FRITZ ENGINEERING LABORATORY
LEHIGH UNIVERSITY
REPORT
Notes on Behavior of "I" and "W" Beams in Shear
TO
BY
C.N. Yang
Table of Contents

Section

Beam in Elastic Range

Shear Strength of T Section

Theoretical Analysis of Beam Behavior After It Has Been Yielded by Shear

An Analysis of Shearing Stress in Beams After Its Flanges Yield by Bending

Shear Problems on 8 WF 40 Beams

Yielding Proceeding of the Shear

Outline on Shear Problems in Beams From 4" I Section Beams Test Results

Shear on 7" Beam Test
Start on a beam in elastic range.

\[
2\int_{0}^{l} f_{s} \, db \, dl + 2m_{1} = (1) \\
2\int_{0}^{l} f_{s} \, y_{0} \, db \, dl + m_{2} = (2)
\]

\[
f_{s} = \frac{VA}{bI} \\
2\int_{0}^{l} f_{s} \, db \, dl + 2m_{1} = \frac{2MQD}{I} + 2m_{1}
\]

\[
2\int_{0}^{l} f_{s} \, y_{0} \, db \, dl + m_{2} = \frac{2MQy_{0}}{I} + m_{2}
\]

(1)+(2) = \frac{2MQ}{I} (D+y_{0}) + 2m_{1} + m_{2}

It is very apparent that the diagram shown above:

\[
\frac{2MQ(D+y_{0})}{I} \text{ equals the moment }
\]

represented by \( \text{ABCD} \).
\[ \Delta M_1 \text{ represent the moment of area } \triangle C' \triangle B' \text{ (times two)} \]
\[ M_2 = \ldots \quad \text{C'D'Ef} \]

The summation of above terms of course represents by areas \( \triangle A'B'HK \), i.e., the moment \( M \) at the action.

This checks the method of analysis is correct, and we can assume the radius of curvature of the action: \[ p = \frac{M}{E} \] and prove it analytically instead of using the diagrams.
\[ I = \frac{b h^3}{12} \]

\[ Q = \frac{(h-y^2)b}{2} \]

\[ \sigma_s = \frac{V \left( \frac{h^2}{2} \right)}{b h^3} \]

\[ \sigma_s = \frac{6V \left( \frac{h^2}{2} \right)}{b h^3} \]

\[ y = 0 \quad \sigma_s = \sigma_{y.p} = \frac{6V}{b h^3} = \frac{6V}{b h^3} \]

\[ V = \frac{\sigma_{y.p} b h}{6} \]

\[ \sigma_s = \sigma_{y.p} \left( 1 - \frac{y^2}{h^2} \right) \]

From Equation (11)

\[ 2\int_{0}^{b} f_s \, db \, dl + 2m = \text{(A)} \]
Where \( f_s = y_0 (1 - \frac{y_0^2}{k^2}) \)

\( f_s = \frac{y_0^2}{k^2} \)

\[ \therefore 2 \int_0^l f_s b d l = 2m = 2 \left( \frac{y_0^2}{k^2} - b d l + m_1 \right) \]

For part (2)

\[ \therefore 2 \int_0^l f_s y_0 b d l + m_2 = \frac{20 y_0 y_0^3}{k^2} b d y_0 + m_2 \]

\( = 2 \frac{y_0 y_0^3}{k^2} b d l + m_2 \]  \( \therefore \)  \( \text{ shear distribution} \)

\[ b \int_0^l y_0 d l = \frac{20 y_0^2}{k^2} \int_0^l f_s b d l + \int_0^l \left( f_s + \frac{df_s}{dy} \right) b d l \]

\( = \frac{2 y_0 ^2 y_0 b d l}{k^2} \]

\[ \therefore \sigma_N = \frac{2 y_0 ^2 y_0 b d l}{k^2} \]
\[ \mathbf{m}_2 = \bar{b} \int \frac{y \, dy}{y_0} \int_0^{y_0} 20y \, dA \, y \, dy = \frac{20 \, y \, p \, d \, l}{k^2} \int_0^{y_0} y^2 \, dy \]
\[ = \frac{20 \, y \, p \, d \, l}{k^2} \cdot \frac{y_0^3}{12} \]
\[ = -\frac{4 \, y \, p \, d \, l}{k^2} \cdot \frac{y_0^3}{12} \]
\[ m_2 = -\frac{8 \, y \, p \, d \, l \, y_0^3}{3k^2} \]
\[ m_2 = -\frac{8 \, y \, p \, d \, l \, y_0^3}{3k^2} \]

\[ (\mathbf{A} \cdot \mathbf{B}) = \mathbf{M} = \mathbf{V} \cdot \mathbf{E} \]

\[ \frac{d \sigma}{d t} = \frac{1}{2} \frac{d \sigma}{d y} = \frac{1}{2} \frac{d \sigma}{d y} \]

\[ \frac{d \sigma}{d y} = 20 \, y \, p \, y_0 \, l / k^2 \]
Suppose \( m_1 = v_1 e \)

\[
\sigma_3 = \frac{6 v_1 \left( D^2 - y^2 \right)}{b D^3}
\]

\[
\frac{d\sigma_5}{dy} = -\frac{12 v_1 y}{b D^3}
\]

\[
\left( \frac{d\sigma_5}{dy} \right)_b = -\frac{6 v_1}{b D^2}
\]

\[
\frac{b D^2}{6 v_1} = \frac{2 \sigma_{y.p.} y_0}{k^2}
\]

\[
v_l = \frac{b k^2 D^2}{12 \sigma_{y.p.} y_0}
\]

\[
M = v_l = \left( A_1 + B \right)
\]

\[
v_l = 2 \left( \frac{\sigma_{y.p.} y_0^2 D b k^2}{k^2} + \frac{b k^2 D^2}{12 \sigma_{y.p.} y_0} \right)
\]

\[
+ 2 \frac{\sigma_{y.p.} y_0^2 b k^2}{12} \phi - \frac{8 \sigma_{y.p.} b k y^3}{3 k^2}
\]

\[
(\text{c})
\]
From equation we know

\[ D = \frac{h - 240}{2} \]

If we know \( V \) we can solve the equation of third degree to solve for \( y_0 \).

By use of \( y_0 \) we'll have the equation of deflection curve by using of equation 1c',

\[ \frac{d^2y}{dx^2} = \frac{h_1}{E I} = \frac{V d}{E I} \]
1. Discussion on I-Beams.

A solve the web part as discussed above.

b. Before the shearing stress getting too high in flanges it can be regarded as separate beams.

2. Discussion on end condition.

Two separate beams rigidly connected at both ends under concentric load. The solution should be one equivalent to a rigid beam.
Shear strength of "I" sections

\[ T_s = \frac{VQ}{2bI} \]

\[ \sigma = \frac{Mc}{I} = \frac{VxLxQ}{I} \quad e = \frac{P}{2} \]

Suppose we want the same bending strength

\[ V = 2kI \quad \text{where} \quad k = \frac{T_s}{IP} \]

\[ \therefore T_s = k \frac{Q}{h_b} \]

Therefore the shear stress is linearly proportion to ratio \( \frac{Q}{h_b} \).
\[ 8 \text{ w} = 40 \]

\[ Q = 41.05 \]

\[ b = 3711 \]

\[ \frac{Q}{b} = \frac{110.5}{8} = 13.8 \]

14 wF30

\[ Q = 0.383 \times 6.733 \times \frac{13.86}{2} + \left( \frac{13.86 - 383}{2} \right) \times \frac{1}{2} \times 27.0 \]

\[ Q = 23586 + 35.8 + 11.5 = 473 \]

\[ b = 0.27 \]

\[ \frac{Q}{b} = \frac{473}{14} = 33.7 \approx 12.5 \]

It is very obvious that 14 wF30 is by much lower shearing strength than that of 8 wF40.
1. Shear strength of "I" beams.
2. Plastic behaviors of beams due to shear failure.
3. Contribution of beam deflection due to shear failure.
4. Shear failure and bending strength of beams.

Some sections above under same moment,

Calculate the relation of bending strength.

5. Consideration of shear failure in plastic design.

a. Relentless. Only happens in changing shape of the section. Rectangular could be considered equivalent to determine structure in bending.

b. Normal "I" shape, the contribution of deflection will be enormous before the section reaches its developed its reluctant shape.

c. Bending strength would be more affected as the shear failure spreads through the web.

d. It seems, shear should be limited below its critical yield strength. The web thickness should be increased in the conventional sections.

In case of non-uniform shear, the problem of changes to a problem of plasticity.
(i.e. Plastic Plastic shear) Assumptions made in coarse plastic shear analysis are not adequate. When only the max. shear reaches the shear yield strength, of course, plastic strength and depth will not be as much affected as in the case of coarse shear.

5. Procedure of shear plastic flow stress hardening in webs of "I" beams.
Theoretical analysis of beam behavior after it has been yielded by shear.
Along AB & CD there is no normal stress inside so after load applied point like E, F do not shift these places.

Assume after loading planes like AE & FK still transform to planes as G E F H. (Fully shearing rigid in elastic region)

The normal stress distribution of course in the same proportion at line GE & FH
From the previous proof we know in the shearing failure region normal stress equal to zero.

General assumptions are made for the solution of the problem.

a. Shear deformation in the elastic region is zero. (Plane remains plane)

\[ \frac{1}{K} = \frac{d^2y}{dx^2} \]  
geometric approximation

Stress distribution diagram under previous assumption.
\[ \sum M = 0 \]

\[ M = 2 \int_{y_0}^{R} \frac{P}{R} \, dy \]

\[ M = \alpha \int_{y_0}^{R} \frac{P}{R} (y - y_0) \varepsilon_y \, dy \]

\[ \left( \frac{\varepsilon_0}{R} = \frac{\varepsilon_y}{y - y_0} \right) \]

When \( \varepsilon_0 = \) fiber strain,

\( \varepsilon_0 = \) fiber stress = \( \sigma_0 \)

\[ M = \frac{4}{3} \int_{y_0}^{R} (y - y_0) \sigma_0 \, dy \]

\[ M = \alpha \left( \frac{I_2 T_0}{R^2} - \frac{\sigma_0 R}{R} \right) = \frac{20}{R} (I_2 - y_0 z_1) \]

Where \( I_2 = \) moment of inertia of area about N.A.

\( z_1 = \) moment of plastic area about N.A.
\[ R = \frac{\frac{E}{h}}{y - y_0} = \frac{J_0}{(h-y_0)E} \quad \text{where} \quad n = \frac{h}{y_0} \]

\[ J_0 = \frac{hM}{(I_{21} - y_0 z)} \]

\[ R = hM \quad \frac{1}{(I_{21} - y_0 z) E (n - y_0)} \]

In case of constant shear:

\[ I_{21}, y_0, z \text{ all constant} \]

Uniform load:

\[ I_{21}, y_0, z \text{ are functions of } x \]

\[ \frac{d^2 y}{dx^2} = \frac{hM}{(h-y_0)(I_{21} - y_0 z)E} \quad \text{where} \quad M = f(x) \]

Solve this equation for general deflection curve.
The distribution of shearing stress

\[ \sigma = \frac{fm}{I_2 - y_0^2} \]

\[ \frac{d\sigma}{dx} = \frac{W_1}{I_2 - y_0^2} \frac{dm}{dx} \quad \text{Suppose} \quad \frac{dm}{dx} = W \]

\[ b \sigma_{sy} \rho = \int_{y_0}^{h} \frac{h}{I_2 - y_0^2} \frac{dm}{dx} \, dA \quad \text{In constant shear} \]

\[ b \sigma_{sy} \rho = \int_{y_0}^{h} \frac{h}{I_2 - y_0^2} \frac{dm}{dx} \, dA = \int_{y_0}^{h} \left( \frac{hW}{I_2 - y_0^2} \right) \, dA = \frac{hW A_0}{I_2 - y_0^2} \]

The only unknown in above eq. is:

\[ y_0 \]

\[ y_0 = -\frac{hW A_0 + b \sigma_{sy} \rho \, I_2}{2 \, b \sigma_{sy} \rho} \]
Boundary Condition Discussion

1. Simply supported.

This can be considered in general case
the moment at the boundary is zero.

2. Fully restrained.

\[ \frac{d^2 y}{dx^2} = -\frac{F_m}{(I_2 - y_0^2)E} (y - y_0) \]

Solve the above eq. put \( \frac{dy}{dx} = 0 \) at

\[ x = 0 \quad \text{and} \quad x = l \]
3. Shear Restrains Boundary

Permanent Elongation = \( \delta_0 \) at \( V \) along

in plastic range

\[
R = \frac{hwx}{SDE} \\
\text{put } (2\frac{h}{2} - V_0) = S \\
(h - y) = 0
\]

\[
\delta_0 = \int_0^L \varepsilon \, dx \quad \varepsilon = R(V - V_0)
\]

\[
\delta_0 = \int_0^L R(V - V_0) \, dx
\]

\[
\delta_0 = \frac{L^2 hwx}{2SDE} (V - V_0)
\]

Suppose the slope at \( x = 0 \) equals \( \delta_0 \).
Elongation \( V = \delta = V_0 \)

Stress \( \sigma = \frac{6E}{\varepsilon} \)

At the elastic range \( \delta_0 = \)

\[
\sigma = \frac{6E}{\varepsilon} = \frac{V'\delta_0}{\varepsilon}
\]

\[
\int_0^\delta \sigma \, dA = 0
\]

\[
2 \int_0^\delta \frac{V'(6-6\delta)}{2} \varepsilon \, dA + \int_0^\delta \frac{V'\delta}{2} \varepsilon \, dA = 0 \quad (\text{A})
\]

\[
2 \int_0^\delta \frac{V_0^2}{2} \varepsilon \, dA + \int_0^\delta \frac{V_0}{2} \varepsilon \, dA = \int_0^\delta V \frac{\varepsilon^2}{2} \, dA
\]

\[
\frac{I_2}{2} + \frac{I_0}{2} = \frac{\varepsilon^2 \varepsilon \varepsilon}{2} (I_{21} - V_0 Z_1)
\]

\[
I_0 = \frac{\varepsilon^3 (I_{21} - V_0 Z_1)}{2 (I_{21} + I_0) (h - V_0) (I_{21} - V_0 Z_1)}
\]

\[
\sigma_0 = \frac{\varepsilon^3 h \varepsilon}{2 (I_{21} + I_0) (h - V_0) E}
\]
\( d_0 \) is determined when \( W, \alpha, \lambda, \) and cross-sectional area of the beam are given.

\[
\frac{d^2 y}{dx^2} = -\frac{h m}{E (h-y)(h-y_2)}
\]

\[
\left. \frac{dy}{dx} \right|_{x=0} = d_0
\]

Equation (A) can be written as

\[
2 \int y_b \delta y_b \frac{dV}{e} dA + \delta \int y' \delta E \frac{dV}{e} dA = M
\]

For any fixed end of moment elastically restrained support.

Shear strain hardening

It is determined by \( \left| \frac{dy}{dx} \right|_{\text{max}} \).
Discussion on I section

1. Make the solution up to the point yielding penetrated to all the web.

2. Consider the two flanges separate beams for additional loads.

3. Boundary condition No. 3 can be solved just in the same way illustrated.

Example in 8 WF 40 restrained, ultimate shear strength.

![Diagram of a beam with a load W applied at one end]
Conclusions from above analysis

1. Bending failure in beams changes the shearing stress magnitude and its distribution.

2. Shear failure in beams changes the normal stress distribution and deflection curves.

3. Max. shear is strength of a member is usually represented by

\[ V = \sigma_s A \]

But actually beams under high shear are weakened first by shear failure than captured by bending.

4. Item 4 & 2 are solved under general beam assumptions for flexural formula.
The shear action near the support local yielded by shear at a very low cost.

The tensile strength is more significantly affected by shear at these sections.
Failed by bending

Failed by shear

End condition discussion

Discussion about resisting moment over the end
More strain should be observed than the central, cons. moment, section
An analysis of shearing stress in beams after the flanges slide by bending.

Jan.
May 3 / 1949
The flanges in section "A" are supposed to be in strain hardening region and the flanges in section "B" are in plastic region but before strain hardening. Everything in section C is in elastic condition.

We want to find the max. shearing stress.
Before strain hardening

\[ f_s \, b \, dx = \int_y^y \, dH \]

\[ \int_y^y \, dH = \frac{b}{2} \, \gamma_{p.p} \, y_1 + \gamma_{p.p} \, b \, (y_2 - y_1) - \frac{1}{2} \, O_{p.p} \, y_2 \]

\[ = \frac{b \, \gamma_{p.p}}{2} \, (y_2 - y_1) \]

Now suppose \[ y_2 = y_1 + \frac{dy}{dx} \, dx \]

\[ f_s = \frac{\gamma_{p.p}}{2} \frac{dy}{dx} \]

\[ \frac{dy}{dx} = f(V, \text{shape of section}) \]

\[ \Delta x \] is of course very difficult to solve.

\[ y = f(x) \] in irregular sections

... but in rectangular \[ \frac{dy}{dx} \] is possible.
b. After strain hardening

Actually before \( \frac{dH}{dx} \) gets too high, in generally structural material sections, the fiber would get strain hardening.

\[
grow(H) = \frac{dH}{dx} \\
\]

It is clear that after strain hardening the plastic region inside of the beam takes shearing stress again. Thus analysis though not impossible it would be too complicated.

I.e. especially I-sections the max shear stress would happen at the center of the web in the section flanges are in plastic range but before strain hardening.
\[ M = 0_y \cdot p \cdot 2 \cdot l + 2b (R_0 - y) S_y \cdot p \cdot x \left[ y + \frac{R_0 - y}{2} \right] \]

\[ + 2 \times \frac{S_y \cdot p \cdot y \times b}{2} \frac{2}{3} y \]

\[ \frac{dM}{dx} = -V = -\frac{2}{3} b \sigma_y \cdot k_y \cdot dy \cdot \frac{dy}{dx} \]

\[ \therefore \frac{dy}{dx} = \frac{3V}{2yb \sigma_y \cdot p} \]

Suppose \( \frac{P}{y_0} = 15 = k \]

(Plastic strain = 15 \times Elastic strain before strain hardening)

\[ \left| \frac{dy}{dx} \right| = \frac{3V \times 15}{2R b \sigma_y \cdot p} = \frac{3kV}{2R b \sigma_y \cdot p} \]

\[ f_5 = \frac{S_y \cdot p \cdot 3 \times 15}{2 \times R b \sigma_y \cdot p} = \frac{45V}{4 \times R b} \text{ or } f_5 = \frac{3kV}{4 \times R b} \]
It shows after the flanges are yielded the max. shear stress equal to

\[ f_s = 0.75 k \frac{V}{hb} \]

Where \[ K = \frac{\text{plastic strain}}{\text{elastic strain}} \]

\[ f_s \text{ function of } h, b, V, \text{ only} \]

Not same thing for any kind of section or rectangular section.
1. Define the symbols:

Suppose: \( A = \text{Area of flange} \)

\[ \frac{f_s}{I_b} = \frac{V \times A_c}{A_c^2 b} \]

\[ \sigma = \frac{M_c}{I} = \frac{KVc}{A_c^2} \]

\[ V = KAC \]

\[ f_s = K \frac{A_c^2}{b A_c^2} = KA \]

It is clear shear yielding is more significant in wide flange sections.

2. Deflection of the beams after the web shear to plastic range seems could be analyzed by the proposed method.

3. The yielding strength and the ultimate strength of beams by bending are apparently raised while shear yield strength is just about the same. It may mean that...
Calc.

Shear Problems

8 WF 40 Beams
Octetoidal Shear Hypothesis

\[ \text{Typ.} = \text{Typ.} \times \frac{1}{N_3} = 39.5 \times \frac{1}{N_3} = 0.395 \text{ kip/ft} \]

\[ W = \frac{\text{Typ.} \times b \times t}{Q} \]

39.5 kip/ft = yield stress of web material

0.391

0.375

I = 143.2

0 = 41.05/2

\[ W = \frac{143.2 \times 37.1 \times 22.8 \times 2}{41.05} = 59.6 \text{ kips.} \]

Initial Yielding load at support = 34.7 k.

Center span = 52.5 k.

Ultimate = 55 k.

\[ W = \frac{34.8 \times 37.1 \times 22.8 \times 2}{41.05} = 52.5 \text{ kips.} \]
Test B.

The road is far from shear yielding line at corners stress concentration is also an factor.

Test B_2

1. We found shear yield lines at a load of 44,000 k. This is due to the residual stress on the rolling section that made the beam web shears earlier than expected.

2. The stress strain relation checks out pretty close i.e. T1 & T2

At load 28 k

\[
T_1 = 1.14 \times 10^7 \times 830 \times 10^{-6} \quad (S = 21, D = 21)
\]

\[
T_1 = 9.5 \text{ K/ft}
\]

\[
T_2 = \frac{28 \times 44.05/2}{143.2 \times 0.371} = 10.5 \text{ K/ft}
\]

3. General shear flow lines are seen at load 47 K > 50 K.
No vertical shear yielding lines seen. This may be due to the shear strength of the web along these two directions are not the same.
Yielding process of the shear web.

The shearing stress in an I section may be distributed as above. But we know

$$\sum_{A} f_s \, dA = F_s$$

Where $F_s$ = Shearing force

i.e. When there is stress concentration in the flanges, the max. shearing stress at the center would be smaller than calculated conventionally.

Due to the stress concentration at flanges (Tens. Pa. give the stress patter for WF section as below) it wouldn't make too much error by assuming the shearing stress distributed uniformly in the web to simplify the method of analysis.

Actually in the web it is just the same way to proceed its yielding as we discussed before.
Suppose the shaded area are yielded by shear at Fig. A. Then the additional shear load will make the section act like two separate beams; its shear stress will be distributed as Fig. B.

Combine the shear stress curve AB and CD. It is clear that the additional load will bring the yielding region from B to C.

The yield by shear is thus proceeded.
Failure of the short high shear beam.

The ultimate shearing strength is

\[ W = A \cdot f_{\text{yp}} \]

Where \( A \) = cross-sectional area

But before the beam develops its full ultimate ultimate strength \( f_{\text{yp}} \) (Shear) it may fail by bending in the following way.

Suppose \( W_0 \) is the load to make the web of the beam yield by shear.

Suppose the fiber stress of the beam at the root still can be calculated by formula

\[ f_1 = \frac{M_c}{I} = \frac{W_0 x f_{\text{yp}}}{I} \]

Now we increase the load \( \Delta W \) and \( \Delta W + W_0 < f_{\text{yp}} A \)

The additional fiber stress would be

\[ f_2 = \frac{\Delta W x f_{\text{yp}}}{2I} \]
Where \( c' = \frac{1}{2} \) thickness of flange

\[ I' = \text{Moment of inertia of the flange about its own centroid axis} \]

\( f_1 + f_2 \) may exceed \( f_y \), before the web get shear strain hardened.

This is of course a function of the shape of the section and end condition at \( A \) and \( A' \).

An easy way to check: (built in)

(assume the end condition)

\[ 15 \times \frac{f_y}{G} = f_1 \text{ (assume constant plastic)} \]

The web yields by shear may give a defect at tip of beam without any strain hardening.

\[ \frac{w' L}{2} \]

Moment = \[ \frac{wL^2}{2} \]

Substitute \( \frac{w' L}{2} \) to the dept. formula to find \( w \). Then you can get moment and find \( f_2 \).

See if \( f_2 \) above would exceed yield stress strength of the plate.
Procedure of strain hardening by shear in beams

\[ W_0 = \text{load to make the web yielded} \]
\[ W = \text{the additional load to make the plane give an angle of } \alpha_0 \text{ at the end} \]
\[ \alpha_0 = 15 \times \frac{J_y p}{G} \]

Suppose the load is still increasing. Then part of the increasing load will be taken by the web at AA' section due to strain hardening. But at BB section, the slope angle \( \alpha \) of the beam at the end will then be \( \alpha_0 \), where \( \alpha > \alpha_0 \). But at section BB, the slope of the flange is \( \alpha_1 \), and it might be:

\[ \alpha_0 < \alpha_1 \]

It is clear that the additional shear taken by web at BB' is smaller than
at A'A section

 shear force at web, due to $w_0$
 shear force at flange

$2 (\delta_1 + \delta_2)$ at any section $= \Delta W$

As $\Delta W$ increase, CC' section moves inward. To the right of CC' section, the web is apparently strain hardened by shear.

We might assume $\delta_2$ load function on flange (straight line function). Otherwise the problem become too complicated to be a special elastic support beam.
AB shows its neutral position.

\[ A'B' \] transformed in \( OA'B' \) after the load is applied and assume that the beam is a rigid piece.

Suppose the mid portion can't take any shear, the reaction would become \( A''C'B'' \) the shearing strain for the mid portion would be \( \frac{cc'd'd'}{cd} \)

\[ J_y = J_0 = J_1, \]

\( J_1 \) is the fiber shearing stress for the beam range beams.

\[ d M = W x d \]

\[ M = w x \frac{L}{2} - \frac{b J_1 x d}{2} \]

(A)
Frictionless

The deflection curve of the beam under pure moment is a function of end condition.

Free  \[ + \]

Restrained

The additional moment is apparently \( dF \).

The end condition becomes

Restrained

\[ \sqrt{3} \] A beam under shear and moment is taken different.

The end condition still determines the moment taken.
When the end is free, it is very apparent that there are no shearing stresses, and the two flanges ends remain straight after bending since the moment of inertia of the section should be considered rigid.

But when taken out, this stress is considered as separate. Two beams, the ends of the two flanges are no longer in one line.

Eq (A) would take care the more zero shearing stress in boundary of a beam.
It is very interesting to see that as a problem above.

When "W" increases the load of the beam AB, of course turns as the beam deflects.

But after the web all yielded by shear, the flanges become two beams like below.

\[ \delta x = \frac{T \times l}{AE} \]

Where

\[ \frac{A_n}{2} \]

The increment of the angle of AB due to SW would be \( \delta x \).

due to M, the upper flange would under tension "T"

\[ T = \frac{M}{R} \]
The above beam, the end conditions are not the same as above. The turning of AB line after the web shear yielded is also a function of the mid portion of the beam.

This end condition changes the moment in shear section to changes the deflection in shear portion.

In this case you can’t just consider flanges as separate beams even if the web is yielded. The end condition at AB must be taken in consideration.
Outline on Shear

Problems in

From 4" I section beams

Tests Results
**4° I Assumed Basic Data**

<table>
<thead>
<tr>
<th></th>
<th>Hand Book</th>
<th>Computer</th>
<th>Calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>6</td>
<td>5.81</td>
<td>6.14*</td>
</tr>
<tr>
<td>E</td>
<td>$2.96 \times 10^7$**</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Q</td>
<td>-</td>
<td>3.325/2</td>
<td>-</td>
</tr>
<tr>
<td>M</td>
<td>2.97 **</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

* Calculated from Test No. 9. Assume $E=2.96 \times 10^7$

** Adopted from a paper by Dr. Johnston.
1. Shearing Strength (by octohedral hypothesis)

\[ T_{y.p.} = 40,300 \text{ kips/in}^2 \text{ Upper } T_{y.p.} \text{ in tensile test} \]

\[ T_{y.p.} = \frac{Q_{y.p.}}{N_3} = 40,300 \times \frac{1}{N_3} = 2.33 \text{ kips/in}^2 \]

\[ V = \frac{F_s \times b \times t}{Q} \]

\[ I = 6 \quad b = 0.19'' \text{ (manual)} \]

\[ Q = 3.125/2 \]

\[ V = 16 \text{ kips} \]

One-third of loading \( W = 32 \text{ kips} \)

2. Max. Shearing Stress and Strain Relations

\[ J = \frac{E}{2(1+\mu)} \epsilon_s = \frac{2.96 \times 10^7}{2(1.297)} \epsilon_s = \frac{1140}{2(1.297)} \epsilon_s \times 10^7 \]

Max. Elastic Strain

\[ \epsilon_s(\text{max, E}) = \frac{23.3 \times 10^3}{1.940 \times 10^7} = \frac{2040}{1.940 \times 10^7} \]
3. Initial yield moment

\[ f_{y.p.} = \frac{M_c}{I} \]

\[ I = 6 \]

\[ C = 2.025 \]

\[ f_{y.p.} = 34.8 \times 10^3 \text{ kips} \text{ upper} \]

\[ f_{y.p.} = 2.34 \times 10^3 \text{ kips} \text{ lower} \]

\[ M = 3.34 \times 10^3 \times 6 / 2.025 = 100 \times 10^3 \text{ ft-lb} \]

\[ A_{m} = 4'' \]

\[ W_{y.p.} = 50 \text{ kips} \]

\[ A_{m} = 6'' \]

\[ W_{y.p.} = 33.3 \text{ kips} \]

\[ A_{m} = 8'' \]

\[ W_{y.p.} = 25 \text{ kips} \]

4. Max. Ultimate Bending Moment

\[ M = f_{y.p.} A \times 33.4 \times 3.325 = 111 \times 10^3 \text{ ft-lb} \]

\[ A_{m} = 4'' \]

\[ W_{u.e.} = 55.5 \text{ kips} \]

\[ A_{m} = 6'' \]

\[ W_{u.e.} = 37 \text{ kips} \]

\[ A_{m} = 8'' \]

\[ W_{u.e.} = 27.8 \text{ kips} \]
Test No. 1

1. Shear

\[ \frac{1}{4} - \frac{3}{4} = -\frac{1}{2} \]

Wp due to shear = 28 k see prob 2 and 4.

Observed strain \( e = 1.75 \times 10^{-6} \) (mean of two tests)

\[ J_1 = G_e = 1.14 \times 10^2 \times 1.75 \times 10^{-6} = 20 \times 10^3 \text{ k/p} \]

\[ J_2 = V/A = \frac{28 \times 5.325 \ell}{40 \times 0.014} = 20.4 \times 10^3 \text{ k/p} \]

a. It yields at a lower value than predicted 23:3 k/p.

b. The distribution of shear stress seems to check the assumption very well. The value of \( J_1 \) and \( J_2 \) are pretty close.

\( J_1 \) could be regarded as the actual shearing stress measured in the beam.

\( J_2 \) could be regarded as the calculated shearing stress.
2. Shear Strains and Beam Deflection

We know elastic shearing strain has very small effect on deflection of a beam. But as soon as it reaches the plastic range, the effect on deflection of the beam becomes more important.

The shearing stress distribution of a wide flange section is as follows:

![Diagram of wide flange section]

When the web is yielded by shearing stress, the flange shearing stress may be ten times below yielding strength.

Suppose the two ends of the beam are in built-in conditions, then any further increase in shearing load to the beam would be taken by the two flanges. The two flanges would act like two separate beams.
Upper surface of flange

Lower surface of flange

Induced strain in web.

We suggest when calculating the "I" of flanged use an effective area as follows:

Effective area

Actual shear stress distribution on a web is as follows:

Actual shear stress distribution on a web is as follows:

due to stress

Conclusion

Of course it is a function of root radius and the shape.

In wide flange section it may like:
The above shear deformations include both vertical and horizontal shearing strain.

In a wide flange section, suppose the shear stress along the web could be regarded as uniformly distributed then after it failed by shear we can take the two flanges as individual beams. But due to continuation of strain in an elastic body, some comp. or tensile strain induced in the web.
Elastic support of individual gage beam

It is very important to note the bilinear streaming strength of a beam section, i.e., the load made the whole section distribute with uniform lower yield of shear at stress.

Some conclusions on test. 1

1. The beam is failed by shear. Theoretical deflection curve.

\[ w \]

- Theoretical deflection curve due to separate shapes.
- Deflection due to bending moment.
- Deflection due to shear force.
- Plastic range.
- Before strain hardening.
- Strain hardening.

2. The shape of deflection curve would be a supported.

3. The initial shear-to-yielding checks the assumption very well.
Test No. 2

No strain gages put on this specimen

The beam yielded at about 28 k.

It went up to 48 kip still held
its yield strength due to bending
Of course due to the span of this
kind of beam it is a little bit too short
The stress distribution may not as
exactly as assumed see (5).
Test No. 3.

& Shear

\[ \begin{align*}
\text{Wp. by shear} & = 30 \text{ kips} \\
\varepsilon_{yfs} & = 20 \times 10^{-6} \\
\tau_1 & = G \varepsilon_{yfs} = 22.8 \text{ kips/in} \\
\tau_2 & = \frac{VQ}{Ib} = 21.9 \text{ kips/in}
\end{align*} \]

1. The initial shearing yielding still close as calculated.

2. Deflection curve shows the beam yields at \( W = 30 \text{k} \) by shear.

3. No normal stress shear. 

4. Highest load we got at \( \text{it is below the initial bending yield strength.} \)
Test 4.

\[ 4'' < \theta < 8'' \Rightarrow \frac{\theta}{2} \]

No. Strain gauges put on this specimen

1. Yielded at 30 k by shear

2. Force went up to 41 k far from yield point. Strength by hand test
Shear

\[ \text{W}_{\text{y,y_s}} = 0.4 \text{ k} \]

\[ \text{E}_{\text{y,y_s}} = 1650 \times 10^{-6} \]

\[ \Gamma_{1} = 1650 \times 10^{-6} \times 1.14 = 18.75 \times 10^{3} \text{ ft}^2 \text{ in}^{-2} \]

\[ \Gamma_{a} = \frac{VQ}{bI} = 17.5 \times 10^{3} \text{ ft}^2 \text{ in}^{-2} \]

Shear yield happens 25% below predicted value.

This may be due to special stress pattern while on those longer moment arms the shear field checkered better (6, 7, 8).

Or may be due to residual stress, the def. curve gives the yield IP load of 32 K.
Test No. 6

\[ W_{yip} = 32 \text{ kip} \]

\[ C_{yip} = 2.153 \times 10^{-6} \]

\[ J_1 = 24.5 \text{ kip} \cdot \text{in} \] \( J_1 = 56 \text{ kip} \cdot \text{in} \)

\[ J_2 = \frac{V_0}{I_b} = 23.3 \text{ kip} \cdot \text{in} \]

1. The deflection curve shows the yielding at 32 k. Probably due to shear.

2. The picture Fig. 1 can see those vertical shear yield lines.

3. The beam should be initially yield by bending at a load of \( W = 33.3 \text{ kip} \)

But normal stress strain gauges show no yield at a load of 40 k.
That may prove that a stress gradient exist the yield strength of the material could be raised. \( \text{(c.a.)} \)

4. The ultimate load should be 37 k.
We got 40 k is apparently higher than estimates.
Those yielded region might have been strain hardened.

Those transition zone may still yield slowly.

Those region still in elastic range and the yield strength is raised by stress gradient.

We have some other expections on this higher ultimate yield strength.

5. The special defl. curve.

6.

Fiber strain at 4350 # (11, 12, 13).

\[ \epsilon = 1.413 \times 10^{-6} \]

\[ \sigma_1 = \epsilon \cdot E = 2.96 \times 10^7 \cdot 1.413 \times 10^{-6} = 4.2 \text{ Kip/in}^2 \]

\[ \sigma_2 = \frac{M_{c}}{I} = \frac{4350 \times 0.025 \times 6}{6} = 43.6 \text{ Kip/in}^2 \]

While Typ. in coupon test is 348 upper.
Test No. 7

\[ \Delta = 8'' \text{ vs } 8'' = \frac{8 - 8}{8} \]

\[ W_{yp} = 28 \text{ k} \]

\[ E_{yp} = 1820 \times 10^{-6} \]

\[ T_1 = E G = 20.7 \text{ kip/ft} \]

\[ T_2 = \frac{E_0}{b^2} = 20.4 \text{ kip/ft} \]

1. Calculated bending yield strength

\[ \sigma = \frac{25}{8} \text{ k} \]

The deflection curve shows yield at 28 k may due to both

2. Vertical shear yield is also clear at

Fig 7 & Fig 8.

3. The cal. UL load is 27.8 k.

We got 30.5 k.

4. Show the special defl. curve.
Test 8

\[ W_y \cdot p = 30 \quad \text{(page 21-24)} \]
\[ b_y \cdot p = 20.23 \times 10^{-6} \]
\[ J_1 = 23.1 \quad \text{Kip/ft} \]
\[ T_2 = 21.9 \quad \text{Kip/ft} \]

1. The deflection curve shows the initial yield commenced at a load of 2.8 k. It is apparently by bendip.

2. Initial bendip strength = 25 k
   Ultimate \( \frac{W_y}{p} = 27.8 \) k
   The beam is, however, over-strength. We reached a load of 33 k.

3. Show the special defl. curve.
Test No. 9

Wp. by bend $S_y = 9.4 K$ (calculator)

1. Shear gage didn't fail.

2. Deflection curve shows a yield at 9.5 K

3. Ultimate $W_u = 8.9 K$
   We got 9.8 K
There is no shear stress gradient exist.

4. We could use the ultimate shearing strength $T_y$ as the design load for shear.

where $T_y = \text{lower shearing yield pt.}$

$$A = \text{cross area of section.}$$

Under such a design load the deflection may be too large before the beam reaches its ultimate shearing strength.

5. The max. shear theory may be able to get a little more consistent result than octohedral shear theory for point 3. discussed.

6. Stress pattern near loading pts and support doesn't seem change much.

The short moment arm didn't change much of the shearing stress pattern either.