there is no evidence that the diagonal tension failure was due to slipping of the web reinforcement or to failure of the welded connection with the longitudinal bar. Fig. 12a shows beam 8B in the testing machine after it had failed and 12b, photographed from the opposite side, shows the inclined bar exposed throughout its length.

Typical examples of the forms of failure for the different beams are shown in Fig. 13, 14 and 15. Fig. 13 shows the beams of
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Fig. 12—Diagonal failure of beam 8B. (a) In testing machine, (b) with exposed inclined bar

equal dimensions but of different strengths of the concrete; Fig. 14 the beams of 2800 lb. per sq. in. concrete and different depths of beams and Fig. 15 beams of 4000 lb. per sq. in. concrete and different depths.

19. Prism Tests. In order to get more definite information regarding the distribution of strains and stresses in the compression section of the concrete area in the beam, two 8 by 8 by 12-in. concrete prisms were made during the manufacturing of
Fig. 13—Failures of beams of different concrete strength
FIG. 14—BEAMS OF 2800 LB. PER SQ. IN. CONCRETE AND DIFFERENT DEPTHS
After all the beams had been made three prisms and three cylinders were made from a mix of concrete similar to that used in group 2 of the beam series. All the prisms were tested at an age of 28 days in such a way that the resultant load and the resultant reaction passed through the edge of the middle one-third of the sectional area of the prism. Fig. 16 which shows one of the prisms after failure indicates the method of applying the loads.
Deformation measurements were taken on two opposite faces of each prism.

The average strains for the two prisms from beams A and B in group 6A are plotted in Fig. 17 by connecting by straight lines the points representing the strains measured three inches away from the opposite faces, that is, assuming a straight line variation of strains across the section of the prism. The deformation curves for low loads all seem to pass through the same point, and show that there was little or no strain at one edge of the prism. This indicates that for these loads the relation between strain and stress is represented very closely by a straight line, since it is only for a straight line stress-strain relation that the strain is zero at the edge of a prism loaded at the edge of the middle third of the sectional area. For larger loads, however, the point of zero strain lay slightly within the prism, indicating that a curved stress-strain distribution existed at high stresses. Fig. 18 shows the average stress-strain diagram from the same data as in Fig. 17 for that edge of the prism which lay nearest to the load-point. It also shows the average stress-strain diagram for the control cylinders from the same batches of concrete. This figure indicates that the modulus of elasticity
obtained for the prism was slightly less than that found on the control cylinders. The average cylinder strength for these two beams represented in Fig. 17 and 18 was 4060 lb. per sq. in.

The deformations found in the additional prisms made after completion of the making of the beams were quite similar to those already discussed.

20. Uniformity of Results. The uniformity in strength results obtained from the tests of three beams in each group was found to be very good. The maximum variations of the individual strengths from the average are shown in Table 5. The greatest of these maximum variations was 13 per cent, and the average for all the groups was 8.2 per cent. The average of the maximum
variations for the beams was slightly higher than the average found for the concrete cylinders. The relatively small variation between the individual specimens indicates that the beams were uniform in quality and that the results represent the actual strength fairly well. A further indication of the reliability of the results is found in the fact that the points showing the beam-cylinder strength ratios for beams of different concrete strengths and for beams of different heights, fall approximately on smooth curves.
21. Relation between Strength of Cylinders and Strength of Prisms. The two prisms made from the same concrete as the beams in group 6A, loaded eccentrically, gave an average compressive strength over the entire area, of 2430 lb. per sq. in. Assuming a straight line stress-strain relation this corresponds to a maximum stress at one face of the prism of 4860 lb. per sq. in., and zero stress at the opposite face. The control cylinder made from the same batches and tested with a uniformly distributed load, gave an average compressive strength of 4060 lb. per sq. in. The apparent strength of the concrete in the prisms was therefore about 20 per cent higher than that of the cylinders.

The three prisms which were made at a later date did not have perfect caps, and therefore failed at lower stresses than might otherwise have been the case. The average compressive strength over the entire area of these prisms was 1675 lb. per sq. in. This corresponds to a maximum fiber stress of 3350 lb. per sq. in. The average strength of the control cylinders from the same batch was 3110 lb. per sq. in. The maximum fiber stress in these prisms was therefore only 8 per cent greater than the cylinder strength of the concrete.

22. Relation between Beam Strength and Prism Strength. This comparison includes only the two prisms made from the same batches of concrete as the beams A and B in group 6A. If a straight-line stress-strain distribution is assumed for the prism, it is found that the prism strength was about 20 per cent higher than the cylinder strength. Using the straight-line formula for computing stresses in the beams, and using the value of \( n \) obtained from the deformation measurements on the control specimens of concrete and of steel, the computed ultimate stress in the beam was about 50 per cent greater than the strength of the cylinders, and 25 per cent greater than the ultimate stress in the prism. The ultimate fiber stress in the prism was therefore considerably less than the fiber stress computed for the beams.

23. Notation and Formulas. For the purpose of studying the test results, an analysis has been made in which the stress curve in the compression part of the beam is assumed to be a parabola, not necessarily of second degree, but of some degree, \( r \). The value
of \( r \) is always the ratio of \( E_c \cdot \varepsilon_c \) to \( f''_c \). See Fig. 19. When \( r \) is one, the stress curve reduces to a straight line and the resulting formulas to the ordinary straight line formulas. When \( r \) is two, the stress curve reduces to the parabola of second degree and the formulas to the ordinary parabolic formulas for stress at the load

\[
F_2 = \frac{E_c \cdot \varepsilon_c}{f''_c}
\]

**Fig. 19—Assumed stress distribution in compression area of beam**

which produces compression failure in the concrete. The analysis could be made applicable for loads below the maximum, but there seems to be no necessity for introducing this complication.

In the formulas used in this report the following notation will be used.

- \( b \) = width of beam
- \( d \) = effective depth of beam, that is, depth from compression surface of beam to center of gravity of sectional area of reinforcing steel
- \( p \) = ratio of reinforcement
- \( n \) = ratio of modulus of elasticity of steel to that of concrete
- \( M \) = bending moment at maximum load
- \( kd \) = vertical distance from compression surface to neutral axis
Compressive Strength of Concrete in Flexure

\[ j d = \text{distance from resultant compressive force to resultant tensile force, that is, moment arm of resisting couple} \]

\[ f'_{c''} = \text{computed compressive stress in extreme fiber of reinforced concrete beam at load causing compressive failure} \]

\[ f'_{c} = \text{compressive strength of concrete as determined from tests on 6 by 12-in. cylinders} \]

\[ f_s = \text{computed stress in reinforcement of concrete beam} \]

\[ r = \text{exponent in a stress-strain equation (assumed to be parabolic) whose graph approximates the stress-strain curve for the control cylinders} \]

The general formulas for \( k, j, f'_{c''}, \) and \( f_s \) are as follows:

\[ k = \sqrt{p n (r + 1) + \left( \frac{p n (r + 1)}{2} \right)^2 - \frac{p n (r + 1)}{2}} \]

\[ j = 1 - \frac{r + 1}{2 (r + 2)} \]

\[ f'_{c''} = \frac{r + 1}{r} \frac{M}{k j d b} \quad \text{and} \]

\[ f_s = \frac{M}{p j d b} \]

The condition for straight-line distribution of stress is obtained when \( r = 1 \). The equation for stress in the concrete at the compression surface then becomes \( f'_{c''} = \frac{2M}{kjdb} \). The condition for parabolic distribution of stress is obtained when \( r = 2 \). The equation of stress, \( f'_{c''} \), in the concrete at the compression surface then becomes \( f'_{c''} = \frac{3M}{2 kjdb} \).

An illustration of the effect of the variation in the assumptions as to the stress distribution on the intensity of the maximum stresses and the position of the neutral axis is given in Fig. 20. In this figure the curves show the stress distribution for straight line, second degree parabola, and fourth degree parabola respectively.

24. Effect of Variation in Strength of Concrete on Beam-Cylinder Strength Ratio. Table 6 shows the extreme fiber stresses in the beams at maximum load as computed by the straight-line formula. In these computations three different values of \( n \) have been used, (1) the values specified by the American Concrete
Table 6—Beam-Cylinder Strength Ratio Using Straight Line Stress Distribution, \( p = 1 \)

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<th>Group No.</th>
<th>Depth, In.</th>
<th>Average Cylinder Strength Lb. Per Sq. In.</th>
<th>Stress Computed By Straight Line Formula; Value of ( n ) According to</th>
<th>Beam-Cylinder Strength Ratio for ( n ) According to</th>
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![Computed compressive stress, \( f_c \), lb per sq in](image)

**Fig. 20—Stresses in Beams According to Various Assumptions as to Distribution of Compressive Stress; \( p = 0.57, N = 10 \)**

Institute Code of 1928,* (2) the values specified in the Joint Committee report of 1924,† (3) the values determined from the deformation measurements on control cylinders and on coupons from the reinforcement steel used in the tests. The compressive strengths of the concrete as obtained from the 6 by 12-in. control

*Reinforced Concrete Building Regulations and Specifications, *Proceedings, American Concrete Institute, Vol. 24 (1928), Sec. 601, page 802.
†Standard Specifications for Concrete and Reinforced Concrete, *Proceedings, American Concrete Institute, Vol. 21 (1925), Sec. 103, page 366.
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cylinders are also given in the same table. A comparison of values given in this table shows that the computed compressive stresses in the beams at maximum load were much greater than the cylinder strengths, the excess over the cylinder strength ranging from 27 to 141 per cent. These results are in agreement with those of the earlier tests shown in Fig. 1 and 2.

![Graph showing the relation of beam-cylinder strength ratio to strength of concrete](image)

**Fig. 21—Relation of beam-cylinder strength ratio to strength of concrete**

Fig. 21 shows the beam-cylinder strength ratio, that is, the ratio of the computed beam stresses to the cylinder strengths for the different values of $n$ used. It is noted from this figure that the ratios were much larger for concrete of low strength than for high-strength concrete. For concrete strength of more than 3000 lb. per sq. in., however, the decrease in the ratio with the increase in strength was very small. When the values of $n$ used in the computations were in accordance with the American Concrete Institute code, the beam-cylinder strength ratio was about 2.0 for 1500 lb. per sq. in. concrete, 1.75 for 2000, 1.55 for 3000, 1.50 for 4000, 1.45 for 5000 and 1.40 for 6000 lb. per sq. in. concrete. For the two other values of $n$ used the relation between
the beam-cylinder strength ratio and the strength of the concrete follows approximately the same trend as the one found using $n$ according to the American Concrete Institute code.

The differences in the ratios for the three different sets of values of $n$ were not large.

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Table 7 shows the extreme fiber stress in the beams at maximum load as computed by the parabolic formula, that is, the formula in which $r = 2$. The values of $n$ used in the computations were the same as were used with the straight line formula. The compressive stresses were considerably less than those found for the straight-line distribution, but for cylinder strengths from 1400 up to 4000 lb. per sq. in. they were greater than the cylinder strengths. For higher cylinder strengths the computed stress in the beam was approximately equal to the cylinder strength.

The beam-cylinder strength ratios plotted in Fig. 21 for parabolic distribution were found to follow approximately the same trend for different strengths of concrete as for the straight-line distribution. The effect of differences in the method of determining $n$ on the beam-cylinder strength ratio was found to be negligible for the stresses computed by the parabolic formula.

25. Effect of Depth of Beam on Beam-Cylinder Strength Ratio. The beam-cylinder strength ratios given in Tables 6 and 7, for beams with different depths, and two strengths of concrete (2800 and 4000 lb. per sq. in.) have been plotted in Fig. 22: The results for the concrete having a strength of 2800 lb. per sq. in. give some
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indication of a slight increase in the ratio with increase in the depth of the beams. However, a curve drawn so as best to fit the points, indicates a higher beam-cylinder-strength ratio for the 10-in. depth than that given in Fig. 21 for the 10-in. beams made with concrete having a strength of 2800 lb. per sq. in. This throws some doubt upon the justification for concluding that there was an increase in the beam-cylinder-strength ratio with an increase in depth of beam. This doubt is emphasized by the fact that the ratios for the 4000-lb. concrete showed no such increase in the beam-cylinder-strength ratio with the increased depth of the beam. From the data obtained, the most logical deduction seems to be that the beam-cylinder-strength ratio was independent of the depth of the beams.

26. Shearing and Diagonal Tension Stresses. Table 5 shows the shearing stresses in the beams developed at maximum load. These shearing stresses were computed by the formula

\[ v = \frac{W}{2bd} \]

where

- \( v \) = shearing stresses
- \( W \) = total load on the beam
The value of \( j \) used in this formula was computed from the straight-line relation between stresses and strains, and the value of \( n \) was that found in the tests. On account of the fact that the strengths of the concrete were considerably higher than the strengths assumed in the design, the shearing stresses were correspondingly higher. However, none of the 10-in. beams failed because of high shearing stresses, nor were diagonal tension cracks found in any of these 10-in. beams. All of the beams having a depth to the center of gravity of the longitudinal reinforcement of 12 and 14 in. respectively, showed diagonal tension cracks in the web at a load equal to, or less than the maximum. Only one of these beams, however, failed in diagonal tension. This was the one in which 11-in. stirrups were attached to the lower layer of bars. The stirrups extended to about half way from the neutral axis to the top of the beam, that is, to about 4½ in. below the top of the beam. At the maximum load a crack extended nearly horizontally at about the elevation of the upper ends of the stirrups, indicating that the stirrups were too short to be fully effective. Anticipating difficulties due to the shortness of these stirrups the other two beams of the same group had the stirrups attached to the upper, instead of the lower, layer of longitudinal bars and the upper and lower layers were connected by means of spacers welded to both layers. This raised the tops of the stirrups 1¾ in., or to within 2¾ in. of the top of the beam. Although diagonal tension cracks were developed in these beams, the beams did not fail in diagonal tension. These facts bring out well the importance of extending the stirrups from the reinforcement to as close to the compression surface of the beams as possible. Evidently extending to a point half way between the neutral axis and top of beam was inadequate, and extending 1¾ in. higher was sufficient to prevent failure by diagonal tension, but probably extending it still higher would have been better. Fig. 11 is a photograph of Beam A of Group 6 which failed in diagonal tension.

Only one of the beams, 8B, shallower than 10 in. (from compression surface to center of gravity of steel) developed diagonal cracks, and this one failed suddenly in diagonal tension as soon as the diagonal cracks appeared. However, this beam carried
approximately the same load as the average of the other two beams of the same group. The average shearing stress for these beams, 8A, 8B, and 8C, was about 280 lb. per sq. in. and the shearing stress assumed in the design was 267 lb. per sq. in. After the test, the concrete was cut away from around the inclined bar and a careful examination showed that there was no slipping of the bar, the welding was still intact, and the steel showed no scaling such as should be expected if it had been stressed beyond the yield point. Fig. 12b shows the photograph of the beam with the concrete chipped away to expose the inclined bar in the region of failure.

The reinforcement for web stresses proved adequate for the purpose of the tests. It was desired to develop compression failure in all the beams. Even though the compressive strength of the concrete and the shearing stresses in the beams at maximum load were higher than the values for which the beams were designed, only two beams failed by diagonal tension. Even in these cases the loads carried were so close to those carried by the companion specimens that the compressive stress in the concrete when failure occurred must have been almost equal to the highest that could have been developed.

27. Stresses in Reinforcement. In Fig. 23 the stresses determined from the strains observed at the level of the center of gravity of the reinforcement are shown as ordinates, and the stresses computed by the straight line formula using λ as specified by the American Concrete Institute’s code, as abscissas. The plotted points represent the results from this series of tests and the solid straight lines are graphs of the equations given in the figure. These equations represent closely the relation between observed and computed stresses in an extensive series of tests carried out by the United States Geological Survey under the direction of Richard L. Humphrey about 25 years ago. On the whole the observed stresses agree very well with the computed stresses, rather more closely in fact than in those of the Geological Survey tests previously mentioned. For the high percentage of reinforcement used in these tests the straight line formula serves with sufficient accuracy for design purposes for computing the stresses in the reinforcement. It is not to be expected that agreement between observed and computed stresses will be as close
with low percentage of reinforcement. This is brought out very well in the paper* formerly referred to.

As illustrated by the beams of group 1, Fig. 23, plotted as triangles, the divergences of the observed stresses from the com-

![Graph](image)

**Fig. 23—Relation between observed stresses in steel and stresses computed according to A. C. I. specifications**

puted stresses was somewhat greater than for any of the other beams. The concrete in these beams had a strength of only 1400 lb. per sq. in., whereas the next highest strength was 2800 lb. per sq. in. However, the amount of reinforcement was not much lower for group 1 than for group 2. This disparity in the amount of reinforcement and the strength of the concrete between this and the other groups may account for a difference in relation between observed and computed stresses. Nevertheless, the difference has not been accounted for in detail.

28. *Stresses in Concrete.* Fig. 24 shows the relation between the so-called observed compressive stresses in the beams and the

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*H. M. Westergaard and W. A. Slater, "Moment and Stresses in Slabs." *Proceedings, American Concrete Institute, Vol. 17, page 480, 1921.*
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Observed stress in concrete, lb. per sq. in.

Fig. 24—Relation between observed stresses in concrete and stresses computed according to A. C. I. specifications

stresses computed by means of the straight line formula. The value of $n$ used in these computations was according to the specification for the American Concrete Institute code. For the strains observed at the extreme fiber of the beam at different loads, corresponding stresses were taken from the stress-strain curve for the control cylinders. These are the so-called observed stresses for the beams and have been plotted as abscissas in Fig. 24. The ordinates in this figure are the computed stresses for the same loads on the beams. The dotted line gives the condition for equal computed and observed stresses. The stress-strain curves do not go as high as the strength of the cylinders because it was necessary to remove the measuring instruments before failure of the cylinder, in order to avoid damage to the instrument. The curves of the observed stresses therefore generally do not extend as high as the cylinder strength. They go far enough, however, to indicate that up to the stresses as high as the strength of the cylinders there is a close agreement between the observed and the computed stresses. The indication is that in a beam at the maximum load the compressive stresses are in agreement with the stresses for corresponding strains in the cylinders up to a height above the neutral axis at which the stress is approxi-
mately equal to the cylinder strength. For points above this height the stresses in the beam are not known, but computations indicated that they must be somewhat greater than the cylinder strengths in order to produce equilibrium between the internal and external moments. This finding is in conformity with the fact previously pointed out that the maximum stress found in the prism tests was about 8 to 20 per cent greater than the cylinder strengths for the same concrete.

29. Deflection. The average deflection for the different groups of beams of the five different strengths included in the series in which the size of the beams remained constant is shown in Fig. 25. For comparison deflections computed from a formula proposed by Prof. G. A. Maney* have also been shown. The solid curves represent the observed deflections, while the dotted curves represent the deflections computed by means of Maney's formula. Fig. 25 shows that a very good agreement exists between the computed and the observed deflections. This indicates that an intimate relation exists between the measured deformations of a reinforced concrete beam and the deflections. The deflection of reinforced concrete beams can therefore be computed as ac-

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It will also be noted from Fig. 25 that the total observed deflection near the maximum load was practically equal for all the beams included in this series.

### TABLE 8—OBSERVATIONS NEAR MAXIMUM LOAD

<table>
<thead>
<tr>
<th>Group No.</th>
<th>Maximum Load Lb.</th>
<th>Load Lb.</th>
<th>Strains in Steel</th>
<th>Strains in Extreme Concrete Fiber</th>
<th>Deflection In.</th>
<th>dL In.</th>
<th>Computed (dL) In.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>32,520</td>
<td>30,000</td>
<td>.00116</td>
<td>.00206</td>
<td>.44</td>
<td>6.4</td>
<td>8.37</td>
</tr>
<tr>
<td>2</td>
<td>44,710</td>
<td>40,000</td>
<td>.00136</td>
<td>.00214</td>
<td>.48</td>
<td>6.6</td>
<td>8.35</td>
</tr>
<tr>
<td>3</td>
<td>63,300</td>
<td>55,000</td>
<td>.00155</td>
<td>.00204</td>
<td>.46</td>
<td>6.1</td>
<td>8.5</td>
</tr>
<tr>
<td>4</td>
<td>74,830</td>
<td>70,000</td>
<td>.00145</td>
<td>.00250</td>
<td>.54</td>
<td>6.3</td>
<td>8.16</td>
</tr>
<tr>
<td>5</td>
<td>87,410</td>
<td>80,000</td>
<td>.00133</td>
<td>.00231</td>
<td>.51</td>
<td>6.4</td>
<td>8.14</td>
</tr>
</tbody>
</table>

In order for the deflections for all beams to be equal, Maney's formula shows that the sum of the unit deformations in the concrete and in the steel must be the same for all groups. Reference to Fig. 23 shows that the observed stresses in the reinforcement were approximately the same for all groups at maximum load, except possibly group 1. Therefore, the deformations in the concrete must also have been nearly the same at maximum load for all groups. That this is approximately true may be seen directly from Table 8.

If equality of the compressive strains and equality of the tensile strains for all groups existed at the maximum loads the arm, \( jd \), of the resisting moment and therefore the value of \( j \), must have been nearly equal for all groups. This also is shown in Table 8 to have been approximately true. It will be seen from the straight line formula for the position of the neutral axis,

\[
k = \sqrt{2pn + (pn)^2 - pn}
\]

that the value of \( k \), and therefore the value of \( j \), depends entirely upon the product \( pn \). In general, the value of \( p \) increased, and the value of \( n \) decreased as the strength of the concrete increased, maintaining the product \( pn \) nearly constant. The resulting computed value of \( j \) in these beams ranges only from .837 to .814. This seems to indicate that for design computations of stresses in the reinforcement of concrete beams the value of \( j \) might well be taken as constant regardless of the values of \( p \) and \( n \).
SUMMARY

30. Summary. (1) In the tests carried out for this investigation all the beams had a width of 8 in. and a span of 9 ft. 6 in. The loads were applied at approximately the one-third points of the span.

(2) The percentage of longitudinal reinforcement varied from 2 to 6. This was sufficient to prevent any failures in tension in the reinforcement. The web reinforcement was designed to prevent failure in diagonal tension.

(3) Two series of tests were included. In one of them 5 different strengths of concrete were used, ranging from 1400 to 5800 lb. per sq. in. as shown by the tests of the 6 by 12-in. control cylinders. In this series the depth from the compression surface to the center of gravity of the reinforcement was 10 in. In the other series, varying depths were used, ranging from 4 to 14 in. In this series two strengths of concrete were used, 2800 and 4000 lb. per sq. in. respectively.

(4) In the mixes for all strengths in both series the fine and the coarse aggregates were 40 per cent and 60 per cent respectively of the total aggregates. The water content was maintained constant at 40 gal. per cu. yd. of the resulting concrete. This gave very nearly the same consistency for all mixes.

(5) In these mixes the quantity of cement ranged from 3.6 to 7.7 sacks per cu. yd. of concrete. The strength of the concrete increased in almost direct proportion to the increase in the cement content.

(6) A high degree of uniformity in the strength of the concrete was secured. This is indicated by the fact that the average of the maximum variations in the concrete strength for the different groups was only 6.4 per cent. The corresponding average of the maximum variations for the beams of the different groups was 8.2 per cent.

(7) All beams, except two, failed in compression of the concrete close to the center of the span. The two exceptions noted were beams which failed in diagonal tension. However, even these beams developed shearing stresses as high as those for which the web reinforcement was designed and the beams were close to failure by compression. The failure by diagonal tension
was due to the development of a higher compressive strength than was anticipated in the design.

(8) The 8 by 8 by 12-in. prisms loaded at the edge of the middle third of the cross-section developed compressive stresses about 20 per cent greater than the strength of the control cylinders from the same mix.

(9) The compressive strength of the concrete in the beams as computed by the straight line formula was about 25 per cent greater than the compressive stress developed in 8 by 8 by 12-in. prisms loaded at the edge of the middle third of the cross-section.

(10) The beam-cylinder strength ratio varied with the strength of the concrete. Using the straight line formula for computing stresses in the beams the ratio varied from more than 2.0 to 1.4 for concrete having a range in strength from 1400 to 5800 lb. per sq. in. Using parabolic stress distribution the beam-cylinder strength ratio varied from about 1.5 to .95 for concrete having a range in strength from 1400 to 5800 lb. per sq. in. For concrete having a strength above 3000 lb. per sq. in. the ratio decreased very little.

(11) Variations in the assumed values of \( n \) had a small effect on the beam-cylinder strength ratio for computations made by the straight line formula, and a smaller effect for computations made by the parabolic formula.

(12) Variations in the depths of the beams showed some effect on the beam-cylinder strength ratio. However, this effect was very small, and not entirely consistent. The data seemed to justify the conclusion that the effect of variation in the depths on the beam-cylinder strength ratio was negligible.

(13) The relations between strength of concrete determined by beam tests and by cylinder tests, found in this investigation confirmed in a general way the results reported by Emperger, Graf and Bach, and by Slater and Zipprodt.

(14) The observed stresses in the steel agreed fairly well with the stresses computed by the straight line formula.

(15) A very close agreement was found to exist between the observed compressive stresses in the concrete of the beams and the stresses computed by the straight line formula up to a stress equal to the strength of the control cylinders. At the top of the
beam the compressive stress seemed to be somewhat larger than the cylinder strength.

(16) The deflections observed near the maximum loads on beams of different strengths but equal dimensions were found to be nearly equal. The deflections observed at different loads agreed very well with the deflections at the same loads computed according to Maney's formula from the observed deformations. This is consistent with the fact that the arm $jd$ of the resisting moment was found to be nearly constant for all the beams.
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