ELASTIC AND INELASTIC BEHAVIOR IN MODEL STEEL COLUMNS

by

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VIII. FIGURES
I. INTRODUCTION

The problem under consideration is the determination of the maximum loading a compression member can carry. This study is limited to the investigation of pin-ended square columns whose centroidal axes are the diagonals of the square. Model steel columns axially loaded in one series of tests, and eccentrically loaded in another series of tests were tested to failure. These experimental results are compared with analytical values computed by a virtual displacement method (5)(7).

II. THEORETICAL ANALYSIS

A large number of variables enter in the solution of this problem and to reduce these the following assumptions are made for this approximate solution:

1. The material under study possesses properties which are idealized as perfectly elastic up to the yield point and then perfectly plastic.
2. Members are originally straight, free from accidental eccentricities, and of uniform cross-section along their lengths.
3. Sections deform only in the plane of the applied moments and will deform in a sine configuration.
4. Sections plane before bending will remain plane after bending.

The behavior of a column in the elastic range is described by the equation, \( y'' = \frac{-P}{EI} (y + e) \). For a column bent
in single curvature the equation of center line deflection is:

\[ \Delta = \frac{e(1-\cos u)}{\cos u} \quad \text{where} \quad \frac{L}{2} \geq \frac{P}{EJ} \]

If no moment is present and only an axial compressive load is applied to the member no solution exists for the above equation. This is to say that the axial thrust, \( P \), for an axially loaded member, can be increased from zero to a critical value, \( P_{cr} \), with the member remaining straight. The slightest increase in load above \( P_{cr} \), however, will cause the column to deform laterally and then collapse. \( P_{cr} \), for the assumed material behavior, is either the yield load, \( P_y \), or the Euler buckling load, \( P_e \).

For the eccentrically loaded column a different type of behavior is observed. In this case the member starts to deflect on the slightest application of thrust. As the load is increased, the member will deform elastically until a load is reached at which yielding takes place. Further increase in load causes yielding to progress across and along the member reducing its resistance to further loading. A point is finally reached at which an increase in load is impossible and the member continues to deflect as the load remains constant. This is the critical load for the eccentrically loaded member.

The upper limit of column strength for the case where bending moments and axial thrusts are present is given by the Simple Plastic Theory. Axial load and moment are computed using a set of assumed plastic stress patterns. Figure 1 shows these values of axial load and moment plotted in the non-
dimensional form of \( \frac{P}{P_y} \) vs. \( \frac{M}{M_y} \).

The ultimate strength of an eccentrically loaded column may best be represented by an \( \frac{P}{P_y} - \frac{L}{r} \) curve. To obtain this curve, a virtual displacement method of determining the stability of beam columns in the elastic and inelastic ranges as described by R.L. Ketter\(^{(5)}\) in a recent paper, was used. To obtain the desired curve yield penetrations as shown in Figure 2 were assumed for the column section. The analysis was divided into two parts, Case A, where yielding only on the compression side of the specimen is considered, and Case B, where yielding has occurred on the tension side as well as the compression side of the specimen. The following relationships were used in this phase of the investigation:

\[
P = \int_A \sigma \, dA \\
M = \int_A \sigma y \, dA \\
\phi = \frac{\epsilon_1 - \epsilon_2}{h} = \frac{\sigma_y}{E_y}
\]

The curvature, \( \phi \), is a function of the strain developed at the section; the axial load, \( P \), and the moment, \( M \), are functions of stress.

Values obtained using the above relationships and the plastic stress patterns shown in Figure 2, are shown in non-dimensional form in Figures 3 and 4. From these curves another set of auxiliary curves (Figure 5) were plotted for \( \frac{M}{M_y} \) vs. \( \frac{\phi}{\phi_y} \) at constant \( \frac{P}{P_y} \). Figure 3 was entered at some value of \( \frac{P}{P_y} \) and values of \( \frac{\phi}{\phi_y} \) determined for each of the assumed plastic stress patterns. These values of \( \frac{\phi}{\phi_y} \) were used to enter Figure 4 to
obtain values for $\frac{M}{M_Y}$ for each of the assumed plastic stress patterns.

Next, two sets of curves (Figures 6 and 7) representing tangent values of the above auxiliary curves were plotted. These curves represent the internal bending stiffness $\frac{\Delta M}{\Delta \theta} \cdot \frac{\theta_Y}{M_Y}$ of a section at a given $\frac{P}{P_Y}$ ratio (Eq. 10, Ref. 5), and more or less correspond to the $EI$ value computed for elastic considerations. The tangent values are plotted as functions of the coordinates $\frac{M}{M_Y}$ and $\frac{\theta_y}{\theta_Y}$. Values of $\frac{\Delta M}{\Delta \theta} \cdot \frac{\theta_Y}{M_Y}$ were selected and corresponding values of $\frac{M}{M_Y}$ and $\frac{\theta_y}{\theta_Y}$ were read from Figures 6 and 7 thus computing $\frac{m_o}{M_Y}$. The equation $\frac{m_o}{M_Y} = \frac{\theta_o}{\theta_Y} = \frac{P}{P_e} \cdot \frac{\theta_Y}{\theta_Y}$ (Eq. 14, Ref. 5) relates the end moment, $m_o$, with the moment and unit rotation quantities at the most stressed center line section ($m_o$ and $\theta_o$) for a given value of internal stiffness $\frac{P}{P_e}$. At this point a family of curves relating $\frac{P}{P_Y}$ and $\frac{L}{r}$ may be computed (Figure 8) since the Euler load, $P_e$, is a function of the slenderness ratio $\frac{L}{r}$.

Since $\frac{m_o}{P_Y} = \frac{ec}{r^2}$ (the eccentricity ratio), a line drawn at a slope $\frac{ec}{r^2}$ proportional to $\frac{m_o}{P_Y}$ (see Figure 8) will intersect the $\frac{m_o}{M_Y}$, $\frac{P}{P_Y}$, $\frac{L}{r}$ curves at the critical load ratio $\frac{P}{P_Y}$. In this manner another family of curves is drawn showing critical load ratios for constant eccentricity ratios. These collapse curves are used to determine the critical load ratio $\frac{P_{cr}}{P_Y}$ for a given eccentricity and slenderness ratio.
III. DESCRIPTION OF EXPERIMENTAL PROGRAM AND TEST RESULTS

Figure A shows a view of the end fixtures before and after assembly. The principal load carrying pieces are made of high carbon steel while the rectangular bearing block is of mild steel. The circular insert is designed to accommodate a specimen of 3/4 inch by 3/4 inch cross-section (maximum size). Smaller sections are fitted by shims. A small pointer attached to the circular insert orients the specimen with respect to the plane of applied eccentricity. Eccentricity is applied by movement of the rocker with respect to the base of the end fixture. This movement is controlled by two knobs which are threaded to the rocker screw. The rocker screw axis is perpendicular to the axis of end rotation. In this study one series of tests was performed with no applied eccentricity while the other series of experiments was made using an \( \frac{\Delta C}{r^2} \) of one quarter. In both series of experiments the centroidal axis of the three quarter by three quarter inch columns was oriented at 45 degrees to the plane of applied eccentricity such that bending occurred about, or parallel to, the diagonal axis of the specimen.

Tension coupons and column specimens were annealed at 1140° F for one and one half hours at temperature and then cooled. Tension specimens were milled to one half such by three quarter inch size for two inch gage lengths. A Huggenberger strain gage set at one half inch gage length was used to determine elastic load deflections. A Peters strain gage was used to record the larger strains encountered beyond the yield point of
the material. A summary of the tension test results and the stress strain curves for the tension specimens is shown on Figure F.

Ten column specimens were chosen for their straightness and freedom from initial twist and were milled to length. A summary of the measurements for these specimens is shown in Tables 1 and 2.

In performing the column tests, the column specimens were first aligned using the alignment apparatus shown in Figure B. The circular wedge plates shown resting on the fixed cross-head of the 60,000 lb. hydraulic testing machine were leveled and then the specimen was geometrically aligned. Load was applied through a spherically seated block which was attached to the movable crosshead. Final alignment for a given eccentricity was made by checking the computed elastic curves and also by checking symmetry of bending. Computed elastic load deflection curves and experimental points are plotted in Figure G. The elastic load deflection curve for specimen E-5, where the eccentricity ratio $\frac{ec}{r^2}$ is 0.16, is shown in Figure H. Deflection readings are referred to the column axis and are independent of the end eccentricity. The adjustable frame used to support the Ames dial gages is shown in Figure C.

After final alignment was obtained for a specimen, load was applied in even increments so as to get about eight load deflection readings within the elastic range. This procedure of loading was continued until the critical load for the column was reached. After the column had buckled, the load was permitted to reach a state of equilibrium. The load was then held constant until the deflection reached equilibrium. This
process of unloading was continued until sufficient data was accumulated for an unloading curve for the specimen. Unloading points were obtained where possible. Sudden and complete collapse occurred at the critical load for specimens having $\frac{L}{r}$ ratios of 100 and 120, and an unloading curve was not obtained for these specimens. Load deflection curves for the axially loaded specimens appear in Figure J. Figure K shows load deflection curves for the eccentrically loaded columns. A summary of predicted collapse loads and actual experimental collapse loads is contained in Tables 1A and 2A.

To determine the effect of friction between the rocker on the end fixture and the bearing block, a loading and unloading cycle within the elastic range was performed for one specimen. The computed elastic curves, as well as the unloading curve for this specimen is shown in Figure H.

IV. DISCUSSION AND SUMMARY

The ten column tests, although far from conclusive, seem to indicate that the virtual displacement method for determining collapse loads for columns is slightly conservative. The largest deviation (5.8%) from a predicted collapse load was noted for the axially loaded specimen with an $\frac{L}{r}$ ratio of 120. It should be noted that, because of some unavoidable initial crookedness in the specimens, it was very difficult to approach a true axial load condition. The eccentrically loaded specimens showed a deviation of about 4% above predicted collapse load. Elastic deformations for these specimens were symmetric and
checked computed deflections reasonably well. Friction between the end fixture and the bearing plate is considered negligible. Eccentricities were set within the accuracy of the end fixtures. When the variation of material properties from specimen to specimen, as compared to the average value used for these material properties is considered, the experimental results can be said to check the theoretical results with reasonable accuracy.
V. ACKNOWLEDGMENTS

The author wishes to express his sincere appreciation to R.L. Ketter for his guidance and counsel throughout the progress of this study. Mr. Ketter is also responsible for the design of the end fixtures.

This project has been carried out at the Fritz Engineering Laboratory as a course in structural research with funds provided by the Institute of Research. The program was under the direction of R.L. Ketter.
**VI. NOMENCLATURE**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Area of cross-section.</td>
</tr>
<tr>
<td>c</td>
<td>Extreme fiber distance from the centroid of a section.</td>
</tr>
<tr>
<td>e</td>
<td>End eccentricity of axial load application.</td>
</tr>
<tr>
<td>E</td>
<td>Young's modulus of elasticity.</td>
</tr>
<tr>
<td>I</td>
<td>Moment of inertia of total cross-section about the principal axis of the section under consideration.</td>
</tr>
<tr>
<td>L</td>
<td>Length of test specimen.</td>
</tr>
<tr>
<td>M</td>
<td>Bending moment.</td>
</tr>
<tr>
<td>M₀</td>
<td>Bending moment at center line section of member.</td>
</tr>
<tr>
<td>m₀</td>
<td>That part of center line moment which is independent of deflection.</td>
</tr>
<tr>
<td>M₀y</td>
<td>Bending moment at which yield is first reached in fixture.</td>
</tr>
<tr>
<td>P</td>
<td>Axial load.</td>
</tr>
<tr>
<td>Pᶜᵣ</td>
<td>Maximum load a member can carry.</td>
</tr>
<tr>
<td>Pₑ</td>
<td>Euler buckling load.</td>
</tr>
<tr>
<td>Pₑ₁</td>
<td>That load beyond which a member deforms inelastically.</td>
</tr>
<tr>
<td>Pᵧ</td>
<td>Axial load corresponding to yield stress level over entire section.</td>
</tr>
<tr>
<td>r</td>
<td>Radius of gyration of a section.</td>
</tr>
<tr>
<td>y</td>
<td>Deflection in the plane of moment application.</td>
</tr>
<tr>
<td>y₀</td>
<td>Center line deflection.</td>
</tr>
<tr>
<td>y''</td>
<td>Second derivative of deflection with respect to longitudinal column axis.</td>
</tr>
<tr>
<td>Δ</td>
<td>Center line deflection.</td>
</tr>
<tr>
<td>σ</td>
<td>Unit normal stress.</td>
</tr>
<tr>
<td>σᵧ</td>
<td>Lower yield point stress.</td>
</tr>
<tr>
<td>ø</td>
<td>Curvature</td>
</tr>
<tr>
<td>øᵧ</td>
<td>Curvature at initial yield.</td>
</tr>
</tbody>
</table>
VII. REFERENCES


Table 1 - Column Specimens (Axially Loaded)

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Length (in)</th>
<th>Avg. Area (in²)</th>
<th>( \ell / r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-1</td>
<td>8.75</td>
<td>0.565</td>
<td>40.5</td>
</tr>
<tr>
<td>A-2</td>
<td>13.00</td>
<td>0.563</td>
<td>60.0</td>
</tr>
<tr>
<td>A-3</td>
<td>17.16</td>
<td>0.562</td>
<td>79.3</td>
</tr>
<tr>
<td>A-4</td>
<td>21.38</td>
<td>0.562</td>
<td>98.7</td>
</tr>
<tr>
<td>A-5</td>
<td>26.00</td>
<td>0.563</td>
<td>120.1</td>
</tr>
</tbody>
</table>

Table 1-A - Summary of Predicted & Actual Column (Axially Loaded) Collapse Loads

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>( P_c ) (Calculated) (lbs)</th>
<th>( P_c ) (Calculated) (lbs)</th>
<th>( P_c ) (exper.) (lbs)</th>
<th>% Deviation from Predicted ( P_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-1</td>
<td>18,070*</td>
<td>112,100</td>
<td>17,500</td>
<td>-3.2</td>
</tr>
<tr>
<td>A-2</td>
<td>18,030*</td>
<td>45,400</td>
<td>18,100</td>
<td>+0.4</td>
</tr>
<tr>
<td>A-3</td>
<td>17,980*</td>
<td>25,890</td>
<td>18,000</td>
<td>+0.1</td>
</tr>
<tr>
<td>A-4</td>
<td>18,000</td>
<td>16,770*</td>
<td>16,700</td>
<td>-0.4</td>
</tr>
<tr>
<td>A-5</td>
<td>18,030</td>
<td>11,340*</td>
<td>12,000</td>
<td>+5.8</td>
</tr>
</tbody>
</table>

* Predicted Failure Load

Table 2 - Column Specimens (Eccentrically Loaded)

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Length (in)</th>
<th>Avg. Area (in²)</th>
<th>( \ell / r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-1</td>
<td>8.75</td>
<td>0.563</td>
<td>40.4</td>
</tr>
<tr>
<td>E-2</td>
<td>13.00</td>
<td>0.564</td>
<td>60.0</td>
</tr>
<tr>
<td>E-3</td>
<td>17.22</td>
<td>0.563</td>
<td>79.5</td>
</tr>
<tr>
<td>E-4</td>
<td>21.45</td>
<td>0.564</td>
<td>99.1</td>
</tr>
<tr>
<td>E-5</td>
<td>25.98</td>
<td>0.562</td>
<td>120.0</td>
</tr>
</tbody>
</table>

Out of Straight Measurements

Readings referred to "X" number stamped on specimen as shown.

<table>
<thead>
<tr>
<th>Spec</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>+0.005</td>
<td>+0.0005</td>
<td>0</td>
<td>+0.001</td>
<td>+0.002</td>
</tr>
<tr>
<td>B</td>
<td>-0.003</td>
<td>-0.002</td>
<td>0</td>
<td>-0.001</td>
<td>-0.002</td>
</tr>
<tr>
<td>E-2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>+0.001</td>
<td>+0.0005</td>
<td>0</td>
<td>-0.001</td>
<td>-0.001</td>
</tr>
<tr>
<td>B</td>
<td>+0.0005</td>
<td>+0.001</td>
<td>0</td>
<td>-0.0005</td>
<td>-0.0015</td>
</tr>
<tr>
<td>E-3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>-0.005</td>
<td>-0.004</td>
<td>0</td>
<td>+0.001</td>
<td>-0.006</td>
</tr>
<tr>
<td>B</td>
<td>+0.002</td>
<td>+0.001</td>
<td>0</td>
<td>+0.003</td>
<td>+0.006</td>
</tr>
<tr>
<td>E-4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>+0.005</td>
<td>+0.0035</td>
<td>0</td>
<td>+0.003</td>
<td>+0.008</td>
</tr>
<tr>
<td>B</td>
<td>-0.005</td>
<td>+0.003</td>
<td>0</td>
<td>+0.001</td>
<td>+0.0005</td>
</tr>
<tr>
<td>E-5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>-0.004</td>
<td>-0.0005</td>
<td>0</td>
<td>+0.003</td>
<td>+0.002</td>
</tr>
<tr>
<td>B</td>
<td>+0.001</td>
<td>+0.010</td>
<td>0</td>
<td>-0.0005</td>
<td>-0.002</td>
</tr>
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</table>
Table 2-A - Summary of Predicted & Actual Column (Eccentrically Loaded) Collapse Loads

<table>
<thead>
<tr>
<th>Specimen No</th>
<th>$P_{el}$ (Calc.) (lbs)</th>
<th>$P_e$ (Calc.) (lbs)</th>
<th>$P_y$ (Calc.) (lbs)</th>
<th>$P_{cr}$ (Calc.) (lbs)</th>
<th>$P_{cr}$ (Experr.) (lbs)</th>
<th>% Deviation from Predicted $P_{cr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-1</td>
<td>13,050</td>
<td>100,070</td>
<td>18,020</td>
<td>16,200</td>
<td>15,500</td>
<td>-4.3</td>
</tr>
<tr>
<td>E-2</td>
<td>11,200</td>
<td>45,390</td>
<td>18,050</td>
<td>15,120</td>
<td>15,750</td>
<td>+4.2</td>
</tr>
<tr>
<td>E-3</td>
<td>8,650</td>
<td>25,900</td>
<td>18,020</td>
<td>13,410</td>
<td>14,000</td>
<td>+4.4</td>
</tr>
<tr>
<td>E-4</td>
<td>6,400</td>
<td>16,670</td>
<td>18,050</td>
<td>11,250</td>
<td>11,750</td>
<td>+4.4</td>
</tr>
<tr>
<td>E-5</td>
<td>6,070</td>
<td>11,330</td>
<td>18,000</td>
<td>9,800</td>
<td>9,950</td>
<td>+1.6</td>
</tr>
</tbody>
</table>

$$P_{el} = \frac{\sigma_y A}{1 + \frac{\epsilon C}{r^2} \sec \frac{1}{2} \sqrt{\frac{P}{E I}}}$$

**Note:**

Average values of $\sigma_y$ & $E$

From Tension Coupons used in calculations.

$\sigma_y = 32,000$ psi

$E = 29.4 \times 10^6$ psi

$$P_e = \frac{\pi^2 E A}{(L)^2}$$

$$P_y = \sigma_y A$$

$P_{cr}$ (Calc.) - Obtained from Fig. 8 using $\frac{\epsilon C}{r^2} = .25$ except for Specimen E-5 where $\frac{\epsilon C}{r^2} = .16$. 
Stress-Strain Curves for Tension Coupons

Fig. F

Tension Coupon A-1

\[ A = 0.371 \text{ in}^2 \]
\[ P_{ult} = 21.35 \text{ kips} \]
\[ \epsilon_{max} = 57.5 \% \]
\[ \epsilon_y = 32.0 \% \]
\[ E = 29.4 \times 10^6 \text{ psi} \]

Tension Coupon A-2

\[ A = 0.374 \text{ in}^2 \]
\[ P_{ult} = 21.4 \text{ kips} \]
\[ \epsilon_{max} = 57.2 \% \]
\[ \epsilon_y = 32.5 \% \]
\[ E = 29.0 \times 10^6 \text{ psi} \]

Tension Coupon A-3

\[ A = 0.374 \text{ in}^2 \]
\[ P_{ult} = 21.2 \text{ kips} \]
\[ \epsilon_{max} = 56.7 \% \]
\[ \epsilon_y = 31.9 \% \]
\[ E = 29.4 \times 10^6 \text{ psi} \]

Tension Coupon A-4

\[ A = 0.369 \text{ in}^2 \]
\[ P_{ult} = 20.8 \text{ kips} \]
\[ \epsilon_{max} = 56.4 \% \]
\[ \epsilon_y = 31.7 \% \]
\[ E = 27.4 \times 10^6 \text{ psi} \]

Tension Coupon A-5

\[ A = 0.373 \text{ in}^2 \]
\[ P_{ult} = 21.5 \text{ kips} \]
\[ \epsilon_{max} = 57.6 \% \]
\[ \epsilon_y = 32.0 \% \]
\[ E = 29.7 \times 10^6 \text{ psi} \]
Fig. G

Elastic Load Deflection Curves

$$\Delta = \frac{e(1-\cos u)}{\cos u} \quad \text{where} \quad u = \frac{L}{2} \sqrt{\frac{P}{EI}}$$

- Experimental Points

\[ P = \text{Load (Kips)} \]

\[ \Delta = \text{Deflection (Inch)} \]
Elastic Load Deflection Curve \((\frac{eC}{P} = 0.16)\)

\[ \Delta = \frac{e(1 - \cos u)}{\cos u} \quad \text{where} \quad u = \frac{1}{2} \sqrt{\frac{P}{E}} \]

- Experimental Points

**Figure H**

Loading & Unloading Curve to Show Effect of Friction at Bearing Surfaces

Specimen E-4 - \(f = 99.1\)

- Elastic Load Deflection Curve
- Loading Curve
- Unloading Curve
Fig. J
Load Deflection Curves for Axially Loaded Specimens

\[ \frac{L}{r} = 40.5 \]
\[ \frac{L}{r} = 60.0 \]
\[ \frac{L}{r} = 79.4 \]
\[ \frac{L}{r} = 98.7 \]
\[ \frac{L}{r} = 120.1 \]

\( P = \text{Load (Kips)} \)

\( \Delta = \text{Deflection (Inch)} \)
Fig. K

Load Deflection Curves for Eccentrically Loaded Specimens

\( \frac{E}{r^2} = 25 \) for Specimens E-1, E-2, E-3 & E-4

\( \frac{E}{r^2} = 16 \) for Specimen E-5

Axis of bending

\( p_{n}, p_{e} \)

\( m_{n}, p_{e} \)

\( L \)

\( e \)

\( \Delta \) - Deflection (Inch)

Spec. E-1

\( \frac{L}{F} = 40.4 \)

Spec. E-2

\( \frac{L}{F} = 60.0 \)

Spec. E-3

\( \frac{L}{F} = 79.5 \)

Spec. E-4

\( \frac{L}{F} = 99.0 \)

Spec. E-5

\( \frac{L}{F} = 120.0 \)
Fig. 1
Simple Plastic Theory

\[ P = \int \sigma_y dA \]
\[ M = \int \sigma_y y dA \]
Stress Patterns at Assumed Yield Penetrations

Fig. 2
Fig. 3
Fig. 5

Max \( \frac{M}{M_y} = 2.0 \sqrt{\frac{P}{P_y}} \)
(Simple Plastic Theory)

\( \frac{\Phi}{\Phi_y} \)
Fig. 7
Fig. 9

Experimental Values for Axially Loaded Specimens

Experimental Values for Eccentrically Loaded Specimens