SHEAR TESTS ON REINFORCED CONCRETE BEAMS

by

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and

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1. General - The investigation reported in this paper gives information on the effect of spacing and angle of bend-up of inclined bars on the strength in diagonal tension of reinforced concrete beams. Furthermore, a study is made of the relations between the deflections computed by means of formulas developed by Professors Maney and Turneaure and the measured deflections.

The spacings used were computed by the three formulas:

(1) \( S = \frac{d}{2}(1 + \cot \alpha) \),

(2) Joint Committee formula, \( S = \frac{45d}{\infty + 10} \),

and (3) \( S = \frac{45d}{\infty} \). The angles of bend-up used were: 12, 21 and 30 degrees with the horizontal. The Joint Committee recommends that an angle of more than 15 degrees be used. One group of beams had no inclined bars.

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a-By spacing is meant the distance between the support and the point of bend-up of the inclined bars.


c-PRINCIPLES OF REINFORCED CONCRETE DESIGN - Turneaure and Maurer, page 112.
A total of 36 beams, 5-1/2 in. wide, 8 in. deep and 76 in. long were made. The design strengths of concrete were 2000 and 3500 lb. per sq.in. Three beams of a kind were made and each beam had three 6 by 12-in. control cylinders. In loading the beams, the span between the reactions was 60 in., giving an overhang of 8 in. which was sufficient for the anchorage of the reinforcement. The load was applied at two points, each point being a distance of \( \frac{d}{2} = 4 \text{ in.} \) from the point of bend-up towards the center of the beam. Table 1 gives general information concerning the beams. Sufficient tension and compression steel was provided in order to insure failure in diagonal tension.

The cement and aggregates were the same as those generally used at the Fritz Engineering Laboratory*. The mixes were designed by the constant water content method and the slump varied between 6-1/2 and 7-1/2 in. Cement-water ratios of 1.25 and 1.61 by weight gave strengths of 2000 and 3500 lb. per sq.in. respectively.

The reinforcement was a mild steel having average yield-point and ultimate stresses of 46,670 and 75,980 lb. per sq.in. respectively. The tension bars were hooked in order to provide anchorage. The reinforcement was welded into a unit before it was placed in the forms. Fig. 1 shows a view of one of the reinforcing units.

* PROCEEDINGS of the American Concrete Institute, 1930, page 838
The beams remained in the forms two days in order to develop sufficient strength for their removal to the moist room. The beams and control cylinders remained in the moist room until they were tested at the age of 28 days.

During the testing the beams were supported on a roller at one end and a spherical bearing block at the other end. The load was transmitted from the head of the testing machine to the beam through a system of helical springs and a steel I-beam on a roller and a spherical bearing block. Fig. 2 is a photograph of the loading arrangement. The spring loading system was used in order to apply the load in such a manner that it closely approximated actual working conditions. The springs cause a continuous application of the load so that when the beam commences to fail the springs follow up the deflections and exert a nearly constant load. This method of loading gave a very abrupt failure of the beams.

Deflection measurements were taken by means of a wire and mirror arrangement on both sides of the beam. Measurements were taken directly under the loads and when feasible, at the center of the beam.

2. Results - Diagonal cracks, generally starting in the region of the neutral axis and rapidly extending upwards toward the loading point and downward toward the support, were observed during the application of the load. A slight change was noted in the deflection curve at load of cracking.
All of the beams failed in diagonal tension. Fig. 3 shows the type of failure obtained. Failure of the beams was very abrupt and complete due to the action of the springs. It was found that the beams having the greatest angle of bend-up and the smallest spacing gave the most warning of impending failure, that is, the greatest difference between ultimate load and load at cracking.

Fig. 4 shows a comparison between the actual ultimate unit shear and the theoretical shearing stress given by the Joint Committee formula: 

\[ \tau = 0.03f' + \frac{f_y A_y}{b s} (\sin \alpha + \cos \alpha). \]

With the 2000-lb. concrete, the observed values were 65 per cent greater than the computed values for the small spacings. For the largest spacing the observed values were 56 per cent greater than the values as given by the formula. The variation for the beams having a designed strength of concrete of 3500 lb. per sq.in. was nearly the same as for the beams having a designed strength of concrete of 2000 lb. per sq.in. This indicates that within the limits of this investigation the trend of the Joint Committee curve is very nearly the same as the trend of the observed curve. The values obtained by the formula are not permitted in this case, as these values exceed the specification, that the last term of the formula shall not exceed 75 lb. per sq.in.
On the basis of working stresses permitted by the Joint Committee the factor of safety varied from 3.90 for the smallest spacing to 2.77 for the largest spacing. From the curves shown in Fig. 4 it would seem advisable to calculate the ultimate stress by multiplying the Joint Committee formula by a factor (in this case, 1.60) and then reducing this ultimate stress to working stress by the use of an adequate factor of safety. It seems logical to use a constant factor of safety over the entire range of spacing. Fig. 4 shows that the unit shear at failure increased with a decrease in the spacing of the inclined rods. Thus the Joint Committee formula: \( S = \frac{45d}{\alpha + 10} \), which resulted in the smallest spacing, gave the highest ultimate shearing strength of the beams.

Fig. 5 indicates that the angle of bend-up affected the unit shear at failure when the same spacing formula was used. The optimum angle of bend-up appears to be between 21 and 30 degrees. The results are to a certain extent made obscure by a variation in spacing, but when using one spacing formula they serve somewhat as a guide for determining the most effective angle of bend-up.

In Fig. 6 the observed deflections have been compared with those computed from the formulas developed by Professors Maney and Turneaure. It is noted from these figures that Maney's formula gave a much better agreement with the observed deflections than did Turneaure's, although the latter gave favorable comparisons at low loads.
3. **Summary** - (1) All beams used in this investigation failed in diagonal tension.

(2) A very abrupt and complete failure of the beams resulted from the use of a spring system for application of the load.

(3) The beams having the greatest angle of bend-up and the smallest spacing gave the most warning of impending failure.

(4) The curve for the observed ultimate shearing stress had the same trend as the curve for the shearing stress as given by the Joint Committee formula: 

\[ F = 0.03f' + \frac{f_{yA_y}}{bs}(\sin \alpha + \cos \alpha) \]. However, the observed values were about 60 per cent greater than the values given by the Joint Committee formula.

(5) The smallest spacing gave the highest ultimate shearing stress.

(6) For a given spacing formula the shearing stress at failure was greatest for angles of bend-up between 21 and 30 degrees.

(7) The deflection computed by the formula proposed by Professor Maney agreed more closely with observed deflections than did the values computed by the formula proposed by Professor Turneaure.
### Table I - General Information

<table>
<thead>
<tr>
<th>Spacing Formula</th>
<th>Spacing</th>
<th>Bend-up Angle</th>
<th>Design Strength of Concrete</th>
<th>Average Cylinder Strength Obtained</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S = \frac{45d}{\infty}$</td>
<td>14.47 in.</td>
<td>0°</td>
<td>2000 lb. per sq. in.</td>
<td>2010 lb. per sq. in.</td>
</tr>
<tr>
<td>$S = \frac{d(1 + \cot \infty)}{\infty}$</td>
<td>19.20 in.</td>
<td>12°</td>
<td>2000 lb. per sq. in.</td>
<td>2170 lb. per sq. in.</td>
</tr>
<tr>
<td>$S = \frac{d(1 + \cot \infty)}{2}$</td>
<td>12.15 in.</td>
<td>21°</td>
<td>2000 lb. per sq. in.</td>
<td>2080 lb. per sq. in.</td>
</tr>
<tr>
<td>$S = \frac{d(1 + \cot \infty)}{2}$</td>
<td>9.20 in.</td>
<td>30°</td>
<td>2000 lb. per sq. in.</td>
<td>2280 lb. per sq. in.</td>
</tr>
<tr>
<td>$S = \frac{45d}{\infty + 10}$</td>
<td>9.20 in.</td>
<td>21°</td>
<td>2000 lb. per sq. in.</td>
<td>2130 lb. per sq. in.</td>
</tr>
<tr>
<td>$S = \frac{45d}{\infty}$</td>
<td>14.47 in.</td>
<td>21°</td>
<td>2000 lb. per sq. in.</td>
<td>2080 lb. per sq. in.</td>
</tr>
</tbody>
</table>
Fig. 1 - Reinforcement Unit Fabricated for Beams

Manufactured in U.S.A.
Fig. 3 - Types of Failure
Fig. 4—Variation of ultimate unit Stress with Spacing of inclined Bars. \( \alpha = 21^\circ \)
Fig. 5—Effect of Change in Angle of inclined Bars. Spacing given by $S = \frac{d}{3}(1 + \cot \alpha)$.
Fig. 6 - Comparison between Observed and Theoretical Deflection of Beams
Strength 3500 lbf/sq.in.