TORSIONAL PROPERTIES
OF FABRICATED I BEAM AND BOX SECTIONS

by I. E. Madsen*

1 - SYNOPSIS

This report gives the results of a series of torsion tests on riveted and welded I-beams, and riveted and welded box girders. Due to the slipping which occurs in the seams, the fabricated girders were neither as strong nor as rigid as the equivalent girder section without longitudinal seams.

On the basis of the tests which were made it was found that a riveted section was about one-third as stiff in torsion as a section in which no slipping occurs. Design methods for computing the stresses and twists of built-up girders are included in this report.

2 - INTRODUCTION

The stresses and twist in a hollow tube may be predicted fairly accurately by the theoretical formulae proposed by R. Bredt. These formulae state that the average shear stress in the walls of the tube due to torque is:

\[ \tau = \frac{M}{2At} \]

The angle of twist per unit length is:

\[ \theta = \frac{M}{GJ} \]

\( \tau \) is the shearing stress in pounds per square inch, \( M \) is the twisting moment in inch-pounds, \( t \) is the wall thickness of the tube, \( A \) is the area of the surface bounded by the center lines of the tube walls, \( G \) is the shear modulus (about 11,500,000 lb per in\(^2\) for steel), and \( J \) is the torsional constant in in\(^4\).

\( J \) is analogous to the polar moment of inertia and for a hollow tube:

\[ J = \frac{4A^2}{\Sigma \frac{S}{t}} \]

where \( s \) and \( t \) are the length and thickness of each side.

The above formulae apply to hollow tubes of all shapes which are bounded by one single line. The formulae are approximate, but give close results when the thickness of the tube wall is small in comparison with the distance of the wall from the center of gravity of the tube. For crane girders this approximation is usually small.

* Assistant Research Engineer, Fritz Engineering Laboratory, Lehigh University, Bethlehem, Pennsylvania

The formulae are derived for one piece tubes, thus they do not necessarily apply to built-up sections since slipping may occur along the longitudinal seams. The torsional strength of a built-up section, loosely bolted together, would be only the sum of the strengths of the individual pieces. This sum is but a fraction of the strength of the section as a tube. In a continuously welded section, on the other hand, no slipping can occur along the seams and the twisting strength should equal that of the solid tube as predicted by the theory. A built-up riveted or tightly bolted section should have properties somewhat less favorable than the welded or solid tube since a limited amount of slip may occur.

This series of tests was made to determine how the actual behavior of built-up box sections compared with that of one-piece sections. In addition, built-up I-beams were also tested to compare their behavior with solid I-beams, for which rational methods of analysis are available*.

Pilot tests were made on a series of small 2 by 3-in. box sections, 50 in. long and made of 1/8-in. plate. The sections consisted of welded and bolted boxes as shown in the drawing of Fig. 1. These boxes were tested in a torsion machine of 24,000 in-lb. capacity. Fig. 2 shows a picture of the bolted box being twisted in the testing machine.

The bolted girders were made to simulate the action of a riveted girder by using bolts which fitted tightly in the holes. Tests were made on the bolted girder with the diaphragms fastened on one, two, and three sides, and with diaphragm spacings varying from three to forty-eight inches.

Pilot torsion tests were also made on two welded boxes, one had diaphragms as shown in Fig. 1, the other had no diaphragms. Three tests were made on the box with the diaphragms. It was first tested with the section shown in Fig. 1, then it was tested with flange angles welded to the box to make the section similar to a riveted section. Finally, the top plate and angle were burned off, leaving a plain rectangular box, and the torsional constant was again found.

The welded box in the pilot test behaved as the theory predicted, so further torsion tests were not deemed necessary on the welded sections. However, the twist and the stresses in the bolted box were so much higher than that predicted by theory, it was thought advisable to make additional tests on riveted sections. Tests were made, therefore, on four I-beam

* STRUCTURAL BEAMS IN TORSION, Inge Lyse and Bruce G. Johnston, Transactions A.S.C.E., 1936, p.1389
Bethlehem Manual of Steel Construction, p.279
and three box sections. The details of the I-beams are shown in Fig. 3. The details of the box girder called T-6 is shown in Fig. 4. The other box girders were similar to T-6 except for the diaphragms. T-5 had no diaphragms and T-6 had four diaphragms equally spaced.

Since these specimens were too large to be tested in an ordinary torsion machine, they were loaded by a special rig attached to a 300,000-lb. Olsen machine*. Fig. 5 and 6 show the details of the loading rig and also show girders T-2 and T-6 being tested.

The twist in the girders was measured by means of level bars. The strains, from which the stresses were determined, were measured with a 2-in. Olsen and a 10-in. Whittemore strain gauge on the outside of the boxes and both sides of the I-beams. The strains were measured longitudinally along the girder and on strain rosettes located on the webs and flange. The shear stresses were found from the strain rosettes. So far as the author knows very few tests have been made in which the stresses have been measured in box beams under torsional loads.

The slip in the seams of the girder was measured by the Whittemore gauge with one end of the gauge on one side of the seam and the other end on the other side of the seam. The locations of the gauge lines are shown in Fig. 7. The gauge lines marked 3, 6, 9, and 12 are a measure of the slip when corrected for the actual strain in the elements themselves.

3 - TEST RESULTS

a. Pilot Tests - Table I presents the results of the twisting tests on the pilot specimens. The torsion constants of the welded boxes agree with the theoretical values of 2.09 in4. The flange angles and outstanding portion of the coverplate have little effect on the results. Neither did the diaphragms have any appreciable effect, since in these girders no slipping occurred to bring the diaphragms into action.

The values of the torsion constants, as shown in Table I, are quite variable for the bolted girder. They are much less than that given by Brodt's theory for a box.

* TORSION TESTING MACHINE OF 750,000 INCH-POUND CAPACITY, Bruce G. Johnston, Engineering News-Record, Vol. 114, No. 9, p.310, February 28, 1935

This is due to the slipping which occurred along the seams of these girders. As the table shows, the diaphragms had an important effect on the twisting of the beam. When the diaphragms were fastened on three sides, they were much more effective than when fastened on two sides. Secondly, increasing the number of diaphragms, when bolted on three sides, increased the stiffness of the girders.

Fig. 8 gives the results of the torsion test on the welded box. The stresses, as measured, are about forty percent greater than the theoretical as computed by Bredt's theory. The twist checks closely. Fig. 9 gives a similar curve for a bolted girder. Two computed shear stresses are shown in this figure. The lower stress curve marked 1 is that given by Bredt's theory, while the higher value marked 2, includes the shear stress due to the twisting of each side of the box. This second factor is small for a box with no slip and can be safely neglected. It is large, however, in the present case.

Fig. 10 gives the results of the torsion test on the bolted girder with the diaphragms spaced at 24 in. It shows in detail the action of the girder. When the girder is first twisted, as shown in the curve marked First Run, the box is initially very stiff due to the friction of the seams. However, as the load is increased, the joints slip, and the friction is gradually broken. The portion of the curve marked CD shows the twist which takes place while the bolts are slipping. Most of the total twist occurs in this range. At point D, the bolts start to bear and the portion of the curve beyond D shows the increased resistance to slip which results. At point F, the load was released, and the section FH on the curve shows the effect of the return friction in preventing the girder from untwisting. If the testing machine had been of such a type that the box could have been twisted in the reverse direction, it is expected that the resultant curve would have a large hysteresis loop.

Fig. 11 shows the variation in the torsional stiffness as the number of diaphragms is increased.

b. I-Beam Tests - The test results on the I-beams were quite similar. T-1 and T-2 were identical in section but T-2 was welded and T-1 was riveted. T-3 and T-4 were similar to T-1 and T-2 except that the sections of the former were 3/8-in. thick and the sections of the latter were 1/4-in. thick.

Since T-2 and T-4 were welded less slip could occur. Therefore, the stresses and twists in these girders should be less than those of girders T-1 and T-3.
Fig. 12 compares the twists of the four I-beam girders under a torque load. The rotation given is the average of the rotation of the flanges and the webs except for T-3 for which the top twist is given. The top twist of T-3 is somewhat less than the average, since the web usually twisted a little more than the flange. The values for the torsion constants are appreciably less than the theoretical values for the equivalent solid I-beam.

Fig. 13 gives the magnitudes of the shear stresses in the web and coverplate of the four I-beam girders. This figure shows that in the same girder section, as for example T-3 and T-4, the measured stresses in the riveted section are about twice that of the welded section. This is the result of the greater twist of the riveted girder. The shear stresses in the flange are greater than those in the web. Theoretically, disregarding stress concentrations, the flange shearing stresses should be twice the web stresses for the girders tested since the flange is twice as thick as the web.

Table II gives a comparison of computed and measured stresses for the 40,000-in-lb. load. The shear stresses were computed by the formula \( \tau = t \cdot G \cdot \theta \), where \( \tau \) is the shearing stress, and \( t \) is the thickness of the section considered. This formula comes directly from the formulae for twist and stress of a narrow rectangular section. The web stresses check closely as shown by the table. The flange stresses are larger than the web stresses but smaller than the theoretical. This shows that some slippage is taking place and that the full thickness of the flange cannot be assumed to be acting as a unit.

Fig. 14 gives curves for the variation of the slip with the twist and the moment for girders T-1 and T-2. Similar results were obtained for T-3 and T-4. The slip given is the total of the slips in all the seams around the girder. It should be noted that for T-1, the twist is proportional to the slip. The slips for T-2 are much smaller than for T-1. The values for T-2 are not actually slips since the seam was welded, but since the welding was intermittent the welds are strained higher than the adjacent plates and this results in a relative strain between the plates.

Secondary longitudinal direct stresses are set up in a twisted I-section or box as a result of the tendency of the fibers farthest from the center of twist to be lengthened. In Fig. 15 are plotted the curves of direct stress in T-1 as actually measured. Table II gives a comparison between computed values of the stress and the measured values at the 40,000-in-lb. load. These stresses
are quite appreciable. The formula* for the direct stress in the I-beam was derived in a manner similar to that which Timoshenko uses for the rectangle in torsion. It should be noted that the stress increases with the square of the twist. Since the built-up beam twists more than the solid beam, these direct stresses are much more important than in the solid beam. The computed stress in the webs checks fairly well with the theory but the stresses in the flange do not check as well. This is probably due to the slipping which occurs, as indicated by the fact that the stress at the edge of the cover plate and at the edge of the flange angle is quite different, showing a relative slip.

c. Box Girder Tests - The three box girders were alike except for diaphragm spacing. T-5 had no diaphragms, T-6 had one diaphragm in each end, and T-7 had four diaphragms equally spaced. After T-7 had been tested the first time, the seams were welded with intermittent welds to prevent slippage and the girder retested. This test was called W-T-7.

Very little difference was found in the tests on the three boxes, so apparently the diaphragms were of little effect in the elastic range. These diaphragms were riveted only to the webs as is the usual practice, and thus are relatively ineffective in preventing slip. Since the results for all girders are so very much alike, the test results will be given only for T-6.

In Fig. 16 is plotted the twist of the box as the moment was increased. In Fig. 17 is shown a number of curves showing the measured slip and measured stresses in the box.

The measured shear in the flange and web are fairly close to the theoretical. The theoretical shear curve marked 1 is the stress as found from Bredt's formula. The other theoretical curve marked 2 is the sum of the stress from Bredt's theory and the stress due to the twisting of the elements of the box.

The shear curves are computed from the strain rosette data and are the shears acting on the cross-section of the girder perpendicular to the longitudinal axis. The rosettes in the portion of the webs and flanges forming the box showed that the direct stresses on these cross-sectional planes were negligible.

In the rosettes on the outstanding portions of the cover plates and flange angles outside the box section, the measured shear stresses were less than the theoretical shear stresses in the box. The direct stresses determined by these rosettes were appreciable, showing that these sections were not subjected to pure shear.

* STRENGTH OF MATERIALS, Vol. 1, S. Timoshenko, p. 89
The direct stress at the edge of the flange as shown in Fig. 17 is appreciable for high loads, but in the working range this stress is zero. This stress is a secondary stress and increases with the square of the twist.

The stress in the diaphragm is rather large. This stress was taken on a gauge line 45° with the horizontal since it was thought that the slip in the girder would stress the diaphragm with a twisting moment. However, this measured stress is much larger than any which were computed.

The slip in the girder is quite appreciable. The slip did not start with the initial torque but at a torque load of about 45,000 in-lb. If an initial tension of 32,000 lb per sq in. is assumed in the rivets and a coefficient of friction of 0.2 is assumed to be acting between the plates, the load at which slippage will occur is 44,000 in-lb.

In making these tests, one has to be careful that the torque is applied equally to the whole girder section. The test on T-5 was made with the torque applied to the webs. Fig. 18 shows that the top flange and web did not twist equally. The average rotation, however, is about the same as that for T-6 and T-7.

The test results for T-7 after the seams were welded are shown in Fig. 19. It is seen that this girder is then much stiffer. The shear curves also check the theoretical computed from Bredt's formulae, because there was no added twist due to slip.

4. DISCUSSION OF RESULTS

a. Pilot Tests - The pilot tests on the box girders showed that the actual twist in a welded box agreed with the theoretical, and that Bredt's formula gives a good value for the torsional constant. These tests also showed that the flange angles, and outstanding edges of the cover plate had little effect on the torsional constant.

The shear stresses on the outside of the box were about forty per cent greater than the theoretical average value from Bredt's formula. This may be due to the eccentric application of the load through the fillet weld. Since the plate is thin, and the box section very small, this effect is emphasized in this test, and it would be picked up by the comparatively long 2-in. gauge length of the strain gauge. Since this effect was not noticed in the larger specimens, it may be a peculiarity of the small specimen.
The pilot tests on the bolted girders probably have no practical significance by themselves. Their chief value is that they exaggerate and emphasize the effects due to slipping in the seams of a riveted girder. As shown by Fig. 11, the torsional stiffness of the girder approaches that of the sum of the component parts as the restraints to slipping are removed.

b. I-Beam Tests - The shear and twist of a long narrow rectangle are given by the formula

\[ \tau = \frac{3M}{bc^2} \]

and \[ \theta = \frac{3M}{bc^3G} \]

and \[ \gamma = cG\theta \]

where \( \tau \) is the shear stress, \( \theta \) is the angle of twist in radians per inch, \( b \) is the width of the rectangle, and \( c \) is the thickness of the rectangle. Since an I-beam is composed of a number of rectangles, an approximate solution for the torsional stresses and twist can be made by dividing the I-beam into a number of rectangles and summing up the sections. However, since the plate thickness enters into the formulae as square and cube terms, it becomes a question as to what thickness to use when several rectangles are put together. Fig. 20a gives a diagramatic idea of the shear stress distribution when slipping is allowed to occur freely. Fig. 20b gives a similar picture for the solid girder in which no slipping occurs. The increased strength due to the greater effective moment arm of the shear stress is apparent. For a riveted girder where some slipping takes place the resultant stress distribution falls between the two distributions shown in Fig. 20a and 20b.

In Table III are given the computed torsional constants of the various girders tested. Two values are given, one is computed assuming free slippage and the other is computed on the basis of no slippage. The measured torsional constant for the riveted girders average a little more than one-third of that for the equivalent solid section. The welded girders average 68 per cent. The welded I-beam girders do not have a factor of 100 per cent, since although the weld does prevent slipping at the toe of the angle, relative strain can occur at the corners of the angles.

For design computations, if a value of the torsion constant of one-third the solid section is used, the computed angles of twist should be fairly close to the actual values. To compute the stresses, these values of \( \theta \) should be used in the formulae. As shown in Table II, the computed stresses are close enough for design purposes. There is little need
to go into any refinements in the calculations since the behavior of the girder will depend so largely on the slip. Since this slip depends on many factors, and is relatively unpredictable, the behavior of the girders may be quite variable.

The direct stresses shown in Table II are quite large. Since the direct stresses increase with the square of the twist, and the twist of the riveted girder is three times that of the solid girder, these direct stresses will be nine times those in the solid girder. The direct stresses are secondary stresses and therefore have little effect on the ultimate load-carrying capacity of the girder. However, a permanent twist will result in the girder, if these stresses exceed the yield point.

c. Tests of Box Girders - The tests on the riveted girders showed that they behaved about as predicted by Bredt's formula, except as this is modified by slip. Table II shows that the torsion constant is about one-third that which is predicted by Bredt's theory. In computing this torsion constant, one must remember that the built-up rectangular section is not a box but a figure more like an H-beam since the stress in the flange must travel outside the box section to the rivet and back again before it is transferred to the web. Fig. 21a shows the section acting in the stress transfer. The dotted line shows the edges of the section used to compute A in Bredt's formula. Fig. 21a also gives a diagramatic picture of the shear stress transfer in a box section where slipping is allowed to occur. Fig. 21b shows the shear stress transfer when no slipping occurs. The reason for the effectiveness of the box section is obvious when one notices the large effective moment arm of the shear stress.

The shear curves for Fig. 17 show that Bredt's theory gives a fair approximation for the average shear stress, and this computation can be made more exact by adding the stress due to the twist of the sides of the box. Bredt's theory assumes that the shear stress is constant through the thickness of the box, and when the thickness of a wall is small in comparison with the distance of the plate to the center of gravity of the box, this approximation is small. Actually, the stress increase from the inside of the box wall to the outside. The correction added to the average shear as given by Bredt's formula gives the maximum stress on the outside of the box. Since the riveted box twists more than the solid box the correction is larger in the former case. In Fig. 19 this correction is negligible since the welded box did not slip resulting in a smaller angle of rotation.

The direct secondary stress in the box as shown by Fig. 17 can be safely neglected in the working range.
The stress in the diaphragms was larger than expected. It may have been affected by local bending and readjustments. The direct stresses for the welded girder were much less than those for the riveted girder.

The measured slip in the joints as given in the reports is the total of the slip in all the seams. The actual slip in any one corner of the box is the plotted slip divided by four. It can be shown* for a box with free slipping seams (no rivets or welds) that the longitudinal motion or warping of any point in the box section is the product of the x and y coordinates of the point multiplied by the angle of twist θ when slipping occurs in all the seams. Since in the same corner, the web and the flange warp in opposite directions, the relative slip is equal to twice this product. Thus the relative slip in the corner of a box of height h and width w would be whθ/2. When the box is riveted, the actual slip is less than the computed value above, and this actual slip is a measure of the torsional constant. For the gauge lines on which slip was measured in the test, the relative slip for loose seams is 428. For girder T-6 the measured slip for the straight line portions of the slip curve is 10.5 Mt(10)^-8 for one seam.

Then the twist angle is given by:

$$\theta = \frac{M}{GJ(\text{no slip})} + \theta \ (\text{due to slip})$$

$$\text{or } \theta = \frac{M}{GJ} + \frac{10.5 \, M(10)^{-8} \, G}{42 \, G}$$

$$= \frac{M}{G} \left( \frac{1}{57.8} + \frac{1}{33.3} \right) = \frac{M}{G} \left( \frac{1}{21.2} \right)$$

The value of 21.2 is the effective torsion constant, and provides a check on the value computed directly from the curve. This check is not absolutely independent, since the value of θ is inherent in the second term.

5 - CONCLUSIONS

The torsion tests herein reported indicate that the following results may be concluded.

1. Fabricated I-beam and box sections do not behave the same as the equivalent solid sections.

2. The slip which occurs in the seams of built-up sections is appreciable under torsional loading and increases the twist and stresses of such sections markedly.

* N.A.C.A. Report No. 502, A THEORY FOR PRIMARY FAILURE OF STRAIGHT CENTRALLY-LOADED COLUMNS
3. The torsional constant of the riveted built-up I and box sections tested in this program was about one-third that of the equivalent solid section.

4. If the twist angle θ is computed using the reduced value of the torsional constant, shear stresses computed from this value will agree fairly well with the actual values.

5. Direct secondary stresses are quite large in the built-up I-section and may be important.

6. Direct secondary stresses are small in a box section when stressed in the working range.

7. The direct stresses vary with the square of the angle of twist.

8. Bredt's theory gives good approximate values for the shear stress in box sections.

9. In applying Bredt's theory to a fabricated box section, one must be careful to use the section bounded by the stress path. This will not be a rectangular section when the corners are formed by riveted angles.

10. Build-up welded I-beams fabricated with flange angles had a torsion constant two-thirds that of the equivalent solid section.

11. Built-up welded box girders had a torsional constant close to that predicted by Bredt's theory.

12. The outstanding legs and parts of a fabricated box section not included in the box have little effect on the torsional properties of the section.

13. The limiting values for the torsion constant of a fabricated beam will be the value for the solid section and the sum of the values of the component parts. The actual value of the constant will be between the boundary values and will depend on the degree of slip.

6 - ACKNOWLEDGMENTS

These tests are a part of a program sponsored and financed by the Association of Iron and Steel Engineers at the Fritz Engineering Laboratory of Lehigh University. The work is being carried out in direct cooperation with the Crane Specifications Committee of the A.I. & S.E. under the chairmanship of Frank W. Cramer. Special acknowledgment is due to Brent Wiley, Managing Director of the Association. The general program is under the direction of Bruce Johnston, Associate Director of the Laboratory. Many thanks are also due Hale Sutherland, Director, and H.J. Godfrey, Engineer of Tests of the laboratory, for their cooperation in the investigation.
### TABLE I

**WELDED GIRDER**

<table>
<thead>
<tr>
<th>Measured Values of J in(^4)</th>
<th>Remarks on Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.17</td>
<td>3-in. Diaphragm Spacing</td>
</tr>
<tr>
<td>2.25</td>
<td>3-in. Diaphragm Spacing (Flange angles added)</td>
</tr>
<tr>
<td>2.03</td>
<td>3-in. Diaphragm Spacing (Flange angles cut off)</td>
</tr>
<tr>
<td>2.09</td>
<td>No diaphragms</td>
</tr>
<tr>
<td>2.09</td>
<td>Theoretical value by Fredt's Theory</td>
</tr>
</tbody>
</table>

**BOLTED GIRDER**

<table>
<thead>
<tr>
<th>Measured Values of J in(^4)</th>
<th>Remarks on Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0391 0.242</td>
<td>3-in. Diaphragm Spacing Diaphragms bolted on 1 side</td>
</tr>
<tr>
<td>0.0393 0.445</td>
<td>3-in. Diaphragm Spacing Diaphragms bolted on 2 sides</td>
</tr>
<tr>
<td>0.1090</td>
<td>3-in. Diaphragm Spacing Diaphragms bolted on 3 sides</td>
</tr>
<tr>
<td>0.0647 0.180</td>
<td>6-in. Diaphragm Spacing Diaphragms bolted on 3 sides</td>
</tr>
<tr>
<td>0.0639 0.229</td>
<td>12-in. Diaphragm Spacing Diaphragms bolted on 3 sides</td>
</tr>
<tr>
<td>0.0222 0.282 1.35</td>
<td>24-in. Diaphragm Spacing Diaphragms bolted on 3 sides</td>
</tr>
<tr>
<td>0.0250 0.131 1.32</td>
<td>48-in. Diaphragm Spacing Diaphragms bolted on 3 sides</td>
</tr>
<tr>
<td>0.0204 0.264 1.02</td>
<td>No Diaphragms</td>
</tr>
<tr>
<td>0.0146</td>
<td>Theoretical value for freely slipping bolts</td>
</tr>
</tbody>
</table>
### TABLE II

<table>
<thead>
<tr>
<th>Girder</th>
<th>Twist, $\theta$ in Radians/in.</th>
<th>Shear in Web</th>
<th>Shear in Flange</th>
<th>Direct Stress in Web, 4 in. from $\xi$</th>
<th>Direct Stress Edge of Flange</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Measured</td>
<td>Computed $f = \frac{E\theta^2}{24} \left( \frac{h^3}{8} + \frac{w^3}{8} - \frac{th^3 + 6h^2tt' + 2t'w^3 + 2t''h^3 - 16t''a^3}{ht + 2t'w + ht'' - 4at''} \right)$</td>
<td>Measured $f = \frac{E\theta^2}{24} \left( \frac{t'}{2} - \frac{th^3 + 6h^2tt' + 2t'w^3}{ht + 2t'w} \right)$</td>
<td>Computed $f = \frac{E\theta^2}{24} \left( \frac{h^3}{8} + \frac{w^3}{8} - \frac{th^3 + 6h^2tt' + 2t'w^3 + 2t''h^3 - 16t''a^3}{ht + 2t'w + ht'' - 4at''} \right)$</td>
</tr>
<tr>
<td>T-1</td>
<td>0.0057</td>
<td>16,800</td>
<td>17,100</td>
<td>26,600</td>
<td>34,200</td>
</tr>
<tr>
<td>T-2</td>
<td>0.0040</td>
<td>12,800</td>
<td>12,000</td>
<td>16,000</td>
<td>24,000</td>
</tr>
<tr>
<td>T-3</td>
<td>0.0024</td>
<td>13,400</td>
<td>10,800</td>
<td>16,000</td>
<td>21,600</td>
</tr>
<tr>
<td>T-4</td>
<td>0.0014</td>
<td>6,200</td>
<td>6,300</td>
<td>7,600</td>
<td>12,600</td>
</tr>
</tbody>
</table>

* Formula for direct stress at edge of flange:

$$f = \frac{E\theta^2}{24} \left( \frac{h^3}{8} + \frac{w^3}{8} - \frac{th^3 + 6h^2tt' + 2t'w^3 + 2t''h^3 - 16t''a^3}{ht + 2t'w + ht'' - 4at''} \right)$$

Formula for direct stress in web:

$$f = \frac{E\theta^2}{24} \left( \frac{t'}{2} - \frac{th^3 + 6h^2tt' + 2t'w^3}{ht + 2t'w} \right)$$

- $h$ = height of web
- $t'$ = flange plate thickness
- $t''$ = thickness of flange angle
- $t$ = web thickness
- $w$ = width of flange
- $\theta$ = angle of twist in radians
- $\xi$ = distance to point in web where stress is computed
- $a$ = distance to bottom of flange angle

All values given for torque load of 40,000 in-lb.
<table>
<thead>
<tr>
<th>Girder</th>
<th>Computed J</th>
<th>Ratio Col.1</th>
<th>Measured J</th>
<th>Ratio Col.4</th>
<th>Ratio Col.4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sum of Individual Parts</td>
<td>Solid Beam</td>
<td>in^2</td>
<td>Col.2</td>
<td>Col.1</td>
</tr>
<tr>
<td></td>
<td>in^4</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>T-1</td>
<td>0.26</td>
<td>1.20</td>
<td>0.22</td>
<td>0.49</td>
<td>1.89</td>
</tr>
<tr>
<td>T-2</td>
<td>0.25</td>
<td>1.20</td>
<td>0.22</td>
<td>0.83</td>
<td>3.20</td>
</tr>
<tr>
<td>T-3</td>
<td>0.88</td>
<td>3.64</td>
<td>0.24</td>
<td>1.10</td>
<td>1.25</td>
</tr>
<tr>
<td>T-4</td>
<td>0.88</td>
<td>3.64</td>
<td>0.24</td>
<td>2.43</td>
<td>2.76</td>
</tr>
<tr>
<td>T-5</td>
<td>0.38</td>
<td>57.80</td>
<td>0.01</td>
<td>20.40</td>
<td>53.70</td>
</tr>
<tr>
<td>T-6</td>
<td>0.38</td>
<td>57.80</td>
<td>0.01</td>
<td>21.30</td>
<td>56.00</td>
</tr>
<tr>
<td>T-7</td>
<td>0.38</td>
<td>57.80</td>
<td>0.01</td>
<td>20.30</td>
<td>53.50</td>
</tr>
<tr>
<td>WT-7*</td>
<td>0.38</td>
<td>57.80</td>
<td>0.01</td>
<td>54.00</td>
<td>142.00</td>
</tr>
</tbody>
</table>

* Girder T-7 with intermittent welds on seams to prevent slipping
Fig. 2 - Bolted Box in Torsion Machine
Torsion I Beam Specimens

Scale: 1" = 1'
Fig. 5 - Torsion Test on Girder T-2
Fig. 6 - Torsion Test on Girder T-6
TOP COVER PLATE OF GIRDER

SIDE OF GIRDER

LOCATION OF STRAIN GAGE LINES

FIG. 7
Torsion Test

Welded Girder With Diaphragms

File 12
Feb. 27, 1940

Fig. 8
**Torsion Test ~ Bolted Girder**

**All Diaphragms ~ Bolted on 3 Sides**

File 12
Feb. 28, 1940

\[ J = \frac{M_t}{Gt} = \frac{12,000}{12,000,000 \times (0.0082)} = 0.109 \text{ in}^4 \]

\[ J = 2.08 \text{ in}^4 \text{ (Computed)} \]
\[ J_{AB} = \frac{6000}{(12,000,000)(0.0037)} = 1.35 \text{ in}^4 \text{ (Initial Friction)} \]
\[ J_{EF} = \frac{2000}{(12,000,000)(0.0069)} = 0.282 \text{ in}^4 \]
\[ J_{FH} = \frac{7000}{(12,000,000)(0.00065)} = 0.900 \text{ in}^4 \text{ (Return Friction)} \]
Torsion Tests ~ Bolted Girder

Variation of Torsional Stiffness with Diaphragm Spacing

Fig. 11

Theoretical for no resistance to slipping in seams.
Fig. 13

TORSION TEST I BEAM
CURVES OF SHEAR STRESSES

WEB
T-4
T-2
T-3 & T-2
T-1

COVER PLATE

TORQUE IN KIP-IN.

STRESS IN KIPS/SQ. IN.
TORSION TEST T-6

MOMENT-ROTATION CURVES

Fig. 16
Torsion Test T-6

Moment Curves

Torque in Kip-In.

Direct Stress at Flap of Flange.

Theoretical.

Shear in Flange.

Shear in Web.

Stress in Diaphragm.

Slip in Inches

0 0.1 0.2 0.3 0.4

0 4 8 12 16

Stress in Kips/Sq. In.
Fig. 19

Torsion Test WT-7

Moments Curves

Torque in Kip-In.

16 Stress in Kips/Sq. In.

Rotation in Rad./In.
(a) APPLIED MOMENT

(b) SHEAR DISTRIBUTION
   I BEAM

FIG 20
SHEAR DISTRIBUTION
BOX GIRDER

(a)

(b)

FIG. 21