RIVETED SEMI-RIGID
BEAM-TO-COLUMN BUILDING CONNECTIONS
PROGRESS REPORT NUMBER 1

BY ROBERT A. HECHTMAN AND BRUCE G. JOHNSTON

AMERICAN INSTITUTE OF STEEL CONSTRUCTION
RESEARCH AT LEHIGH UNIVERSITY

COMMITTEE ON STEEL STRUCTURES RESEARCH
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NOVEMBER, 1947
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FOREWORD

THE restraining effect of beam-to-column connections, in frame action, and the potential economy that might result from the recognition of partial continuity in beam design, are subjects which have commanded the attention of investigators for many years. Because of the numerous variables involved in the functioning of these connections, progress toward the promulgation of a semi-rigid design technique has been slow, although the complete logic for such a technique has been freely admitted.

In approaching the problem, the American Institute of Steel Construction and the authors of this report have been guided by the considerations that:

1. To be of any real practical value the technique should be simple of application, and
2. That it must give "safe" results.

The first consideration suggests that the limiting, or critical, conditions of loading should be investigated and that simple general formulas, which will yield "safe" results even for these limiting cases, should be devised—the alternate being a solution by continuous frame analysis, modified for the semi-rigid characteristics of the several joints.

The second consideration requires that the actual moment-rotation of each approved type of connection be investigated experimentally to determine (1) the relationship between these variables for any condition of loading, size of beam, and required span length, and (2) to insure the presence of a dependable minimum factor of safety under any condition of service.

These two considerations are the basis for the following report.

Mineographed copies of an earlier edition of this report were circulated in April 1942, when the war interrupted plans for a general publication at that time.

In the opinion of the American Institute of Steel Construction Committee on Steel Structures Research, the authors have made a most valuable contribution in the field of the semi-rigid design of
steel frame buildings—one which can now be put into practice even though the experimental research to date has not developed a qualified semi-rigid connection for every size beam the designer may have occasion to use. Those connections which have met the necessary requirements will be found to cover the bulk of the beam tonnage usually specified in tier building construction. The design procedure developed in this report is one which can be applied to any one, or any number, of the beams in a frame otherwise designed as Type 2 construction, as defined in the A.I.S.C. Standard Specification, without affecting the rest of the beam framing.

T. R. HIGGINS

Director of Engineering

American Institute of Steel Construction, Inc.
NOMENCLATURE

d Depth of a beam in inches.

E Modulus of elasticity—taken as 29,000,000, lb. per in.²

F Redesign coefficient, applicable to a particular beam as loaded.

I Moment of inertia of a member.

K Ratio, $I/l$, of any member in a frame.

$K_B$ Ratio, $I/l$, for a beam in a frame.

$K_C$ Ratio, $I/l$, for a column in a frame.

l Length of a beam or column between joints, in inches.

$M$ Moment in a member at a joint.

$M_e$ Positive moment in a restrained beam.

$M_R$ Moment at the ends of a beam which would be induced by a given loading if the connections were fully continuous and the supporting columns did not rotate. (Full fixed end moment.)

$M_r$ Moment at the ends of a beam which would produce a specified tensile stress in the tension rivets in its semi-rigid connections.

$M_s$ Simple span moment in a beam resulting from given gravity loading.

P A concentrated load on a beam.

p Percentage of rigidity of a semi-rigid connection.

S Section modulus of a member.

w Uniform load per unit length of a beam.

$\phi$ Angle of rotation at the end of a beam; also the angle of rotation, produced by $M$, within a semi-rigid connection.

$\phi_s$ Angle of rotation at end of a beam, due to simple span loading.

$\sigma_r$ Specified unit tensile stress for rivets.

$\sigma_w$ Specified unit flexural stress for beams.

$\theta$ Angle of rotation of a column at a joint, due to frame action. Subscript designates joint; thus $\theta_A$ indicates the rotation of the column at joint A.
I—INTRODUCTION

This is a report on tests of semi-rigid beam-to-column connections, such as may permit the weight of beams to be reduced as compared with results of usual design practice in which end supports are assumed to be without bending restraint. With this purpose in view, tests of forty-seven riveted beam-to-column connections were made at the Fritz Engineering Laboratory of Lehigh University. The test results are interpreted and a simple design procedure is developed. The investigation was sponsored and financed by the American Institute of Steel Construction, which, through its Committee on Steel Structures Research authorized the initial work to begin in September 1939. Actual testing was completed in June 1941, and the study of the test data was virtually completed by January 1942, but the war interrupted plans for publication.

The design requirements for a beam-to-column connection in a steel building frame will include the following:

1. The connection must have vertical shear strength sufficient to carry safely the vertical beam end reaction.

2. If lateral forces on the building, such as wind loads, are to be considered, the connection must have moment strength as well as shear strength.

3. If the connection is designed only for vertical loads it must either be (a) flexible, or non-moment resisting, or (b) moment resisting with moment strength inversely proportional to its flexibility.

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† Professor of Civil Engineering and Director, Fritz Engineering Laboratory, Lehigh University, Bethlehem, Pennsylvania.
4. The connection should be economical in design and convenient for erection.

Building connections may be classified under three different headings with respect to their moment-rotation characteristics.

1. "Rigid" Connections are those in which the relative rotation between the end of the beam and the column is reduced to a minimum by the use of stiff connections, as in the case of continuous frames where full continuity is assumed in the analysis. (See Fig. 1(a)).

2. Flexible Connections are those which are capable of carrying the end shear, but which allow relatively free rotation between the end of the beam and the column, as illustrated in Fig. 1(c). A flexible connection approaches the common assumption of pin end supports, in which case the beams are designed for full simple beam moment. This has been the general practice in the case of standard riveted building connections.

3. Semi-Rigid Connections are intermediate between rigid and flexible connections and transmit appreciable bending moment, with some rotation between the end of the beam and the column. (See Fig. 1(b)). Many connections assumed as "flexible" are inherently "semi-rigid", thereby developing end moments which have not been considered in the design.

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The design of the connections that were tested followed standard practice as closely as possible and the details were checked by a fabricator's drafting department.

In each semi-rigid beam-to-column connection test the relative rotation at the connection, between the end of the beam and the adjacent column, was recorded for successive increments of applied...
connection moment. Such data permit evaluation of the actual moments developed at the ends of beams so connected in a building frame. This evaluation makes possible the design of beams in building frames, using somewhat less than the maximum simple beam moments and thereby saves weight.

A considerable background of information on semi-rigid connections has been developed, in this country and in England, by work on (1) tests, (2) analysis and (3) design.

Tests of six riveted connections of a variety of types were made at the University of Illinois\(^1\) in 1917, and a few tests of riveted building connections at the University of Toronto\(^2\). During 1931-1936 the Steel Structures Research Committee of the Department of Scientific and Industrial Research of Great Britain\(^3\) tested about thirty-five riveted connections. During the same period eighteen riveted connection tests sponsored by the American Institute of Steel Construction were made at the College of the City of New York\(^4\). Tests also have been made in Great Britain to study the effect of concrete encasement\(^5\).

Methods of analysis are fundamental to the development of generalized design procedures. The application of both the slope-deflection and moment-distribution methods to the analysis of frames with semi-rigid connections was made by the British\(^6,7\). Similar methods of analysis have been presented by one of the authors\(^8\).

The question of both beam and column design was considered in great detail by the British Steel Structures Research Committee\(^3\). A “Joint Committee of the Institution of Civil Engineers and the Institution of Structural Engineers” (Great Britain) concluded that

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\(^1\) W. M. Wilson and H. F. Moore, “Tests to Determine the Rigidity of Riveted Joints in Steel Structures”, Bulletin 104, Engineering Experiment Station, University of Illinois.


\(^3\) First, Second and Final Reports of the Steel Structures Research Committee of the Department of Scientific and Industrial Research, Great Britain, 1931-1936.


the British Steel Structures Research Committee design procedures were "far more laborious than those in general use". This Joint Committee recommended instead, the making of minor reductions in calculated simple-span maximum moment, using top and seat angle connections.

The authors\(^9\) have suggested a simplified design procedure applicable to a wide range of connection stiffness. The National Building Code of Canada also has formulated simplified design procedures\(^10\) based on the work of the British Steel Structures Research Committee. The present tests have been made because it was felt that previous investigations were not sufficiently complete to furnish data covering a wide range of beam depths, beam sections and span lengths. Extensive tests of connections for beams of both light and fairly heavy weight from 12 to 18 in. in depth were carried out, and exploratory tests made on 21 and 24 in. beams.

The following are the more important factors which have been investigated experimentally:

1. Variation of beam depth.
2. Variation of top angle thickness.
3. Variation of rivet diameter.
4. Effect of approximately doubling the beam flange thickness.
5. Difference between like connections to the column web and to the column flange.
6. Variation in identical connections fabricated in different shops.

The following information has been obtained from the test results:

1. Moment, angle-change relationship of each connection.
2. Reaction value of typical connections.
3. Observation of initial and final failure of each connection.
4. Location of the center of rotation, and division of rotation into component contributing parts, in the case of 28 tests.
5. Suitable design range and moment restraint values of each connection.

The results have been summarized, correlated and certain recommendations for design procedure have been developed.


II—TEST PROCEDURE

A typical set-up to test the moment and rotation requirements of a connection is shown in the diagram, Fig. 2, and by photograph in Fig. 3. The test assemblage, consisting of two beam stubs riveted to a column stub, is supported in an inverted position. The supports apply shear and moment at the connection approximately equivalent to the shear and moment at the end of a building beam framed at each end to a column. After a preliminary determination of the moment-rotation characteristics of the connection at low loads, the moment arm length "a" can be adjusted so that the ratio between
Fig. 3.—Photograph of typical set-up.
moment and shear will simulate any desired beam length and load condition. This ratio of moment to shear would remain constant if there were a linear relation between moment and rotation during the test.

Fig. 2 shows the rotation bars used to measure relative rotation between beam and column by use of a 20 in. level bar of the same type used and described in a previous investigation. This level bar was sensitive to changes in angle of 1/20,000th of a radian. Fig. 2 also shows 1/1000 in. dial gages in position to measure horizontal movement of the beam flanges relative to the column face, thereby locating

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Fig. 4.—Comparison of constant-load and constant-maximum-stress beam lines applied to a typical test.

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the center of rotation. Additional dial measurements were taken for the purpose of evaluating the contribution of rivet slip and connection angle flexure to rotation.

From the rotation readings obtained by the level-bar, curves have been drawn showing the relation between connection moment "\( M \)" and span-end rotation "\( \phi \)" between the beam and the column. Fig. 4 shows typical moment-rotation, or \((M, \phi)\) curves for Test No. 16. Since each test assemblage consisted of two connections, two curves were obtained from each test.

The connection passes through three stages: first, an initial stage with moment approximately proportional to rotation; second, a gradual spread of yielding in the connection; and third, a stage of accelerated rotation, finally resulting in either failure or very excessive deformation.

![Diagram](image)

As the connections began to yield, the rotations increased disproportionately to the moments. In an actual beam framed in a building the non-linear increase in rotation would result in an increasing ratio of vertical reaction to connection moment. In some of the tests the load point was moved in toward the column to simulate this condition and to study the effect of a change in the moment-shear relationship.
By removing the applied moment from the connection, at various stages of the tests, the unloading and reloading ($M, \phi$) curves and the amount of initial set and rivet slip were found as shown in Fig. 4.

As a routine check of their adequacy to take reaction, four connections were so tested, after the moment-rotation tests were completed. The ends of the beam stubs were supported (see Fig. 5) and the load applied close to the connection. The top angles of top and seat angle connections were removed. stiffeners were used at the reaction point to prevent beam web crippling. The results of this study are reported under "Failure of Connections in Reaction", on page 37.

III—GENERAL TEST PROGRAM

Forty-seven separate test assemblages were fabricated and tested. Except for a series of five similar assemblages made by different fabricators from the same detail drawing, each assemblage was of different design. Each assemblage, however, consisted of two identical connections tested under the same conditions. Two test curves are reported for each assemblage, the maximum reported load being that at which one or both connections of the assemblage failed.

Table I shows in summary the general program of connection tests, indicating the factor investigated, beam depth, type of connection, column face connected to, and rivet diameter.

Shop details of the various assemblages are shown in Appendix B of the report. Fabrication was carried out in regular fabricating shops under conditions simulating shop and field practice. The following notes were listed on the drawings as furnished the fabricator:

**Material:** All material to meet A.S.T.M. Standard Specification A7-39 for Building Steel and to be free from rust, with mill scale intact. All sections of same size should be cut from material of same heat and rolling, and a 1 ft. 4 in. extra length for laboratory coupons shall be provided.

**Holes:** Punch $\frac{1}{16}$ in. dia. for $\frac{3}{4}$ in. dia. rivets. Punch $\frac{1}{8}$ in. dia. for $\frac{3}{8}$ in. dia. rivets.

**Rivets:** $\frac{3}{4}$ in. dia. and $\frac{7}{8}$ in. dia. as noted. All rivets to be driven by pneumatic hammers by methods corresponding to best field driving practice.

**Paint:** No paint.

**Shop Note:** Clip angles to be riveted to the column sections before beam stubs are placed. Rivets, through the beam flanges and outstanding legs of angles, to be driven last, are shown as open holes.
<table>
<thead>
<tr>
<th>Test No. for Col.</th>
<th>Factor Investigated</th>
<th>Beam Depth of Type of Face Riv.</th>
<th>Introduced</th>
<th>Flange</th>
<th>Web</th>
<th>Flange</th>
<th>Web</th>
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<tr>
<td>12 14 16 18 21 24</td>
<td>Beam Depth</td>
<td>35</td>
<td>2 20 22 9</td>
<td>Top and Seat Angles</td>
<td>a  &quot; a &quot; &quot; a &quot;</td>
<td>3/4</td>
<td>&quot; a &quot;</td>
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<tr>
<td>35 9 24 12 16 18</td>
<td>Top Angle Thickness</td>
<td>1</td>
<td>1 5 22 20</td>
<td>Top and Seat Angles</td>
<td>a  &quot; a &quot; &quot; a &quot;</td>
<td>3/4</td>
<td>&quot; a &quot;</td>
</tr>
<tr>
<td>17 11 23 12 19</td>
<td>Beam Flange Thickness</td>
<td>14</td>
<td>17 15 25 26 32</td>
<td>Web 3/4 Flange 3/4</td>
<td>&quot; a &quot; &quot; a &quot; &quot; a &quot;</td>
<td>3/4</td>
<td>&quot; a &quot;</td>
</tr>
<tr>
<td>20 22 9 18 16 14</td>
<td>Column Face Connected to</td>
<td>10</td>
<td>14 16 17 10 6 14</td>
<td>Top and Seat Angles Top Story</td>
<td>a  &quot; a &quot; &quot; a &quot; a  &quot;</td>
<td>3/4</td>
<td>&quot; a &quot;</td>
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On this page, Test Nos. on same horiz. line have top details cut from same section.
### Table 1—(Cont'd)
#### TEST PROGRAM OF CONNECTIONS

<table>
<thead>
<tr>
<th>Factor Investigated</th>
<th>Test No. for Beam Depth of</th>
<th>Type of Connection</th>
<th>Col. Face Conn. to</th>
<th>Riv. Diam. in.</th>
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<tr>
<td></td>
<td>12</td>
<td>14</td>
<td>16</td>
<td>18</td>
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<tr>
<td>Fabricator</td>
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<tr>
<td>Rivet Diameter</td>
<td>2</td>
<td>23</td>
<td>9</td>
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<tr>
<td></td>
<td>35</td>
<td>40</td>
<td>24</td>
<td></td>
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<tr>
<td>Web Angle Size</td>
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<td></td>
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<td></td>
<td>4</td>
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<tr>
<td>Conn. to One Side Col. Web</td>
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<tr>
<td>T &amp; Seat Angle</td>
<td>34</td>
<td>33</td>
<td></td>
<td></td>
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<tr>
<td>T-Connection</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>T-Conn. and Web Ls</td>
<td></td>
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<td></td>
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<tr>
<td>Length of Vert. Leg of Top L</td>
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<tr>
<td>Top, Seat, and Web Ls</td>
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<tr>
<td>Top Story Connections</td>
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<td>47</td>
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The specimens were ordered in six different groups, successive groups being designed after the results of the preceding tests had been studied. All details conformed as closely as possible with standard shop details and were checked for this feature by the fabricator.

The forty-seven test assemblages can be classified as follows:

- Top and seat angles................................. 34
- Top and seat angles to one side of column web....... 2
- Top, seat and web angles............................. 2
- Standard web angle connections....................... 4
- Tee and seat angle..................................... 2
- Web clip and seat angles............................. 1
- Tees on both beam flanges........................... 1
- Tees on both beam flanges and web angles............ 1

In the case of top and seat angle, tee and seat angle, and tee connections, one-eighth inch shims were used under the top beam flange detail. All holes were punched and rivets were driven by pneumatic hammers.

Wide-flange sections were used for both beams and columns, the beam depths varying from 12 in. to 24 in., the columns from 8 in. to 14 in. Connection tests using beams of 12 to 18 in. in depth were quite numerous, while only exploratory tests using 21 and 24 in. beams were made. Twenty-eight of the connections were made to the column flange and nineteen to the column web. Three-fourth in. rivets were used on twenty-eight connections and \( \frac{3}{8} \) in. on nineteen. Both light and medium weight beam sections were included in the program, and the following is a summary of the number of tests on each beam size:

- 10—12 WF 25
- 2—12 WF 50
- 4—14 WF 34
- 7—16 WF 40
- 2—16 WF 78
- 15—18 WF 47
- 3—18 WF 85
- 1—21 WF 59
- 1—21 WF 108
- 1—24 WF 74
- 1—24 WF 120

The tests on 12 WF 25 and 18 WF 47 sections covered the range of connection variables. The results of these tests provided the basis for designing the other specimens.

All specimens except assemblages No. 27, 28, 29 and 30 were fabricated in the same shop. The latter were fabricated in other shops, and together with No. 22, constituted the "Fabricator Series", in which the differences in materials and fabricating practice were studied.
IV—EVALUATION OF SEMI-RIGID CONNECTIONS WITH RESPECT TO BEAM DESIGN REQUIREMENTS

When a beam is framed with semi-rigid connections to columns of multi-story buildings, the magnitude and location of the maximum bending moment in the beam will depend both upon the behavior of the frame as a whole and upon the characteristics of the semi-rigid connections. The maximum moment in the beam could be either a negative moment at the end or a positive moment at or near the mid-span. The effect of frame action as a whole will be considered later in the report. For the present, columns will be assumed as fixed against rotation at the beam connections, and consideration will be given to the effect of the connection alone.

Since the \((M, \phi)\) relationship of a semi-rigid connection is non-linear, the amount of restraint afforded by such connections will vary, in a non-linear fashion, for any given span of a particular beam, as the loading is increased. Hence it is necessary, in order to determine the amount of restraint actually afforded, to consider each beam size, beam span and loading as a separate problem. With the \((M, \phi)\) characteristics of a particular semi-rigid connection determined experimentally, a curve representing this relationship may be plotted with \(\phi\) values as abscissa and \(M\) values as ordinates, as explained on page 18. In order to determine the specific \(M\) and \(\phi\) values which will obtain when a semi-rigid connection is used for a particular beam, span and loading, a graphical construction, first developed by C. Batho\(^3\), could be used. The constant-load-beam line illustrated in Fig. 4 is Batho’s construction. In this report a “constant-maximum-stress beam line”, also shown in Fig. 4, has been employed.

The intersection of the constant-load-beam line with the experimental \((M, \phi)\) curve, determines the actual amount of end moment and rotation which a given connection will develop for a given beam on a given span under a given load. This method was followed in a recent study of welded beam connections\(^1\). The most economical beam size sufficient to support a particular load, on a particular span, restrained at the ends by semi-rigid connections, can be determined

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\(^3\) First, Second and Final Reports of the Steel Structures Research Committee of the Department of Scientific and Industrial Research, Great Britain, 1931-1936.

only if the amount of restraint afforded by these connections is known. However, using the constant-load-beam line construction, the load assumed for a given beam and span length probably will not be the one which would stress the beam to the allowable design stress. The constant-load-beam line construction, therefore, is at best a cut-and-try method.

The constant-maximum-stress beam line method, on the other hand, considers the case where the beam size, the span and the load are so related that the mid-span bending moment will have a constant value equal to \( \sigma_w S \) (allowable bending stress times section modulus). This method leads to a design procedure whereby the permissible load may be increased as the connection stiffness increases, or, stated another way, using this method, the beam size for a given load may be decreased as the connection stiffness increases. It is better adapted to the usual design problem, since the object is to determine the most efficient beam size which the specified bending stress for the beam, and the actual restraint from the connections, will afford. The method is, of course, limited to cases in which the connection stiffness will produce end moments no greater than the moment at mid-span, because, with greater fixity, the point of critical moment in the beam would be transferred from mid-span to the supports. For uniform loading the limiting case is one in which the connection provides not more than 75\% of full end fixity; for a single concentrated load at mid-span the end moments in no case will exceed the mid-span moment.

For any beam symmetrically loaded, with identical semi-rigid connections at its ends, let the release from imaginary fixed-end moment, \( M_R \), to the actual end resisting moment, \( M \), permit rotation of each end of the beam through the angle, \( \phi \). Then, since the resulting change of slope from end to end of the beam equals the area of the \( M/EI \) diagram,

\[
2\phi = \frac{(M_R - M)}{EI} \text{ or } M = M_R - \frac{2EI\phi}{l}
\]  

(1)

For any given span length and symmetrically disposed given loading, \( M_R \) is a constant whose value may be readily calculated. For any given beam of given span, \( E, I \) and \( l \) are also constants. Therefore Eq. (1) represents a straight line in terms of the variables, \( M \) and \( \phi \). This line has been defined as the "constant-load-beam line". It
expresses the relationship of $\phi$ to $M$ for all combinations, from the case where $\phi = 0$ and $M = M_R$ to the case where $M = 0$ and $\phi = \phi_*$, where $\phi_*$ is the rotation angle at the ends of the beam when completely unrestrained (simple span). When $M = 0$ it may be seen from Eq. 1 that

$$\phi_* = \frac{M_R}{2EI}$$

As is shown in Fig. 4, both the line for Eq. 1 and the test results for a given connection may be plotted to the same coordinates—each set of coupled $(M, \phi)$ values obtained from the test yielding one point on the test curve. The intersection of this curve with a beam line defines a connection moment and rotation $(M, \phi)$ exactly consistent with the $I, l$ and load assumed for the beam. The $(M, \phi)$ value at this intersection point is the only value relevant to the given problem.

Thus far $M_R$ has been treated as a constant, which is the case when the magnitude of the loading is fixed. However, in the case of the constant-maximum-stress beam line, $M_R$ is variable, because the amount of load which a given beam may carry and still retain a constant bending stress at the mid-span is dependent upon the degree of restraint afforded by the end connections.

To derive the equation for the constant-maximum-stress beam line, the center span moment, $M_c$, is held equal to $\sigma_w S$ (allowable bending stress in beam times its section modulus).

Considering first the condition of uniform loading and noting that $M$ is the actual restraining end moment,

$$M_c = \frac{wL^2}{8} - M$$

where $\frac{wL^2}{8}$ is the equivalent bending moment on a simple span beam. Since the fully fixed end moment, $M_R = \frac{3}{2}$ of this value, $M_c = \frac{3}{2}M_R - M = \sigma_w S$, and

$$M_R = \frac{2}{3}(M + \sigma_w S). \quad (2)$$

Substituting this value of $M_R$ in Eq. 1

$$M = \frac{2}{3}M + \frac{2}{3}\sigma_w S - \frac{2EI\phi}{l} \quad \text{or}$$

$$M = 2\sigma_w S - \frac{6EI\phi}{l} \quad \text{or}$$

$$M = 2\sigma_w S - \frac{3ES\phi}{l/d} \quad (3)$$
which is the equation for the constant-maximum-stress beam line for the condition of uniform loading.

Similarly, for a concentrated mid-span load,

\[ M_c = \frac{P}{{\frac{4}{3}}} - M. \]

In this case the magnitude of \( M_R \) is one-half that of the equivalent simple span bending moment,

\[ M_R = \frac{1}{2} (M + \sigma S). \]  \hspace{1cm} (4)

and, substituting this value in Eq. 1,

\[ M = \sigma S - \frac{4EI\phi}{l} \] \hspace{1cm} or \hspace{1cm} \[ M = \sigma S - \frac{2ES\phi}{l/d}. \]  \hspace{1cm} (5)

Eq. 3, 5 or a similarly derived equation for any other type of loading, may be plotted for any number of arbitrarily chosen values of \( l/d \) for a given beam. These lines will radiate downward to the right from the common point \((\phi = 0, M = M_R)\). At the intersection point of these lines with a connection test curve, the \((M,\phi)\) values obtained will measure the moment-rotation characteristics of that connection for the chosen \( l/d \) ratio for the given beam, and the magnitude of loading will be just sufficient to produce the specified bending stress, \( \sigma_s \), at mid-span. The end moments thus derived can be expressed, in terms of the full fixed end moment \( M_R \), as a percentage of rigidity. Thus the "percentage of rigidity" will be

\[ p = \frac{M}{M_R} \times 100. \]

The percentage of rigidity is proportional to the dependable \( M \) from the case where \( p = 100 \) when \( M = M_R \), to the case where \( p = 0 \) when \( M = 0 \).

On diagrams employing the constant-maximum-stress beam line, \( M_R \) is not constant;

for uniform load \[ M_R = \frac{2}{3} (M + \sigma S) \] \hspace{1cm} (Eq. 2)

and

\[ p = \frac{100M}{\frac{2}{3}M + \frac{2}{3}\sigma S} = \frac{150}{1 + \frac{\sigma S}{M}} \] \hspace{1cm} (6)
For concentrated mid-span load \( M_R = \frac{1}{2}M + \frac{1}{2}\sigma_wS \) (Eq. 4)

and

\[
p = \frac{100M}{\frac{1}{2}M + \frac{1}{2}\sigma_wS} = \frac{200}{1 + \frac{\sigma_wS}{M}} \tag{7}
\]

In this case the percentage of rigidity is not linearly proportional to \( M \).

The beam and its connections should be designed so as to have an adequate factor of safety. Consider the case of a beam designed for a particular percentage of connection rigidity and a maximum working stress \( \sigma_w \) equal to 20 k.s.i. Assume that the connection test curve crosses the design constant-maximum-stress beam line at a moment \( M' \) (see Fig. 6) at a point exactly corresponding to the percentage rigidity assumed in design. If a linear relationship existed between \( M \) and \( \phi \) up to a connection moment of 1.65 \( M' \), it would be possible to

![Diagram](image-url)

**Fig. 6.—Typical test No. 16 showing construction to obtain permissible percentage rigidity for a particular beam.**
put 1.65 times the design load on the beam before stressing the beam up to the specified minimum yield point of structural steel, viz., 33 k.s.i. But since the connection has a non-linear \((M,\phi)\) relationship, loads corresponding to 1.65 \(M'\) would produce an end moment no greater than \(M''\). Therefore the bending moment at the mid-span would be larger than that required to stress the beam to 33 k.s.i.

In order to insure in all cases a load factor of safety equal to the usual stress factor of safety of 1.65, a straight line will be drawn in Fig. 6 from point A to point C. Point C is the intersection of the test curve and a constant-maximum-stress beam line obtained by substituting 33 k.s.i. for \(\sigma_w\) in Eq. 3 (or in Eq. 5, as the type of loading may indicate). The intersection at B of Line A-C with the constant-maximum-stress beam line for the condition \(\sigma_w\) equals 20 k.s.i., gives a reduced value for the end moment which is \(\frac{1}{1.65}\) times the end moment, \(M''\), that would obtain if the beam were loaded 1.65 times the intended design load. The percentage rigidity represented by point B (40% in this case), since it will afford the 1.65 load factor of safety, will be used instead of the percentage of rigidity as determined by the end moment \(M'\) (50%) actually produced by the connection at the specified working stress for the beam.

The horizontal lines \((p = 30\%, p = 40\%, \ldots)\) are plotted on the diagram in accordance with Eq. 6.

Pursuing the same line of reasoning, a similar limitation can be placed on the tensile stress in the rivets connecting the top fitting to the column. While this tensile stress usually will not be a governing factor in the determination of the percentage of rigidity, it must be investigated and, when the \(p\)-value derived by the straight line construction outlined above, is larger than one based upon a limiting rivet tensile stress, the latter should control.

The tensile stress in the rivets may be expressed as

\[
\sigma = \frac{M}{d' n A}
\]

where

- \(d'\) = the vertical distance from the center line of the tension rivets to the top of the seat angle
- \(n\) = number of tension rivets
- \(A\) = area of one rivet.

Let \(M_r\) be the end restraining moment which will produce a stress \(\sigma_r\) in the rivets, 1.65 times the permitted design rivet stress, when the beam is loaded 1.65 times its design load.
Then

\[ M_r = \sigma d' n A \]

This moment is represented in Fig. 7 by a horizontal line intersecting the test curve at \( C' \).

If, as in Case I, Fig. 7, \( C' \) should fall to the right of point \( C \) (the intersection of the test curve and the constant-load beam line for the condition of 65% overload), rivet stress will not be the governing factor.

Consider now Case II, where \( C' \) falls to the left of point \( C \).

\[ M_R = M + \frac{2EI\phi}{l} \quad \text{(Eq. 1)} \]

Substituting \( M_r \) and \( \phi_r \), the particular values at point \( C' \), for \( M \) and \( \phi \) in Eq. 1, \( M_R \) will then have a definite value in terms of \( M_r \) and \( \phi_r \).

Thus:

\[ M_R = M_r + \frac{2EI\phi_r}{l} \]
A line from this value of $M_R$ at $\phi = 0$ through point $C'$, where $M = M_r$ and $\phi = \phi_r$, establishes the constant-load beam line, defined by Eq. 1, representing the case where the load is of just sufficient magnitude to produce the limiting rivet tensile stress with given values for $I$ and $l$. For a given beam having a moment of inertia $I$ and span length $l$, the fully-fixed end moment, $M_R$, will be directly proportioned to the magnitude of the loading. If a load factor of safety of 1.65 is to be maintained with respect to the load which will produce 1.65 times the limiting rivet tensile stress, it may be applied directly to the foregoing expression for the fully-fixed end moment. The general expression for $M$ and $\phi$, as determined by permissible rivet stress, then takes the form, from Eq. 1, of

$$M + \frac{2EI\phi}{l} = \frac{1}{1.65}(M_r + \frac{2EI\phi_r}{l})$$

But, if the design bending stress, $\sigma_w$, at the mid-span of the beam is 20 k.s.i.,

$$M + \frac{6EI\phi}{l} = 40S$$

(Eq. 3)

which is the general expression for $M$ and $\phi$ as determined by maximum allowable bending stress in the beam.

Solving these two expressions as simultaneous equations, each containing to the left of the equality sign, the value for $M$ and $\phi$ which will satisfy both the condition of load safety factor and maximum design bending stress in the beam, $\phi$ may be eliminated, leaving $M$ in terms of $M_r$ and $\phi_r$.

$$M = \frac{3}{2 \times 1.65}(M_r + \frac{2EI\phi_r}{l}) - 20S.$$  

Substituting this expression for $M$ in Eq. 6,

$$p = 100\left[1.50 - \frac{33S}{(M_r + \frac{2EI\phi_r}{l})}\right]$$

where $p$ is the largest percentage of rigidity which will provide a load factor of safety of 1.65 when tensile stress in the rivets is the criterion.

For the loading condition of a concentrated load at mid-span, the corresponding equation for percentage of rigidity, when rivet tensile stress governs, will be found to be

$$p = 100\left[2 - \frac{33S}{(M_r + \frac{2EI\phi_r}{l})}\right]$$
If $p_u$ is the maximum percentage of rigidity that can be used for a given beam designed for uniform loading on a given span, when rivet stress is the criterion, and $p_c$ is the corresponding maximum percentage when the same beam, using the same semi-rigid connections, is designed for a concentrated load at mid-span, it can be shown that

$$p_u = \frac{3\left(M_r + \frac{2EI\phi_r}{l}\right) - 66S}{4\left(M_r + \frac{2EI\phi_r}{l}\right) - 66S} \cdot p_c$$

from which it will be seen that the condition of uniform loading requires the larger reduction in allowable percentage of rigidity when rivet tensile stress is the limiting factor.

V—TEST RESULTS

The quality of material used in the connections was checked by means of sixty-three tensile coupon tests, made in the direction of rolling, from samples of all material used. All of these tests satisfied the ductility requirements for A.S.T.M. A7 Specification for Structural Steel. Only three samples out of 63 had upper yield points below the specification requirement of 33 kips per sq. in., and most of the upper yield points were between 35 and 40 kips per sq. in. Most of the steels had nominal ultimate strengths between 60 and 65 kips per sq. in. Fifteen out of 63 samples had nominal ultimate strengths below 60 kips per sq. in., several of these being in the neighborhood of 55 kips per sq. in. The single sample with the lowest strength had an upper yield point of 29.8 kips per sq. in., and a nominal ultimate of 53.9 kips per sq. in. The sample with the highest strength had a yield point of 42.7 kips per sq. in. and a nominal ultimate of 71.2 kips per sq. in.

Results of representative connection tests are presented in the Appendix B. With each test the following information is given:

a. Brief record or "log" of test.
b. Shop details.
c. Photograph(s) after test.
d. Moment-angle change relationship in design range, correlated with constant maximum stress beam lines which indicate the range of beam design requirements.
e. Graphical subdivision of moment-angle change diagram (tests 20 to 47) into components assignable to the following causes:
   1. Bending of top angle and column flange, and extension of tension rivets.
   2. Slip of top beam flange rivets.
   4. Slip of bottom flange rivets.
## TABLE No. II
### General Summary of Test Results

<table>
<thead>
<tr>
<th>Line No.</th>
<th>Test No.</th>
<th>Beam Name</th>
<th>Column Name</th>
<th>Beam Flange DT</th>
<th>Column Flange DT</th>
<th>Beam Top</th>
<th>Beam Seat</th>
<th>Column Top</th>
<th>Column Seat</th>
<th>Web Angle Conn</th>
<th>Web Angle Conn</th>
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### Details on Beam Flanges
- **Top Detail**
- **Seat Detail**

### Web Angle Conn
- **Angle Size**
- **Bolt Size**
- **Flange Size**

---

**Note:** The table contains data on various beam-to-column connections, including test numbers, beam and column names, and various dimensional details such as size, bolt, and flange details for both the top and seat of the connections along with the web angle connection details.
## TABLE No. II.—(Cont’d)
### General Summary of Test Results

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<th>TYPE OF FAILURE</th>
<th>PERCENTAGE RIGIDITY FOR UNIFORM LOAD AND LOAD FACTOR OF 1.65</th>
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<th>Roy At Des Mem</th>
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**PROGRESS REPORT NUMBER ONE**
Essential information regarding details of connections, together with the more significant test results, are summarized in Table II. The test results presented in this table have been rearranged to start with the lightest weight 12 in. WF section tested and end with the heaviest 24 in. WF section. Columns 3 to 17 give essential details regarding the type and make-up of the connections tested.

*Moments and Type of Failure*—Columns 18, 19 and 20 give the dependable moment values of the connection, for span-depth ratios of $l/d = 10, 20$ and $30$, respectively (calculated as shown in Fig. 6), based on the straight-line construction to give a safety factor of 1.65 (see page 28). The real factor of safety is the total reserve of connection rotation beyond the rotation at design load. Because of the straight-line construction there will be the same apparent factor of safety for moment at working load as for rotation. Actually, at working loads the true connection moment will be larger and the connection rotation will be slightly smaller than indicated by the straight line construction, as the constant-maximum-stress beam line will intersect the $(M,\phi)$ curve above and to the left of its intersection with the straight construction line. Nevertheless, this is of secondary importance, since the ratio of beam load at connection failure to working load will be equal to or greater than the chosen factor of safety. The amount of moment calculated at working load by the straight line procedure determines the usable restraint value of the connection. Blank spaces in columns 18, 19 and 20 are cases of connection rotation factors of safety insufficient to satisfy the foregoing construction.

Column 21 of Table II indicates the maximum test moment either at failure or at a deformation so excessive that the test was stopped. In the latter case the test load was usually increasing at a very slow rate. In some cases failure of one or more rivets occurred at loads below the maximum test load, without producing a general failure of the connection as a whole.

Column 22 indicates the type of failure. In summary, eighteen connections failed by excessive deformation; one because of fracture of the top angle; the remaining twenty-eight because of general tension failure of the rivets. All rivets failed near the middle of the shank except in the case of connections No. 5 and 40, where failure was just under the rivet head.

All failures of the top-and-seat-angle type of connections were in the top angle or top angle rivets. Seat angles were adequate in all the test specimens and showed signs of yielding only where extreme rota-
tion and/or high reactions were developed. Where connections were made to light column flanges, the column flanges bent considerably but the upstanding leg of the top angle tended to remain comparatively straight, throwing greater tension into the rivets nearer the centerline of the column. With connections on both sides of the column web, a more equal division of the tension took place among the rivets of the upstanding leg of the top angle. In the case of a connection to one side of the column web only, the outer rivets were most severely stressed. In the case of light beam flanges the top angle rotated as a whole and caused considerable deformation of the beam flange at high moments. The greatest deformation of the beam flanges occurred in the connection having the greatest thickness of top angle, and the least in those having the least thickness of top angle. Considerable slip occurred in the rivets fastening the top angle to the beam flanges. Two shims were used between the flange and angle in twenty-five connections and one shim in nine connections. Relatively little slip occurred in the rivets connecting the beam to the seat angle.

Standard web angle connections failed by excessive deformation in the case of web angle thicknesses of three-eighths inch; by fracture of the end rivet in tension for a web angle thickness of five-eighths inch. Slip of the rivets connecting the angles to the 12 inch beam webs were observed at moments within the working range of the connection. However, in the deeper connection to the eighteen inch beams, no such slip occurred to a degree observable by eye.
All tee connections failed by fracture of rivets in the flange of the
tee, even when fewer rivets were in the stem.

Columns 23, 24 and 25 indicate the dependable percentage of rigidity
of the connection as compared with the actual moments tabulated in
columns 18, 19 and 20, assuming no column rotation, and based on
the straight-line load factor of 1.65.

Maximum Connection Rotation—Column 26 gives the maximum
measured or estimated rotation between the end of the beam and the
column for each test, based on the minimum of the two connections.
In a few cases, where the rotation at failure was not obtained, a tangent
to the \((M, \phi)\) curve was extended from the last measured value up to
the moment at failure. This procedure would be expected to give less
than the actual maximum rotation because of the normally convex
upward shape of the usual \((M, \phi)\) curve, and hence yields conservative
results.

Connection Factor of Safety—In evaluating semi-rigid connection
behavior, a factor of safety with respect to rotation is of greater signifi­
cance than one with respect to the moment producing rotation. For
any given beam and system of applied loading, the actual amount of
end rotation is limited to something less than that which would take
place if no end restraint were present. The ability of a semi-rigid
connection to rotate more than this amount, as demonstrated by
actual tests, is of far more importance than the magnitude of the
moment which would have to be employed—or the relationship of
this moment to any actual end moments at service loads.

Columns 27 and 28 give the factor of safety of each connection,
for \(l/d = 20\) and 30, at working load. In the case of Test No. 36
(line 39) the insufficient factor of safety may be explained by the fact
that the connection is made to only one side of the column web. The
thin web has little flexural stiffness and most of the tensile force is
carried by the outside rivets adjacent to the column.

Components of Rotation—The division of the rotation into com­
ponents (due to deformation of various parts of the assemblage) was
made from data obtained by 1/1000 in. dial readings. Since these
data vary as much as fifteen percent from the more accurate level-bar
readings, they were adjusted to agree with the latter. Typical figures
for Tests 20 to 47, in the appendix, show the plotted components of
rotation. The curve nearest the bottom of the page is the mean
\((M, \phi)\) curve of the two connections in each assemblage.

These curves indicate that only a small part of the rotation is due
to bending of the seat angle and slip of the bottom beam flange rivets.
Most of the rotation is due to bending of the top detail and slip and extension of its rivets. Also, these curves show that in the case of top and seat angle connections, and tee and seat angle connections, the design of the top detail, for all practical purposes determines the rigidity and strength characteristics of the connection.

The five test assemblages, Numbers 22, 27, 28, 29 and 30, of same design but fabricated by different companies, had moment-rotation curves that did not differ greatly, as shown by Fig. 9.

![Graph](image-url)
### TABLE No. IV
Center of Rotation

<table>
<thead>
<tr>
<th>Comm No.</th>
<th>Type of Connection</th>
<th>Column Face Conn. to</th>
<th>Beam Section</th>
<th>Center of Rotation at</th>
<th>Conn.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>First Load</td>
<td>Q2 Upl. Mon</td>
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<tr>
<td>9</td>
<td>Top &amp; Seat Angles</td>
<td>Flange</td>
<td>18W47</td>
<td>M</td>
<td>g</td>
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<td>188</td>
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<td>0.24</td>
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<td>Flange</td>
<td>216</td>
<td>0.92</td>
<td>214</td>
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</tbody>
</table>

---

*a-distance from top beam flange to center of rotation  d - beam depth*
VI—ADAPTABILITY OF THE SEMI-RIGID CONNECTIONS TESTED TO BEAM DESIGN

The connection details will now be considered in relation to applicability to semi-rigid design as determined by permissible percentage of rigidity. The principal factors affecting the behavior of a semi-rigid connection have been outlined in Table I. In the evaluation of a particular variable it is preferable to make a series of tests in which only that variable is changed. In view of the many different factors affecting the test results, the complete experimental study of any one particular type of connection would require a great many more tests than could be made in this program. However, enough tests have been made to arrive at definite conclusions as to the adaptability of the top and seat angle type of riveted connection to semi-rigid design, within a specified range of beam sizes.

Standard Beam Web Angle Connections—The results of standard web angle type connection tests are tabulated in Table II (Tests 3, 4, 7 and 8). The percentages of rigidity developed by these connections were lower than results obtainable with a similar number of rivets in top and seat angle connections.

By increasing the rivet size to \( \frac{3}{8} \) in. diam. about 25 per cent rigidity, for \( l/d = 20 \), probably could be allowed in the case of the 12 WF 25 beam, scaling downward to about 15 per cent for the 18 WF 47 beam. Further reduction in allowable percentage of rigidity would have to be made for smaller \( l/d \) ratios or for heavier beam sizes.

It was evident from the first series of web angle tests that the top and seat angle connection offered greater end restraint effectiveness and the remainder of the program was devoted to this type connection or some variation thereof.

Top and Seat Angle Connections—Thirty-eight tests were made on this type of connection to investigate the principal variables.

The beam depth was the principal variable in two series of tests which will be denoted as Series A and B.

Series A (Tests 20, 22 and 9) had the following factors maintained constant:

1. Lightest or next to lightest weight beam for each particular beam depth.
2. Connection to flange of 12 WF 65 column.
3. \( 6 \times 4 \times \frac{3}{8} \times 12 \) in. top angle.
4. Four \( \frac{3}{4} \) in. diam. rivets in each leg of top angle.
For Series B (Tests 21, 23 and 12) the preceding four items were maintained as follows:

1. Same as Series A.
2. Connection to web of 12 WF 58 column.
3. $6 \times 4 \times \frac{5}{8} \times 10$ in. top angle.
4. Same as Series A.

**Series A and B**

Allowable Percentage Rigidity for $l/d = 20$

<table>
<thead>
<tr>
<th>Beam Size</th>
<th>Series A</th>
<th>Series B</th>
</tr>
</thead>
<tbody>
<tr>
<td>14 WF 34</td>
<td>49.7</td>
<td>51.8</td>
</tr>
<tr>
<td>16 WF 40</td>
<td>46.8</td>
<td>49.1</td>
</tr>
<tr>
<td>18 WF 47</td>
<td>42.4</td>
<td>43.8</td>
</tr>
</tbody>
</table>

Tests Numbers 25, 26, 31 and 32 provide a study of relatively deep beams in both light and medium weight sizes. The tests are grouped together on the last four lines of Table II.

The thickness of the top angle was the variable in four different groups of tests, namely:

- 3 tests of 12 WF 25 beam connections to column flange (No. 1, 16, 2)
- 2 tests of 12 WF 25 beam connections to column web (No. 17 and 19)
- 4 tests of 18 WF 47 beam connections to column flange (No. 5, 9, 10, 6)
- 4 tests of 18 WF 47 beam connections to column web (No. 11, 12, 13, 14)

The results of these tests are presented graphically in Fig. 10, which shows that connections on each side of the column web, when tested against each other, were somewhat stiffer than similar connections to the column flange. The columns used in the tests were relatively light sections, and the bending of the column flanges contributed to the difference in behavior. Hence these two types of test represent two extremes of condition. A column with very heavy flanges would increase the stiffness of a top-angle-to-column-flange connection, as compared with the lighter weight column sizes used, but not more than the amount indicated for connections in pairs on each side of a column web.
Fig. 10.—Effect of variable top angle thickness—percentages of rigidity shown are for $l/d = 20$. 

- 12" WF 25 Beam with 6"x4"x6\frac{3}{4}" Top Angle to Column Web 2-\frac{3}{4}" Rivets
- 18" WF 47 Beam with 6"x4"x1'0" Top Angle to Column Flange 4-\frac{7}{32}" Rivets
- 18" WF 47 Beam with 6"x4"x6\frac{3}{4}" Top Angle to Column Flange 4-\frac{7}{32}" Rivets
Tests Numbers 36 and 37 (Lines 39 and 40 in Table II) simulated a connection to the web of an outer column in a building where the connection frames to one side only. The specimens were identical except for the addition of a reinforcing angle on the outside of the column web in Test No. 37, as shown in the details. Test No. 36 had the poorest relative strength of any test in the whole program, whereas Test No. 37 was considerably above average. These two tests indicate the desirability of reinforcement for the column web for web connections to one side of a column.

The effect of approximately doubling the ratio of connection shear to connection moment was evaluated in the case of Assemblies Nos. 32, 33, 35 and 37. Considerable increase in shear increased the slope of the \((M, \phi)\) curve by only a small amount. In the case of heavily loaded beam connections, an unstiffened seat angle may be insufficient to carry the reaction, while a stiffened seat in some cases may not be permissible because of building clearance requirements. Beam web connecting angles may be provided in this case to furnish the reaction capacity. Tests 41 and 43 are of this type. (Lines 24 and 42 in Table II.)

The rivet pattern of the tension rivets in the vertical leg of the top angle is a factor influencing the behavior of the connection. In most of the tests the vertical leg was the four-inch leg of a 6 by 4 angle and the tension rivets were all in one line. On a small column it may not be possible to get four rivets in one line. Two lines of rivets require either a 6 by 6 top angle, or a structural tee connection. Tests 41, 42 and 43 (Lines 24, 37 and 42 in Table II) made use of two lines of rivets in the vertical leg of a 6 by 6 angle. Of these, only test 42 may be compared to a similar test using a 6 by 4 top angle. (Test 24, Line 36, is exactly the same as Test 42 in other respects.) Tests 24 and 42 both gave good results, although 42 was somewhat stronger than 24. The rivet pattern in the vertical leg of the top angle in Test 42 should be noted. Two rivets with the short gage length are placed at the ends of the angle where the greatest column flange flexibility obtains. An improvement in equilization of tensile rivet stress is obtained by this pattern. It seems probable that a reversed rivet pattern to that used in No. 42 would be preferable in a column web connection when two rows are necessary.

Connections With Structural Tee Replacing Top Angle—The structural tee is particularly adapted to connections to a column web, where space will not allow a sufficient number of efficiently placed tension rivets in a top angle. It also has possibilities in connecting heavier
beams when a top angle becomes inadequate to the task of developing an appreciable semi-rigid moment.

Four tests were made using a structural tee at the top, namely, Tests 34, 44, 33 and 45, reported on Lines 9, 25, 38 and 43 of Table II. All of these connections were to the column web. Their safety factors, with respect to rotation, are summarized below from the data in Table II. Test 45 had beam web angles in addition to the structural tees.

<table>
<thead>
<tr>
<th>Line in Table II</th>
<th>Test No.</th>
<th>Beam Size</th>
<th>Tee Size</th>
<th>No. Tensile Rivets</th>
<th>No. Shear Rivets</th>
<th>Rotation Factor of Safety $l/d =$</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>34</td>
<td>12 W 25</td>
<td>10 W 41 × 7 1/2</td>
<td>4</td>
<td>4</td>
<td>2.9 2.0</td>
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<tr>
<td>25</td>
<td>44</td>
<td>16 W 78</td>
<td>12 W 37 × 11 1/4</td>
<td>8</td>
<td>6</td>
<td>6.2 4.3</td>
</tr>
<tr>
<td>38</td>
<td>33</td>
<td>18 W 47</td>
<td>10 W 41 × 7 1/2</td>
<td>4</td>
<td>4</td>
<td>7.4 5.1</td>
</tr>
<tr>
<td>43</td>
<td>45</td>
<td>18 W 85</td>
<td>12 W 37 × 11 3/4</td>
<td>8</td>
<td>6</td>
<td>4.3 2.9</td>
</tr>
</tbody>
</table>

The tests made by Rathbun⁴ include a number of tee connections, most of which were of a wind-bracing type, somewhat heavier and with more rivets than those tested in this program.

**Roof Connections**—Three assemblages were designed with the top of the beam flush with the top end of the column, so as to simulate the usual conditions at roof framing. Test 38 (Line 10, Table II) made use of a seat angle with beam web angles rear the top of the beam. This test indicated a rather low factor of safety. Two tests, Nos. 46 and 47 (Lines 15 and 16, Table II) had inverted 8 by 6 top angles. These tests had good factors of safety but developed somewhat less rigidity in proportion to strength than most of the other connections, this being particularly true of the column flange connection. The flanges of the column bent rather easily at the end of the column.

**Development of Semi-Rigid Connection Standards**—This program of tests provides the necessary data for the preparation of Tables V to VIII, giving dependable restraint values, $p$, for the five types of top and seat angle connections shown in Fig. 11, within the range of beam sizes covered by this investigation. In the preparation of these data the estimate of dependable restraints has been based on the

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NOTES—

- RIVETS FOR TYPE I & TYPE III CONNECTIONS.
- 5/8" BOLTS AS NOTED, AND 3/8" RIVETS, FOR TYPE II CONNECTIONS.
- 7/8" RIVETS FOR TYPE III AND TYPE V CONNECTIONS.

Fig. 11.—Suggested standard semi-rigid connections.
straight-line construction of Fig. 6, with an over-load factor of 1.65. As previously discussed in connection with Fig. 7, the values have been reduced where necessary so that the average rivet tensile stress will, in no case, exceed 1.65 times 15 k.s.i. when the beam is subjected to a 65% overload. The rivet value of 15 k.s.i. has been chosen to provide an additional factor of safety, with respect to the working value recommended by the American Institute of Steel Construction, in order to cover uncertainties as to individual rivet loading where the permitted reduction in beam size is directly a function of rivet stress.

In one type of connection (Type II) where rivet stress is frequently the determining factor, dependable restraint values are based on the use of high strength bolts meeting the requirements of A.S.T.M. Specification A-261, with the proviso that such bolts shall be tightened to a specified torque. The amount of this torque has been determined from a study of the experimental work of the British Steel Structures Research Committee².

A 6 × 4 × ½ in. angle has been adopted as a standard in the proposed design tables. This is ½ in. thinner than might have been selected on the basis of the majority of the tests, but it has been chosen to provide added flexibility (even at the sacrifice of somewhat greater possible restraint percentages), thereby minimizing the possibility of too low a rotation.

The shear rivets of the seat angle (and of web angles if they are required by the beam reaction) must, of course, provide sufficient beam reaction capacity, based on the specified allowable working stress in shear.

A suggested supplement to the A.I.S.C. Standard Specification covering the use of these semi-rigid beam-to-column connections follows:

Section 1. Application

(a) Specific Citation Required

Where the "Specification for the Design, Fabrication and Erection of Structural Steel for Buildings" of the American Institute of Steel Construction has been adopted by citation in a Building Code or Ordinance, or is used in the design and construction of a building, this Tentative Supplement is not in force, except by specific citation.

(b) Reference

When this Supplement is in force, beams framed to columns by means of approved semi-rigid connections may be designed on the basis of partial end restraint, as specified in Section 14 (b) and limited by Section 21 (e) of the Standard Specification.

Section 2. Semi-Rigid Connections

(a) Types

Only the five types of semi-rigid beam-to-column connections shown in Fig. 11 are approved under this supplement.

(b) Percentage of Rigidity

The maximum amount of end restraint which may be used in computing the moment, for which a semi-rigidly connected beam may be designed for the given loading, shall not exceed the percentage of rigidity given in Tables V to VIII, for the beam and connection selected. If dissimilar connection types are employed at opposite ends of a beam, the percentage of rigidity used in the computations shall not exceed the value tabulated for the more flexible type.

* This supplement is included to show the reader how the proposed design technique might be made to operate within the framework of the present A.I.S.C. Specification, and to invite comment. It does not constitute a design recommendation of the Institute at this time, as it has not been studied by the Committee on Specifications.
(c) **High Strength Bolts**

If high strength bolts are substituted for the tension rivets, as in Type II connections, they shall conform to A.S.T.M. A-261-44T. Their holes may be either drilled or punched. Bolts used in lieu of tension rivets shall be tightened in the field to a torque of not less than 125 lb. ft.

(d) **Beam Reactions**

Beam seats, unstiffened or stiffened, and/or web angle connections, shall be adequate to provide for the beam reactions, using the allowable unit stresses prescribed in Section 15 of the Standard Specification.

(e) **Connections on One Side of Column Webs Only**

When beams are not located on opposite sides of a column web so as to permit the use of the same tension rivets or bolts to connect the top angle of both connections, a reinforcing angle shall be placed, on the side of the column web away from the beam connection, in line with the top angle of the connection.

**Section 3. Supporting Members**

(a) **Alignment**

Columns, used to support beams designed on the basis of partial restraint by virtue of the approved semi-rigid connections, shall have their axes approximately in line with these beams.

(b) **Other Requirements**

A semi-rigid connection shall not be considered as affording restraint at the end of a beam when the flange of a column to which it frames is less than one-half inch in thickness, unless the column flange is stiffened to resist an outward deflection caused by the pull of the tension rivets or bolts.

A semi-rigid connection shall not be considered as affording restraint at the end of a beam unless the top of the column extends above the center line of the tension rivets a distance at least equal to the width of the column flange.
### Table V.

DEPENDABLE PERCENTAGE OF RIGIDITY, $p$, FOR
TYPE I & TYPE II SEMI-RIGID CONNECTIONS
(See Fig. 11 for Details)

- $2 - \frac{3}{4}''$ Tension Rivets (Type I)
- $2 - \frac{3}{8}''$ Tension Bolts (Type II)

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Upper Lines Give $p$-Values for Type I.
Lower Lines Give $p$-Values for Type II.
Table VI.

DEPENDABLE PERCENTAGE OF RIGIDITY, \( p \), FOR
TYPE III SEMI-RIGID CONNECTIONS
(See Fig. 11 for Details)

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2 - \( \frac{3}{8}'' \) Tension Rivets
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Table VIII.
DEPENDABLE PERCENTAGE OF RIGIDITY, $p$, FOR
TYPE V SEMI-RIGID CONNECTIONS
(See Fig. 11 for Details)

$4 - \frac{7}{8}''$ Tension Rivets

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VII—PROPOSED METHOD OF DESIGN FOR BEAMS
WITH SEMI-RIGID CONNECTIONS

It has been shown (page 13) that, for uniformly loaded beams, a connection rigidity of 75 percent is needed to produce negative moments at the ends of a beam equal to the positive moment at mid-span, and that a connection rigidity of 100 percent is similarly required for beams supporting a concentrated load at mid-span. In the method of design proposed, only connections which will function with considerably less rigidity than 75 percent will be used, so that, in every case, the beam may be proportioned for the positive moment. This moment will be less than the one which would exist if no end restraint were present, the extent of the reduction being determined by the amount of dependable restraint afforded by the semi-rigid connections used, and by the contributory effect of frame action.

To obtain the section moduli required for beams framed by connections having dependable amounts of restraint, "redesign coefficients" will be derived for critical conditions of loading with respect to frame action, and for various types of beam loading. The "redesign coefficients", multiplied by the section modulus required for the condition of no end restraint, will immediately give a "safe" reduced section modulus, based on the dependable effect of the semi-rigid connections under the least favorable frame action likely to exist. While this reduced section modulus will always be greater than the one which might be derived by an accurate analysis of the moments in the given frame containing the particular beam under consideration, the sacrifice of some of the possible economy can be justified by the considerable reduction in the design computations involved.

Six factors influence the effect of frame action on the positive moment produced by gravity loading in any semi-rigidly framed beam in a tier building. These factors are:

1. The relative length of adjacent spans.
2. The relative size of adjacent beams.
3. The relative size of adjacent columns.
4. The relative rigidity of the several end connections.
5. The symmetry or asymmetry of loading on each beam.
6. The arrangement of loading of adjacent spans—both at the level of the beam under consideration and also at adjacent levels.
Assuming, first, a complete symmetry of frame with respect to the first five of these factors, the arrangement of span loadings shown in Fig. 12 (a) is the most critical one possible for A-B, an interior beam in a frame several bays wide and several stories high. Although its occurrence is highly improbable, this arrangement is frequently assumed, in analyzing the effect of live loading on continuous-frame tier buildings, to obtain results that are bound to be on the safe side.

Joints A and B, as well as joints C, D, E and F, are restrained against rotation by the stiffness of the column sections above and below the joint. They are also restrained by the stiffness of the unloaded beams in the adjacent bays, but this restraint will be diminished, in comparison with the condition of full continuity, by reason of the flexibility of the connections framing these beams to the columns. If this latter restraint were assumed to be removed completely, joints A, B, C, D, E and F would rotate somewhat more, and the positive moment in beam A-B would thereby be increased. Since the results will err on the "safe" side without too much loss of economy, such a simplifying assumption will be made. This assumption will also have the effect of largely eliminating (1) and (2) from the list of factors which otherwise would have to be investigated in analyzing the effect of frame action.

A requirement that the same connection type be used in framing both ends of a given beam will eliminate (4) as a factor. If different types of connections are used at the ends of a beam and the beam size is determined by the rigidity given for the more flexible type of connection, (4) will also be eliminated as a factor.

At first it will be assumed that columns CAE and DBF (Fig. 12 (b)) are the same size, and that the loading on beam A-B is symmetrically disposed about the center of span.

The dotted lines in Fig. 12 (c) indicate the actual moment diagram; the solid lines show the moments which would result if the unloaded beams were removed (assumed as taking no moment).

In Section IV a method of evaluating the dependable percentage of rigidity of semi-rigid connections was developed, based on the assumption of no rotation of the columns. As joints A and B rotate through angles $\theta_A$ and $\theta_B$ (Fig. 12 (b)) the restraining end moments are reduced (shifted to mid-span) and, therefore, the connections actually are less heavily loaded than previously assumed. Because of the non-linear $(M,\phi)$ relationship of semi-rigid connections, any reduction in the assumed connection moment results in a proportionately smaller angle
Fig. 12.—Analysis of semi-rigidly connected frame for critical loading condition.
of rotation of the connection, thus increasing its actual percentage of rigidity under this condition of frame action.

Note, in Fig. 12 (c), that the moments induced in the columns are of constant magnitude between joints, and that the negative moments at the ends of the loaded beams are also of equal magnitude at each joint, although opposite in sense at the two ends of any one beam.

Because of the symmetry of the frame and loading, a direct distribution of the moments induced by frame action can be made in one operation in accordance with the Hardy Cross procedure, with no "carry-over". Under this procedure the joints are first assumed as locked against rotation and the fixed-end moments produced by the beam loading are assigned to the joints where the beam ends frame to the columns. As the joints are simultaneously "unlocked", the locked-up unbalanced moments at the joints are distributed to each member in proportion to its "bending stiffness" and in the case of the loaded beams, subtracted from the fixed-end moments to obtain the actual moments induced by the frame action. Bending stiffness may be defined as the ratio of applied moment to corresponding angle change, i.e., the amount of moment required to produce a unit angle change—$M/\phi$. Since the moments induced by frame action in this problem are of constant magnitude throughout the entire length of each member (beams and columns), the change in slope from one end of the member to the other is, by the moment-area principle, $ML/l$, where $l$ is the length of the member. And, since the change of slope is of equal magnitude at both ends of the member, it may be expressed, for either end, as

$$\phi = \frac{ML}{2EI}$$

or

$$\phi = \frac{M}{2EK}, \text{ (equals } \theta_a \text{ in Figure 12 (b)}$$

where $K = I/l$.

Then the bending stiffness, $M/\phi$, equals $2EK$.

This value is correct for the columns and the total bending stiffness of the column (above and below joint A) may be written as $2EEK_c$.

The actual bending stiffness of the beams, taken together with their connections however, by reason of the connection flexibility, will have to be modified. Such a modification may easily be made if the $(M, \phi)$ relationship of the semi-rigid connection is assumed to be constant.
Let $M_1$ be end moments applied to the connections at each end of a beam, such that $M_1$ is constant throughout the length of the beam. If the connections are 100% rigid ($p = 100$%) the angle change $\theta_1$, at the ends of the beam, will be, by the moment-area principle

$$\theta_1 = \frac{M_1}{2EK_B}$$

where $K_B = \frac{I}{l}$ for the beam.

Now, if the connections are "semi-rigid" ($p < 100\%$), let $M_2$ be the end moment required to rotate each end of the beam through an angle $\phi_2$, such that

$$\phi_2 + \phi_c = \theta_1$$

where $\phi_c$ is the angle of rotation produced in the connection by reason of its flexibility, between the end of the beam and the point of application of $M_2$. If the connections were very flexible and approached the condition of zero stiffness ($p \to 0$), the rotation $\phi_c$ could be produced by a moment $M_2$ that approaches zero, and the ends of the beam in the limiting case would not rotate, i.e., $\phi_2 = 0$. The "percentage of rigidity" of the connection as defined in Section IV, may now be expressed as the percentage of $\phi_2$ to $\theta_1$ or $M_2$ to $M_1$, i.e.,

$$\frac{p}{100} = \frac{\phi_2}{\theta_1} = \frac{M_2}{M_1}$$

substituting $M_1 = 2EK_B \theta_1$ (Eq. 9)

into Eq. 10,

$$\frac{p}{100} = \frac{M_2}{2EK_B \theta_1}$$

and the stiffness of the member consisting of the beam and its semi-rigid connections, becomes

$$\frac{M_2}{\theta_1} = \frac{p}{100} (2EK_B).$$

The unbalanced moments produced by unlocking the joints can now be distributed to the ends of the members forming the joints. Since the "distribution factor" is the ratio of the bending stiffness of any particular member to the sum of the bending stiffnesses of all the members entering the joint, the distribution factor for beam A-B in Fig. 12 (b) will be
But, due to the flexibility of the connections, the moment at the ends of the beam, when the joints were "locked" against rotation, was not the full fixed-end moment, but \( \frac{P}{100} \cdot M_R \). Unlocking the joints, this moment will be lessened by the proportional part of the unbalanced moment distributed to it, i.e., by

\[
\frac{pM_R}{100} \left( \frac{\frac{p}{100} \cdot K_B}{\Sigma K_c + \frac{p}{100} \cdot K_B} \right)
\]

and the net moment remaining at the end of the beam will be

\[
M_A = \frac{pM_R}{100} - \frac{pM_R}{100} \left( \frac{\frac{p}{100} \cdot K_B}{\Sigma K_c + \frac{p}{100} \cdot K_B} \right) = M_R \left( \frac{1}{\frac{p}{100} + \frac{K_B}{\Sigma K_c}} \right)
\]

The positive moment at the mid-span of the beam equals

\[
M_e = M_s - M_A
\]

where \( M_s \) = the moment produced by the same loading on a simple span.

The desired redesign coefficient then equals

\[
F = \frac{M_e}{M_s} = 1 - \frac{M_A}{M_s} = 1 - \frac{M}{M_s} \left( \frac{1}{\frac{p}{100} + \frac{K_B}{\Sigma K_c}} \right)
\]

For any particular type of loading (uniform, concentrated load at mid-span, etc.) \( \frac{M_R}{M_s} \) is a readily derived constant. Then the only variables affecting \( F \) for a given type of loading are (1) the percentage of rigidity of the connections used and (2) the relationship of beam stiffness to the total stiffness of the column.

The adequacy of frames designed in accordance with the foregoing procedure has been tested by recomputing the bending stresses in both exterior and interior beams in several frames designed by this procedure, where the ratio of bay widths was varied through a considerable range. In recomputing these stresses the more accurate
**SEMI-RIGID BEAM-TO-COLUMN CONNECTIONS**

**Fig. 13.**—Calculated maximum beam stresses in frames designed by proposed design procedure.
but more tedious moment distribution method, as modified for use in semi-rigidly connected frames\textsuperscript{7,8} was employed.

The results of this study are given in Fig. 13 and indicate surprisingly good agreement. In every case the stresses derived by means of the more precise moment distribution analysis prove to be less than those obtained by use of the proposed method. This may be largely accounted for by reason of the actual restraining effect of the unloaded beams, which is ignored in the proposed design method, but which was included in making the moment distribution analysis. In the case of the three equal 24' spans the difference in stress by the two methods is practically constant for all parts of the frame. Even when the ratio of interior to exterior span lengths was changed by as much as 100\%, the true stresses resulting from inequality of span length, in no case exceeded or even quite equalled the stresses derived by the proposed method. The fact that all of these stresses range from a low of 17.1 to a high of 19.2 k.s.i., instead of being 20 k.s.i., is of course explained by the fact that the available beam sizes which most nearly meet the design requirements have section moduli slightly in excess of actual requirements. This is typical of the condition encountered in every day design practice and, where it occurs, provides an additional factor of safety which is frequently overlooked.

Curves based on Eq. 13, for various percentages of connection rigidity and various types of loading, are plotted as solid lines in Figs. 14, 15 and 16.

If all the spans were equal; all the story heights were the same; and every span were equally loaded (as, for example, by dead load) there would be no rotation of the joints (i.e., no frame action) at the ends of interior beams. Then the term $\frac{K_b}{\Sigma K}$ would not appear in Eq (13). Assume such a frame with one-third of the total loading caused by dead load. Let $F_D$ be the redesign coefficient associated with this dead loading, and let $F_L$ be the redesign coefficient associated with the live loading arranged to give the critical condition assumed in the earlier discussion. Then, dividing the problem into two parts (dead load and live load), and applying the principle of superposition, the


redesign coefficient expressing the effect of the combined loading would be

\[ F = \frac{1}{3} F_D + \frac{2}{3} F_L \]

where

\[ F_D = 1 - \frac{M_R}{M_s} \left( \frac{1}{100} \right) \]  \hspace{1cm} (Eq. 13)

and

\[ F_L = 1 - \frac{M_R}{M_s} \left( \frac{1}{100} + \frac{K_B}{\Sigma K_C} \right) \]  \hspace{1cm} (Eq. 13)

Then

\[ F = 1 - \frac{M_R}{3M_s} \left( \frac{1}{100/p} \right) - \frac{2M_R}{3M_s} \left( \frac{1}{100/p} + \frac{K_B}{\Sigma K_C} \right) \]

\[ = 1 - \frac{M_R}{M_s} \left( \frac{1 + \frac{pK_B}{300\Sigma K_C}}{\frac{100}{p} + \frac{K_B}{2K_C}} \right). \]  \hspace{1cm} (14)

Curves based on Eq. 14 are shown as dotted lines on Figs. 14, 15 and 16. For frames having nearly equal bay widths, “safe” F-coefficients for interior beams may be read directly from the appropriate dotted-line curves. If F-coefficients are read from the solid line curves for exterior bay beams, and for interior beams when the bays are unequal by more than a moderate amount, it is obvious that, since the balancing effect of dead load is entirely neglected, a wide margin of safety is afforded.

Assume now that column CAE (Fig. 12 (a)) is stiffer (larger) than column DBF. Since the conditions at beam A-B are no longer completely symmetrical, a precise solution of the problem by the Hardy Cross method involves a “carry-over” and cannot be completed in one cycle. However, if the stiffness of column CAE is assumed equal to the lesser stiffness of column DBF the resulting redesign coefficient will err on the “safe” side and, within the usual working range of \( \frac{K_B}{2K_C} \) the error will be small.

Analysis of the case of unsymmetrically loaded beams requires a somewhat different attack. It is given in Appendix A. It will be seen from this analysis that, for asymmetrical loading, F-values taken from
Fig. 14.—Redesign coefficient curves—uniform and third-point loading.
Fig. 15.—Redesign coefficient curve—concentrated load at mid-span.
Fig. 16.—Redesign coefficient curves—quarter-point loading.
Fig. 15 provide redesign coefficients which are on the safe side, and
that, within the usual working range, any consequent error will be too
small to warrant further refinement.

When the bending moment in a beam is produced by a combination
of two or more types of loading, an adjusted redesign coefficient may
be obtained by weighting the coefficients for each type of loading (read
from the curves) in proportion to the contribution of each type of
load to the total moment.

Assume for example, that a total moment is 40% due to uniform
loading and 60% due to a concentrated load. Also assume that, for
uniform loading, $F = .81$ and, for the concentrated load, $F = .86$.

For the combined loading
\[ F = .4 \times .81 + .6 \times .86 = .84. \]

From Figs. 14, 15 and 16 it will be seen that the greatest reductions
apply to the condition of uniform loading, and that the least reductions
apply to the condition of a single concentrated load at mid-span—
with the case of equal concentrated loads at the three quarter-points
intermediate between these limits. A little study of these curves will
enable the designer to select, by inspection, a reduction factor which
will be very close to, and on the safe side of, the actual value, even
for a complicated system of loads.

The proposed method of design may be summarized in the following
rules of procedure:

1. Design the beam for maximum bending moment, assuming
simple supports (no restraint).

2. Calculate the stiffness constants for the next lighter beam
size and the less stiff, or minimum $\Sigma K_c$, of the two supporting
columns, and then compute the ratio $K_B / \Sigma K_c$. ($K_B = \frac{I}{l}$ for
the beam, and $\Sigma K_c = \Sigma \frac{I}{l}$ for the column above and below the
beam.)

3. Select the type of semi-rigid connection to be used and obtain,
from the appropriate Table V, VI, VII or VIII, the dependable
percentage of rigidity for this next lighter beam size, on the
required span.

4. Choosing the curves appropriate for the required type of
loading, enter Fig. 14, 15 or 16, with the computed $K_B / \Sigma K_c$
and move vertically upward to an intersection with the de-
dependable percentage of rigidity obtained from the tables,
interpolating between the plotted curves when necessary. Move horizontally to the left and read the redesign coefficient, \( F \), from the vertical scale. For interior spans in frames having nearly equal bay widths, the dotted curves may be used; otherwise use the solid line percentage curves. If the bending is produced by a combination of uniform loading and concentrated loads a weighted \( F \)-coefficient may be derived from the \( F \)-values obtained individually, from Figs. 14 and 15, or Fig. 14 and 16. A single concentrated load may be assumed as located at mid-span, regardless of its actual location, for the purpose of obtaining its \( F \)-coefficient.

5. Multiply the required section modulus previously calculated for the simple span design, by the redesign coefficient, and select the lightest beam which will provide a section modulus equal to the product of this multiplication.

6. When a particular beam is repeated under identical loading conditions a number of times in a frame, so as to justify the extra work, and the size determined by step (5) is less than that assumed in step (2), a further economy may sometimes be effected by repeating the procedure until \( K_B \) in step (2) exactly agrees with the beam size actually used.

VIII—ACKNOWLEDGEMENTS

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ILLUSTRATIVE EXAMPLES

Example A

Given: Uniformly loaded beams, framing between columns spaced 26 feet on centers. Total load on each beam, 30 kips (45% live, 55% dead). Story heights 10 feet. Columns, 12 WF 53 at the sixth floor, and 12 WF 79 at the fourth floor. Beams will frame to column flanges.

Required: Size of beams, at the sixth and fourth floors, semi-rigidly framed to columns.

\[ M_s = \frac{30 \times 26}{8} = 97.5 \text{ kip-feet} = 1170 \text{ kip-inches} \]

\[ \text{Req. } S = \frac{1170}{20} = 58.5 \]

Required simple beam, 16 WF 40 \((S = 64.4)\) or 14 WF 43 \((S = 62.7)\).

Since the simple beam requirements would almost be satisfied with the next lighter section, the redesign computations will be made at once using the second lighter section.

At the sixth floor, Type II semi-rigid connections will be specified. From Table V, \(p = 33\%\) for a 14 WF 34 beam on a 26 foot span.

\[ K_B = \frac{339.2}{26 \times 12} = 1.09 \quad \text{and} \quad \Sigma K_c = \frac{2 \times 426.2}{10 \times 12} = 7.10 \]

\[ \frac{K_B}{\Sigma K_c} = \frac{1.09}{7.10} = .153 \]

Interpolating between the solid line curves for \(p = 30\%\) and \(p = 40\%\) in Fig. 14, a redesign coefficient, \(F = .79\), is obtained (note that the maximum effect of frame action in this problem, as indicated by the relationship \(\frac{K_B}{\Sigma K_c}\), is so small that there is little difference between \(F\)-values derived by the solid and dotted curves; hence the more conservative solid line value may be taken for both interior and end bays).

\[ \text{Req. redesign } S = 58.5 \times .79 = 46.2 \]

use 14 WF 34 \((S = 48.5)\)

Saving in weight, 15%
Saving in depth, 2 inches.
At the fourth floor, Type V semi-rigid connections may be used. A 14 WF 30 beam will be tried. From Table VII, $p = 56\%$ for a 14 WF 30 beam on a 26 foot span.

$$K_B = \frac{289.6}{26 \times 12} = .93$$

$$\Sigma K_C = \frac{2 \times 663.0}{10 \times 12} = 11.05$$

$$\frac{K_B}{\Sigma K_C} = \frac{.93}{11.05} = .084$$

Interpolating between $p = 50\%$ and $p = 60\%$ in Fig. 14, an $F$-value of .64 is obtained.

Req. redesign $S = 58.5 \times .64 = 37.4$

use 14 WF 30 ($S = 41.8$)

Saving in weight, 25%.

**Example B**

Required: To design a line of girder beams, each spanning 24 feet and supporting 31 kip concentrated loads at its third points. Story height 10 feet. Exterior beams frame to flange of 12 WF 65 wall columns. Interior columns, 12 WF 79 with webs normal to webs of girder beams.

$$M_s = 31 \times 8 = 248 \text{ kip-feet} = 2976 \text{ kip-inches}$$

Req. $S = \frac{2976}{20} = 148.8$

Required simple beam, 21 WF 73 ($S = 150.7$)

Type V semi-rigid connections will be specified. From Table VIII, $p = 30\%$ for a 21 WF 68 beam on a 24 foot span.

$$K_B = \frac{1478.3}{24 \times 12} = 5.15$$

The lesser $\Sigma K_C = \frac{2 \times 216.4}{10 \times 12}$ (minor axis of 12 WF 79) = 3.61

$$\frac{K_B}{\Sigma K_C} = \frac{5.13}{3.61} = 1.43$$
From Fig. 14,

\[ F = 0.86 \text{ for end bays (from solid lines)} \]
\[ F' = 0.84 \text{ for interior bays (from dotted lines)} \]

For end bays, Req. redesign \( S = 148.8 \times 0.86 = 128.0 \)
Use 21 WF 68 (\( S = 139.9 \)).

For interior bays, Req. redesign \( S = 148.8 \times 0.84 = 125.0 \)
Use 21 WF 62 (\( S = 126.4 \))

(Actually, for a 21 WF 62 beam, \( p = 32\% \); \( K_B = 4.64 \); \( \frac{K_B}{\sum K_C} = 1.28 \);
\[ F = 0.83 \text{ instead of } 0.84 \); and Req. redesign \( S = 123.5 \))

Saving in weight, 7% in end bays.
Saving in weight, 15% in interior bays.

**Example C**

Required: To design a semi-rigidly connected beam to carry a uniform load of 1.7 kips per linear foot, and a concentrated load of 22 kips located 7 feet from one end, when the c. to c. of supporting columns is 20 feet. Columns just below the beam are 12 WF 72 and story height is 12 feet; columns immediately above the beam are 12 WF 65 and this story height is 10 feet. Beam will frame to webs of supporting columns.

\[ M, \text{ occurs at point of concentrated load.} \]
\[ M_c = \frac{22 \times 13 \times 7}{20} = 100.1 \text{ kip-feet} \]
\[ M_u = (1.7 \times 10 \times 7) - (1.7 \times 7 \times 3.5) = 77.3 \text{ kip-feet} \]

Total \( M_c \)

\[ = \frac{177.4 \text{ kip-feet}}{2129 \text{ kip-inches}} \]

Req. \( S = \frac{2129}{20} = 106.5 \).

Required simple beam = 18 WF 60 (\( S = 107.8 \)).

If Type IV semi-rigid connections are specified, \( p = 29\% \) for an 18 WF 55 beam on a 20 foot span.

For an 18 WF 55, \( K_B = \frac{889.9}{20 \times 12} = 3.70 \)
\[ \Sigma K_C = \frac{174.6}{10 \times 12} + \frac{195.3}{12 \times 12} = 2.81 \]
\[ \frac{K_B}{\Sigma K_C} = \frac{3.70}{2.81} = 1.32 \]
Moment due to uniform loading = \( \frac{100.1}{177.4} = 0.57 \) of total moment.

From Fig. 14, \( F = 0.86 \) for uniform loading.
Moment due to concentrated load = 0.43 of total moment.

Although the concentrated load is not at mid-span, the \( F \)-value obtained from Fig. 15 (\( F = 0.90 \)) will err on the "safe" side (see p. 64).

A weighted \( F \)-value, for the combination of uniform and concentrated loading, may be computed as

\[
F = 0.57 \times 0.86 + 0.43 \times 0.90 = 0.88.
\]

Req. redesign \( S = 106.5 \times 0.88 = 93.7 \).

Use 18 WF 55 (\( S = 98.2 \)).

Saving = 8\% = 100 pounds of plain material.

Had Type V semi-rigid connections been specified, \( p \) would equal 36\% for an 18 WF 50 beam on a 20 foot span, instead of 29\% for the 18 WF 55 beam; \( \frac{K_p}{2K_c} \) would have been 1.18 instead of 1.32; the weighted \( F \)-value would have been 0.86; and the required redesign \( S \) would have been 91.6. Thus it will be seen that a 12\% increase in the rigidity of the connections, resulting from the use of \( \frac{3}{8}\)" \( \phi \) instead of \( \frac{3}{4}\)" \( \phi \) tension rivets, would produce but a 2\% decrease in the required section modulus, which would be insufficient to permit the use of this next lighter beam. Had the lighter beam proven to be adequate using the larger rivets, the added saving in main material would have been 100 pounds. Such a saving would probably not have warranted changing to \( \frac{3}{8}\)" \( \phi \) tension rivets here, if the balance of the fabrication was based on using \( \frac{3}{4}\)" \( \phi \) rivets.
APPENDIX A

REDESIGN COEFFICIENTS FOR UNSYMMETRICAL LOADING

On page 26 the percentage rigidity, $p$, of a connection was defined in reference to a symmetrically loaded beam. On pages 52 to 60 the redesign coefficient, $F$, was derived, in terms of $p$; the load distribution; and the relative beam and column stiffness. The redesign coefficient, $F$, for an unsymmetrically loaded beam, as for example the case of a single concentrated load placed anywhere on a single bent span as shown in Fig. 17, can also be derived. It will be shown in the following derivation for this case, involving both sidesway and unsymmetrical loading, that, while the expression for $F$ is too involved for frequent use in design problems, nevertheless a handy design short cut is available—that the $F$-values, already given in Fig. 15 for the special case when this load is placed at mid-span (symmetrical loading), can be substituted for the true values to give results which are on the safe side and, at the same time, are but slightly in error.

![Fig. 17.—Single bent with concentrated load—P at any location.](image)
The slope deflection procedure of frame analysis, modified to take into consideration the rotational effect of semi-rigid column connections, will be used in the derivation.

Fig. 18 shows the deflected and bent shape of any beam, AB. Identical semi-rigid connections are assumed at A and B. The following notation, indicated in Fig. 18, will be used:

\[ \theta_A, \theta_B = \text{Rotation of column axes, at A and B, respectively, at their intersection with the beam axis. (Positive when clockwise.)} \]

\[ \Delta = \text{Deflection of B relative to A, taken transversely to the beam axis. (Positive when causing clockwise rotation of a straight line between A and B.)} \]

\[ M_A, M_B = \text{Final moments acting on the ends of the beam (and through the semi-rigid connections, on the columns) at A and B, respectively. (Positive when acting clockwise on end of beam.)} \]

\[ V_A, V_B = \text{Shears at A and B, respectively. (Positive when tending to rotate the beam clockwise.)} \]

\[ \gamma = \text{A constant (assumed) ratio between the moments transmitted by the semi-rigid connections and the consequent rotations, between the ends of the beam and adjacent columns, produced by these connections. (Note: Identical connections are assumed at A and B.)} \]

By these definitions, \( \gamma M_A \) is the rotation within the connection at A and \( \gamma M_B \) is the rotation within the connection at B, as shown in Fig. 18.

It has been shown (page 24) that when no column rotation takes place, the moment at the ends of a symmetrically loaded, semi-rigidly connected beam will be

\[ M = M_R - \frac{2EI\phi}{l} \quad (\text{Eq. 1}) \]
But, by the definition of $\gamma$, which assumes a linear relationship between $M$ and $\phi$,

$$\phi = \gamma M, \text{ and }$$

$$M_E = \frac{100M}{p}$$

Hence

$$M = \frac{100M}{p} - \frac{2EIM\gamma}{l}$$

Cancelling out $M$,

$$\gamma = \frac{1}{2EK} \left( \frac{100}{p} - 1 \right) \quad (15)$$

Since the tabulated “dependable” values for $p$ (Tables V to VIII inclusive), for a given semi-rigid connection have been determined by the type and limiting magnitude of the symmetrically disposed loading that may be placed on a beam of given span and moment of inertia, the angles of rotation within the connections at ends A and B in Fig. 18 may be expressed, respectively, as

$$\gamma M_A = \frac{M_A}{2EK} \left( \frac{100}{p} - 1 \right) \text{ and }$$

$$\gamma M_B = \frac{M_B}{2EK} \left( \frac{100}{p} - 1 \right)$$

Equations for $M_A$ and $M_B$, in terms of $\theta_A$, $\theta_B$ and $\Delta$, commonly called the “slope-deflection” equations, can now be derived. It is assumed that the reader is familiar with the “moment-area” procedure for obtaining slopes and deflections given in most texts on strength of materials and indeterminate structures. By this procedure the change in slope between any two points along a straight member may be obtained directly as the area of the $\frac{M}{EI}$ diagram between these two points.

In Fig. 18 the slope of $AB$ at A is seen to be

$$\theta_A = \frac{M_A}{2EK} \left( \frac{100}{p} - 1 \right)$$

At B it is

$$\theta_B = \frac{M_B}{2EK} \left( \frac{100}{p} - 1 \right)$$
Hence, the change in slope between A and B is equal to

$$\theta_A - \theta_B = \left(\frac{100}{p} - 1\right) \left(\frac{1}{2E} \right) (M_A - M_B)$$

The bending moment anywhere along beam AB may be obtained by adding algebraically the three separate parts shown in Fig. 19, i.e.,

(a) the moment for a simply supported beam, plus

(b) the proportionate contribution of end moment $M_A$, as indicated in the triangle having $M_A$ as the ordinate at the left end, plus

(c) the proportionate contribution of end moment $M_B$.

If beam AB has a uniform cross-section, the total area of the $\frac{M}{EI}$ diagram between A and B equals

$$\frac{1}{EI} \left[ A_s + \frac{1}{2} (M_A - M_B) \right]$$

where $A_s$ = the area of the simple beam moment diagram.

This expression for the change in slope may be equated to the expression, in terms of $\theta$ and $M$, already derived, giving, after minor rearrangements,

$$M_A - M_B = \frac{2pE}{100} (\theta_A - \theta_B) - \frac{2pA_s}{100l}$$

(16)

To obtain separate expressions for $M_A$ and $M_B$ a second equation containing $M_A$ and $M_B$ is needed. This equation may be written by applying the moment-area procedure for determining deflections. The deflection of any point, B, along the axis of a deflected beam, with reference to the tangent through any point A, is equal to the moment of the area of the $\frac{M}{EI}$ diagram between points A and B, taken about point B.

As shown in Fig. 18, the tangent to the deflected beam axis at A makes an angle equal to

$$\theta_A - \frac{\Delta}{l} = \gamma M_A$$

with respect to the straight line connecting A and B. Substituting the expression for $\gamma$ given in Eq. 15, the deflection of B, with reference to the tangent at A, may be expressed as

$$l \left[ \theta_A - \frac{\Delta}{l} - \frac{M_A}{2E} \left(\frac{100}{p} - 1\right) \right]$$
Referring to Fig. 19, the algebraic sum of the moments of the $\frac{M}{EI}$ diagrams, about point B, is equal to

$$A_x \vec{e} + \left(\frac{M_A}{2}\right) \left(\frac{2l}{3}\right) - \left(\frac{M_B}{2}\right) \left(\frac{1}{3}\right)$$

When this expression is equated to the previously derived expression for the deflection at B, we get, after rearrangement,

$$MA \left(\frac{300}{p} - 1\right) - MB = 6EK \left(\frac{\theta}{\ell} - \frac{\Delta}{\ell}\right) - \frac{6A_x \vec{e}}{\ell^2} \quad (17)$$

Simultaneous Equations 16 and 17 may now be solved for $M_A$ and $M_B$, giving the slope-deflection equations for a beam having semi-rigid connections. Thus

$$M_A = \frac{2pEK}{300 - 2p} \left[ \left(3 - \frac{p}{100}\right) \theta_A + \frac{p}{100} \theta_B - \frac{3\Delta}{\ell} - \frac{A_x \vec{e}}{EI} \left(\frac{3\vec{e}}{\ell} - \frac{p}{100}\right) \right] \quad (18)$$

$$M_B = \frac{2pEK}{300 - 2p} \left[ \left(3 - \frac{p}{100}\right) \theta_B + \frac{p}{100} \theta_A - \frac{3\Delta}{\ell} + \frac{A_x \vec{e}}{EI} \left(\frac{3(\ell - \vec{e})}{\ell} - \frac{p}{100}\right) \right]$$
When \( p = 100 \), Eq. 18 reduce to the familiar equations for a fully restrained (fixed-ended) beam.

\[
M_A = 2EK \left[ 2\theta_A + \theta_B - \frac{3\Delta}{l} - \frac{A_T(3\bar{x})}{EI} \left( \frac{3l}{l} - 1 \right) \right] \\
M_B = 2EK \left[ 2\theta_B + \theta_A - \frac{3\Delta}{l} + \frac{A_T(3(l - \bar{x})}{EI} - 1 \right] 
\]

For the beam AB in Fig. 17, the vertical displacement of B, with respect to A, may be assumed equal to zero, i.e., \( \frac{\Delta}{l} = 0 \). For the case of the single load, \( P \), the maximum simple beam moment is equal to \( \frac{P_{ab}}{l} \).

Hence

\[
A_s = \left( \frac{P_{ab}}{l} \right) \left( \frac{1}{2} \right) = \frac{P_{ab}}{2}
\]

and the distance, \( \bar{x} \), from end B to the centroid of the simple beam \( \frac{M}{EI} \) diagram, may be written

\[
\bar{x} = \frac{1}{3} (2l - a)
\]

For this special case, Eq. 18 becomes

\[
M_{AB} = \frac{2pEK_B}{300 - 2p} \left\{ \left( \frac{3 - p}{100} \right) \theta_A + \frac{p}{100} \theta_B - \frac{1}{2EK_B} \left[ \left( 2 - \frac{p}{100} \right) P_{ab} b^2 + \left( 1 - \frac{p}{100} \right) P_{ab} b^2 \right] \right\}
\]

\[
M_{BA} = \frac{2pEK_B}{300 - 2p} \left\{ \left( \frac{3 - p}{100} \right) \theta_B + \frac{p}{100} \theta_A + \frac{1}{2EK_B} \left[ \left( 2 - \frac{p}{100} \right) P_{ab} b^2 + \left( 1 - \frac{p}{100} \right) P_{ab} b^2 \right] \right\}
\]

The first subscript after \( M \) indicates the end of the member under consideration and the two subscripts, together, identify the member.

For columns AC and BD, full continuity is assumed and Eqs. 19 apply. Since no transverse (lateral) loads are applied, the last term in Eqs. 19 drop out. Also the rotations at the column bases (\( \theta_C \) and \( \theta_D \)) equal zero. With these assumptions, the four slope-deflection equations for the two columns in Fig. 17 become

\[
M_{AC} = 2EK_C \left( 2\theta_A - \frac{3\Delta}{h} \right) \\
M_{CA} = 2EK_C \left( \theta_A - \frac{3\Delta}{h} \right) \\
M_{BD} = 2EK_C \left( 2\theta_B - \frac{3\Delta}{h} \right) \\
M_{DB} = 2EK_C \left( \theta_B - \frac{3\Delta}{h} \right)
\]
These expressions for moments (Eqs. 20 and 21) contain three unknown deformation quantities, $\theta_A$, $\theta_B$ and $\Delta$. By writing three equations based on conditions of static equilibrium, these deformation quantities may be evaluated in terms of the applied loading, the properties of the members, and the semi-rigid characteristics of the beam connections. These values may then be substituted in Eqs. 20 and 21 to obtain usable expressions for all of the moments.

Two conditions of equilibrium can be obtained by equating the sum of the moments acting at joints A and at B to zero. A third condition can be obtained by equating the moments acting on the columns to zero. These equilibrium equations would be

$$
M_{AB} + M_{AC} = 0 \\
M_{BA} + M_{BD} = 0 \\
M_{AC} + M_{CA} + M_{BD} + M_{DB} = 0
$$

To reduce the labor of solving Eqs. 22, the following substitution of constants will be made:

$$
C_1 = \frac{p}{300 - 2p} \\
C_2 = 2EK_B C_1 \left(3 - \frac{p}{100}\right) \\
C_3 = \frac{2pEK_B C_1}{100} \\
C_4 = C_1 \left(2 - \frac{p}{100}\right) \\
C_5 = C_1 \left(1 - \frac{p}{100}\right)
$$

The substitution of Eqs. 20 and 21 into Eqs. 22 now gives

$$
(C_1 + 4EK_c) \theta_A + C_4 \theta_B - 6EK_c \frac{\Delta}{h} = C_4 \frac{Pab^2}{b^2} + C_4 \frac{Pab^2}{b^2} \\
C_1 \theta_A + (C_1 + 4EK_c) \theta_B - 6EK_c \frac{\Delta}{h} = -C_4 \frac{Pab^2}{b^2} - C_4 \frac{Pab^2}{b^2} \\
\theta_A + \theta_B - \frac{4\Delta}{h} = 0
$$
The solution of these simultaneous equations gives

\[ \begin{align*}
\theta_A &= \left[ C_4C_4 - EK_C (1.5C_4 - 2.5C_4) + C_4C_5 \right] \frac{Pa b^2}{l^2} + \\
&\quad \left[ C_4C_4 + EK_C (2.5C_4 - 1.5C_4) + C_4C_5 \right] \frac{Pa b^2}{l^2} \\
\theta_B &= -\left[ C_4C_4 + EK_C (2.5C_4 - 1.5C_4) + C_4C_5 \right] \frac{Pa b^2}{l^2} + \\
&\quad -\left[ C_4C_4 - EK_C (1.5C_4 - 2.5C_4) + C_4C_5 \right] \frac{Pa b^2}{l^2} 
\end{align*} \]

(23)

\[ \Delta = \frac{h}{4} (\theta_A + \theta_B), \]
but does not need to be evaluated since it does not appear in Eqs. 20 for \( M_{AB} \) and \( M_{BA} \), which are the only joint moments required for the derivation of the redesign coefficient, \( F \).

The evaluation of \( M_{AB} \) and \( M_{BA} \) is now possible by substituting the foregoing expressions for \( \theta_A \) and \( \theta_B \) (Eqs. 23) into Eqs. 20. However, since these expressions are lengthy, and are but an intermediate step in the derivation of the redesign coefficient, they will be omitted here.

Referring to Fig. 19, it may be seen that the moment causing compression in the top flange ("positive" moment, according to the usual beam convention) at any point along the beam shown in Fig. 17 can be written

\[ M = M_s + \frac{M_{AB}b}{l} - \frac{M_{BA}a}{l} \]

In all problems where the proposed design procedure may be advantageously applied, the maximum moment in the semi-rigidly connected beam will be equal, or very nearly equal, to the moment at the point where \( M_s \), the simple beam moment, is also maximum. Then

\[ M_{max} = M_{s(max)} + \frac{M_{AB}b}{l} - \frac{M_{BA}a}{l} \]

The redesign coefficient, as previously defined on page 57, equals

\[ F = \frac{M_{max}}{M_{s(max)}} = 1 + \frac{1}{M_{s(max)}l} (M_{AB}b - M_{BA}a) \]

or, in the special case of the single concentrated load in Fig. 17,

\[ F = 1 + \frac{1}{Pa b} (M_{AB}b - M_{BA}a) \]

(24)
The substitution of $M_{AB}$ and $M_{BA}$, obtained from Eqs. 23, into Eq. 24 finally leads to the expression

$$F = 1 - \left[ \frac{4(K_B + 200)(a^2 - a)}{(p - l)} + \frac{13K_B + 800}{p} - 4 \right]$$

As $a$ varies with respect to $l$, the location of the load, $P$, giving a maximum or minimum value for $F$ is determined by setting the first derivative of Eq. 25 equal to zero.

$$\frac{dF}{da} = 0$$

The only part of Eq. 25 affected by $a$ is the first term in the numerator within the brackets, from which it is readily apparent that a maximum or minimum value for $F$ occurs when

$$a = \frac{l}{2}$$

or, when the load is at mid-span.

Furthermore, it is also evident that the second derivative of $F$, with respect to $a$, is always negative. Hence $F$ is always maximum when the load is at mid-span. For example, letting $p = 50$ and $\frac{K_B}{K_C} = 2.0$, the following values of $F$ are obtained from Eq. 25:

<table>
<thead>
<tr>
<th>$a/l$</th>
<th>$F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.813</td>
</tr>
<tr>
<td>0.2</td>
<td>0.822</td>
</tr>
<tr>
<td>0.3</td>
<td>0.828</td>
</tr>
<tr>
<td>0.4</td>
<td>0.832</td>
</tr>
<tr>
<td>0.5</td>
<td>0.833</td>
</tr>
</tbody>
</table>

When $a/l = 0.5$ there is no sidesway and the case is similar to the conditions which are the basis for Fig. 15, where $F = 0.833$ when $p = 50$ and $\frac{K_B}{\Sigma K_C} = 1.0$. In making a comparison of the two cases it should be noted that the columns in Fig. 17 are fixed at their bases. A column so fixed is effectively twice as stiff at joint A as it would be in a limitless frame as is, for example, column AE in Fig. 12 (b).

Since $F$ is maximum for the symmetrical case, when the concentrated load is at mid-span, use of Fig. 15, in the design of a beam for any asymmetrical positioning of the load, will give the least reduction in required section modulus that need be considered and the design will err on the safe side.
APPENDIX B
TEST NO. 2 (PILOT SERIES)

LOG OF TEST NO. 2

<table>
<thead>
<tr>
<th>MOMENT (IN.-KIPS)</th>
<th>SHEAR (KIPS)</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>504</td>
<td>15.75</td>
<td>Strain lines inside column flange near top angle rivets.</td>
</tr>
<tr>
<td>640</td>
<td>20.00</td>
<td>Top angle pulling away with vertical leg remaining nearly straight.</td>
</tr>
<tr>
<td>737.6</td>
<td>23.05</td>
<td>Fracture through horizontal leg of top angle, at fillet.</td>
</tr>
</tbody>
</table>

GENERAL NOTES: The fracture of the top angle is of particular interest. A complete lack of ductility was noted and the fracture took place along the fillet where stress distribution is uniform rather than through the rivet holes, where stress concentrations are localized and the net sectional area less. The connection had good rotation prior to fracture and developed between forty and fifty per cent rigidity.
TEST NO. 2 (PILOT SERIES)

M-φ Curve for Conn. No. 2 related to Moment-Rotation Beam Design Requirements
LOG OF TEST NO. 5

<table>
<thead>
<tr>
<th>MOMENT (IN.-KIPS)</th>
<th>SHEAR (KIPS)</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>522</td>
<td>13.05</td>
<td>Scaling in column fillet opposite seat angle.</td>
</tr>
<tr>
<td>1040</td>
<td>26.00</td>
<td>Scaling in column face above top angle opposite column fillets.</td>
</tr>
<tr>
<td>1080</td>
<td>27.00</td>
<td>Scaling in column face below top angle opposite column fillets.</td>
</tr>
<tr>
<td>1180</td>
<td>29.50</td>
<td>Two rivets broke in vertical leg of top angle.</td>
</tr>
</tbody>
</table>

GENERAL NOTES: Most of the bending in the top angle. Good rotation factor of safety and between thirty and forty per cent rigidity.
TEST NO. 5 (PILOT SERIES)

M-Ø CURVE FOR CONN. NO. 5 RELATED TO
MOMENT-ROTATION BEAM DESIGN REQUIREMENTS

Test No. 5—Top Angle

Test No. 5

Top and Seat Angles

Uniform Load

To Flange of 12WF65 Col.
Top L = 6 x 4 x ½ x 1'-0
Seat L = 6 x 6 x 7½
Rivets - ½''

Max. Mom = 1,200 k'
Max Rot. ø > 0.260 ø Rad

Concentrated Load at Midspan

Rotation between End of Beam and Column in Radians

Connection Moment in Kip Inches

Test No. 5—Top Angle
LOG OF TEST NO. 9

<table>
<thead>
<tr>
<th>MOMENT</th>
<th>SHEAR</th>
<th>REMARKS</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>706</td>
<td>17.65</td>
<td>Scaling in column fillets opposite heel of seat angle.</td>
<td></td>
</tr>
<tr>
<td>850</td>
<td>21.25</td>
<td>Opening of 1/6 in. between beam and top angle.</td>
<td></td>
</tr>
<tr>
<td>1706</td>
<td>42.65</td>
<td>Rivets sheared between beam and West top angle.</td>
<td></td>
</tr>
</tbody>
</table>

**GENERAL NOTES:** Good rotation factor of safety with between thirty-five and forty-five per cent rigidity. Distortion balanced between bending in top angle and column flanges.

LOG OF TEST NO. 10

<table>
<thead>
<tr>
<th>MOMENT</th>
<th>SHEAR</th>
<th>REMARKS</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>542</td>
<td>13.55</td>
<td>Scaling in column fillets opposite heel of seat angle.</td>
<td></td>
</tr>
<tr>
<td>900</td>
<td>22.50</td>
<td>Scaling at center column flange face above top angles.</td>
<td></td>
</tr>
<tr>
<td>1850</td>
<td>46.25</td>
<td>Rivets sheared between beam flange and East top angle.</td>
<td></td>
</tr>
</tbody>
</table>

**GENERAL NOTES:** Most of the bending in beam and column flanges. Good rotation factor of safety with thirty-five to fifty per cent rigidity, nearly the same as Test No. 9 with lighter angle. Detail of connection same as that for Test No. 9.
M-Φ Curve for Conn. No. 10 Related to Moment-Rotation Beam Design Requirements

M-Φ Curve for Conn. No. 9 Related to Moment Rotation Beam Design Requirements
TEST NO. 11 (SECOND SERIES)
SEE NEXT PAGE
TEST NO. 11 (SECOND SERIES)

LOG OF TEST NO. 11

<table>
<thead>
<tr>
<th>MOMENT</th>
<th>SHEAR</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>(IN.-KIPS)</td>
<td>(KIPS)</td>
<td></td>
</tr>
<tr>
<td>1546</td>
<td>38.0</td>
<td>Maximum Load.</td>
</tr>
<tr>
<td>1360</td>
<td>34.0</td>
<td>Tension Fracture of one rivet in vertical leg of top angle.</td>
</tr>
</tbody>
</table>

GENERAL NOTES: Nearly all bending in top angle. Good rotation reserve with thirty-five to fifty per cent rigidity. Compare with Test No. 5, similar except connected to column flange.
Test No. 11

Test No. 11

M-ϕ Curve for Conn. No. 11 Related to Moment-Rotation Beam Design Requirements
TEST NO. 16 (SECOND SERIES)

LOG OF TEST NO. 16

<table>
<thead>
<tr>
<th>MOMENT (IN.-KIPS)</th>
<th>SHEAR (KIPS)</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>616</td>
<td>19.25</td>
<td>Tensile fracture of one rivet in vertical leg of top angle.</td>
</tr>
</tbody>
</table>

GENERAL NOTES: Most of bending in top angle. Excellent rotation reserve and between thirty and forty-five per cent rigidity.
Top and Seat Angles To Flange of 10WF49 Col.
Top L - 6 x 4 x 3/8 x 6
Seat L - 6 x 6 x 1/2 x 6
Rivets - 3/8

Test No. 16

M-ϕ Curve for Conn. No. 16 Related to Moment-Rotation Beam Design Requirements

Test No. 16 (Second Series)
TEST NO. 17 (SECOND SERIES)

LOG OF TEST NO. 17

<table>
<thead>
<tr>
<th>MOMENT (IN.-KIPS)</th>
<th>SHEAR (KIPS)</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>538</td>
<td>16.80</td>
<td>Tensile fracture of one rivet in vertical leg of top angle.</td>
</tr>
</tbody>
</table>

**GENERAL NOTES:** Most of bending in top angle. Good rotation reserve with between thirty-five and forty-five per cent rigidity.
Top and Seat Angles
To Web of 10WF49 Col
Top L = 6 x 4 x 3/8 x 0.39
Seat L = 6 x 6 x 5/8 x 0.39
Rivets = 3/8".

Test No. 17

Concentrated Load
at Midspan

Max. Mom = 538 k";
Max. Rot. = > 0.051 Rad.

M-ϕ Curve for Conn. No. 17 related to
Moment- Rotation Beam Design Requirements
**TEST NO. 18 (SECOND SERIES)**

**LOG OF TEST NO. 18**

**MOMENT**  **SHEAR**  **REMARKS**

<table>
<thead>
<tr>
<th>(IN.-KIPS)</th>
<th>(KIPS)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>568</td>
<td>17.75</td>
<td>Tensile failure of one rivet in vertical leg of top angle.</td>
</tr>
</tbody>
</table>

**GENERAL NOTES:** Between twenty and thirty per cent rigidity. Good rotation reserve.
M-Ø Curve for Conn. No. 18 Related to Moment-Rotation Beam Design Requirements
TEST NO. 20 (THIRD SERIES)

LOG OF TEST NO. 20

<table>
<thead>
<tr>
<th>MOMENT (IN.-KIPS)</th>
<th>SHEAR (KIPS)</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1305</td>
<td>36.25</td>
<td>Ultimate load. No failure.</td>
</tr>
</tbody>
</table>

GENERAL Notes: Excellent rotation reserve with forty to fifty-five per cent rigidity. Test stopped because of excessive deformations, with balanced bending in column flange, beam flange, and seat angle.
TEST NO. 20 (THIRD SERIES)

M-θ Curve for Conn. No. 20 Related to Moment-Rotation Beam Design Requirements

M-θ Curve for Connection No. 20 showing Components of Rotation

Test No. 20
[99]
**LOG OF TEST NO. 22**

<table>
<thead>
<tr>
<th>MOMENT (IN.-KIPS)</th>
<th>SHEAR (KIPS)</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1494</td>
<td>41.50</td>
<td>Ultimate load. Test stopped prior to failure.</td>
</tr>
</tbody>
</table>

**General Notes:** Good rotation reserve with between forty and fifty per cent rigidity. Bending distributed between column flange, horizontal leg of top angle, and top of beam flange.
TEST NO. 22 (THIRD SERIES)

M-φ Curve for Conn. No. 22 Related to Moment-Rotation Beam Design Requirements

Test No. 22

[ 101 ]
TEST NO. 23 (THIRD SERIES)

LOG OF TEST NO. 23

MOMENT SHEAR REMARKS
(IN.-KIPS) (KIPS)

1233 34.25 Combine shear and tensile failure of outer rivets in horizontal leg of top angle.

GENERAL NOTES: Rotation reserve is ample, but considerably less than companion Test No. 22 to column flange. Deformation largely due to deformation of rivets in horizontal leg of top angle and tension deformation in vertical leg rivets.
Test No. 23

[103]
TEST NO. 24 (THIRD SERIES)

LOG OF TEST NO. 24

<table>
<thead>
<tr>
<th>MOMENT (IN.-KIPS)</th>
<th>SHEAR (KIPS)</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1946</td>
<td>48.65</td>
<td>Near maximum load. Test stopped.</td>
</tr>
</tbody>
</table>

General Notes: Compare with Test No. 9 which is same except for rivet size. Good rotation factor of safety and between forty and fifty-five per cent rigidity.
TEST NO. 24 (THIRD SERIES)

M-θ Curve for Conn. No. 24 Related to Moment-Rotation Beam Design Requirements

Test No. 24
[105]
LOG OF TEST NO. 25

<table>
<thead>
<tr>
<th>MOMENT (IN.-KIPS)</th>
<th>SHEAR (KIPS)</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1914</td>
<td>43.50</td>
<td>Scaling at bearing toe of seat stiffener angles.</td>
</tr>
<tr>
<td>2486</td>
<td>56.50</td>
<td>Shear failure of outer rivets in horizontal leg of top angle.</td>
</tr>
</tbody>
</table>

GENERAL NOTES: Good rotation reserve, but deformation largely in column flange. Between forty and fifty-five per cent rigidity.
TEST NO. 25 (THIRD SERIES)

M - θ Curve for Conn. No. 25 related to Moment-Rotation Beam Design Requirements

Test No. 25
[107]
LOG OF TEST NO. 26

<table>
<thead>
<tr>
<th>MOMENT (IN.-KIPS)</th>
<th>SHEAR (KIPS)</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>2640</td>
<td>55.00</td>
<td>Scaling at bearing toe of seat stiffener angles.</td>
</tr>
<tr>
<td>2904</td>
<td>60.50</td>
<td>Shear failure of rivets in horizontal leg of top angle.</td>
</tr>
</tbody>
</table>

General Notes: Good rotation reserve. Between thirty and forty-five per cent rigidity.
TEST NO 26 (THIRD SERIES)

M-# Curve for Conn. No. 26 related to Moment-Rotation Beam Design Requirements
TEST NO. 31

LOG OF TEST NO. 31

<table>
<thead>
<tr>
<th>MOMENT (IN.-KIPS)</th>
<th>SHEAR (KIPS)</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>2060</td>
<td>43.00</td>
<td>Tension rivets in vertical leg of top angle near failure.</td>
</tr>
<tr>
<td>2064</td>
<td>172.00</td>
<td>(New lever arm).</td>
</tr>
<tr>
<td>2755</td>
<td>229.40</td>
<td>Maximum load. Tensile failure of rivets. Exact load at failure unknown.</td>
</tr>
</tbody>
</table>

GENERAL NOTES: Although moment-rotation curve shows no rotation reserve, there was some degree of unmeasured rotation beyond the measured values. Nevertheless, this connection is apparently deficient in rotation reserve, while developing between twenty-five or more percent rigidity.
**TEST NO. 31**

M - θ CURVE for CONN. No. 31 RELATED to 
MOMENT - ROTATION BEAM DESIGN REQUIREMENTS

Test No. 31

[ III ]
TEST NO. 32

LOG OF TEST NO. 32

<table>
<thead>
<tr>
<th>MOMENT (IN.-KIPS)</th>
<th>SHEAR (KIPS)</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>3105</td>
<td>2823</td>
<td>Tensile fracture of rivet in vertical leg of top angle.</td>
</tr>
</tbody>
</table>

General Notes: Good rotation reserve with between twenty-five and forty per cent rigidity. Rotation largely accounted for by bending in column flange and slip of beam flange rivets.
TEST NO. 32

M-Φ Curve for Conn. No. 32 Related to Moment-Rotation Beam Design Requirements

Test No. 32 [113]
TEST NO. 35

LOG OF TEST NO. 35

<table>
<thead>
<tr>
<th>MOMENT (IN.-KIPS)</th>
<th>SHEAR (KIPS)</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>560</td>
<td>17.50</td>
<td>Column flange scaling at top angle.</td>
</tr>
<tr>
<td>835</td>
<td>41.75</td>
<td>Tensile failure of rivet in vertical leg of top angle.</td>
</tr>
</tbody>
</table>

GENERAL NOTES: Fair rotation reserve with forty-five to fifty-five per cent rigidity. Rotation primarily due to bending of top angle and extension of tension rivets.
TEST NO. 35

M-# Curve for Conn. No. 35 related to Moment-Rotation Beam Design Requirements

M-# Curve for connection No. 35 showing components of rotation

Test No. 35

[115]
LOG OF TEST NO. 36

Top and Seat Angle Connection to One Side of Column Web

<table>
<thead>
<tr>
<th>MOMENT SHEAR (IN.-KIPS)</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>640 16.0</td>
<td>Strain lines due to bending of column web adjacent to ends of top angle.</td>
</tr>
<tr>
<td>744 18.6</td>
<td>Tensile rivet fracture in top angle at end.</td>
</tr>
<tr>
<td>988 24.7</td>
<td>Second tensile rivet failure in top angle.</td>
</tr>
</tbody>
</table>

GENERAL NOTES: Insufficient rotation reserve. Flexibility of column web throws all tensile load in outer or end rivets of top angle. The desirability of a reinforcing angle on the opposite side of the column is indicated.

LOG OF TEST NO. 37

Top and Seat Angle Connection to One Side of Column Web with Web Stiffened

<table>
<thead>
<tr>
<th>MOMENT SHEAR (IN.-KIPS)</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1800 90.0</td>
<td>Scaling on vertical leg of seat angles adjacent to fillets.</td>
</tr>
<tr>
<td>1900 95.0</td>
<td>Scaling in beam web above seat.</td>
</tr>
<tr>
<td>2457 122.85</td>
<td>Tension failure of all four rivets in top angle.</td>
</tr>
</tbody>
</table>

GENERAL NOTES: Good rotation reserve with between forty and fifty-five per cent rigidity. Comparison with No. 36 shows the marked improvement produced by stiffening the opposite side of the column web, thereby forcing bending to take place in top angle and distributing load between top angle rivets.

Details of this connection similar to that of Test No. 36, except for web reinforcement.
TESTS NO. 36 AND NO. 37

Test No. 36

Test No. 37

[117]
TESTS NO. 36 AND NO. 37

M-ø Curve for Conn. No. 36 Related to Moment-Rotation Beam Design Requirements

M-ø Curve for Connection No. 36 Showing Components of Rotation

M-ø Curve for Conn. No. 37 Related to Moment-Rotation Beam Design Requirements

M-ø Curve for Connection No. 37 Showing Components of Rotation