PLASTIC DESIGN OF STEEL STRUCTURES

by

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The research on steel structures has been focused on the behavior of these structures beyond their yield points. In a program which has brought to light new knowledge concerning the behavior of these structures beyond their yield points, the results of these years of research and endeavor are now available in this latest work. The research conducted at Lehigh University and the Pennsylvania State University on the design of steel structures in the recent years has been focused on the solution of the structural problem of a group of members interconnected to satisfy the geometric and material properties of the system. The work will be detailed more in and off this paper the selection of the best possible design for the problem at hand. In the design of steel structures, such factors as the reserve strength manifested during such plastic deformation are considered but not more the yield point. However, it is possible to load members for a stress beyond the elastic limit under the assumption that they may deform plastically without failure. Furthermore, it is possible to utilize the intact behavior of the region where they may deform plastically and not beyond the yield point. However, it is possible to load members to be stressed only in the elastic region of safety, assuming that the elastic limit is far above the yield point. This means that the steel members are stressed up to the allowable stresses which incorporate a factor of safety against the elastic limit. The present methods of determining nearly all steel structures are based upon allowable stresses which incorporate a factor of safety against the elastic limit.
Strange to say, as plastic design is put into practice the "sidewalk superintendent" will see nothing different in the structure being erected than beforehand. The building frame will look just the same as one designed by conventional elastic methods. To the engineer, however, it will mean a difference because he will have designed it in less time; and to the owner it will mean economy because plastic design requires less steel. Authorities charged with the responsibility for checking design should see a difference, too, since designs can be checked more rapidly. Thus, although it may not be evident in a dramatic way to the average individual, plastic design will mean that a better use will be made of the world's natural resources and of engineering manpower.

Although improved design based on a better understanding of structural behavior is a desirable goal, a new technique will usually find adoption only if it can be proven to be economical. It is from that point of view that this paper is prepared.

Iron and steel once had to themselves the field of structures too large or difficult for timber construction; and even when timber was a competitor, steel "had the edge" because of fire resistance and long life. Competition in those days was not so much between rival materials but rather between rival fabricators of structural steel. Few cities, states or even railroads had engineering staffs so competent to design their bridges as the staffs of the fabricators. Numerous scanners of bridges and industrial buildings were contracted for through lump sum bidding based on competitive designs prepared by the fabricators.
Early in this century reinforced concrete entered as a serious competitor. Instead of fabricator against fabricator it became the steel fabricator against the cement maker.

This competition called for stabilization for safety's sake. Independent engineers took over more and more of the designing. The American Railway Engineering Association enacted to set minimum standards for bridges, the Portland Cement Association for concrete construction, and the American Institute of Steel Construction for buildings. Missionaries from the steel industry vied with missionaries from the cement industry in the effort to convince the now independent designing engineers of the superiorities of the one or the other material.

Within the last ten years "pre-stressing" of concrete has been developed, a procedure by which the steel reinforcement is placed under tension before the structure goes into service. This new procedure is gaining at a rapid rate and is going to replace more and more tonnage of structural steel, giving back only a minor replacement in bars and wire.

From this introduction it will be appreciated that it becomes necessary more and more to establish methods of design which will keep, or make, steel fully competitive without resorting to over-design. The steel industry is now discovering that plastic design means part of this need. Although plastic design cannot reduce the weight of all steel structures, in a particular and rather important category it can and will do just that. An "average" saving for a practically designed structure will run from 15 to 20% of the weight of the steel frame as compared with one designed by the current "elastic" design methods. It should be noted at this point that even though the tonnage per job will thus be less,
the number of steel jobs will increase. Lowering the cost of steel in a given structure will make it possible to build more structures out of steel.

**Structural Design**

The objective of the designer of a steel structure is to select members which, when joined together, will safely support the load and will incorporate maximum possible economy. In realizing this goal, one of the designer’s biggest problems is to compute the forces that exist in the structure. The ease or the difficulty with which these calculations are made depends to a great extent on the type of structure.

Engineers divide steel structures into two categories: "simple" and "continuous". In the former the members are supported in such a way that the ends are free to bend; so a simply-supported beam might be termed a "free" beam, being discontinuous at the supports. Continuous structures are so-named because the beams are not free at the supports, but one is a continuation of the other. Fig. 1 illustrates this distinction for a series of beams that must carry load across several spans. In case (a) as load is applied to the left span the ends are free to bend at both supports. In case (b) the two members are joined together into one continuous unit, the member to the right helping the other to carry the load by bending. As a consequence, the deflection of the left span is less (the dotted line shows the behavior of the "free" or discontinuous beam), and it follows that a lighter beam would support the given load.

In case (a) of Fig. 1, the load can be carried down to the supports by only one path, and the computation of the resulting...
forces is very simple. In case (b) the left-hand beam is effectively "fixed" or clamped to the right-hand beam at the center support. In this latter case the magnitude of the support forces cannot be computed easily because a part of this load is also supported by the fact that one beam is clamped to the other. Thus, although continuous structures will be lighter than discontinuous ones, the penalty in conventional elastic design is a more complicated set of calculations to determine the support forces.

Figure 2 shows a similar comparison, but for a building frame. Structure (a) represents the "simple beam" type of construction, the beams resting on cantilevered to the columns. Of course, additional framing angles would be added to prevent the beams from twisting off their supports, and diagonal bracing keeps the columns vertical. Beam sizes may be determined simply. The beams in the lower structure (b), on the other hand, are not only supported at the ends but are rigidly attached by welding or otherwise to the columns, giving continuity to the whole structure and enabling the designer not only to omit the bracing but also to use lighter member sizes. However, since all parts of this frame are attached together, each member participates in supporting every load; and thus the forces at the connections are unknown. Therefore, in using the existing method of elastic design the designer must set up likely trial sizes for all the elements, calculate the stresses in them by rather elaborate methods, and then correct the design if the calculated stresses are too large, or too small, for the members assumed.
European engineers have in general preferred to use statically
designed continuous structures, being willing to spend the extra time to
make the calculations which according to the preceding discussion appear
to be quite involved. One of the arguments has been that if a member or
support in a discontinuous structure is damaged by an accident, there is
no path left to carry the load and the structure must collapse. On the
other hand, damage to a continuous structure is less catastrophic because
there is more than one path to carry the load. While it may be grossly
overstressed, the building will probably stand up until repairs can be made.

Figure 3 gives graphic evidence of this advantage of continuity. Even
though both damage knocked out an entire corner column, continuity saved
the rest of the structure until that column could be replaced. Figure 4 (a)
shows in a dramatic way the failure that would have been expected had
the beams been freely supported at their ends. The beams and the loads
they support would simply collapse in a pile of rubble. Fig. 4 (b) shows
on the other hand how the adjoining parts of a continuous structure
defend to prevent the collapse that otherwise would occur due to failure
of one member. Now, even the most cautious engineer would not have
designed the building to stand up without the support of a corner column.
So it is evident that our conventional design methods can be wasteful of
steel. And this is one of the reasons that engineers have been exploring
the practicability of plastic design.

Even though it requires more material, American engineers on the
other hand have rather favored "one-path" (that is, "simple") structures.
This is because they could be designed in a fraction of the time. Today,
in the interest of security and efficiency, this preference bids fair to
The argument for plastic design arises from just the argument cited above for continuous structures. If one possible stress path or element of resistance is overstressed and commences to yield, let it yield. It will only throw more load, from that time forward, onto another path or element; but it will keep right on carrying the forces under which it yielded. For that is the great security factor in structural steel; after it reaches yield point stress, with additional loading it will gradually flow, but without any change in the stresses that made it yield. Keep increasing the load until all the possible stress paths or stress-resisting elements have yielded, and only then the ultimate failure load will be reached.

Under past concepts of steel design it was considered unsafe to increase the load beyond that at which the first element commenced to yield. This is entirely appropriate for a structure such as the building in Fig 1a; under any further loading, nothing comes to the aid of the yielding element, it continues to flex without limit, and the member will collapse. Such a concept is not correct, however, for continuous structures like the building frame of Fig 2(b). As long as there are critical elements that are not yet yielding, the ultimate or failure load has not been reached. Plastic design permits the engineer to utilize this substantial increment of strength that exists between the yield point and the ultimate load. In other words the plastic design criterion is the ultimate load the structure will carry as distinct from the point of first yield.

Plastic design is therefore a concept that increases the "tolerable" load on a continuous structure because of the finite margin of plastic strength that exists in it. Therefore the result is lighter members than otherwise would be required. Of course, there is no intention of reducing the factor
of economy. A look at any of the older continuous or semi-continuous structures in civil engineering clearly shows that the economy of using steel in these structures can be kept in the same relative order as in the past for simple types of structures.

Even the economy of material that can be gained by using increasing the ultimate load through a consideration of plasticity would not greatly appeal if the cost of designing were so great as has previously been the case for continuous structures. The saving required for plastic design is that the computations turn out to be very crude, but this saving that with plastic design and no less accurate, usually the reason is that the critical stresses are singled out and assumed to be carrying just enough force to cause their yield. Instead of merely making these the actual points of load force and force that resulting post tension the continuous structure behaves as if it were a "single" one and is subject to the simpler type of calculations.

Thus the civil engineer has been stimulated to study the plastic strength of steel structures and to apply this knowledge to design. Not only because it is a more logical design but also not only because it is more economical in the use of steel, but also because it requires a substantial saving of time in the design office.

Thus far the foregoing has dealt with steel structures created in some detail.

Creasing the ability to produce test of a continuous structure based on the rolled angles, the resulting stress-Strain curves for both in Fig. 2. The deflection are very small up to the strain point, following the "Living of beam" the deformation continues even after the maximum load.

Thus the material will not fail by bending and the ultimate strength of the steel is not reached.
"tensile strength" as shown in the figure.

If the first portion of the stress-strain curve is examined in
closer detail (up to 2% elongation), the picture will look like Fig. 5a.
Here again, the stress is plotted vertically against the strain horizontally.
Following the elastic range, one sees the flat "plateau" that is characteristic
of the ductility of structural steel. The stress in this plastic range
is called the lower yield or "yield stress level," and the stress continues
at this value until the strains are about 15 times the strain at the yield-
point. Afterwards, increased strength is exhibited as the material
strain-hardens.

Ultimate load calculations in plastic analysis are made on the basis
of this yield stress level which, for structural steel, is taken to be the
specification tension of 33,000 psi. The increase in strength above the
plateau of ductility due to strain-hardening is neglected. Thus, according
to plastic design, at ultimate load the stresses in the member will not
s appreciably exceed the lower yield point of the steel. Also, at ultimate
load on a structure, the maximum strain will not have exceeded about 1.5%
elongation. For ordinary structural steel, final failure by rupture occurs
only after a specimen has stretched about 20 times the maximum strain that
takes place before strain-hardening sets in (Fig. 5a), and therefore a major
portion of the ductility still remains as an added reserve.

It now remains to be shown how this ductility, manifested in a tension
test, will manifest in a structure which utilizes beams and columns to support
their loads.

Figure 6 shows the different loads that may act on beams: namely,
a tension force, a compression force, a shear force, and finally a bending
moment. In the last case, a short length of beam is shown clamped at the
left end and is bent by the action of two equal and opposite forces, W,
separated by a distance j. The product of the force F times lever distance d produces bending and is termed a "moment"; it is usually designated by an arrow showing the direction in which the end of the member is being bent. Our attention is focused upon the bending moment; although tension, compression and shearing forces are also usually present, beams support their loads primarily by bending.

In considering the action of a beam under bending moment, we will first simplify the problem and, as shown in Fig 7a, will use a rolled I-beam in which the top and bottom flanges contain all of the bending moment. It is assumed that the web does not participate in supporting the load. As the bending moment is applied to a length of this beam (Fig 7b), the compression flange will shorten and the tension flange will lengthen an equal amount as shown in Fig 7c. (For simplicity the flanges are shown straight instead of curved). Since the force acting on the tension flange is exactly like that which exists in the tension test specimen, its behavior will be similar to that which was shown in Fig 3b. Therefore, the force in the top flange may be plotted against the elongation as shown in Fig 7d, neglecting the upper yield point and strain-hardening. Remembering that equation 3 is valid for forces three times leverage distance (F x d), the curve of moment versus angle of bending may now be plotted, Fig 7e. In this figure the angle of bending, \alpha, is a measure of the amount of bending action and is directly related to the flange elongation, Fig 7c. Thus ductility in a bending member reflects a behavior that is very similar to that in the tension test specimen.

Considering next an actual rolled WF or Z-beam, its action under bending moment is shown by the heavy solid line in Fig 8. By comparison with the moment-arc-angle of bending curve of the simplified I-beam which only consisted of flanges (reproduced as a dotted line in Fig 8 as obtained from Fig 7a), it is evident that the assumption was not so unrealistic as might
First as supposed. Although yielding of a typical W beam actually
starts at about 90% of the ultimate value, no plastic deformation continues
its at each rapidly otaining the "full plastic moment," $M_p$. Thereafter
rotation continues at this constant moment.

Figures 2b, 3b, and 4c show how this short length of a real beam
bends at the bottom of the beam gradually increases and also shows
(below) the corresponding strains. Since the deformations are very
small, the bending is shown on exaggerated scale. In an early stage of
loading (Fig 4b) the eccentric couple creates a pull, or tensile stress,
above the center of the beam, and a thrust or compressive stress below the
center. These stresses vary in intensity as shown by the sloping line
in the stress pattern that0at works at the center to maximum at top and bottom.
In Fig 4b it is assumed that the moment has been increased until the two
maximum stresses have reached the yield point of the steel. As the loads
are still further increased, these top and bottom fibers will continue
to stretch and shorten, respectively, but they will not accept any stress
because they have passed the yield point. The stresses are in the plastic
region and the magnitudes of the stress is equal to the yield stress level.
Therefore further increase of load will result must be picked up by the interior
fibers which have not yet yielded. The process of yielding will continue
then, from top and bottom of the beam towards the center (an intermediate
stage being shown dotted in Fig 4c), until the final condition of Fig 4c
is reached. All of the fibers are now under the same stress, namely yield
point stress, and they cannot take any more. The corresponding moment is
the full plastic value, $M_p$, and rotation is constant around this moment.
Plastic Hinge

One of the fundamental concepts in the plastic theory is the "plastic hinge." This concept is suggested by the moment-rotation (angle of bending) curve shown in Fig. 6 where in there is no change in moment as the plastic angle of bending increases. The comparison is illustrated in Fig. 9. If the member is supported by a frictionless pin, then theoretically no moment is required to rotate it; the behavior is that of an ordinary hinge as shown in sketch (a) in that free rotation is possible ($M = 0$ equals zero). If the member is clamped into a wall, then the rotation depends on the amount of moment that is applied, sketch (b). When the moment reaches the limiting value of $M_p$, the member is once more free to rotate as shown in sketch (a), except that it is restrained by the plastic moment.

So the plastic hinge may be defined as a yielded zone in a structural member which rotates at $M_p$ pinned except that it is restrained by the plastic moment $M_p$. With the aid of "whitewash" to reveal the fiction of wall ends when the yield point is reached, plastic hinges also may be represented pictorially. An example is given in Fig. 10, showing a 16W36 beam connected to an I133 column. Since the moment is greatest at the column connection, the plastic action (dark area) in the greatest there, decreasing as the distance from the column increases.

It should now be evident why the attainment of yield-point stress does not correspond to failure. At the point where the moment is greatest, a zone of yielding develops, a plastic hinge occurs, and the less heavily-stressed portions are called upon to carry further increase in load. Eventually, when enough plastic hinges form, the structure will reach the ultimate load and fail by continued deflection at that load.
Not only do the hinges form at known predictable points, but (as is shown by Fig. 2), the magnitude of the plastic hinge moment is also a fixed, known quantity and this fact simplifies the analysis. These plastic moment values have been tabulated for each rolled WF and I-shape, (1, 3).

The argument will now be summarized in an illustration embodying the continuation of the calculations by a large-scale laboratory test. Figure 11 is typical of a popular type of beam and column structure known as a rigid frame, much used for warehouses, drill halls, and auditoriums. Its joints are so fabricated as to be rigid, and the structure is thus continuous at the stresses in any one member depending on the relative sti/1ness of the other members.

The indicated loads cause bending action (measures) at each of the connections between the sloping girders and the posts. As the intensity of the loads gradually increases, the moment in the beam just outside the right-hand corner (Section 10) increases, and its stresses change just as was indicated in Fig. 6a, 6b, and 6c. Until stage 6c is approached, the corresponding deformations will scarcely be visible to the eye. After this point, further increase of load will not encounter any more resistance at the right connection. If it had been a simple or non-path structure (like the beam of Fig. 10), under further increase of load it would simply deflect without limit and then be completely useless. But in this continuous frame, further loads will be resisted by the bending resistance at other points in the frame where the moment is not yet up to the magnitude needed for yielding.

As a matter of fact, the ultimate load for the frame of Fig. 11 will be reached then, and not until then, plastic hinges have formed at both connections at the beam, at the right-hand column base, and in the members near the ridge. The corresponding positions, 4, 10, 21, 6, and 3 have been circled in Fig. 11.
Figure 12 shows an actual test of this same rigid frame, the frame being at the ultimate load condition when the photograph was taken. The ultimate load, that under which the structure continued to deflect while the dial showed no further increase of load, was within two percent of that computed beforehand by plastic analysis from coupon tests of the steel. The corresponding load-versus-deflection curve obtained during the test is shown in Fig 13. It was noted earlier that an adequate margin of safety is specified for plastic design, and this value is selected on the basis that a plastically designed structure will be at least as safe as the corresponding elastic design of a structure using simply-supported beams. The resulting working load for plastic design of this frame is shown in Fig 13. When compared with the elastic design working load, it is quite evident that more load may be allowed by the plastic method. It is also clear that the deflections of the plastically-designed frame are well under control.

Another continuous frame is shown under test in Fig 14. Rolled shapes were used for this two-span structure. The left span was 20-ft. long, and the right span was 30-ft. in length.

An illustration of some of the concepts described up to this point is presented in Fig 15. A beam is to be designed to support a total load of 21,000 lbs. on a span of 30'. If the ends are freely supported (case 1) an I3WF50 shape is required, an 18-in. wide-flange beam weighing 50 lbs. per foot. The corresponding load-deflection curve is also shown. If, now, the beam is made continuous (represented by the "built in" condition), then
past elastic design procedures would prescribe a smaller beam, namely a 16x36 shape. Quite evidently, if the freely supported beam has an adequate reserve of strength (compare \(P_w\) with \(P_T\)) then there is an unnecessary reserve of strength and corresponding waste of material for the elastic design (II). Finally, if plastic design is used and advantage is taken both of continuity and plasticity, then the beam size may be reduced even further to a 16x36 shape (case III) with its corresponding load-deflection curve. The things to note about the results are:

1. There is a weight saving of 20% for the plastic design as compared with the elastic design of the continuous beam, and a 67% saving as compared with the simple beam design.

2. The plastic design (III) is just as safe as the simple beam design (I) that has proven satisfactory from long years of experience.

3. At working load \(P_w\) the plastically-designed beam is still in the elastic range.

4. The deflection at working load for the plastic design (III) is less than that of the simply-supported beam (X) and is well within the "specification" limit shown in the figure.

Plastic design is therefore an advantageous replacement for conventional elastic design as applied to statically loaded steel structures. These include rigid-jointed frames, continuous or restrained beams and girders, and in general those structures that are stressed primarily in bending. Obviously it is not intended that plastic design
will replace all elastic design. For example, the procedure is not recommended for structures that are essentially pin-connected; nor would it be appropriate for structures in which the total load is applied, removed, and then reapplied a large number of times producing fatigue failure. However, an important percentage of building frames are such that plastic design will be both safe, economical, and efficient when applied to them.

Prior to considering a few of the many structures that have already profited from the plastic design method, a word should be said about the so-called "limitations". No progress can be made in developing a new design technique by glossing over some factor that might be a deterrent to its application. It will be noted, for instance, that nothing has been said in this discussion about buckling of columns, about the influence of shear force, or about the influence of axial force. Of course, such factors must be taken into account, but it should be kept in mind that the necessity for considering them is no different in principle from present elastic design procedures. After the engineer has selected the required sizes of members he must always check the design for such things as shear force, axial force, buckling, a deflection limit (if one exists), and must proportion connections to transmit the required moments. In the research at Lehigh University prime importance has been given to evaluating all such "secondary design considerations", and as a result guides have been developed for the designer's use in making certain that the structure will perform its intended function.1, 2, 3 These guides have been verified not only theoretically but also by laboratory tests.
About six hundred industrial single-story frames have been designed in England by the plastic method—also a school building, a four-story and a five-story office building. (7) Figures 16 and 17 show two of these. On this continent the first building to be designed plastically was in Canada. It was a two-story frame with beams continuous over 6 spans.

At least a score of plastically-designed structures have been built in this country despite the handicap of very outdated building code provisions. Figure 18 shows one of these during erection. The span of the frames of this warehouse in Sioux Falls, South Dakota, is 80-ft. The completed warehouse is shown in Fig 19. The plastic design required a 240RS4 shape, uniform throughout. By comparison, a conventional elastic design would have required a 3039108 shape showing a saving of about 15% in structural steel in favor of the plastic design.

Although plastic design presently finds its main application in structures with rolled shapes, it is also applicable to structures built up of plate material. Figure 20 shows one such application in the captured haunch connection. The haunches consist of portions of the frame that are built up from plate material in such a way that the thickest or deepest portions are located in the region of maximum forces. This makes it possible to use lighter members in the rest of the frame than otherwise would be required. The project at Lehigh University has produced a means for designing these haunches, taking plastic behavior into account. As confirmed by tests, the strength of these built up members can be correctly predicted by the plastic design developed.
In ships such as the USS Ranger (Fig. 21) some application of these concepts has already been made, and the results of current research should lead to even further applications. In the older carriers that proceeded the Forrestal class, the plastic design approach was used in the girder work supporting the flight deck. Aircraft were much smaller and lighter when the ships were first built; and in modernizing the carriers to accommodate the heavier planes, plastic design concepts were used to determine how much structural reinforcement was necessary.

The transverse frames of a ship are highly complex structures; but the methods of plastic analysis may be applied successfully to predict the required thickness of plating and stiffener sizes. Figure 22 shows the corner of such a transverse frame being tested at approximately half scale at the David Taylor Model Basin. The proportioning of diagonal stiffeners to assure adequate strength and stiffness has been one of the products of the research at Lehigh University. Plastic analysis will also predict the most effective location for access openings; and fortunately this coincides with the best location from an operational point of view (close to the deck).

There have been many instances in the past where the complexity of the structure was such that the only way to estimate the ultimate load was to perform a test. The failure load so obtained was then used as a basis for finding the safe working load. This is, in a sense, an application of plastic design principles but without the benefit of the theory. Examples of such a technique in the past are the design of submarine pressure hulls and the design of protective bulkheads in ships to absorb underwater shock.
Since the plastic methods of structural analysis form the only basis for determining the true ultimate load, we can expect that more and more of the large scale testing will be done to substantiate theory. Engineers, in turn, will find more and more opportunity to utilize knowledge of the plastic behavior of steel structures in each solution of their design problems.

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FIGURE LEGENDS

Figure 1. Two types of structures: (a) "simple" or free to bend, and (b) "continuous" or joined together at the supports. In comparison with type (b), type (a) is discontinuous.

2. Building frame with (a) simple beam (discontinuous) type of construction and (b) fully continuous construction.

3. View of building with entire corner support knocked out by bomb blast. Collapse of overhanging structure was prevented by virtue of continuity. (Photograph, American Institute of Steel Construction)

4. (a) Diagrammatic representation of complete failure of overhanging structure when simple beams are used; note two fallen beams.

(b) Representation of action of a continuous structure to prevent collapse due to loss of support.

5. (a) Stress-strain curve of structural steel showing yield point and tensile strength.

(b) Initial portion of stress-strain curve of structural steel showing elastic, plastic, and strain-hardening curves.

6. Forces that may exist in beams.

7. Simplified action of a beam under bending: (a) simplified I-beam (not drawn to scale); (b) forces acting on flanges; (c) change of length of tension and compression flanges; (d) force-vs-elongation curve for tension flange; and (e) idealized moment-vs-angle change relationship.
Figure 2: Typical moment-rotation curve for WF beam. Under gradual increase of moment, stresses are first elastic (sketch a), reach the elastic limit at extreme fibers (b), and eventually all stresses are at yield value (sketch c).

Action of a plastic hinge compared with that of an ordinary hinge.

Photograph of a beam-column connection showing plastic hinges (dark areas) in the beams. Use of whitewash reveals flaking of mill scale as a result of plastic yielding.

Welded rigid frame as loaded to simulate actual conditions of vertical and wind loading. Circles indicate locations of plastic hinges. (4)

Welded rigid frame being tested to confirm plastic theory. Although frame is shown at ultimate load, deformations are not severe. Span is 40 ft. and shape is 126036. (4)

Load-ve-deflection curve of welded rigid frame, showing the remarkable agreement between observed and predicted ultimate load. (4)

Test of two-span continuous beam. (5)

Load-deflection relationship of three different designs to support a 22,000-lb load. The plastic design (III) uses the lightest beam, yet has the same strength as the simple beam (I) and exhibits less deflection at working load. (6)

Steel frame for British research laboratory designed by
Figure 17  Industrial building frame including large crane runway designed by the plastic method.

18  Frame of plastically designed warehouse in Sioux Falls, South Dakota hoisted into position after field welding. (3)

19  Completed warehouse. The span is 38 Ft. (3)

20  Plastically designed steel frame using braced connections built up of plate material with deeper portions accommodating large stress, these braces permit use of lighter beams.

21  Aircraft carrier USS Ranger. Some of the concepts of plastic design have been used in such vessels for deck reinforcement and transverse frame design (Official U. S. Navy Photograph).

22  Corner of transverse ship frame shown under test. Plastic analysis permits engineer to proportion members and locate bulkhead openings for greatest efficiency. (Official U. S. Navy Photograph).