PLASTIC STRENGTH AND DEFLECTION
OF A GABLED FRAME

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ABSTRACT

The method of analysis and results of a test of a full size, single span gabled frame are presented in this report. The frame was subjected to combined vertical and wind loads. It was of welded construction and had a span of 40 ft., column height of 10 ft. and a roof slope of 1:3.

Design procedure and fabrication of joints are first described. A step-by-step method for predicting the behavior of the frame is then discussed. Also included is an example showing a procedure for calculating deflections and rotations at the ultimate load by slope-deflection equations.

Results of the test, including deflections and rotations at plastic hinges and column bases are compared with theoretical predictions at successive stages. A discussion on lateral buckling and the forces in the purlins is also included.

The results obtained from this study lend further assurance to the applicability of the plastic theory to the analysis and design of structures.
1. INTRODUCTION

This report deals with the analysis, design and test of a welded-single-span gabled frame. It was tested as a demonstration during the Summer Course on PLASTIC DESIGN IN STRUCTURAL STEEL given at Lehigh University in September, 1955.

Previous tests conducted at Lehigh were made to study the behavior of rectangular portal frames subjected to vertical and horizontal loads with either pinned or fixed bases (11,12). This test carried the program a step further to study the behavior of a gabled frame subjected to combined loading. A similar gabled frame of smaller size (span length 16 ft.) has been tested by Baker and Eickhoff at Cambridge.(1)

In an earlier report, covering all the demonstration tests in the Summer Course, sufficient results of the frame test were presented to verify that the maximum load and mechanism could be predicted by simple plastic theory(4). In this report, there will be presented, not only more complete details of the test results, but also a description of the design procedure and analysis of the frame.
An approximate procedure for the calculation of the step-by-step loads, moments, and deflections of the frame during formation of a mechanism is featured in the theoretical analysis. A procedure for calculating directly the deflections at maximum load without resorting to the more tedious step-by-step procedure is also given.
2. DESIGN OF FRAME

While the frame was designed primarily for a specimen to be tested under conditions of combined horizontal and vertical loading, it was desired to meet the conditions for design of an industrial building. It could be assumed that the span, height, bay length and slope of the rafters had been selected by architectural considerations.

2.1 Working Loads

A total vertical load of 60 psf was selected as a reasonable approximation of dead and snow load. Wind loads were selected as 20 psf according to A.I.S.C. specifications.

With 40 ft. span and 18 ft. bent spacing, the total vertical load was $40 \times 18 \times 0.06 = 43.2$ kips per frame. The overall height was 13 ft. 4 in. The total horizontal load would be $18 \times 13.3 \times 0.02 = 4.8$ kips per frame. This combination of loading would put the ratio of total horizontal load to total vertical load at 1:9.

In arranging a test set up, it was convenient to apply four equal concentrated vertical loads instead of a uniformly distributed load. The side load could also be more easily applied in the form of two concentrated loads. By making each of the horizontal loads equal to one-
fourth of one of the vertical loads, an easier combination was achieved for test purposes, but the ratio of horizontal to vertical load was thus altered to 1:8. (See Figure 1 for the loading arrangement.)

2.2 Design Loads

A frame of this type is designed either for dead plus live load alone with a load factor of 1.88 or with dead plus live plus wind load in which case the load factor may be reduced to 1.41.*

The design full load for the first case was established by multiplying the working loads by 1.88.

\[
4 P_u = 1.88 \times 43.2 = 81.2 \text{ kips} \\
R_u = 20.3 \text{ kips}
\]

For the case including wind, the design full load was

\[
4 P_u = 1.41 \times 43.2 = 61.0 \text{ kips} \\
R_u = 15.2 \text{ kips}
\]

The corresponding horizontal loads were then 3.8 kips each.

2.3 Design Moments and Selection of Section

To determine the design moments for the frame it is necessary to select the correct mechanisms for both the presence and absence of horizontal load. However

* Current recommendations suggest load factors of 1.85 and 1.40 respectively.
for this particular case it will be shown that essentially the same mechanism controls each design.

Taking first the case including wind, after studying the possible combinations of mechanisms, two critical mechanisms are obtained each having the same ratio of $M_p$ to ultimate load (Figs. 2 and 3). An equilibrium check shows that the same moment diagram is obtained for each mechanism (Fig. 4), and that the plasticity condition is not violated.\( ^{(2)} \) This means that the two plastic hinges in the beam form simultaneously, a special case that occurs frequently in structures of symmetrical shape. It is noted that the horizontal load does no work in the formation of the mechanisms shown. By this it is implied that with vertical load alone the same mechanism might form.

It might be expected that analysis of the frame for vertical load alone would result in a symmetrical mechanism as shown in Fig. 5. However, such a mechanism is very unstable, and structures as they are actually built will generally lapse into an unsymmetrical mechanism to one side or the other as shown in Fig. 6. Generally, the load consistent with the symmetrical and unsymmetrical mechanisms will be the same. For this particular frame, it is noted
that once $M_p$ has been selected, the total ultimate vertical load will be the same whether side load is absent or present. This would be true for values of total side load from zero to $8/3 P$. Since the equation for $M_p$ is $PL/3$ for either case, the case involving no wind will control the selection of the members because of its larger full load value.

$$M_p = \frac{PL}{3} = \frac{20.3 \text{ k} \times 240 \text{ in.}}{3} = 1624 \text{ kip-in.}$$

The plastic section modulus necessary to develop this moment is:

$$Z = \frac{M_p}{\sigma_y} = \frac{1624 \text{ kip-in.}}{33 \text{ ksi}} = 49.2 \text{ in.}^3$$

For 14WF34, $Z = 54.5$

12WF36, $Z = 51.4$

The most economical structural shape would be the 14WF34. However, since a sufficient quantity of 12WF36 was readily available, this section was selected instead of the 14WF34 shape.

To test the design condition of this frame, it should be subjected to vertical load only. However the direction of sway under vertical load would be undetermined and could cause difficulty with test measurements. By applying a small side load, the direction of sway would be controlled
without changing the ultimate vertical load. Thus the frame would be tested in almost the design condition but under slightly more severe loading. By fixing the ratio of the small side load to vertical load the same as for the design wind, the behavior of the frame under that ratio of combined loading could be studied without altering the failure mechanism produced by vertical load alone.

2.4 Design Details and Fabrication

Having selected the rolled section, and having the moment diagram for the frame (Fig. 4), it was then necessary to design the welded connections to join the members and to connect the column bases to their base plates. The forces on the welds at each joint were deduced from the moment diagram.

All joints were welded for full continuity, the welds being proportioned so that the nominal stresses at the ultimate load of the frame would be 33,000 psi on the net area of butt welds, and 22,400 psi on the throat area of fillet welds. (6)

The most complex of the joints were the knees joining the sloping rafters to the columns. In the detailing of these knees, advantage was taken of the results of earlier research on square corner connections (5, 13). By making
slight changes to take into consideration the sloping rafters, the procedures of References 5 and 13 were used to detail the knee joints. Fig. 7 shows the details of one of the knees. The essential features of the knee include: diagonal stiffeners to limit distortion due to shear, vertical stiffeners to prevent web crippling and flange buckling where the column inner flange meets the rafter, and an end plate to transmit the column outer flange force into the rafter web and diagonal stiffener. This type of connection was selected because its square counterpart in the earlier studies had shown adequate strength and rotation capacity while being economical to fabricate in terms of material and welding and cutting time.

Fig. 7 also shows the type of welded joint used to connect the columns to the base plates which would anchor the columns to their moment-resisting foundations. In this joint fillet welds were used on the webs and butt welds on the flanges.

The third type of joint was the ridge joint connecting the two rafters. (Fig. 8) In this joint the two rafters were welded to the stiffener plate, applying fillet welds to the webs of the rolled sections, and using single-bevel butt welds at the flanges.
3. LOAD-DEFLECTION ANALYSIS

Two types of load-deflection analysis will be discussed. The first is the rather complex calculation of the predicted load-deflection curve. The second is the more direct calculation of the deflection as the structure first reaches its ultimate load.

3.1 Determination of Predicted Load-Deflection Curve

The predicted load-deflection curve (Fig. 9) is approximated by a series of linear steps occurring between the successive formation of enough plastic hinges to cause a mechanism to form. Reference 9 outlines the step-by-step deflection calculation for a simple frame. The procedure will be illustrated here in more detail with the aid of several tables and figures.

The initial step necessary is the determination of the moment diagram and equations for deflection of the frame in the elastic range. Any text on indeterminate structural analysis presents a number of methods for determination of the moments and reactions. Most texts do not, as such, illustrate the calculation of deflections of rigid frames, but halt with the determination of redundants. The principles
of deflection calculations are illustrated with problems on beams. However, all indeterminate solutions are dependent on some consideration of deformation. Once the redundants are determined, they may be substituted into deformation expressions from which they were derived and used to determine deflections. Examples of the solution of gabled frames are given in references 8 and 10.

An elastic analysis was made for the frame of Fig. 1, which will be designated in this discussion as Structure I or the primary structure. Both moments and deflections were determined in the analysis. For the step-by-step analysis, moments and deflections are tabulated in tables 1 through 5, and moment diagrams are given in Figs. 10 through 15 and 19 through 21. The moments for Structure I are given in the first line of Table 1, in terms of the unit load $P$ and the span length $L$, and are plotted in Fig. 10. Positive moments are those causing tension on the outside of the frame. The deflections for Structure I are given in Table 4. The only deflections which will be tabulated are the horizontal deflections of the knees, $\delta_{H4}$ and $\delta_{H10}$, and the vertical deflection of the ridge, $\delta_{V7}$. 
The load-deflection analysis for Structure I gives the deflection corresponding to three important loads, the working load, the yield load and the load at which the first plastic hinge forms. In Fig. 9 is plotted the predicted load-deflection curve for the gabled frame. Loads are plotted as decimal parts of the ultimate load \( P_u \). The dashed line I, which coincides with the initial portion of the predicted load-deflection curve is the elastic load-deflection curve of Structure I. The important events on this elastic line are designated. The letter \( W \) indicates the working load, at which the total vertical load equals the predicted ultimate load divided by 1.88. The letter \( Y \) indicates the predicted yield load, at which the maximum moment \( 0.417 P_1L \) at the lee knee (point 10) equals \( M_y \) of the structural shape (see Fig. 10). The number 1 indicates the load at which the first plastic hinge forms at the lee knee. Then the moment \( 0.417 P_1L \) equals \( M_p \). Corresponding to this stage, the load is \( 2.397 M_p/L \). Since the ultimate load \( P_u \) is \( 3 M_p/L \), the load \( P_1 \) at which the first plastic hinge forms is \( 0.799 P_u \). At that load, the vertical deflection \( \delta_{y7} \) of the ridge is \( 0.0428 P_1L^3/EI \) (as given in Table 4) or \( 0.0342 P_uL^3/EI \) when the magnitude of load \( P_1 \) is substituted (Table 5).
Once the greatest moment is equated to $M_p$, all other moment may be expressed as a fraction of $M_p$. For example, the moment at the lee base which is $-0.406 \, P_1L$ becomes $-0.974 \, M_p$, and so on around the frame as given in Fig. 11 and in the first line of Table 2 for the first hinge. Table 3 lists the loads and reactions at the formation of each plastic hinge.

Once the first hinge has formed at the lee knee, the structure no longer behaves elastically. The moment at the lee knee cannot increase. The best example of a structure in which the moment at the lee knee cannot increase, is a structure with a pinned joint at that knee. In sketch II of Fig. 9, just such a structure is depicted. By analyzing the elastic behavior of this auxiliary structure II, data for the next increment of behavior of the primary structure may be obtained. An elastic analysis of structure II gives moments shown in the second line of Table 1 and Fig. 12, and gives deflections shown in Table 4. The slope of the elastic load-deflection relationship for structure II is given by dashed line II in Fig. 9.
The solid line 1-2 for the second stage of the behavior of the primary structure is parallel to dashed line II in Fig. 9. The length of line 1-2 is determined by how much additional load is required to cause a second plastic hinge to form. In Fig. 11, it is shown that the moment was 

-0.974 M_p at the lee base just as the first plastic hinge formed. If this moment were altered by an amount -0.026 M_p, a plastic hinge would form at the lee base. The moment at the lee base for auxiliary Structure II is -0.874 P_{II}. If the moment -0.874 P_{II} is placed equal to the moment -0.026 M_p and solved for P_{II}, the result is the amount \( \Delta P_2 \) by which the load of the primary structure exceeds P_1, the load to cause one plastic hinge.

\[
\Delta P_2 = \frac{0.026}{0.874} \frac{M_p}{L} = 0.030 \frac{M_p}{L}
\]

Since \( M_p = P_u L/3 \), \( \Delta P_2 \) is also equal to 0.010 \( P_u \), then

\[
P_2 = P_1 + \Delta P_2 = 0.799 P_u + 0.010 P_u = 0.809 P_u.
\]

Increments of deflection functions between the formation of the first and second plastic hinges may be calculated by substituting the value of \( \Delta P_2 = 0.010 P_u \) for \( P_{II} \) in the
expressions given in Table 4. Thus,

\[ \Delta \delta_{V7} = 0.1114 \frac{P_{II}L^3}{EI} = 0.0011 \frac{P_uL^3}{EI} \]

In Table 5 are shown the increments of deflections in terms of \( P_u \) and the cumulative deflection at each step in terms of \( P_u \).

The increments of moment at every point in the frame are determined at the same time as the moment at the lee base. Each value of moment in the auxiliary structure II, Fig. 12, is multiplied by the same factor, \( 0.030 \frac{M_p}{P_{II}L} \) to give an appropriate increment for moment in terms of \( M_p \) (Fig. 13). The sum of the moment diagrams in Fig. 11 and Fig. 13 is Fig. 14 which shows the total moment diagram when the second plastic hinge has formed. Only the moments at the top and bottom of the lee column are as great as \( M_p \), indicating that the proper hinges were used in adjusting. Successive increments and cumulative totals of moments at every step are given in terms of \( M_p \) in Table 2.

Next an auxiliary structure must be considered with hinges at both the top and bottom of the lee column as in sketch III of Fig. 9. The elastic analysis of Structure III by the method of moment distribution is given in Fig. 16.
to 18. The analysis shown is the same type as was used for Structures I and II. To readers familiar with the use of moment distribution with corrections for sidesway, the figures should be self-explanatory. However, reference to texts such as Ref. 8 and 10 may prove helpful. The most important points to observe for the problem being studied are the simultaneous equations for correction of sidesway in Fig. 18c. The use of the sidesway corrections in calculating the horizontal deflections of the knees is illustrated in Fig. 18e. The final moments of Fig. 18d are used to obtain all moments for Structure III in Table 1 and Fig. 15. Similarly the deflections of Fig. 18e and 18f are listed in Table 4.*

Having the elastic solution for structure III, data are then available for determining the increments of all functions of the primary structure between the formation of the second and third plastic hinges. As in the previous step, the increments are determined by the amount of load necessary to cause an additional plastic hinge to form.

* A small apparent discrepancy in the numerical results of Fig. 20 and Tables 1 and 3 is due to a difference in the number of significant figures carried in the arithmetical work. The values tabulated are the more accurate.
The moment at joint 4, the windward knee, at the formation of the second plastic hinge is \( +0.967 \, M_p \) (Fig. 14). This moment must be increased by \( +0.033 \, M_p \) to reach \( M_p \). Equating \( +0.033 \, M_p \) to the moment \( +0.592 \, P_{III} \) at that joint in Structure III (Fig. 15) gives the ratio \( \frac{0.056 \, M_p}{P_{III}} \) by which all functions of structure III must be multiplied to obtain the increments on the primary structure as caused by the load \( \Delta P_3 \). These increments and their cumulative effects are included in Tables 2, 3 and 5, and in Fig. 19 and 20.

Auxiliary Structure IV in Fig. 9 is used to obtain the increments of behavior for the last step, 3-4, in forming a mechanism. The results are included in Tables 2, 3 and 5, and the final moment diagram is repeated in Fig. 21.

For a complete picturization of the auxiliary structures in this problem, Structure V is included in Fig. 9. This structure is a mechanism and would deflect without load. Therefore, its load-deflection curve is the deflection axis of Fig. 9 and as a superimposed increment of deflection on the primary structure, it gives the solid horizontal line 4-5.
3.2 Deflection at Ultimate Load

Having determined the collapse mechanism and obtained an ultimate moment diagram, a solution to the deflections and rotations of the structure at ultimate load can be made by a simpler procedure without going through step-by-step calculations. By assuming that yielding is concentrated at the plastic hinge, considering that a moment of $M_p$ is applied at the hinge point, it is possible to get the deflections at different points on the frame by elastic methods, such as moment-area, conjugate beam, virtual work and slope-deflection. In this report a slope-deflection method is adopted, and the equation is given as follows:

$$\theta_{NF} = \theta_{NF}' + R_{NF} + \frac{1}{3EI} \left( M_{NF} - \frac{1}{2} M_{FN} \right). \ldots \ldots \ldots (1)$$

$\theta_{NF}$ = Slope of near end of member

$\theta_{NF}'$ = Slope of near end of similarly loaded member when simply supported = $\frac{F_{ab}}{2EI}$ (see Fig. 22 for notation). \ldots (2)

$R_{NF}$ = Rotation of a chord between ends of members.

= Deflection of one end of a member with respect to the other divided by the distance between them = $\Delta/l$
\[ l = \text{Length of member} \]
\[ M_{NF} = \text{Moment at near end of member} \]
\[ M_{FN} = \text{Moment at far end of member} \]

The sign convention used here is that slope angles are defined as positive when the rotations are clockwise, and end moments are defined as positive when acting in the clockwise sense. Once the bending moments for a structure are known, the slope deflection equations for each end of each member can be written in the form of Eq. (1). The unknowns in these equations are \( \Theta \) and \( R \) terms. Additional equations will be needed to solve the problem. These may be derived by considering the compatibility of the \( R \) terms. From previous analysis it is noted that the last hinges are at points 6 and 8, and there is continuity at those points when the structure just attains its ultimate load. Solution of the unknown \( \Theta \) and \( R \) terms gives the deflection and rotation at each end of each member.

3.3 Sample Calculation of Deflection at Ultimate Load

Referring to Figs. 1 and 4, \( \Theta' \) values are given by the following formulas using Eq. (2):
Continuity at point 7 gives: $\Theta_{7-4} = \Theta_{7-10}$

From Eq. (1),

$$\Theta_{7-4} = -\frac{\sqrt{10}}{32} \frac{PL^2}{EI} + \frac{3 \delta_7}{\sqrt{10}} L + \frac{\sqrt{10}}{9EI} (M_{7-4} - \frac{M_{4-7}}{2}) \ldots (5)$$

$$\Theta_{7-10} = \frac{\sqrt{10}}{32} \frac{PL^2}{EI} - \frac{3 \delta_7}{\sqrt{10}} L + \frac{\sqrt{10}}{9EI} (M_{7-10} - \frac{M_{10-7}}{2}) \ldots (6)$$

Solving (5) and (6) gives:

$$\delta_7 = \frac{5}{48} \frac{PL^3}{EI} + \frac{5}{27} \frac{L^2}{EI} (M_{7-10} - M_{7-4} + \frac{M_{4-7}}{2} - \frac{M_{10-7}}{2}) \ldots (7)$$

The vertical component of the deflection of the ridge is:

(See Fig. 23)

$$\delta_V7 = \frac{3}{10} \delta_7 \ldots \ldots \ldots \ldots \ldots (8)$$
and the horizontal component is

$$\delta H7 = \frac{1}{\sqrt{10}} \delta 7$$

(9)

The condition that there is no angle change at point (1) gives:

$$\Theta_{1-4} = \frac{1}{144} \frac{PL^2}{EI} + \frac{2\delta H4}{L} + \frac{L}{6EI} \left( \frac{M1-4 - \frac{M4-1}{2}}{2} \right) = 0 \ldots (10)$$

Therefore

$$\delta H4 = -\frac{1}{288} \frac{PL^3}{EI} - \frac{L^2}{12EI} \left( M1-4 - \frac{M4-1}{2} \right) \ldots (11)$$

$$\delta H10 = \delta H4 + 2\delta H7 \ldots (12)$$

Substituting the moment values from Fig. 4 into these equations, \(\delta V7\), \(\delta H4\) and \(\delta H10\) (Fig. 23) can readily be obtained. For example, from (7):

$$\delta 7 = \frac{5}{48} \frac{PuL^3}{EI} + \frac{5}{27} \frac{L^2}{EI} \left( \frac{2}{3} Mp + \frac{2}{3} Mp + \frac{-Mp-Mp}{2} \right) \ldots (13)$$
Since \( M_p = \frac{P_uL}{3} \) then,

\[
\delta 7 = 0.1248 \frac{P_uL^3}{EI} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (14)
\]

\[
\delta v7 = \frac{3}{\sqrt{10}} \cdot 0.1248 \frac{P_uL^3}{EI} = 0.1184 \frac{P_uL^3}{EI} \ldots \ldots \ldots (15)
\]

This value for the deflection at ultimate load is the same as obtained in the step-by-step method of Section 3.1 and corresponds to point 4 in Fig. 9. It is quite evident that it is much easier to compute the deflection at ultimate load using the slope-deflection equations than it is to obtain the entire load-deflection relationship.
4. DESCRIPTION OF TEST

4.1 Test Program

The purpose of this frame test was to provide a verification of load and deformation theory, to obtain information on lateral support forces, and to demonstrate behavior of a gabled frame under load.

Verification of the theory in Chapters 2 and 3 was to be accomplished by applying loads to the structure and gradually increasing them all in the same proportion until all desired data had been obtained. The relationship between applied load and the deflection and rotation of points on the structure was to be obtained by taking proper measurements during the application of the load. The measurements to be taken during the test were the ultimate load, the horizontal deflections of the tops of the columns, the vertical deflection of the ridge, and several rotations. The rotations were to be measured at plastic hinges or at points where plastic hinges might form if the same structure were loaded differently. Therefore, the rotations were to be measured at the two knees, the column bases and the two innermost vertical load points.
Prior to testing, measurements were made of the cross sectional dimensions of the 12WF36 shapes used in fabricating the specimen. The measured properties are given in Table 6 and compared with the dimensions given in the AISC Steel Construction Manual. In conjunction with these measurements, tensile coupon tests were conducted giving the actual physical properties listed in Table 7.

4.2 Test Apparatus

A sketch of the test apparatus is shown in Fig. 24.

Testing Bed

The test frame was erected on laboratory testing bed which is a massive, reinforced concrete slab. Numerous threaded anchorages are provided on the surface of the test bed. The column base plates of the test frame were rigidly bolted to the bed at these anchorages. The mountings and brackets which provided anchorage for the vertical and horizontal jacks were also bolted to this floor.

Loading Assembly

Three manually operated hydraulic jacks of 50,000 pounds capacity were used to apply load to the frame, two
in tandem for the vertical loads and one for the horizontal forces. The load in each jack was measured by an aluminum tube dynamometer. The vertical load was distributed from each jack by a horizontal beam with vertical ties at each end to apply two concentrated loads on each roof member. For simplicity, horizontal load was transmitted from the jack to two points on the windward column through diagonal ties, omitting a distributor beam. It was realized that the compression in part of the column which resulted from the vertical components of the forces in the diagonals would have a negligible effect on the strength of the frame.

All joints in the loading assemblies were pinned. The stroke of each jack was 9 inches; the pumps were capable of providing a load of up to 50 kips each and the dynamometers had a capacity of 85 kips. (See Fig. 24).

Deflection Measurements

The three points on the frame at which deflection was measured are shown in Fig. 25 (a). The vertical deflection at the center of the gable was taken by measurements made with a surveyor's transit sighted on a graduated scale clamped to a vertical rod suspended from the ridge of the frame.
Horizontal deformations were measured at each of the columns by means of plumb wires damped in water baths. The wires were suspended from rods attached to each column and level with the eaves of the frame. Behind each wire was a fixed horizontal scale and along one edge of each scale was attached a strip of mirror as indicated in Fig. 25 (b). When taking a reading, the wire could be made to coincide with its reflected image to ensure the elimination of parallax. In addition, a "plotting board" was used to give a continuous record of vertical load P as a function of the center vertical deflection of the frame. This equipment and its use on this test is described in Ref. 4.

**Rotation Measurements**

Rotation indicators were used to measure the angle changes at the knees, column bases and the curvature over a finite length at the inner vertical loading points on the rafters. Their arrangement and the method of calculation of rotations are indicated in Fig. 26.

**Lateral Support**

Nine 6B12 beams with a spacing of one-fourth of the rafter length were used as purlins (Fig. 23). The function
of the purlins was merely to provide lateral support, preventing the frame from deflecting out of its loading plane. The purlins were fastened to the top flange of the rafters at one end and to the laboratory wall at the other.

A turnbuckle to allow initial lateral adjustment and a flex bar to allow free vertical movement of the frame were inserted between the end of each purlin and the beam on the wall (Fig. 27).

Small cylindrical dynamometers were inserted in series with the turnbuckles of five of these purlins. SR-4 strain gages were mounted on each dynamometer, and readings were taken after each load increment to record the lateral force holding the frame. Diagonal braces were welded between the end purlins and the inner corners of the knees to prevent lateral buckling of the compression flanges. (Fig. 28)

4.3 Testing Procedure

Before the application of any load, a set of zero readings was taken. This included readings of all the rotation dials, lateral and vertical deflection scales, lateral buckling and loading dynamometers. Load was then applied to the specimen by increments in the ratio of 4:1 for each vertical force to each horizontal force. This proportion of horizontal to vertical load was maintained constant throughout the test.
To attain any desired load on the frame, the strain indicators connected to the dynamometers which had been previously pumped until a balance was obtained on the indicator.

After each load increment, rotations, deflections and lateral forces readings were taken. (See Fig. 28). Simultaneously, load versus rotation curves for the knees were being plotted manually on a large display graph; load versus deflection at the center of the frame was being recorded on the automatic plotting board.

The behavior of the frame in the elastic range was not of primary interest and therefore the minimum number of load increments required to establish the various curves through this range were taken. Five increments were made before bringing the frame to the predicted yield point. After a further increment of load, the actual yield point became apparent and the first plastic hinge began to form. When this stage was reached, it was evident that further increments of load would produce deflections of rapidly increasing magnitude. It was decided, therefore, to continue the test by increments of deflection. Vertical deflection at the center of the
frame was used as the criterion from there up to the formation of the fourth and final plastic hinges when the ultimate load had been reached.

Deflections became appreciable when the test was continued beyond the maximum load, and the forces in the lateral support at the windward knee also increased greatly. The frame was then unloaded.
5. TEST RESULTS AND DISCUSSIONS

5.1 General Behavior

The frame under test behaved as predicted in the elastic and plastic ranges.

The frame carried the predicted yield loads and approached the theoretical ultimate value very closely through the successive formation of plastic hinges in the sequence predicted. The first hinge became evident at about 13 percent above the predicted yield load, occurring at the east knee joint, point 10. The second and third hinges formed in the sequence predicted forming respectively at the east base and the west knee joint, points 11 and 4, Fig. 1. Considerable deformation of the frame and some increase in load then occurred before the fourth and fifth hinges formed at the two inner vertical load points, 6 and 8. At this stage the frame had reached a load slightly greater than the predicted ultimate load and a mechanism was formed. After considerable straining at this constant load, the test was stopped.

Thus the frame showed an ability to carry loads slightly higher than that predicted by the simple plastic theory,
which neglects the effects of direct and shearing stresses.

The purlin system was effective in preventing lateral buckling. Stability was maintained at the plastic hinge locations up to the formation of the failure mechanism. Though some lateral deformation out of the plane of the frame was visible at certain hinges, it did not appear to affect the ultimate load capacity.

5.2 Deflections

The vertical deflection at the center of the frame together with the horizontal deflection of the windward and leeward knees plotted against load are shown in Fig. 30. The predicted curves for the deflections at these locations are also given on the same figure. Due to the effects of residual stresses and stress concentrations, the experimental results deviate from the theoretical curve in the elastic and initial plastic ranges. However, upon increasing the load to the ultimate, experimental load became very close to that predicted. The ultimate load was about 1.4 per cent higher than predicted. The whole structure was capable of carrying the ultimate load through large deformations after the formation of the failure mechanisms. The
load vs. vertical deflection curve shows that after the completion of the test, the deflection was about 9 times that at the elastic limit and twice that at ultimate load. As can be seen from Fig. 29, the frame showed no serious local buckling and distortion at the final stage.

5.3 Rotation of Joints

Curves of load versus rotation at the plastic hinge locations and column bases are plotted in Figs. 31 to 36 inclusive. Rotations at two knees and column bases were measured over a finite length at these points. They are expressed in radians. Those at points 6 and 8 are plotted in the unit of radians per inch to express curvature.

The first plastic hinge was developed at the lee knee (point 10); therefore the rotation capacity of this joint was important. The joint should have sufficient ability to maintain a constant moment while rotating through a finite angle until plastic hinges at 6 and 8 were formed, resulting in a mechanism. Fig. 31 shows that the load-rotation curve agrees well with the theoretical curve obtained from the ste-by-step method described before and that the rotation capacity of this connection was excellent. The joint was
able to rotate through an angle about twice as much as that needed to form a mechanism. No evident local crippling was seen at the joint after the test. (see Fig. 37).

The experimental curve at the east column base, where the second hinge formed, shows a fair agreement with the theoretical predictions (Fig. 32). For the west column base, the two curves do not agree (Fig. 33). This is probably due to the fact that the base plates were not anchored to the floor firmly enough. Measurement on a later test using the same type of anchorage indicated that there definitely was rotation at the column bases before the full moment was developed. For this later test, the experimental rotations agreed well with theory after a correction for rotations of the base plate was made. This fact indicates that the ultimate load of a structure, as determined by the plastic theory, would not be reduced by small accidental support rotation.

Fig. 34 shows the behavior of the windward knee throughout the test. This knee was the location of the third hinge. Yielding occurred at the joint, but it suffered no local buckling (Fig. 38).
The rotations of the last two plastic hinges (point 6) and 8 are shown in Figs. 35 and 36. Before reaching the ultimate load, rotations are elastic. As soon as the $P-\theta$ curve reached the leveling point, a failure mechanism was formed. Photographs taken after the test show the degree of yielding in the flange at these two points. (Figs. 39 and 40).

5.4 Lateral Support Forces

The magnitude of the forces in the purlins gives an indication of the tendency toward lateral buckling at points along the rafter. The possibility of such instability is increased when portions of the rafter reach the plastic range. In order to prevent the frame from lateral buckling, the purlin system should be so designed that the lateral movement of the beam is minimized.

The bracing forces in some of the purlins were measured by SR-4 strain gages. In Fig. 41 the measured forces at the apex, leeward knee, and the middle point between 8 and 9 are plotted against force $P$. The total applied vertical load on the frame was $4 \, P$ (see Fig. 1).
The purlin force at the lee knee (location of the first plastic hinge) increased approximately in proportion to the applied load. When the frame just reached its ultimate load, the measured force was 1.47 kip, about 1.5 per cent of the total load or 0.4 per cent of the yield load of the rafter (yield stress x cross sectional area). This fact indicates that the support forces needed to insure good plastic action of the partially yielded structure up to the formation of its failure mechanism were relatively small. More important is the stiffness of the supporting structure. After the structure reached the computed ultimate load, the bracing forces increased at a greater rate and lateral distortion became evident.

Supporting forces in the purlins at the lee knee and the middle point between 8 and 9 show that the rafter had a tendency to buckle in opposite directions at these two points (Curve 1 and 2). The apex of the frame was stable throughout the test. (Curve 3).
6. SUMMARY AND CONCLUSIONS

A brief discussion of the Plastic Method used to analyze and design a single-span gabled frame is presented. Complete load-deflection curves for the apex and eaves for this frame are obtained. These curves served as a guide to evaluate the behavior of the whole frame during the test. A method for determining the deflected shape of the structure at ultimate load is developed. Using the continuity condition at the last plastic hinge, solution of simultaneous slope-deflection equations is made possible, which yields the rotations and deflections of the frame when the mechanism is about to form. A sample calculation is included to show the general procedure.

Results obtained from a gable frame test are analyzed and compared with the theoretically predicted values.

The following statements sum up the results:

1. The frame carried the predicted yield load and reached its maximum load through the successive formation of plastic hinges in the sequence predicted.

2. The frame was able to carry loads 1.4 percent higher than the predicted ultimate load through large deformations (see Fig. 30).
3. The predicted load-deflection curves were satisfactory to use as a measuring control. Residual stresses and stress concentrations had considerable influence on the deformation of the structure especially when the load was beyond the yield value (see Fig. 30).

4. The knee joints of the frame were capable of carrying a plastic moment greater than that predicted without showing any signs of failure. Their rotation capacity was sufficient for the development of the failure mechanism (see Figs. 31, 34, 37, and 38).

5. The lateral support forces needed to insure good plastic action of the partially yielded structure were relatively small. The largest measured force in the purlins was only about 1.5% of the total load at failure. (see Fig. 41).

6. There was no evidence of local flange buckling at the knees or in the beam, when the ultimate load was reached (see Figs. 39 and 40).

In conclusion, the test results have given further indication of the applicability of plastic method in structural design.
7. ACKNOWLEDGEMENTS

This work was conducted at Lehigh University in Fritz Engineering Laboratory of which Professor W.J. Eney is Director. The test and results reported herein form part of an investigation on "Welded Continuous Frames and Their Components". This project is sponsored by the Welding Research Council and the United States Navy Department. Funds are supplied by the American Institute of Steel Construction, American Iron and Steel Institute, Office of Naval Research, Bureau of Ships and the Bureau of Yards and Docks. The investigation is being carried out under the direction of Prof. L. S. Beedle.

The authors wish to express their sincere appreciation to Messrs. A.W. Huber, Y. Fujita, G. Haaijer, G. Heimberger, S.J. Errera, D.L. McCullough, and M.W. White, who provided valuable assistance for the test. Narration during the demonstration test was provided by F.W. Schutz, Jr., of Georgia Institute of Technology. Special thanks go to Mr. K.R. Harpel and his staff and to Mr. I.J. Taylor for their work in fabrication and set-up of the specimens and instrumentation.
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30, (8)
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Part III - Discussion of Test Results and Conclusions,
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9. NOMENCLATURE

\( \delta \) = Deflection

\( E \) = Young's Modulus

\( f \) = Shape of factor

\( I \) = Moment of inertia

\( l \) = Length of member

\( L \) = Half span length = 20 feet

\( M \) = Bending moment

\( M_{NF} \) = Moment at near end of member

\( M_{FN} \) = Moment at far end of member

\( M_p \) = Plastic moment

\( P \) = Load

\( P_u \) = Ultimate load

\( R \) = Rotation of a chord between ends of members

\( S \) = Elastic section modulus

\( Z \) = Plastic section modulus

\( \sigma_y \) = Stress at yield point of material

\( \Theta \) = Slope of deflection curve or rotation angle

\( \Theta_{NF} \) = Slope of near end of member

\( \Theta'_{NF} \) = Slope of near end of similarly loaded member when simply supported

\( \varphi \) = Curvature
10. TABLES AND FIGURES
### TABLE 1 - BENDING MOMENTS IN AUXILIARY STRUCTURES

<table>
<thead>
<tr>
<th>Structure No.</th>
<th>M/PL At Station Number (see Fig. 1 for station number)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Structure I</td>
<td>-0.302</td>
</tr>
<tr>
<td>Structure II</td>
<td>-0.234</td>
</tr>
<tr>
<td>Structure III</td>
<td>+0.717</td>
</tr>
<tr>
<td>Structure IV</td>
<td>+0.125</td>
</tr>
</tbody>
</table>

Positive moment causes tension stresses on outside fibers.

### TABLE 2 - BENDING MOMENTS OF FRAME AT SUCCESSIVE STAGES

<table>
<thead>
<tr>
<th>Load Increment on Frame</th>
<th>M / M_p At Station Number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1st Hinge P_1</td>
<td>-0.723</td>
</tr>
<tr>
<td>Δ P_2</td>
<td>-0.007</td>
</tr>
<tr>
<td>2nd Hinge P_2</td>
<td>-0.730</td>
</tr>
<tr>
<td>Δ P_3</td>
<td>+0.040</td>
</tr>
<tr>
<td>3rd Hinge P_3</td>
<td>-0.690</td>
</tr>
<tr>
<td>Δ P_4</td>
<td>+0.065</td>
</tr>
<tr>
<td>4th Hinge P_4</td>
<td>-0.625</td>
</tr>
</tbody>
</table>

Positive moment causes tension stresses on outside fibers.
TABLE 3

LOADS AND REACTIONS OF FRAME AT SUCCESSIVE STAGES

<table>
<thead>
<tr>
<th>Load Increment on Frame</th>
<th>( A\Delta P_n L = B M_p )</th>
<th>( \Delta P_n = \frac{B M_p}{A L} )</th>
<th>( P )</th>
<th>( \frac{P}{F_p} )</th>
<th>( H_1 )</th>
<th>( \frac{V_1}{M_p/L} )</th>
<th>( H_{11} )</th>
<th>( \frac{V_{11}}{M_p/L} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Hinge ( P_1 )</td>
<td>0.417</td>
<td>1.000</td>
<td>2.397</td>
<td>0.799</td>
<td>+2.749</td>
<td>+4.769</td>
<td>+3.947</td>
<td>+4.818</td>
</tr>
<tr>
<td>( \Delta P_2 )</td>
<td>0.874</td>
<td>0.026</td>
<td>0.030</td>
<td>0.010</td>
<td>+0.038</td>
<td>+0.068</td>
<td>+0.053</td>
<td>+0.053</td>
</tr>
<tr>
<td>2nd Hinge ( P_2 )</td>
<td>2.427</td>
<td>0.309</td>
<td>0.030</td>
<td>0.069</td>
<td>+2.787</td>
<td>+4.837</td>
<td>+4.000</td>
<td>+4.871</td>
</tr>
<tr>
<td>( \Delta P_3 )</td>
<td>0.592</td>
<td>0.033</td>
<td>0.056</td>
<td>0.019</td>
<td>-0.028</td>
<td>-0.012</td>
<td>0</td>
<td>+0.096</td>
</tr>
<tr>
<td>3rd Hinge ( P_3 )</td>
<td>2.483</td>
<td>0.828</td>
<td>0.517</td>
<td>0.172</td>
<td>+2.759</td>
<td>+4.966</td>
<td>+4.000</td>
<td>+4.967</td>
</tr>
<tr>
<td>( \Delta P_4 )</td>
<td>1.000</td>
<td>0.517</td>
<td>3.000</td>
<td>1.000</td>
<td>+2.501</td>
<td>+6.000</td>
<td>+4.000</td>
<td>+6.000</td>
</tr>
</tbody>
</table>

TABLE 4

DEFLECTIONS OF AUXILIARY STRUCTURES

<table>
<thead>
<tr>
<th>Structure</th>
<th>( \frac{P L^2}{EI} )</th>
<th>( \delta_{h4} )</th>
<th>( \delta_{l10} )</th>
<th>( \delta_{v7} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structure I</td>
<td>-0.0121</td>
<td>+0.0165</td>
<td>+0.0428</td>
<td></td>
</tr>
<tr>
<td>Structure II</td>
<td>-0.0015</td>
<td>+0.0728</td>
<td>+0.1114</td>
<td></td>
</tr>
<tr>
<td>Structure III</td>
<td>+0.0809</td>
<td>+0.2800</td>
<td>+0.2986</td>
<td></td>
</tr>
<tr>
<td>Structure IV</td>
<td>+0.0069</td>
<td>+0.3071</td>
<td>+0.4501</td>
<td></td>
</tr>
</tbody>
</table>

TABLE 5

DEFLECTIONS OF FRAME AT SUCCESSIVE STAGES

<table>
<thead>
<tr>
<th>Deflection ( \frac{P_u L^2}{EI} )</th>
<th>( \delta_{h4} )</th>
<th>( \delta_{h10} )</th>
<th>( \delta_{v7} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Hinge ( P_1 )</td>
<td>-0.0096</td>
<td>+0.0132</td>
<td>+0.0342</td>
</tr>
<tr>
<td>( \Delta P_2 )</td>
<td>0.0000</td>
<td>+0.0007</td>
<td>+0.0011</td>
</tr>
<tr>
<td>2nd Hinge ( P_2 )</td>
<td>-0.0096</td>
<td>+0.0139</td>
<td>+0.0353</td>
</tr>
<tr>
<td>( \Delta P_3 )</td>
<td>+0.0015</td>
<td>+0.0053</td>
<td>+0.0056</td>
</tr>
<tr>
<td>3rd Hinge ( P_3 )</td>
<td>-0.0081</td>
<td>+0.0192</td>
<td>+0.0409</td>
</tr>
<tr>
<td>( \Delta P_4 )</td>
<td>+0.0012</td>
<td>+0.0529</td>
<td>+0.0775</td>
</tr>
<tr>
<td>4th Hinge ( P_u )</td>
<td>-0.0069</td>
<td>+0.0721</td>
<td>+0.1184</td>
</tr>
</tbody>
</table>
### Table 6 - Section Properties of 12WF36 Shape

<table>
<thead>
<tr>
<th>Material</th>
<th>Area (in²)</th>
<th>Depth (in.)</th>
<th>Flange Width (in.)</th>
<th>Flange Thickness (in.)</th>
<th>Web Thickness (in.)</th>
<th>Axis x-x</th>
</tr>
</thead>
<tbody>
<tr>
<td>Handbook</td>
<td>10.59</td>
<td>12.24</td>
<td>6.565</td>
<td>0.540</td>
<td>0.305</td>
<td>280.8</td>
</tr>
<tr>
<td>Measured</td>
<td>10.78</td>
<td>12.30</td>
<td>6.625</td>
<td>0.514</td>
<td>0.337</td>
<td>282.1</td>
</tr>
<tr>
<td>% Variation</td>
<td>+1.79</td>
<td>+0.49</td>
<td>+0.91</td>
<td>-4.81</td>
<td>+10.50</td>
<td></td>
</tr>
</tbody>
</table>

|               |            |             |                    |                        |                     |          |
|               |            |             |                    |                        |                     |          |

### Table 7 - Summary of Tensile Coupon Test Results

<table>
<thead>
<tr>
<th>Coupon Number</th>
<th>Material</th>
<th>Location</th>
<th>Average Coupon Test Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-2</td>
<td>12WF36</td>
<td>Flange</td>
<td>σ_y (ksi)</td>
</tr>
<tr>
<td>P-4</td>
<td>12WF36</td>
<td>Flange</td>
<td>33.7</td>
</tr>
<tr>
<td>P-9</td>
<td>12WF36</td>
<td>Web</td>
<td>34.2</td>
</tr>
<tr>
<td>P-10</td>
<td>12WF36</td>
<td>Web</td>
<td>37.7</td>
</tr>
</tbody>
</table>
$EI = \text{constant}$
$L = 20 \text{ feet}$

FIG. 1. LOADS & DIMENSIONS OF FRAME

$M_P \frac{\theta}{2} \left[ \frac{5}{4} + 2 + \frac{7}{4} + 1 \right] = PL \theta \left[ \frac{5}{4} \cdot \frac{5}{4} + \frac{5}{4} \cdot \frac{5}{4} + \frac{5}{4} \cdot \frac{5}{4} + \frac{5}{4} \cdot \frac{5}{4} \right]$

$6M_P = \frac{32}{16} PL$
$M_P = \frac{1}{3} PL$

FIG. 2. MECHANISM 1
\[ M_p \Theta \left[ \frac{3}{4} + 2 + \frac{9}{4} + 1 \right] = PL \Theta \left[ \frac{3}{4} + \frac{3}{4} + \frac{5}{4} + \frac{1}{4} + \frac{3}{4} + \frac{3}{4} \right] \]

\[ 6 M_p = \frac{32}{16} PL \]

\[ M_p = \frac{1}{3} PL \]

**FIG. 3. MECHANISM 2**

**FIG. 4. ULTIMATE LOAD MOMENT DIAGRAM**
\[ M_p \theta (1 + 2 + 3) = PL \theta \left( \frac{1}{4} \cdot 2 + \frac{3}{4} \cdot 2 \right) \]

\[ 12M_p = 4PL \]

\[ M_p = \frac{1}{3} PL \]

**FIG. 5 - SYMMETRICAL MECHANISM FOR VERTICAL LOADS**

\[ M_p \theta (\frac{5}{4} + 2 + \frac{7}{4} + 1) = PL \theta \left( \frac{5}{4} \cdot \frac{1}{4} + \frac{5}{4} \cdot \frac{3}{4} + \frac{3}{4} \cdot \frac{3}{4} + \frac{1}{4} \cdot 1 \right) \]

\[ 6M_p = \frac{32}{16} PL \]

\[ M_p = \frac{1}{3} PL \]

**FIG. 6 - UNSYMMETRICAL MECHANISM FOR VERTICAL LOADS**
FIG. 7 - WELD DETAILS OF KNEES AND COLUMN BASE PLATES

FIG. 8 - WELD DETAILS OF RIDGE JOINT
FIG. 9 - PREDICTED LOAD-DEFLECTION CURVE OF THE RIDGE
$0.417 \frac{P_1}{L} = M_p$

$P_1 = 2.397 \frac{M_p}{L}$

**FIG. 10 - ELASTIC RANGE MOMENT DIAGRAM IN TERMS OF PL**

**FIG. 11 - MOMENT DIAGRAM WHEN FIRST PLASTIC HINGE IS REACHED AT LEE KNEE**
0.874\Delta P_L = 0.026 M_P

\Delta P_L = 0.030 \frac{M_P}{L}

FIG. 12 - MOMENT DIAGRAM OF STRUCTURE II

IN TERMS OF PL

FIG. 13 - INCREMENTS OF MOMENTS FOR STRUCTURE II IN TERMS OF \( M_P \)
FIG. 14 - TOTAL MOMENT DIAGRAM WHEN THE SECOND HINGE IS REACHED AT LEE BASE

0.592 \Delta P_3 L = 0.053 M_P
\Delta P_3 = 0.056 \frac{M_P}{L}

FIG. 15 - MOMENT DIAGRAM OF STRUCTURE III IN TERMS OF PL
FIG. 16 MOMENT DISTRIBUTION SOLUTION OF STRUCTURE III (PAGE 1 of 3)
FIG. 17 MOMENT DISTRIBUTION SOLUTION OF STRUCTURE III (PAGE 2 of 3)
Let $A = \text{Multi. Factor of (a) Forces}$. Let $B = \text{Multi. Factor of (b) Forces}$.

\[\sum H_4 = 0 \quad 3.038P + 8.386PA - 3.340PB = 0\]
\[\sum H_{10} = 0 \quad -3.570P - 3.346PA + 2.041PB = 0\]

\[
\begin{align*}
A &= 0.963 \\
B &= 3.329
\end{align*}
\]

(c) **Simultaneous Equations for Restraining Force Corrections**

<table>
<thead>
<tr>
<th>Moments Due To Loads</th>
<th>Joint 1</th>
<th>4</th>
<th>7</th>
<th>4-7</th>
<th>7-10</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) x A</td>
<td>0.019</td>
<td>-0.122</td>
<td>-0.251</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b) x B</td>
<td>-1.451</td>
<td>+0.976</td>
<td>-0.586</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Sigma) Mom. PL</td>
<td>-0.713</td>
<td>-0.587</td>
<td>+0.708</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(d) **Final Corrected Moments**

}\[
\begin{align*}
\text{When } \Delta x \text{ was induced at Joint 4,} \\
\text{F.E.M. } &= 2PL = \frac{24EI}{L^2} \Delta x \\
\therefore \Delta x &= \frac{PL^3}{12EI} \\
\text{Actual } \delta_{H4} &= A \cdot \Delta x = 0.963 \cdot \frac{PL^3}{12EI} \\
\delta_{H4} &= 0.0802 \frac{PL^3}{EI} \\
\delta_{H10} &= 0.277 \frac{PL^3}{EI} \\
\end{align*}
\]

(e) **Corrected Horizontal Deflection**

From triangles of displacements (previous page), vertical deflection of ridge:

\[
\delta_v7 = \frac{3}{2} (\Delta y - \Delta x) = \frac{3}{2} (0.277 - 0.080) \frac{PL^3}{EI}
\]

\[
\delta_v7 = 0.296 \frac{PL^3}{EI}
\]

(f) **Vertical Deflection of Ridge**

FIG. 18 MOMENT DISTRIBUTION SOLUTION OF STRUCTURE III (PAGE 3 of 3)
FIG. 19 - INCREMENTS OF MOMENTS FOR STRUCTURE III IN TERMS OF $M_p$.

FIG. 20 - TOTAL MOMENT DIAGRAM WHEN THE THIRD HINGE IS REACHED AT WINDWARD KNEE.
FIG. 21 - TOTAL MOMENT DIAGRAM WHEN THE LAST HINGES ARE FORMED AT INNER LOADING POINTS

\[ \theta_{NF} = \theta_{NF}^' + R_{NF} + \frac{1}{3EI} (M_{NF} - \frac{1}{2} M_{FN}) \]

For sloping member use inclined length for a and b and the component perpendicular to the member for F in the formula for \( \theta_{NF}^' \).

FIG. 22 - NOMENCLATURE FOR SLOPE-DEFLECTION EQUATION
Note: $\delta_{H4}$ in the actual structure is to the left and would be negative. Assumed deflection is shown positive.

FIG. 23 - DEFLECTION AT ULTIMATE LOAD
FIG. 24 - TEST SET-UP
25(a) Enlarged View of Mirror Gage

FIG. 25 - DEFLECTION MEASUREMENT
Rotation = \frac{R_1 + R_2}{d} \text{ radians}

Curvature = \frac{R_3 + R_4}{bd} \text{ radians/inch}

FIG. 26 - ROTATION MEASUREMENT

FIG. 27 - LATERAL FORCE MEASUREMENT
FIG. 28 - DEFLECTION AND ROTATION MEASUREMENT AFTER EACH LOAD INCREMENT

FIG. 29 - DEFLECTED SHAPE OF THE FRAME WHEN ULTIMATE LOAD WAS REACHED
FIG. 30 - COMPARISON OF LOAD-DEFLECTION CURVES
FIG. 31 - ROTATION AT LEE KNEE
FIG. 32 - ROTATION AT LEE BASE

FIG. 33 - ROTATION AT WINDWARD BASE
Plastic Design
Working Load

Elastic Design
Working Load
($\sigma_{\text{max}} = 20 \text{ ksi}$)

Predicted Yield Load

FIG. 34 - ROTATION AT WINDWARD KNEE
FIG. 35 - ROTATION AT POINT 6

Rotation in Radians per Inch

FIG. 36 - ROTATION AT POINT 8

Rotation in Radians per Inch

Load P in kips

0 0.0005 0.001 0.0015

Last Hinge

Last Hinge
FIG. 37 - AFTER THE TEST, THE LEWARD KNEE (FIRST PLASTIC HINGE) SHOWED CONSIDERABLE YIELDING BUT NO BUCKLING

FIG. 38 - THE WINDWARD KNEE AFTER TESTING
FIG. 39 - YIELDING WAS ALSO VISIBLE AT THE LEEWARD INNER LOADING POINT (ONE OF THE TWO LAST PLASTIC HINGES)

FIG. 40 - THE WINDWARD INNER LOADING POINT AFTER TEST
FIG. 41 - MEASURED LATERAL FORCES IN PURLINS