EFFECT OF RIGID BEAM-COLUMN CONNECTIONS ON COLUMN STRESSES
by Inge Lyse* and E. H. Mount°

INTRODUCTION
A study of the design and behavior of the top angle beam connection designed for end restraint appeared in the October 1936 and October 1937 issues of THE WELDING JOURNAL®. The present paper contains the results of a study of the localized column stresses produced by this type of connection. The investigation was sponsored by the Structural Steel Welding Committee of the American Welding Society and was carried out at the Fritz Engineering Laboratory of Lehigh University.

The column moments induced by partial rigidity in the beam connection may be determined by modification of any one of a number of methods of rigid frame analysis. This investigation was, therefore, confined to a study of those localized column stresses of a secondary nature. The object of the investigation was to determine whether these secondary stresses acting simultaneously with axial column loads were of sufficient magnitude to cause local buckling of the column when subjected to vertical loading.

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/ PROGRESS REPORT ON WELDED BEAM-COLUMN CONNECTIONS /
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EFFECT OF WELDED TOP ANGLES ON BEAM-COLUMN CONNECTIONS /
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A series of beam-column connections with column size, beam size and top-angle size as variables were investigated. Analysis of the data obtained indicates that these localized stresses are of a negligible order within the proposed limits of the top angle connection.

ACKNOWLEDGMENT

The investigation was carried out under the direction of the Structural Steel Welding Committee of which Mr. L. S. Moisseiff is Chairman and Mr. W. Spraragen the Secretary. Acknowledgment is due all members of the Committee for their interest in the work and their assistance in formulation of the test program. Acknowledgment is also due Mr. Roger Fluck of the Bethlehem Steel Company for welding of specimens and Mr. H. J. Godfrey and other members of the Fritz Engineering Laboratory staff for their assistance in carrying out this investigation.

TEST PROGRAM

The original program provided that the principal testing be carried out using a simplified specimen consisting of a plate simulating the tension flange of a beam, welded to the column flange. Direct tension was applied to the plate which in turn transferred the tension to the column flange and web. Pilot tests, however, indicated that this type of specimen could not be made to duplicate column flange stresses produced
by the welded top angle connection. The program was, therefore, revised and a cantilever specimen of the type shown in Fig. 1 was substituted. The program in its final form provided for the testing of fourteen specimens of this general type with beam size, column size, and top angle size varying as indicated in Table I.

**FABRICATION OF TEST SPECIMENS**

The stub columns were cut to size and mounted horizontally on a jig. The seat angle was then welded in place. The beam was set in place and shimmed to allow 1/8 to 1/4-in. clearance between beam end and column face. The beam was tack-welded to the seat angle and the shims removed. The top angle was tack-welded in place and the beam checked finally for perpendicularity to column face. The top angle welds were completed and the jig rotated through 90° to allow placing of the final seat angle welds in the horizontal position. The specimen was then reversed in the jig and the procedure repeated in attaching the second beam to the opposite column face. Top angle welds were placed in two ways. The weld metal was placed in a series of beads until the desired weld size was reached, or the jig was rotated through 45° and the weld metal floated in by a series of flat horizontal beads. No appreciable difference in behavior between welds made by these two methods was noted in testing and, therefore, no differentiation is made in the test data between specimens welded by the two methods.
All welds were made by a qualified welder and upon failure showed excellent fusion and absence of inclusions or blow holes. All large welds were made with 3/16-in. Murex heavy mineral coated electrode and all tack-welds and small finishing beads were made with 5/32-in. Airco No. 87 coated electrode.

**METHOD OF TESTING**

All specimens were tested to failure in a 300,000-lb. Olsen testing machine. Loading was applied as shown in Fig. 1. The beams were supported on rollers placed 32 in. from the column faces and the load applied to the center of the column through a spherical bearing block.

The rotation of each beam at the face of the column was measured by means of four Ames dials reading to 1/1000-in. attached to the beams with plungers bearing against the column face as shown in Fig. 1. Gage plates were tack-welded on the centerline of the column web opposite both seat and top angles. Three rows of gage holes were spaced at varying distances from the column web. Corresponding gage holes on the inner surface of the column flange permitted measurement of column flange deflections relative to the column center line. Gage holes were spaced at 2-in. intervals in the rows and permitted observation of deflections four inches above and below the weld connecting top angle to column and two inches above and below the heel of the seat angle. An Ames dial reading to 1/1000-in. and modified as shown in Fig. 2.
was used in measuring the flange deflections.

The specimens were whitewashed before testing and observation of scaling aided in determining the location of critical points.

TEST DATA

General information concerning specimen size, design loads, ultimate strength, apparent factors of safety and flange deflections is presented in Table I. The factors of safety listed in Table I are factors of safety against end failure. It should be remembered that the flexibility of this type of connection is such that failure by overstressing at the center of the beam will occur prior to ultimate failure of the connection. The tabulated flange deflections are those observed at the flange edge at the point of attachment of the top angle and at the heel of the seat angle. Deflections of slightly greater magnitude than those recorded at the top angle were observed at a point on the flange edge two inches below the point of attachment of the top angle. This phenomenon was the result of the moment applied to the column flange by the top angle and disappeared as loads were increased above the design load, so that the maximum deflections occurred at the point of attachment of the top angle. The shift in the point of maximum deflection may be accounted for by the fact that yielding of the top angle weld at or slightly above design load allows the moment applied to the flange to increase at a
much lower rate, while the direct pull continues to increase with the increased load.

Flange deflections observed for angles smaller than 3 by 3 by 1/2-in. and having a length less than 8 in. were very small and erratic.

Typical transverse flange deflection curves for larger angles are shown in Fig. 3 and 4. In each case the left half of the figure shows the observed deflections at the point of attachment of the top angle and the right half of the figure shows corresponding curves for a point two inches below the top angle connection. Typical deflection curves of the flange edge are presented in Fig. 5. These curves show the local nature of the deflections. In all cases the plotted deflections are the average of observed deflections for corresponding points on opposite sides of the column web. The average was used in order to eliminate the effect of any eccentricity in the loading of the specimen.

Flange deflections at the seat angle were at all times small and erratic. The general trend, however, seemed to be in the direction of uniform deflection for all points on a transverse section which indicates that the major portion of the deflection was the result of elastic and plastic deformation of the web.

The measured end rotations of the beams were used in calculating heel deflections of the top angles and in the
determination of the rigidity of the connection. A typical load-deflection curve is shown in Fig. 6. The results were plotted for three angles of the same size, 3 by 3 by 1/2-in. but with varying lengths of 6, 8, and 10 in. The fact that the points fall so closely together indicates that variation in length has no effect upon the flexibility of the connection. However, increase in the length of the angle over six inches showed a tendency to decrease the efficiency of the connection, i.e., decrease the ultimate load expressed in pounds per inch length of top angle. This tendency became particularly pronounced in angles as large as 3 by 3 by 7/8 in. and 3 by 3 by 1 in. Fig. 7 shows the type of failure which occurred in a 3 by 3 by 1 in. top angle 10 in. in length. This failure is typical of angles of this size. Failure started by tearing of the weld at the root in the center of the column flange and progressed toward the throat and in both directions from the center to the ends of the welds.

Elastic and plastic deformations of the web contribute materially to the center deflections of the flange as shown in Fig. 3 and 4. The critical points are shown by the scaling of the whitewash in Fig. 8. The scaling at the top angle is inclined at 45° indicating that the principal force acting is direct tension applied at the point of attachment of the top angle to column flange and acting in a direction perpendicular to the column flange. The scaling at the seat angle is typical of web crippling due to bearing loads.
DISCUSSION AND ANALYSIS OF TEST RESULTS

Before any attempt was made to analyze the column stresses it was necessary to establish an upper limit of top angle size which in turn established the maximum load which need be considered in the analysis of column stresses and deflections.

The rigidity of the cantilever connection was computed as shown in Fig. 9. The cantilever was assumed to be an end portion of a partially restrained beam having an identical connection at the opposite end, the point of inflection being coincident with the point of application of load to the cantilevered beam. The load, end rotations and point of inflection being fixed by test procedure and results, it was possible to calculate the length of this imaginary beam and to compare its free end rotation with the observed rotation of the cantilevers. This comparison gave a measure of the rigidity developed by the connection under various load conditions. Fig. 10 presents a comparison of rigidities developed by three sizes of angle, namely; 3 by 3 by 1/2 in.; 3 by 3 by 5/8 in., and 3 by 3 by 3/4 in. The length of these angles was also a variable but Fig. 6 showed that the length has no effect upon the flexibility or its reciprocal, rigidity. In calculating the rigidity of the connection it was necessary to use a beam size which when coupled with the connection in question gave a balanced design. Since Table I shows that beam size has no effect upon
the action of the top angle it was necessary to consider the balanced design only in computation of the rigidity. The dotted curve in Fig. 10 intersects the rigidity curves at the per cent fixity developed at design load. These curves are typical of others plotted for the remainder of the specimens. They show that in all cases the rigidity developed at design load is well above the minimum allowable rigidity of fifty per cent. They also show that design load rigidity tends to increase with angle size rather rapidly for those sizes equal to or greater than 3 by 3 by 3/4 in. This increase in rigidity is caused by two factors. First, general yielding throughout the entire weld is improbable in the larger size welds. Second, the ductility of the top angle transverse to the direction of rolling was found by free bend tests to decrease rather markedly in angle sizes above 3 by 3 by 3/4 in. As the flexibility of the connection depends upon the yielding of the top angle weld and the ductility of the angle itself, these two factors tend to increase the rigidity as mentioned above. This increase in rigidity of the connection is accompanied by a comparable decrease in the factor of safety against end failure. The flexibility of the connection might be increased by increasing the length of leg of the top angle but this in turn decreases the allowable load on the connection. Therefore an increase in the amount of material used in the connection at this point is not accompanied by a corresponding increase in load-carrying ability.
The drop in efficiency of the connection accompanying the use of lengths over six inches in the larger angles is explained by consideration of the type of failure pictured in Fig. 7. It is evident that at loads near the ultimate all portions of the top angle weld are not contributing full value at the same time. This condition can occur only in extreme cases and is eliminated entirely if an upper limit to top angle size be set at a point such that the largest beam used in a balanced design would require a top angle of length not in excess of six inches. The above considerations led to the establishment of an upper limit to top angle size of 3 by 3 by 7/8 in.

Examination of the transverse deflection curves of Fig. 3 and 4 reveals the fact that the deflections may be separated into two parts, that due to bending of the flange and that due to yielding of the web. Such a division can be only approximate as no actual measurements of web deformations were made.

Consideration of that portion of the deflection due to bending of the flange revealed that the transverse deflection curves were very similar to the deflection curves of a simple cantilever uniformly loaded. A method of analysis proposed in an article by Gregor, appearing in DER PRAKTISCHE STAHLHOCHBAU, Band IV, 1932, and reviewed in LA SOUDRE A L'ARC ELECTRIQUE, was used with certain modifications. A portion of
the column flange was assumed to act as a cantilever with its fixed end at the column web and loaded uniformly with the design load of the top angle. The dimensions of the cantilever were assumed as follows: width equal to the width of column flange, and length equal to one-half the length of the column flange. Since the top angle contributes to the stiffness of the cantilever, the depth of the cantilever was taken equal to the thickness of the column flange plus three-eighths the thickness of the top angle. A comparison of computed and observed deflections is shown in Fig. 11.

Examination of rolling tolerances led to the establishment of a maximum allowable edge deflection of the flange relative to the center of the flange of 0.1 in. Substituting this value together with the design load of the 3 by 3 by 7/8 in. top angle in the standard deflection equation for the free end of a cantilever provides the worst possible case with regard to flange deflection. For the following rotations we have:

\[ \Delta = \text{deflection in inches} \]
\[ w = \text{load in pounds per inch} = 5100 \text{ in-lb}. \]
\[ l = \text{length of cantilever} = \frac{1}{2} \text{flange width} = \frac{F}{2} \]
\[ I = \frac{1}{12} bd^3 \]
\[ b = \text{flange width} = F \]
\[ d = \text{flange thickness} + \frac{3}{8} \text{top angle thickness} = t + 0.328 \]
\[ \Delta = \frac{I}{8EI} \]
\[ 0.1 = \frac{5100(F)^4}{8 \times 30,000,000 \times \frac{1}{12} F(t+0.328)^3} \]

Solving this equation gives:

\[ \frac{F}{t+0.328} \leq 18.4 \] as the limiting ratio of flange width to flange thickness. Examination of a standard table of H and column sections shows that all are well within this limit. That portion of the flange deflection which was caused by bending is, therefore, negligible.

The portion of the deflection caused by yielding of the web was confined to safe limits by limiting the web stress at critical points. Observation of scaling of the whitewash showed that the seat angle at all times gave a better distribution of load over the column flange and web than the top angle. The critical point is, therefore, located at the point of attachment of the top angle. Limiting of web stress at this point should make the connection amply safe.

Computation of the web stress was treated as a bearing problem with a tensile force substituted for the bearing load. The total top angle design load was considered to be distributed at 45° through the flange into the web giving a stressed area equal to the web thickness times the length of leg of top angle weld plus twice the flange thickness. For:
t = flange thickness
T = web thickness
P = top angle load = pounds per inch of top angle times length of top angle
f = length of leg of top angle weld
F = flange width = top angle length,
we have:
\[
\frac{P}{(f+2t)T} = 24,000^* \text{ p.s.i.}
\]
Substituting the load and dimensions of the 3 by 3 by 7/8 in. top angle gives a limiting ratio of flange width to web thickness of:
\[
\frac{F}{T} \leq 94.1 (0.437 + t)
\]
Examination of a large number of H and column sections showed all to be well within this limit.

**SUMMARY AND CONCLUSIONS**

1. The rigidity of welded top angle connections at design load increases with increase in angle thickness and is well above the minimum allowable end restraint of fifty per cent.

2. Practical consideration of economy, rigidity and safety limits the top angle size to a maximum of 3 by 3 by 7/8 in.

* Allowable secondary stress.
3. The flange deflections produced by the welded top angle connection may be separated into two parts, that due to flange bending and that due to web deformation. The bending deflections may be computed by assuming a portion of the flange to act as a simply uniformly loaded cantilever. The web deformations may be held within reasonable limits by analyzing the web stresses in a manner similar to bearing stress computations.

4. The localized stresses and deformations produced by the welded top angle connections are of a negligible order of magnitude providing the upper limit of top angle size is not exceeded. If further assurance of this fact is required, one need only consider the fact that the column is always laterally supported at the connection. Therefore local stresses and deformations of the order produced by the connection can not possibly cause column failure under primary loading.
# TABLE I

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<th>Top Angle Size</th>
<th>Column Size</th>
<th>Beam Size</th>
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Δta = Design load edge deflection of flange at point of attachment of top angle
Δsa = Design load edge deflection of flange at heel of seat angle

The 3 x 3 x 7/8 in. top angle failed in the fillet of the top angle as a result of a flaw in the steel. The data from this test therefore are omitted from the report.
Fig. 2

Gage Used In Measuring Flange Deflections
FIG. 3 - TRANSVERSE BENDING OF COLUMN FLANGE
3\times3\times\frac{1}{2} TOP ANGLE 10'' LONG
Fig. 4 - Transverse Bending of Column Flange
3 x 3 x \( \frac{1}{2} \) Top Angle 8" Long
FIG. 5 - LONGITUDINAL BENDING OF COLUMN FLANGE
3 x 3 x 1/2 TOP ANGLE 10" LONG
Fig. 6 - Load Deflection Curve for Top Angle

- $3 \times 3 \times \frac{1}{16}$ Top Angle 10" Long
- $3 \times 3 \times \frac{1}{16}$ Top Angle 8" Long
- $3 \times 3 \times \frac{1}{16}$ Top Angle 6" Long

Load (lb per inch of top angle)

Heel Deflection (thousand of inch)
Fig. 7
Failure Of Large Size Top Angle Connection

Fig. 8
Web Yielding Indicated By Scaling Of Whitewash
\[
M_a = \frac{P L}{4} - \frac{PL}{4}
\]

\[
M_a = 2EI \left(2\theta_a + \theta_b\right) - \frac{PL}{4}
\]

\[
-16P = 2EI \theta_a - \frac{PL}{4}
\]

\[
L = 64 + 4\sqrt{256 + \frac{EIL\theta_a}{P}}
\]

\[
\theta_u = \frac{PL^2}{16EI}
\]

\[
R = 1 - \frac{\theta_a}{\theta_u}
\]

**Fig 9 - Method of Calculating Rigidity**
Fig. 10 - Variation in Rigidity with Load
Deflection of Flange Relative to Flange Center Line
(thousandths of inch)

Distance from Flange (inches)

3\times3\times\frac{1}{2} Top Angle 10' Long
Column Flange 10'' \times 0.558''

3\times3\times\frac{1}{2} Top Angle 8' Long
Column Flange 10'' \times 0.558''

3\times3\times\frac{3}{4} Top Angle 10'' Long
Column Flange 10'' \times 0.558''

3\times3\times\frac{3}{4} Top Angle 8' Long
Column Flange 8'' \times 0.618''

Fig. 11 - Comparison of Observed and Computed Flange Deflections