THE MODULAR RATIO IN REINFORCED CONCRETE DESIGN

by Inge Lyse*

I. INTRODUCTION

The paper on THE MODULAR RATIO by Dr. Hajnal-Konyi in the January, February and March issues of "Concrete and Constructional Engineering" is a very opportune contribution on one of the most important problems before the engineering profession. The progress of research in concrete and reinforced concrete has brought forth enlightened understanding of the actual behavior of structures built of this composite material, but at the same time it has also re-emphasized the necessity of vastly more experimental study in this field. Recent experimental evidences such as those presented by Dr. Hajnal-Konyi have shown that the prevailing methods of analyzing reinforced concrete members on the basis of definite working stresses do not give results which correspond to the facts and that some more rational method of design must be found.

Dr. Fritz Emperger should be given special credit for contributing very effectively in bringing this matter before the profession at large and in promoting international discussion of this problem. The time seems ripe for a thorough reconsideration of the design of reinforced concrete structures and an international exchange of views should be highly beneficial for the development of more rational methods.

* Research Associate Professor of Engineering Materials
Lehigh University, Bethlehem, Pennsylvania
Concrete is not as elastic material in the same sense as steel. The apparent modulus of elasticity is affected by a large number of factors such as the speed of testing, moisture condition of concrete at test, age of concrete, composition of concrete and type of aggregates and cements used. In a structure the members will continually be subjected to the dead load and generally for a considerable length of time the live load is also present. Because of the large change in modulus with time, which is generally termed plastic flow, there is a continuous change in the modular ratio during the first few months the structure is under load. Consequently the modular ratio is of little if any value in ascertaining the actual stress condition in reinforced concrete members. The time-honored conception of designing for working stresses thus becomes inadequate. Modern viewpoint in design tends toward abolishing the working stress conception and introducing the use of definite factors of safety against failure of any part of the structure. With this conception it becomes important to establish the ultimate strength of the various members which constitute the reinforced concrete structure. In this discussion both the compressive members such as columns and the flexural members such as beams and slabs will be considered.
II. Columns

Experimental studies have shown quite conclusively that the modular ratio has no place in the strength of reinforced concrete columns. The strength of reinforced concrete columns with no spiral reinforcement may be obtained from the formula:

\[ F = 0.85 f'_c A_c + f_y A_s \]  

(1)

and for spirally reinforced concrete columns:

\[ F = 0.85 f'_c A_c + f_y A_s + k f'_y A'_s \]  

(2)

where:

- \( F \) = strength of column
- \( f'_c \) = strength of 6x12-in. concrete control cylinders
- \( A_c \) = cross-sectional area of concrete in column
- \( f_y \) = yield-point stress of longitudinal reinforcement
- \( A_s \) = cross-sectional area of longitudinal reinforcement
- \( k \) = effectiveness factor of spiral reinforcement
  (may be taken equal to 2.0)
- \( f'_y \) = yield-point stress of spiral reinforcement
- \( A'_s \) = equivalent cross-sectional area of spiral reinforcement

It should be kept in mind that the concrete protective shell outside the spiral of the column cannot act simultaneously with the spiral reinforcement. In other words, the protective shell must fail before the spiral can develop its effectiveness. Thus it becomes necessary either to neglect the protective shell in computation of the strength of the column, or
to reduce the effectiveness factor in such a manner that only
the strength of spiral in excess of the strength of shell be
used. Because of the fire protective function of the shell,
it seems logical that it be considered of no load-carrying
value and that the net area of concrete (that is, the area en-
closed by the spiral) be used in the design calculations.

The factor of safety to employ in each case will nat-
urally depend upon the type and function of the structure.
Various factors may be desired for various percentages of re-
inforcement and for different grades of concrete. It is the
function of the Building Code Authorities to establish the
proper factors of safety.

III. FLEXURAL MEMBERS

The conventional method of designing reinforced con-
crete beams is based on linear distribution of compressive
stress in the concrete, no tensile strength of concrete, and
steel stress in accordance with a given modular ratio. This
method assumes a straight line stress-strain relation for
concrete. As early as 1905 Professor A. N. Talbot* proposed
that the stress distribution in the concrete should follow a
parabolic trend. However, the straight line basis of design
was generally recognized as giving reasonable values within
the range of stresses permitted under working conditions. The
initial modulus of elasticity of the concrete was assumed as

* Bulletin No. 4, University of Illinois
suitable for the modular ratio and as information became available regarding the relationship between initial modulus and strength of a concrete, a variable modular ratio was accepted in many building codes. The American Concrete Institute in its building Regulations for Reinforced Concrete, assumed the initial modulus of elasticity of concrete to increase in direct proportion to the increase in the strength of concrete. Thus a variable modular ratio given by the formula:

\[ m = \frac{30,000}{f'_c} \]  

was introduced in the 1928 regulations. This formula for the modular ratio is retained in the tentative 1936 Regulations. Experimental results have shown that the initial modulus of elasticity of concrete is not proportional to the strength of concrete. In a recent series of tests at Lehigh University the strength of the concrete was varied from 1000 to 6000 lb. per sq in. The modulus of elasticity of this concrete varied only from about 2,200,000 to about 4,500,000 lb per sq in.*. When such factors as plastic flow and moisture condition of the concrete are considered, the fallacy of the above formula for modular ratio is apparent. Since the effect of the value of the modular ratio is relatively small and since the straight

*See Fig. 12 of the paper: A STUDY OF THE QUALITY, THE DESIGN AND THE ECONOMY OF CONCRETE by Inge Lyse, published in the Journal of The Franklin Institute, April, May, June, July 1936
line method of design is at best very approximate, it seems advisable that a constant average modulus ratio be used in building code regulations until a more rational method of design has been developed.

In considering rational methods of design it is well to recognize the fact that concrete beams will generally give strength well above that computed by means of the conventional straight line stress distribution method. Particularly, beams with so large percentage of reinforcement that failure takes place in the concrete show strength far above the computed value. Also beams with very small percentage of reinforcement show strength far above the computed values. In order to study over-reinforced concrete beams, an extensive investigation was carried out at Lehigh University in 1930*. The variables studied were the effect of the strength of concrete and the effect of depth of beams. For five groups of beams the average strengths of concrete control cylinders were 1390, 2790, 4070, 4800, and 5740 lb per sq in. The effect of this variation in strength is illustrated in Fig. 1 where the moment at failure is plotted against the strength of the concrete. It is noted that except for the very weak concrete, the moment increases in direct proportion to the increase in strength of concrete for these beams which all failed due to crushing of the concrete. In the paper referred to, it is shown that the

* COMPRESSION STRENGTH OF CONCRETE IN FLEXURE AS DETERMINED FROM TESTS OF REINFORCED CONCRETE BEAMS
by W. A. Slater and Inge Lyse, Proceedings, American Concrete Institute, Vol. 26, 1930, p.831
observed moments at failure were far above those computed by
the ordinary straight line method. On the assumption of a
parabolic stress distribution in the concrete, the computed
results agreed more closely with the test data. For para-
bellic stress distribution the ratio of the distance between
the centers of pressure to the effective depth of the beam
is given by:

\[ j = 1 - \frac{r+1}{2(r+2)} k \]  \hspace{1cm} (4)

where \( r \) is the power of the parabola and \( k \) is the ratio of
the distance from the compression surface to the neutral
axis to the full effective depth of the beam. The test re-
sults indicated that the neutral axis remained approximately
at the same depth for all the beams. Assuming the neutral
axis to be located at the center of the beam, that is \( k=1/2 \),
the degree of the parabola which will give computed moments
of approximately the same magnitude as the observed moments
may readily be ascertained. A fifth degree parabola was
found to give very satisfactory results. In Fig. 1 the dotted
straight line represents the moments computed on the
basis of a fifth degree parabolic stress distribution in the
concrete. It is noted that except for the very weak concrete,
the agreement between computed and observed moment is very
good. The design formulas for this condition become:

\[ j = 1 - \frac{5+1}{2(5+2)} \cdot \frac{1}{2} = 1 - \frac{3}{14} = \frac{11}{14}. \]
\[ M' = \frac{5}{6} \frac{1.11}{28} f_c' b d^2 = \frac{55}{168} f_c' b d^2 \] 

(5)

or approximately:

\[ M' = \frac{1}{3} f_c' b d^2 \] 

(6)

where \( M' \) is the maximum moment at failure. The maximum stress in the steel is computed in the ordinary manner:

\[ f_s = \frac{M_{\text{max}}}{J.d.A_s} = \frac{M'}{\frac{11}{14}d.A_s} \] 

(7)

A comparison between the stress in the steel computed in this manner and the stresses observed in the beams, is made in Table I in which the observed values are for loads close to the ultimate loads carried by the beams. The observed stresses

TABLE I - STRESS IN REINFORCEMENT

<table>
<thead>
<tr>
<th>Group No.</th>
<th>Concrete Strength lb/sq in</th>
<th>Maximum Load lb</th>
<th>Measurements at Last Observation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Strain in Steel</td>
<td>Observed Stress lb/sq in</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Load lb</td>
<td>Computed Stress lb/sq in</td>
</tr>
<tr>
<td>1</td>
<td>1390</td>
<td>32,520</td>
<td>30,000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.00116</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>33,700</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>38,400</td>
</tr>
<tr>
<td>2</td>
<td>2790</td>
<td>44,710</td>
<td>40,000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.00136</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>39,400</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>38,400</td>
</tr>
<tr>
<td>3</td>
<td>4070</td>
<td>63,300</td>
<td>55,000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.00135</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>39,200</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>39,900</td>
</tr>
<tr>
<td>4</td>
<td>4800</td>
<td>74,830</td>
<td>70,000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.00145</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>42,000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>40,000</td>
</tr>
<tr>
<td>5</td>
<td>5740</td>
<td>87,410</td>
<td>80,000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.00138</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>38,600</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>38,400</td>
</tr>
</tbody>
</table>

were obtained by multiplying the observed strains with a modulus of elasticity of steel equal to 29,000,000 lb per sq in. It is noted that the agreement between observed and computed stress in
the steel is very satisfactory, particularly so for beams in which the strength of the concrete control cylinders exceeded 2000 lb per sq in. The simple method of design here presented, in which the modular ratio was omitted, thus served very well for the group of over-reinforced beams studied in this investigation.

In beams designed by the ordinary straight line method the failure is always due to the yielding of the reinforcement and the subsequent raising of the neutral axis until the compression area becomes so small that the concrete is crushed. Professor Saliger presents a method of design for this type of beams which gives values agreeing better with test results than does the common design practice. However, many variables have to be considered, such as the tensile contribution of the concrete and the extensibility of concrete, factors which vary with the type and moisture conditions of the concrete and therefore very elusive of evaluation. It therefore seems to the writer that with the lack of general evidence still prevailing the Saliger method should be considered as an interesting solution worthy of further study. Any analysis which tends to indicate accuracies beyond the limits of a variation in the materials themselves is questionable from a practical standpoint.

A simple basic conception which gives results within a reasonable range of dependability would be preferred until the time arrives when concrete is a material with more definite
properties than at present, and when more is known of the actual behavior of reinforced concrete beams in flexure. One simple method of analysis of beams of low percentage of reinforcement where the yield point of the steel is reached before the concrete fails in compression, may be based on rectangular stress condition in the concrete at failure. This assumed stress condition and the method of analysis are illustrated in Fig. 2. The moment at failure is given by:

\[ M' = \frac{1}{2} f_y A_s (d + d') \]  

(8)

and

\[ d - d' = \frac{f_y A_s}{f_c' b} \]  

(9)

A comparison between the maximum moments computed by this simple method and the observed moments at failure is shown in Fig. 3. It is noted that all the observed moments are somewhat higher than the computed moments. The difference is nearly constant, thus making the percentage of difference much greater for low than for high percentage of reinforcement. This is logical because the deformation of the reinforcement at the cracks will be greater for small percentage of steel and therefore the actual stress in the reinforcement at time of failure is considerably above the yield-point stress. Considering all the uncertainties which enter into any reinforced concrete beams the simple method here presented seems to offer a satisfactory solution for present day conditions. As a matter of fact, it gives results which
agree better with the experimental results than does the ordinary design method which employs the variable modular ratio. This simple method may also be applied to T-beams.

The writer is therefore hesitant about advising a complicated design method of the Saliger type until an abundance of supporting experimental results have shown that such a refinement in the analysis of reinforced concrete beams is justifiable.
Fig. 1 - Effect of Strength of Concrete on Strength of Over-Reinforced Beams.

\[ M' = \frac{5}{6} \cdot \frac{11}{28} \cdot bd^2 \cdot f_c' = \frac{55}{168} \cdot bd^2 \cdot f_c' \]

\[ f_s = \frac{M'}{\frac{11}{14} A_s \cdot d} \]
Fig. 2 - Stress Condition at Failure of under-reinforced Beams.

\[ T = C = f_y A_s = f'_c (d - d') b \]
\[ M' = T (d - \frac{1}{2} (d - d')) = \frac{1}{2} T (d + d') = \frac{1}{2} f_y A_s (d + d') \]
\[ d - d' = \frac{f_y A_s}{f'_c b} \]
Fig. 3 - Comparison between Computed and Observed Moments.