INTRODUCTION

The investigation reported herein was undertaken as a study of the design and behavior under uniform load of two-way reinforced concrete slabs. Originally it was planned to conduct all tests on concrete slabs freely supported on rollers on all four sides and loaded uniformly by means of a water tank with thin flexible bottom. It was found, however, in trying this set-up that uniform bearing was very difficult to achieve even though several layers of celotex were placed between the slab and the rollers. The material itself was a lessened the chance of variable factor and Mitigated against the discovery of relevant factors above the action of the slab under load, because the observations could not be duplicated at different times even though no change had been made in the set-up.

It was finally necessary to revert to an elastic material used in model form to obtain results from which definite conclusions could be drawn. After experimenting with celluloid, and aluminum and brass plates, brass was selected as the material to be used. With the use of small scale models, the field of the investigation was widened considerably. The effect of beams on the slabs could now be effectively studied, since the slabs were built with beams. Moreover, if the beam had enough torsional rigidity to produce about seventy per cent of end fixity in the slab, the effect of continuity would be unimportant.

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The thickness of plate decided on was 0.042 in. which gives appreciable deflection but does not leave a permanent set in the plate within the loads used in the experiments. The size of panels was taken as 5 by 5 in., 5 by 7-1/2 in., and 5 by 10 in., thus giving ratios of 1 to 1, 1 to 1-1/2, and 1 to 2 which were the same as those selected in the original investigation. The thickness was not reduced proportionately to the length of slab because of the difficulty in obtaining deflection of any significant magnitude.

Naturally, in order to apply the results to concrete, we have to prove that there is some similarity in the behavior of concrete and brass under load. We will therefore test some concrete slabs, particularly those with beams, and try to correlate the data obtained from the model and concrete studies. We also have the results of tests made at Dresden, Germany, under the supervision of W. Gehler and H. Amos in a similar problem with which we expect to correlate our model data. We have checked some of the data already and find that the results are reasonably consistent.

PRESENT THEORY AND DESIGN

Many attempts have been made to solve the problem of two-way slabs theoretically. The outstanding contribution of America to its solution is Professor Westergaard's work*

* Proceedings, A.C.I. Vol. XVII, 1921 page 415 - 538
Vol. XXII, 1926 page 26 - 44
which is based on certain assumptions which are probably justified for a general solution. Among other noteworthy contributions are those by Neilsen in Denmark, Marcus in Germany, and by Galerkin in Russia.

One method of design, developed by various men with Nielsen* as one of the leaders, is by difference equations. We find that our observed deflections of the models check the calculated values by this method well within ten per cent. However, the use of difference equations involves great care in the mathematical manipulation of the equations to be solved and also required too much time for the ordinary engineer's problem.

From his theoretical solution of the problem, Professor Westergaard derived certain empirical relations, which on substituting the proper values for length of span and continuity, a moment coefficient could be obtained that gave approximately the same result as did the theoretical solution. Most of the building codes in existence in America have accepted Westergaard's work as the basis for their own codes, but have applied their own little changes for various reasons. The result is that every code has its own variation and no standard code has been adopted. Fig. 1 and 2 show the moment coefficient given by the various codes plotted against the ratio of sides of the slab. The discrepancies in the various codes are shown very well in the figure.

* "Spændinger i plader" by Nielsen (Danish publication)
* Proceedings, A.C.I., Vol. XXX, 1934 page 498 - 503
We do not expect to find a theoretical solution of the problem that would be any simpler than those already in existence, but rather do expect to develop some rational empirical method of design that will agree with our observed results.

In the tests conducted on the square aluminum model we were able to plot the deflection along the center line and along the diagonal. From the Dresden data for a square two-way slab simply supported, we plotted the deflection curves for center line and diagonal to a scale which made the center deflection of the concrete slab and our model fall on the same point. The result is shown in Fig. 3 and is the basis of our optimism in assuming that from the results of our tests on models we will be able to draw definite conclusions as to the action of concrete slabs under load.

Naturally, we took the deflection of the concrete slab at a load for which there was no cracking and full torsion moment was in effect. After cracking the curves would not check very well because the effect of the torsional moment would be eliminated since in a two-way slab there is no reinforcing that would maintain the torsional moment after cracking. However, it is our belief, that after cracking design on the basis of fourth-power distribution of the load would be safe and probably be as accurate as could be desired. Fourth-power distribution of the load means that the total load is divided up in proportion to the fourth power of the length of the radii of the panel with the short direction carrying more than the long direction.
The danger of cracking of concrete in two-way slabs is negligible, under ordinary working loads as has been proven by the Dresden tests as well as at Fritz Laboratory during the present investigation. The German tests showed no cracking under three times the cracking load for two-way slabs and we found no cracking up to two and one-half times the cracking load which was the limit of our tests. Therefore it probably will be safe to say that under ordinary conditions we are able to depend on the torsional moment carrying part of the load.

We found in our calculations that the torsional moment carried approximately fifty per cent of the total load in the square slab which checks up very well with Marcus' theoretical solution of the problem. Naturally we would like to take advantage of this load-carrying property and we might even suggest that part of the steel be placed radially so as to make the torsional moment effective even after cracking.

As the ratio of sides increases the load-carrying capacity of the torsional moment decreases rapidly and soon becomes negligible. As to the limiting ratio we have not gone far enough into the investigation to say as yet.

To date we have investigated all the non-continuous slabs. We expect to go on to the partly continuous and fully continuous slabs as planned in the original outline. Fig. 4 shows the outline of brass slabs to be tested. The short side is to be five inches in each panel.
The set-up for the tests is as shown in Fig. 5, which shows the five by ten-inch simply-supported brass plate, and Fig. 6 which shows the five by ten-inch brass plate supported on beams. The beams are designed on the basis of equal maximum stress in the beam and in the slab as computed by the now-existing design formulas. The moment coefficients of the slab are those given by Westergaard's empirical formulas and the load distribution to the beams is taken to be as follows:

Each long beam is said to carry the load superimposed on one of the trapezoids and each short beam carries the load in the triangle. However the short beams are made the same size as the long beams, which is standardized practice.

The concrete slab on beams is designed in exactly the same way. Incidentally we assume full continuity in our design for slab on beam, so that if cracks should appear we still would be able to carry the negative moment by the steel and obtain results comparable to those of the brass models.

The Ames dials used, read to ten thousandths of an inch. The mirrors used for securing rotation of the beam are shown in Fig. 6. A transit is set up about fifteen feet from the plate and scales divided into fiftieths of an inch are pasted to a solid foundation. Readings are taken for various loads on plate and the angular rotation is computed from these readings.
The load is applied by means of water measured by a graduated cylinder. The maximum load per square inch on any plate is 120 grams or 0.264 lb. The load is added in three equal increments. However, the zero loading represents the weight of the tank plus approximately 25 gr per in. or about 0.053 lb per sq in. The reason for using this zero load is obvious, and since deflections are directly proportional to the load within reasonable experimental errors, no error is involved in so doing.

The dials are moved around under the slab, and under the beam when beams are used, and can be placed to the nearest quarter of an inch. The column prevents our getting very close to the corners of the slab but the deflection curves are so regular that we are able to project them into the corner without much fear of error.

Fig. 7 shows the deflection curves along the center line of the 5 by 5-in. brass plate. Fig. 8 shows the deflection curve along the center line of the 5 by 5-in. brass plate on beams. The tendency towards fixity is markedly shown here by reference to the deflection curve of the simple beam plotted with same end and center deflection. The calculated deflection by difference equations* are plotted also. Here the deflections are greater than the calculated value which is in agreement with conditions since we computed our difference equations on the basis of full continuity. We also applied

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* Nielsen "Spændinger i plader"
our method of calculation which at present is in an embryonic stage in that we can work from observed data, but we cannot as yet work directly from load. However, we are working at the problem of torsion in the beam and expect to arrive at some method for starting with the load and arriving at the stresses and deflections.

At present the method is as follows: From the center deflection of the simply-supported slab we determine the load required to produce that deflection in a simple one-way slab. That is:

\[
\frac{W}{EI} = \frac{384M'}{5L^4}
\]

and substitute in the elastic equation of deflection:

\[
y = \frac{384y'x^4}{24EI} - \frac{WLx^3}{12EI} + \frac{WL^3x}{24EI}
\]

which gives, when \( y' \) is center deflection:

\[
y = \frac{384y'x^4}{120L^4} - \frac{384y'x^3}{60L^3} + \frac{384y'x}{120L}
\]

\[
= \left[ 3.2\left(\frac{x}{L}\right)^4 - 6.4\left(\frac{x}{L}\right)^3 + 3.2\left(\frac{x}{L}\right) \right] y'
\]

\[
\frac{y}{y'} = 3.2 \left[ h^4 - 2h^3 + h \right] \quad \text{when} \quad h = \frac{x}{L}
\]

We then can compute the deflection at any point. From the center deflection of the plate on beams which we take as equal to the center deflection minus the deflections of the beams, we compute the moment required to lift our simple beam up to that deflection.
\[ y_m = y' - y'' + y''' \]

where \( y' \) = center deflection, simple slab

\( y'' \) = center deflection on beam

\( y''' \) = beam deflection

\[ \frac{M}{EI} = \frac{8ym}{L^2} \]

Substituting this value of \( \frac{M}{EI} \) in the elastic deflection equation which is given below:

\[ y = \frac{Mx^2}{2EI} - \frac{MLx}{2EI} \]

we have,

\[ y = \frac{8ymx^2}{2L^2} - \frac{8ymLx}{2L^2} = \frac{4ymx^2}{L^2} - \frac{4ymx}{L} \]

\[ \frac{y}{ym} = 4 \left[ \left( \frac{x}{L} \right)^2 - \left( \frac{x}{L} \right) \right] = 4(h^2 - h) \]

where \( h = \frac{x}{L} \)

Adding algebraically the \( y \)'s computed from various points from simple deflection and moment deflection equations, we obtain deflections which have been plotted on the various curves and noted as calculated deflections.

From the elastic equations we can compute the end slope which gives us the angle of rotation. However, in the case of the plate on beams, if we compute the end slope using the I of the plate above we will naturally find too large rotations. We may be able to compensate for this error by using a weighted average for the I for the plate and the I for the beam. At present we have not gone far enough in the investigation to be able to say that this weighted average will always work.
We also have computed the end moment from the elastic equations and using this moment as the torsional moment, we can compute the rotation of the beam in torsion and check our computed values from the same equation. The torsion equation* is:

\[ T = KG\theta \]

where \( T \) = torsional moment
\( K \) = torsion constant
\( G \) = shearing modulus
\( \theta \) = angular rotation

\( K \) may be found by the equation:

\[ K = \frac{b^3 b}{3} - 2Vh^4 \]

where \( b \) = length of section
\( h \) = width

\( V \) = factor depending on \( \frac{b}{h} \)

equal to 0.105 from \( \frac{b}{h} > 3 \)
and not very much different from \( \frac{b}{h} < 3 \)

We assumed first that the moment was distributed along the beam as shown below in (a) and then as shown in (b). The true distribution is probably somewhere between the two.

* STRUCTURAL BEAMS IN TORSION by Inge Lyse & B.G. Johnston
Proceedings, A.S.C.E., April 1935
The data obtained thus far do not warrant our going into a more detailed analysis of the torsion in the beams.

However, in order to apply our method to concrete, we must know something of the torsional properties of concrete. We are therefore going to conduct an investigation into the torsional properties of reinforced concrete beams. The other figures included are self-explanatory.

At present our final method of design is outlined more or less definitely in our minds as follows:

We will determine the part of the load carried in each direction in the slab from curves between percentages of load carried and ratio of sides. From this load we can compute the moment to be carried at the center of the span in each direction.

As to computing the fixidy moment we are not yet prepared to state definitely any method that can be applied rigidly. However, as we continue our work on the plate and on torsion in concrete we expect to be able to correlate the two and arrive at a rational solution of the problem.

It would be appreciated very much, if the Portland Cement Association would send suggestions and criticisms regarding the continuation of the work in order that we might be guided in our next year's investigation.
Fig. 1
Proposed Program for Two-Way Reinforced Concrete Slab Investigation

No Continuity

Number of Test Panels

Partly Continuous

Fully Continuous

Total 45

Fig. 4
Fig. 7

Deflections Along Center Line
5" x 3" Brass Plate
Deflections Along Center Line
5" X 5" Brass Plate

Deflection
Simple Stress

Observed Deflection Curve

a. Observed
b. Difference Equations
c. Calculated

Fig. 8
Deflections Along The Short Center Line
1/2" x 3" Brass Plate.
Deflections Along The Short Center Line
1/2 x 5' on beams
- Observed
- Calculated

Fig. 11
Deflection Along The Long Center Line
7/8" x 5" on Beams

- Observed
- Calculated

Fig. 12
Fig 13

Deflections Along The Short Center Line
10' x 5' Brass Plate
Deflections Along The Long Center Line
10" x 5" Brass Plate

Fig. 14
Deflections Along The Short Center Line
10 x 5' on Beams
- Observed
- Calculated

Fig. 15
Deflections Along The Long Center Line
10" x 3" on Beams
O Observed
O Calculated

Fig. 16