RESIDUAL STRESS AND THE COMPRESSIVE PROPERTIES OF STEEL

Progress Report

RESIDUAL STRESSES IN WIDE-FLANGE BEAMS AND COLUMNS

by

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SYNOPSIS

The scope of the investigation upon which this report is based was to determine whether the magnitude and distribution of cooling residual stresses in wide-flange beams and columns are a function of shape, material properties and manufacturing conditions. Further the residual stresses due to cold bending were investigated, as well as the cooling residual stresses and the plastic deformation during fabrication.

This report shows that, within satisfactory engineering accuracy, the prediction of residual stresses is possible. Further, data on the residual stresses are presented.
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1. INTRODUCTION

When a structural member such as a WF-column or I-beam is subjected to uneven cooling, cold-bending or welding during production the finished product will have internal stresses, although no external forces are applied. If a member which contains residual stresses is loaded progressively, stresses due to the loads and residual stresses are superposed and certain portions of the cross-section will reach the yield point at a lower load than would be the case if no residual stresses were present. In the load-deformation relationship, the effect of residual stresses will be expressed by a higher rate of increase in deformations.

So far the observations have been very general. It has been shown that the ultimate strength of beams is not affected by the presence of residual stress, although deflections are increased near the ultimate load.\(^{(1,4)}\)

However, in columns, residual stresses lower the ultimate strength and for this reason their evaluation is of great importance.\(^{(1)}\) \(^{(9)}\) \(^{(11)}\)

There are several types of residual stresses:

(a) Cooling stresses in hot rolled members.

(b) Stresses due to fabrication operations such as cold-bending, ("gagging"), cambering, welding, etc.

In this paper the study will be limited to the formation of residual stresses due to permanent deformation during cooling and plastic deformation during fabrication, and to their magnitude and distribution. Cold bending processes
and their effects will be considered, and data will be shown for the estimation of the magnitude and distribution of the resulting residual stresses. Their influence on axially loaded columns will be considered in a later report. The reader is also referred to the list of references.*

*Beams are discussed in Ref. (1) (4) (5); Columns in Ref. (1) (9) (10) (12)
2. THEORY

2.1 General Derivation of Residual Stress Relationships in Beams

(a) Linear Stress - Strain Relation

If the fibers of a long beam are subjected to a non-linear strain distribution, an internal stress system is necessary to satisfy both equilibrium and compatibility. The corresponding strains, $\varepsilon$, could be due to such causes as uneven temperature distribution, or deformations beyond the elastic limit of the material. A solution will be sought only for the interior of the beam.

The assumptions for the derivation are the same as used in the strength of materials i.e. the material is homogenous and isotropic, the fibers follow the same stress-strain curve as the material exhibits in simple tension and compression, previously plane cross-sections remain plane after deformation of the beam and the stress-strain relationship is linear and identical in tension and compression.

Let us consider the fibers of a beam of arbitrary cross-section as shown on Fig. 1a. If the fibers are free to expand individually due to any of the causes listed above, the condition of continuity at the ends will be violated.

To make all fibers equally long, stresses will have to be introduced at the ends to counteract the various elongations. The strains will be constant along the length of each fiber (Fig. 1b). Now the fibers should be considered to act together. At the ends of the beam, however, boundary conditions require zero magnitude of stresses. To remove the
violation of the boundary conditions, stresses of opposite sign have to be superposed, which is a legitimate procedure in the theory of elasticity (5) (Fig. lc). According to St. Venant's principle these superimposed boundary stresses will become linearly distributed a sufficient distance away from the ends. (5) For the residual stress in the interior of the beam we may then write the following equation:

\[ \sigma_r = E \left[ \varepsilon - (a + bx + cy) \right] \]  

(1)

where the second term is the expression for a plane strain distribution. Equilibrium requires that the following set of equations be satisfied:

\[
\begin{align*}
\int_A \sigma_r \, dA &= 0 \\
\int_A \sigma_r x \, dA &= 0 \\
\int_A \sigma_r y \, dA &= 0
\end{align*}
\]

(2)

With the aid of the equations (2) we can find the three unknowns \( a, b, \) and \( c \) in equation (1).

\[
\begin{align*}
\int_A \sigma_r \, dA &= \int_A E \left[ \varepsilon - (a + bx + cy) \right] \, dA = 0 \\
\int_A E \varepsilon \, dA - a \int_A E \, dA - b \int_A E x \, dA - c \int_A E y \, dA &= 0
\end{align*}
\]

If we choose the coordinate system such that the last two integrals become zero (for \( E \) constant for all fibers the origin would/identical with the center of gravity of the section), "a" can be determined directly from the expression

\[ a = \frac{\int_A E \varepsilon \, dA}{\int_A E \, dA} \]

and \( O = \int_A E x \, dA = \int_A E y \, dA \)

From the second of equations (2)
\[
\int \sigma_r \times dA = \int A E \left[ \epsilon x - (a x + b x^2 + c y x) \right] dA = 0
\]

\[
\int A E \epsilon x dA - c \int A E x^2 dA - c \int A E y dA = 0
\]

If \( E \) is constant for all fibers, the last two integrals in equation (3) express simply the moments of inertia multiplied by \( E \).

Using the third of equations (2), another equation similar to equation (3) is obtained. Writing \( J_x = \int A E y^2 dA \), \( J_y \), \( J_{xy} \), the following simultaneous equations can be solved for \( b \) and \( c \).

\[
\int A E \epsilon x dA - b J_y - c J_{xy} = 0
\]

\[
\int A E \epsilon y dA - b J_{xy} - c J_x = 0
\]

Finally substituting \( a, b, \) and \( c \) back into equation (1)

\[
\sigma_r = E \left[ \epsilon - \frac{\int A E \epsilon dA}{\int A E dA} - \frac{(x J_x - y J_{xy}) \int A E \epsilon x dA + (y J_y - x J_{xy}) \int A E \epsilon y dA}{J_x J_y - J_{xy}^2} \right]
\]

Equation (4) is very similar to the general beam bending equation. For axial symmetry of the section and of the strain distribution, \( \epsilon \), only the first two terms of equation (4) remain, which means that no rotation of the strain plane occurs (Fig. 1d).

If \( \epsilon \) is linear of the form \( \epsilon = d + ex + fy \) equation (1) takes the form

\[
\sigma_r = E \left[ (d - a) + (e - b)x + (f - c)y \right]
\]

substituting into equation (2) \( a, b, \) and \( c \) can be determined. \( a = d \), \( b = e \), \( c = f \).

This result is important because it shows that no residual stresses will result from a linear distribution
of strains. There must be non-linearity of applied strain in order that residual stresses may form.

An important engineering material, steel, has a stress-strain relation that agrees well with the assumptions made. However, beyond the yield point the strains increase while the stress remains constant (Fig. 2). Unloading follows again a linear law at the same modulus of elasticity as in loading.

Next, the strain, $\varepsilon$, in equation (1) will be assumed to be such that the yield point was exceeded. Then equation (1) takes the form

$$\sigma_r = E \left( \varepsilon - (a' + b'x + c'y) \right) \quad E \varepsilon \leq \sigma_y$$

$$\sigma_r = \left[ \sigma_y - E (a' + b'x + c'y) \right] \quad E \varepsilon \geq \sigma_y$$

Equations (5) together with the equilibrium equation (2) will be sufficient to solve the problem. A closed general solution cannot be given without specifying the strain, $\varepsilon$, and the shape of the beam.

If $\varepsilon$ is assumed to be a linear bending strain, $E$ to be constant for the elastic region, and the x and y-axes to be axes of symmetry for the section, a relatively simple solution results (Fig. 3). Considering the applied stress to be made up of an elastic stress, $E \varepsilon$, minus the stress that is in excess of the yield point, $E \varepsilon'$, we can rewrite equation (4) as follows:

$$\sigma_r = E \varepsilon - \frac{Y}{I_x} (\int_A E \varepsilon \gamma dA - \int_{A_2} E \varepsilon' \gamma dA) \quad (\varepsilon \leq \varepsilon_y)$$

$$\sigma_r = \sigma_y - \frac{Y}{I_x} (\int_A E \varepsilon \gamma dA - \int_{A_2} E \varepsilon' \gamma dA) \quad (\varepsilon \geq \varepsilon_y)$$

(6)
where the first of equations (6) applies in the area, $A_1$, and the second in the area, $A_2$, of the cross-section. The fibers in the area, $A_2$, have been deformed permanently to the strain, $\varepsilon'$. Equation (6) can be simplified further, when it is recognized that:

$$\frac{y}{I_x} \int_A E \varepsilon' y dA = E \varepsilon$$

Since the left side represents elastic bending stress which is equal to $E \varepsilon$, then

$$\sigma_r = \frac{y}{I_x} \int_{A_1} E \varepsilon' y dA$$  \hspace{1cm} (\gamma \leq \gamma_c)

$$\sigma_r = -E \varepsilon' + \frac{y}{I_x} \int_{A_2} E \varepsilon' y dA$$  \hspace{1cm} (\gamma \geq \gamma_c) \hspace{1cm} (7)

Equation (7) indicates that only the permanent strain, $\varepsilon'$, causes residual stress. This result is important and will be useful in later developments.

(b) Non-linear Stress-Strain Relation

In order to study the influence of the stress-strain relation on the formation of residual stresses, due to bending, a general stress-strain law will be considered in this section. The basic assumptions are the same as before. Loading follows a non-linear law, but unloading is again assumed to be under a constant slope, $E$, a behavior typical for materials in engineering use. The discussion will be limited to beams where the $y$-axis is an axis of symmetry (Fig. 4). Similar to the previous development, we can write the residual stresses as the summation of applied stress, $\sigma(\varepsilon)$, axial stress, $\sigma_a$, and bending stress $\sigma_b$

$$\sigma_r = \sigma(\varepsilon) - \sigma_a - \sigma_b$$
where the axial stress is given by:

\[ \sigma_a = \frac{1}{A} \int_A \sigma(\varepsilon) \, dA \]

and the bending stress must satisfy the equation:

\[ \int_A \sigma(\varepsilon) \gamma \, dA = \int_A \sigma_b \gamma \, dA \]

In accordance with the assumption of linear bending strain distribution, the axial and bending strains are linear.

In the special case of beam bending followed by unloading, equation (8) can be written in a more specific form because \( \sigma_b \) will be linear, and

\[ \sigma_r = \sigma(\varepsilon) - \frac{1}{A} \int_A \sigma(\varepsilon) \, dA - \frac{\gamma}{I_x} \int_A \sigma(\varepsilon) \gamma \, dA \]  

The problem that was solved by equation (7) could also be solved by equation (9) since the stress-strain relation of Fig. 2 is non-linear for strains greater than \( \varepsilon_y \).

Application of the equations developed in this section will be made in the following sections.

2.2 Formation of Cooling Residual Stresses

Hot rolled steel beams and columns are widely used in many types of engineering structures. It is also known that such members contain residual stresses. Some of these stresses are due to non-uniform cooling during the production process. Others are due to fabrication operations, such as the stresses due to cold bending which will be treated in Section 2.3.
The behavior of a plate during cooling will now be analyzed. This will be followed by a presentation of typical examples of cooling residual stresses.

(a) Formation of Residual Stresses in a Plate

A long plate (cross section shown in Fig. 5) is cooled from a uniform "high" temperature, $T_0$. Residual stresses will be set-up due to the temperature gradient in the material. These stresses will be a function of time. Of particular interest is the distribution of residual stress after the cooling of the plate to a uniform temperature.

At first the temperature distribution must be known for any time, $t$. The assumption is made that the thermal diffusivity, $a^2$, and thermal conductivity, $k$, are independent of temperature. It is further assumed that Newton's law of cooling is applicable i.e. the rate of heat emission is proportional to the difference in temperature between the surface and the surroundings. (6) For convenience the latter can be taken as zero, since only relative temperatures are required.

The partial differential equation of non-stationary heat flow is

$$\frac{\partial T}{\partial t} = a^2\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$

(10)

The boundary conditions are

$$\left(\frac{\partial T}{\partial n}\right)_s = -\frac{c}{k} T_s = -h T_s$$

(11)

where $c$ and $h$ are constants, and $n$ is a direction normal to
the surface. The subscript \( s \) refers to the surface of the plate. The rate of heat emission per unit area is given by (6)
\[
\frac{dq}{dt} = c T_s = -K \left( \frac{\partial T}{\partial n} \right)_s
\]
For one-dimensional heat flow equation (10) can be solved readily. For two-dimensional flow a product solution will be used, which is made up of the solutions of one-dimensional flow for the \( x \) and for the \( y \) direction. We can then write
\[
T(x, y, t) = A T_1(x, t) T_2(y, t)
\]
where \( A \) is a constant.
Taking the partial differential of Eq. 12 on the one hand and on the other, substituting back into equation (10) we obtain:
\[
\frac{\partial T_1}{\partial t} T_2 + \frac{\partial T_2}{\partial t} T_1 = a^2 \left( \frac{\partial^2 T_1}{\partial x^2} T_2 + \frac{\partial^2 T_2}{\partial y^2} T_1 \right)
\]
Since \( \frac{\partial T_1}{\partial t} = a^2 \frac{\partial^2 T_1}{\partial x^2} \), and \( \frac{\partial T_2}{\partial t} = a^2 \frac{\partial^2 T_2}{\partial y^2} \), Eq. 13 is identically satisfied and equation 12 is a solution.
From equation (12) for \( t = 0 \) \( T_0 = AT_0^2 \) and \( A = \frac{1}{T_0} \)

A solution for one-dimensional flow is given by the series:
\[
T_i = \sum_{n=1}^{\infty} A_n \cos \omega_n x e^{-\omega_n^2 \alpha^2 t}
\]
In reference (11) the solutions for one-dimensional heat flow in infinitely long plates are tabulated. With the aid of these tables the temperature for any point can be obtained with relative ease from Eq. (12). In order to simplify
subsequent calculations the temperature distribution across the thickness was replaced by the mean temperature.

Now that the temperature distribution as a function of time has been determined the magnitude and distribution of residual stresses due to cooling can be investigated. Due to the temperature differences in the plate, non-linear strains will be produced which may be computed from the expression:

\[ \varepsilon = \int \alpha \frac{T}{T_0} \, dT \]

Of course, the material properties such as the modulus of elasticity, \( E \), the coefficient of linear expansion, \( \alpha \), and the yield point, \( \sigma_y \), must be known as functions of temperature. It is also assumed that stresses and strains are proportional up to the yield point and that from here on the material is ideally plastic. For the particular plate that is analyzed below, the assumed temperature relationships of the material properties are shown in Figures 6b, 6c and 6d. Equation (15) is readily evaluated for the linear relationship of \( \alpha \) shown in Figure 6c. From the temperature distribution, as plotted for a particular plate in Figure 6e, the strains, \( \varepsilon \), can be computed from equation (15). Figure 6f shows a plot of the strain distribution over one half of the plate width. The basic equation (4) for residual stresses was derived in section 2.1. This equation may now be used to calculate the residual stresses in the plate for any desired time. At first the strain distribution as shown in Fig. 6f
is expressed with the aid of Fig. 6d and 6g in terms of stresses as shown in Fig. 6h. Application of equation (14) will give a residual stress distribution as shown in Fig. 6i where the stress may exceed the yield value at some points. In this case a correction must be made which will result in a small shift of the base line. The procedure is the same as the one used for the derivation of equation (7). The stress exceeding the yield point is designated by $\Delta \sigma$ and the residual stress is then given by:

$$\sigma_r = (\sigma - \Delta \sigma) - E(x, y, t) \frac{\int \sigma \Delta \sigma \, dA}{\int E \, dA}$$  \hspace{1cm} (16)$$

The actual evaluation of the integrals for the example was made by a graphical integration of the stresses in Fig. 6h resulting in a residual stress distribution as for example shown in Fig. 6k. Because $\Delta \sigma$ is not known exactly a trial and error procedure is involved, however, it will hardly be necessary to go further than the first approximation. After cooling the maximum values of $\Delta \sigma$ will determine the residual stress distribution. Then

$$\sigma_r = -\Delta \sigma_{\text{max}} + \frac{1}{A} \int \Delta \sigma_{\text{max}} \, dA$$  \hspace{1cm} (17)$$

We are now in a position to calculate the residual stress distributions in a particular steel plate at different times during cooling. The dimensions of the plate and the assumed heat flow constants are given in Fig. 6a. The initial temperature was assumed to be 1300° F. The assumed material properties are given in Figs. 6b, 6c, and 6d. We can summarize the required steps in the calculations as follows:
1. Evaluate the temperature distribution at any point in the plate. (Fig. 6e) Note that the mean temperature through the thickness was used in order to simplify calculations.

2. Evaluate equation (15) giving the strain distribution in the plate (Fig. 6f).

3. From Figs. 6b and 6e the auxiliary curves of Fig. 6g are obtained, giving the values of the modulus of elasticity in the plate. These values will be necessary for the evaluation of equation (16).

4. The strain distribution of Fig. 6f is converted with the aid of Fig. 6g to a stress distribution as shown in Fig. 6h.

5. For each desired time the stress distribution of Fig. 6h is used to find the residual stress distribution according to equation (16).

A graphical integration was used in this example and the resultant stress distributions are shown on Fig. 7 for one-half of the plate width. The magnitude of residual stress after cooling is quite small (Fig. 7, t = ∞). Because of the various assumed values of material properties and the assumptions involved in the heat-flow analysis it cannot be expected that the calculated stresses will equal exactly actual stresses. However, the general process of the formation of residual stresses during cooling was exemplified.
In the following section the cooling stresses in other shapes will be investigated proceeding from assumed strain distributions.

(b) **Examples of Residual Stresses on WF Shapes**

In the previous section the residual stresses in a plate were investigated during cooling to room temperature. A similar analysis of structural shapes would be much more complex. However, it was shown that for the final residual stress distribution only the strains greater than the strain corresponding to the yield point are of importance. In this section the effect of various assumed strains on the residual stress distribution will be investigated. It must be kept in mind, however, that the strain distribution is also a function of cross-sectional dimensions and this makes a direct comparison between beams of the same shape but of different size somewhat unrealistic.

Stresses have been computed by equation (4). The assumed strains and the results of the calculations will now be described.

A symmetrical strain distribution of parabolic and linear shape with a maximum value of $\varepsilon_0$ as shown in Fig. 8a is assumed. The residual stress distribution is obtained from equation (4) as follows:

$$\frac{\sigma_r}{E \varepsilon_0} = \frac{\sigma_r}{\sigma_{r_0} - \sigma_{r_c}} = \frac{\varepsilon}{\varepsilon_0} - \frac{4 + \frac{\omega d}{t_b} (3 + 2 \chi)}{3 (2 + \frac{\omega d}{t_b})} \text{ Parabolic Distribution} (18)$$
where \( \sigma_r \) is the residual stress at an arbitrary point that has the initial strain, \( \varepsilon \), and \( \alpha \) is a factor which is either positive or negative. The stress distribution is shown in Fig. 8b.

Equations 18 and 19 were used to plot the graphs shown in Fig. 8c. The ratios of residual stress at the flange tips \( (\sigma_{rc}) \) and flange center \( (\sigma_{ro}) \) are plotted as a function of the web-flange thickness ratio for \( \alpha = 1 \) and \( \alpha = -1 \). Test results are also shown in Fig. 8c. These will be discussed later.

It would be of interest to compare residual stresses resulting from these assumed strain distributions with actual tests. Suppose residual strain measurements had been made. Then, dividing equation (4) by \( E \) and omitting the bending term:

\[
\varepsilon_r = \varepsilon - \frac{1}{A} \int_A \varepsilon \, dA
\]

where \( \varepsilon_r \) has been measured. The strain is

\[
\varepsilon = \varepsilon_r + c
\]

where \( c \) is an arbitrary constant, since

\[
\varepsilon_r = (\varepsilon_l + c) - \frac{1}{A} \int_A (\varepsilon_l + c) \, dA \equiv \varepsilon_l
\]
This means then that the strain distribution, $\varepsilon$, must have the same shape as the measured strains and the magnitude differs only by a constant.

Let us now consider an unsymmetrical strain pattern as shown in Fig. 9. The resulting residual stresses are also shown in this figure. The pattern is of interest because the residual stresses are symmetrical although the strains were unsymmetric. However, the initially straight beam would take on a permanent curvature after introduction of the residual stresses. This curvature is obtained from the bending term of equation (4):

$$\phi = \frac{1}{I_y} \int_A \varepsilon_x dA = \frac{\varepsilon_y}{b}$$  

(21)

While this indirect method of residual stress analysis gives some qualitative results, we must depend on actual measurements to obtain the residual stress distribution in structural members.

The solution for residual stresses in a beam derived earlier (eq. 4) is only valid in the interior of long beams. It is desirable to know the distance from the ends of a beam where this solution becomes valid. For a solid section (rectangular beam for instance) this distance would be approximately equal to the larger cross-sectional dimensions according to St. Venants' principle. For structural steel WF shapes, the distance may also be assumed to be equal the
larger cross-sectional dimension. When the flange and web thickness becomes very small with respect to the other cross-sectional dimensions the distance becomes larger. The problem of finding this distance is similar to the so called "shear-lag" problem of stiffened panels. (8) (12)

2.3 Residual Stresses Due to Plastic Deformation

In this section residual stresses due to mechanical deformations (for instance gagging) will be investigated in steel beams. In Section 2.1 it was stated that residual stresses are the result of permanent deformations of parts of a beam section.

Consider a rectangular steel beam initially without residual stress (Fig. 10). Bending which results in yielding of some fibers will leave residuals in the beam (5).

Since the strain distribution is linear, direct use may be made of equation (7). We can therefore write:

\[
E \varepsilon' = \sigma_y \left( \frac{y}{y_o} - 1 \right) \quad (y > y_o)
\]

and

\[
\sigma_r = \frac{24y}{d^3} \int_{y_o}^{d} \sigma_y \left( \frac{y}{y_o} - 1 \right) y \, dy \quad (y \leq y_o)
\]

or after evaluation of the integrals:

\[
\sigma_r = \sigma_y \left( \frac{y}{y_o} - \frac{3y}{d} + \frac{4y \, y_o^2}{d^3} \right) \quad (y \leq y_o)
\]

\[
\sigma_r = \sigma_y \left( 1 - \frac{3y}{d} + \frac{4y \, y_o^2}{d^3} \right) \quad (y > y_o)
\]

(22)
Equations 22 give the residual stress for any fiber as a function of $\gamma_0$, which is a measure of the plastification of the section.

Next let us consider a beam of rectangular cross-section that contains initial residual stresses, $\sigma_{ri}$, which are symmetrically distributed. (Fig. 11a). If the beam is bent beyond the elastic limit, the applied stresses may be obtained as shown in Fig. 11b. In this figure the initial residual stresses have been subtracted from the yield points in tension and compression; thereby the limit at which yielding is reached is determined as indicated by the yield limit lines in Fig. 11b. Due to the initial residual stresses, the yielding process is asymmetric. The total stresses in the beam before removal of the bending moment are given by the addition of initial residual stresses, $\sigma_{ri}$, and applied stresses, $\sigma(\varepsilon)$, (Fig. 11c). The bending stresses, $\sigma_b$ due to the removal of the bending moment must be subtracted from the total stresses in the beam before unloading in order to obtain the final residual stress. The resulting stresses are hatched in Fig. 11c and give the final residual stress distribution which is redrawn in Fig. 11d.

In the same way let us consider axial deformation in a rectangular beam containing the same initial stress pattern as before (Fig. 12a). The yield limit is indicated in Fig. 12b. Superposition of a uniform stress will result in a total stress distribution shown in Fig. 12c and the
final stresses are shown in Fig. 12d. By sufficient plastic
deformation it is possible to wipe out completely the initial
stress pattern.

Now let us consider another example of an H-beam,
an example that was used in section 2.2 (Fig. 9). Due to
the unsymmetrical residual stress pattern, bending of the
beam resulted, and the corresponding curvature amounted to
\( \phi_0 = \frac{E_0}{b} \). To obtain a straight beam a permanent curvature of
the opposite sign must be introduced. In production mills
this is done by "gagging", a straightening process done by
application of a set of concentrated forces. From the
M-\( \phi \) relationship of the beam with initial residual stresses
the required curvature and bending moment to insure straightness
after deformation can be obtained (1) (Fig. 13a). From
the required bending moment the applied stress \( \sigma(\varepsilon) \) may
be determined (Fig. 13b). The unloading bending stress \( \sigma_b \)
is elastic. The final residual stress distribution is obtained
from the addition of \( \sigma(\varepsilon) \), \( \sigma_b \) and the initial residual
stress (Figs. 13c and 13d).

If an infinite curvature would be applied the
cooling stresses would be completely wiped out and only cold
bending residuals corresponding to a full plastic moment
would remain. (1)
In this section it was shown how residual stresses due to cold-bending can be calculated. It was also shown how the presence of initial curvature and initial residual stresses can be taken into account.
3. EXPERIMENTAL INVESTIGATION

While cold-bending residual stresses on WF shapes can be calculated with relative ease and accuracy, it is not possible to obtain the residual stresses due to cooling in the same manner as pointed out earlier. Therefore, it was the aim to determine by a simple method residual stresses on a number of representative WF beams and columns.

All measurements and tests were conducted at the Fritz Engineering Laboratory, Lehigh University. Residual stresses were measured by the sectioning method. Only as-delivered and annealed WF sections of steel rolled to ASTM A-7 specifications were used in the experiments.

3.1 Description of Residual Stress Measurements

The sectioning method (14) was adopted for the measurement of residual strains because of its simplicity (Fig. 14). Longitudinal strains were measured over a 10" gage length by a 1/10,000 Whittemore strain gage on a series of holes. A standard 10 inch mild steel bar was attached to the specimen to observe changes due to temperature between readings. The average error in these measurements corresponds to about ±600 psi. Following an initial set of readings on drilled and reamed holes serving as gage points, a second set of readings was taken after relaxation due to sawing. In one case (8WF24) strain readings were taken over a 1" gage length
with SR-4 gages. Strips of about 1/2-1" width, each containing one pair of gage holes, were severed from a one foot section in the earlier tests. Later it was found that the strips could be made wider without impairing results. Each strip contained several pairs of drilled and reamed gage holes. In such cases checks on strips of small width were also made. The errors in partial sectioning is for most cases small. A theoretical estimate of the error is made in Appendix 8.2. The stresses were calculated from the strains using an elastic modulus, E, of $30 \times 10^3$ ksi.

3.2 Test Results

The following is a brief presentation of results. The latter are discussed and compared with the theory in Chapter 4. Table 1 gives a summary of shapes tested, the residual stresses at flange tops, flange and web centers, and their classification as to type. Three types of residual stress distribution have been observed (Fig. 15). A summary of measured residual stress distributions in WF shapes is shown in Fig. 18. The residual stresses computed from stress measurements are plotted in Figs. 17 to 38. Most stress distributions can be considered to be symmetric. Some measurements were made on sections that were cold-straightened. These results are presented on Figs. 33 to 37.

The differences resulting from partial versus strip cutting may be seen in Figs. 18, 30 and 31. An indication of
the accuracy of the sectioning method is Fig. 27 where the stress differences of the results obtained independently by two investigators are plotted. How cooling rate influences the magnitude and distribution of residual stresses may be seen in Fig. 29. While one 14 WF 43 was allowed to cool on the cooling bed together with other sections, the other was set aside and allowed to cool at a faster rate. In Fig. 23 the stresses along the length of a 8 WF 31 beam are plotted for three different sections. In Fig. 38 the average stresses at the flange tips and the flange centers are plotted along the same 8 WF 31 beam. The average residual stresses for the flange tops and the flange centers at a section i-j which is located on the center part of the 81 WF 31 beam are shown in Fig. 39 as a function of beam length.
4. DISCUSSION

In this chapter some comments will be made on the theory presented, the tests and their correlation.

4.1 Residual Stresses Due to Cooling

The plastic deformations during cooling are dependent on various factors, the most important ones being cooling rate, temperature distribution, material properties and specimen dimensions; and all of these factors are interrelated.

The behavior of the plate that was analyzed in Section 2.2 can serve as a guide in the qualitative analysis of other shapes. The portions of a section which cool most rapidly are at first under residual tension while the slower cooling portions are under compression (Fig. 7). Yielding may occur either in tension or compression, or both, depending on the shape, temperature distribution and the magnitude of the yield point. The non-elastic deformations during the cooling process determine the magnitude and distribution of the residual stresses after a uniform temperature has been obtained. As a rule, the fastest cooling portions of the section will be under final compression stresses and the slowest cooling parts under tension stresses.

If this reasoning is applied to a wide-flange section, for instance a 36 WF 150 (Fig. 16), the flange tips
and the web center would cool more rapidly than the flange centers. As another example let us consider a 14 WF 426 (Fig. 16). Because of the shallow depth and large web thickness, the flange tips would be cooling the fastest while the web and flange centers will not have a large difference in cooling rate. Finally let us consider an I-beam with very small flanges. In that case the web center will cool more rapidly and the flanges will be the slowest to cool. Thus, the I-beam may have tensile residual stresses in the flanges. A check of the other sections shown in Fig. 16 will confirm the rule cited above.

Since cooling conditions are variable, residual stresses may have different distributions and magnitudes even in the same shape. This is shown by the measurements on several shapes. Fig. 22 shows the residual stress distribution in three 8 WF 31 beams, from the same heat. Fig. 23 shows the distribution in another 8 WF 31 beam which came from a different heat and also had a greater difference in the stress distribution than the other three beams. In Fig. 29 the residual stress distribution is plotted for two 12 WF 43 beams which came from the same heat and rolling; however, one beam was cooled separately and therefore cooled much faster than the other which remained on the cooling bed with the rest of the rolled sections. Fig. 18 shows the stress distribution in a 5 WF 18.5 which was allowed to cool in the middle of the other members of the rolling. Fig. 36 shows the residual stress distribution on the edge
member on the same cooling bed. The edge beam also retained a permanent bend after cooling while the other beam remained straight.

The variation of residual stresses through the thickness is relatively small in thin walled section (Figures 19, 20, 26, 27), but becomes appreciable for thick walled shapes (Figs. 30, 31, 32). In accordance with the rule expounded above, the stresses on the outside flanges are smaller than the stresses on the inside (if tension stresses are taken positive), because of the faster cooling of the outside. The web shows no residual stress gradient because of equal cooling conditions on both sides (Figs. 19, 20, 24, 26, 27, 30, 31, 32).

The measurements of the residual stresses were made by a simple method. In order to save time and costs complete sectioning was not employed to all shapes. In Appendix 8.2 the probable error is estimated. However, some strips were usually isolated in order to have also an experimental check. (Figs. 18, 25, 26, 30, 31)

The personal factor on the strain measurements with the Whittemore gage was found to be very small. Fig. 28 shows stress difference of independent measurements made by two different investigators.

In one test on a 8 WF 24 the residual stresses as measured by the Whittemore gage over a 10" gage length were checked
by SR-4 gages of a 1" gage length. In consideration of the stability of SR-4 gages in long range testing (several days) the check was very good (Fig. 20).

The residual stresses in the interior of long beams and columns should be constant along each fiber according to the theory. The comparison between the 1" and 10" gage length mentioned above does not show any substantial difference. Measurements were also made on a 9 ft. long 8 WF 31 beam on a gage length of 10". These tests confirmed the theoretical prediction (Figs. 23 and 38a). The results of these measurements could also be used to show that at the end of beams the residual stresses show a variation only over a length equal to about the greater cross-sectional dimension (Fig. 38a) or in other words that in the center of a short beam length (minimum about 3d) residual stresses still have their full magnitude (Fig. 38b).

In Fig. 8c the results of the residual stress measurements are plotted as function of the web-flange thickness ratio for shapes of a depth-width ratio of about unity. The factor varied between +1 and -1 for these sections. On the basis of the theoretical studies and the test results Table III has been prepared for an approximate estimate of the residual stress distribution. The difference between parabolic and linear distribution is small in comparison to the uncertainty of the cooling conditions (see 8 WF 31 Fig. 8c). Since the majority of shapes contained residual
stresses of approximately parabolic distribution, Eq. (18) in conjunction with Table III may be used for a rough estimate.

Welding residual stresses can be treated theoretically in a very similar manner. However, the sequence of welding and the shape of the elements making up the section will determine the final residual stresses.

4.2 Residual Stresses due to Cold-Bending

The measurements reported in the previous section were made at locations in a beam that showed no evidence of cold-bending. Measurements were also made at locations that showed such evidence in the form of flaked mill-scale (Figs. 33, 34 and 35). The amount of plastic stressing could be estimated roughly from the lengths of the yield lines. It appears that all sections were straightened by bending about the weak axis of the section. The results indicate that the flanges were sometimes subjected to a different amount of stressing. This was also the case in the laboratory for a controlled bending test of a 5 WF 18.5. The initially curved beam was placed in a testing machine with the convex side up. Loads were applied near the ends subjecting the major portion of the beam to a constant moment (Fig. 38a). The residual stresses were measured on an adjacent piece of the beam before the test (Fig. 36), and after straightening of the beam (Fig. 37b). The procedure for straightening of the beam was similar to the one which was used in the theoretical analysis.
of a WF beam (Fig. 13). However, actual deflections were used to determine the required amount of straightening instead of the beam curvature (Fig. 13a). Although the conditions assumed in the theoretical analysis differed from the actual conditions, the qualitative agreement between theoretical residual stresses (Fig. 13d), and the stresses in 5 WF 18½ beam after bending (Fig. 37b), is very good.

4.3 Effect of Residual Stresses on Structural Behavior

Only a few brief remarks will be made here. Members subjected to bending reach the proportional limit of the material at a lower load and afterwards have increased deflections. Generally the ultimate load is not affected by the residual stresses. (1, 4) A short specimen in compression acts very similar (Fig. 39a). By comparison with an annealed specimen (Fig. 39b) the effect is clearly demonstrated. (9) On the other hand unannealed steel columns show an appreciable effect as shown in Fig. 40. (9, 10, 12) This is due to the loss of bending stiffness when portions of the cross-section have yielded prematurely due to compressive residual stresses. However, annealing would not be an effective method to eliminate the effect of residual stress because it also lowers the yield point of the material. This variable has been eliminated in Fig. 40. Residual stresses have no influence on slender steel columns i.e. for total stresses applied plus residual stresses below the proportional limit.
5. SUMMARY

The principal results of this report may be summarized as follows:

(1) Residual stresses are formed as a result of permanent deformation of certain fibers during the cooling process, or are due to plastic deformation during fabrication. (Residual stresses due to non-linear temperature distributions are not permanent but disappear when the temperature distribution becomes either linear or uniform.)

(2) The magnitude and distribution of cooling residual stresses depend on shape, initial temperature, cooling conditions and material properties.

(3) Cooling residual stresses of axial symmetric sections will also be symmetric under uniform cooling conditions.

(4) Cooling residual stresses are constant along the length of beams and columns except for a distance about equal to the larger cross-sectional dimension at the ends.

(5) The sign of cooling residual stresses can be estimated from the relative cooling rate of various parts of a section. The fastest cooling portions of the section will be under final compression stresses and the slowest cooling parts under tension stresses.
(6) Residual stresses due to cold-bending (gagging) can be predicted with reasonable accuracy. The maximum and minimum stresses are of the same order of magnitude as the cooling residual stresses.

(7) The measurements of residual stresses made on a large number of WF shapes should be helpful for an estimate of the magnitude and distribution of residual stresses likely to be encountered.

(8) The following maximum, average, and minimum values of cooling residual stresses were measured in WF shapes (tension +, compression -), the average being shown only where warranted by sufficient test data.

<table>
<thead>
<tr>
<th></th>
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<th>WEB CENTER</th>
</tr>
</thead>
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<td>av.</td>
<td>min.</td>
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</tr>
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<td></td>
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<td>-15.5</td>
</tr>
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<td></td>
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</table>
6. ACKNOWLEDGMENTS

This report presents a part of the theoretical and experimental studies made during the course of a three year research program on the influence of residual stress on column strength carried out at the Fritz Engineering Laboratory, Lehigh University, Bethlehem, Pennsylvania, of which William J. Eney is director.

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Messrs. Yuzuru Fujita, Tadahiko Kawai and Lambert Tall assisted in the tests. The report was typed with care by Miss Carolyn Smith. Messrs. Lambert Tall and Robert Wagner also assisted in the preparation of the tables and figures.

Their cooperation is gratefully appreciated.
7. REFERENCES


8.1 Nomenclature

- \( a^2 \): thermal diffusivity
- \( A \): cross-sectional area
- \( A_w = wd \): approximated web area
- \( b \): flange width
- \( c \): plate dimensions, a constant
- \( d \): depth of H section between center lines of flanges
- \( E \): Young's modulus of elasticity
- \( h = c/k \): a constant
- \( I \): moment of inertia
- \( J_x \): 
  \[ J_x = \int x^2 dA \]
- \( k \): thermal conductivity
- \( q \): heat emission per unit area
- \( t \): flange thickness, time
- \( T \): temperature
- \( w \): web thickness
- \( x_0, x_1 \): distances from the center to the beginning of the yielded area
- \( \alpha \): coefficient of linear expansion
- \( \varepsilon \): unit strain
- \( \varepsilon_0 \): reference strain
- \( \varepsilon_y \): strain corresponding to the yield point
- \( \sigma \): normal stress
- \( \sigma_r \): residual stress
- \( \sigma_{rc} \): residual stress at flange edges
- \( \sigma_{ro} \): residual stress at flange centers
- \( \sigma_{rw} \): residual stress at web center
- \( \sigma_{rt} \): residual stress at web edges
- \( \sigma' \): yield stress level; average stress in the plastic range
8.2 Some Theoretical Considerations in Respect to Measuring Technique

(a) Residual Stress Gradient

If a strip of width, s, is cut, a stress gradient will result in bending of the strip (Fig. 42). The curvature is given by \( \gamma = -\frac{\Delta \epsilon_r}{s} \) and the center line of the curved strip has the equation:

\[
y = \frac{\Delta \epsilon_r}{2s} x (L-x)
\]

The error in axial strain measurement is then given by

\[
\delta \epsilon = \frac{1}{L} \int (ds - dx) \approx \frac{1}{L} \int_0^L \frac{1}{2} y^2 dx = \frac{\Delta \epsilon_r L^2}{2s^2} \quad \text{(A.1)}
\]

Taking typical values: \( \Delta \epsilon_r = 100 \times 10^{-6} \), \( L = 10^4 \), \( s = .5 \); \( \delta \epsilon = 1.7 \times 10^{-6} \) which is negligible.

(b) Partial Cutting

Time and costs are saved by cutting larger sections instead of strips. In a plate of length, \( L \), and thickness \( 1 \), the residual stress distribution be given by

\[
\sigma_r = \left( 10 - 120 \left( \frac{y}{s} \right)^2 \right) \sin \frac{\pi x}{L} \quad \text{(A.2)}
\]

The measured average value over \( L \) is then

\[
\sigma_{r(\text{av})} = \frac{2}{L} \left[ 10 - 120 \left( \frac{y}{s} \right)^2 \right] \quad \text{(A.3)}
\]

If the plate is cut in the center a rotation and an axial deformation will take place causing a partial release of residual stresses. To calculate the final stresses it is only necessary to substitute the initial residual stresses for \( E \sigma_r \) in Eq. (4). Then the remaining average residual stress, \( \Delta \sigma_r \), in the plate is given by:

\[
\Delta \sigma_r = \frac{2}{n} \left[ -s + 60 \frac{y}{s} - 120 \left( \frac{y}{s} \right)^2 \right] \quad \text{(A.4)}
\]
Both the initial stress distribution and the distribution after the cut are shown for the center of the plate \( x = \frac{L}{2} \) in Fig. 42.

For a linear distribution of initial stress \( \Delta \sigma \) would be zero. The error in axial stress measurements is given by Eq. (A.1). For \( \Delta \bar{\varepsilon} = \frac{2}{\pi} \frac{3\sigma}{E} = 0.637 \times 10^{-3} \), \( L = 10'' \), \( \delta \varepsilon = 0.7 \times 10^{-6} \), a very small value and certainly negligible.
8.3 TABLES

TABLE I

Cooling Residual Stresses in WF Shapes
(Average Values)

<table>
<thead>
<tr>
<th>SHAPES</th>
<th>w/t</th>
<th>d/b</th>
<th>( \sigma_{rc} )</th>
<th>( \sigma_{ro} )</th>
<th>( \sigma_{rw} )</th>
<th>TYPE</th>
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<td>-2.0</td>
<td>6.5</td>
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<td>3.2</td>
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### TABLE II

Residual Stresses in Cold-Bent WF Shapes

<table>
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<tr>
<th>SHAPE</th>
<th>w/t</th>
<th>d/b</th>
<th>τ_{min}(F) (compr)</th>
<th>τ_{max}(F) (Tens.)</th>
<th>τ_{max}(min) (Web)</th>
<th>Remarks</th>
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<td>1.018</td>
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<td>8.0</td>
<td>-0.5</td>
<td>cold-bent under uniform moment (laboratory test)</td>
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<td>1.000</td>
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<td>8.0</td>
<td>-6.0</td>
<td>as delivered: evidence of heavy cold-bending (measured near end of beam)</td>
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<tr>
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<td>.616</td>
<td>1.088</td>
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<td>7.0</td>
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</tr>
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<td>-5.5</td>
<td>9.0</td>
<td>-11.5</td>
<td>as delivered: evidence of mild cold-bending</td>
</tr>
</tbody>
</table>

Note: τ_{min}(F) = Maximum compressive stress in the flanges  
τ_{max}(F) = Maximum tensile stress in the flanges  
τ_{max}(min) = maximum stress (compression or tension) in the web. Fillet stresses excluded

### TABLE III

Residual Stress Types in Dependence on Dimensions  
Values for Residual Stress Estimate

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<th>III</th>
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<td>14&lt;(E_{\varepsilon_0}&lt;26)</td>
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<td>5&gt;(\alpha&gt;1)</td>
<td>(\alpha&gt;5)</td>
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</tbody>
</table>

\(E_{\varepsilon_0}\) in Ksi 
\(d\) in inches

Note: The small number of tests, the unknown cooling conditions, etc. permit only a very rough estimate of the approximate residual stress distribution
<table>
<thead>
<tr>
<th>Shape</th>
<th>TYPE</th>
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<th>$\sigma_{ro}$</th>
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</tr>
<tr>
<td>14WF426</td>
<td>II</td>
<td>-16.0</td>
<td>8.0</td>
<td>14.0</td>
</tr>
<tr>
<td>36WF150</td>
<td>I</td>
<td>-10.8</td>
<td>14.3</td>
<td>-15.0</td>
</tr>
</tbody>
</table>
FIG. 1: BEAM SECTION & INTERNAL STRAINS

FIG. 2: IDEALIZED STRESS-STRAIN CURVE

FIG. 3: PARTIALLY YIELDED SECTION

FIG. 4: SECTION WITH GENERAL STRESS-STRAIN RELATION

FIG. 5: PLATE DIMENSIONS
Constants for steel

\[ h = 0.1 \]
\[ a^2 = 1.18 \text{ in/min} \]

\( (a) \)

\( (b) \)

\( (c) \)

\( (d) \)

\( (e) \)

**FIG. 6: DEVELOPMENT OF RESIDUAL STRESSES IN A STEEL PLATE**
FIG. 6: CONTINUED
FIG. 7: RESIDUAL STRESS DISTRIBUTIONS
FOR VARIOUS TIMES
VALUES ARE RATIOS OF $\frac{\sigma_A}{E\varepsilon_0}$

Figure 9

COLD-BENDING STRESSES IN STEEL BEAM

FIGURE 10
Figure 11

Figure 12
Figure 13

Figure 14
Type I
12WF50, 12WF55, 16WF43, 6KI5.5, 36WF150, 12JI4 SWF11

Type II
4WF3, SWF3, 14WF428, SWF18-1/2, GLC15.5, SWF26

Type III
SWF24, SWF67, SWF18-1/2, SWF31

Type IV (not measured)

TYPE OF RESIDUAL STRESS DISTRIBUTION

Figure 15
FIG: - 16. RESIDUAL STRESS DISTRIBUTIONS IN WF-SHAPES
FIG: - 16 CONTINUED
Figure 16 continued
FIG. - 16 CONCLUDED
FIGURE 17

- Outer Surface - Complete Sectioning
- Inner Surface - Complete Sectioning
- Estimated Mean Curve - Complete Sectioning
© Outer Surface - Complete Sectioning
O Inner Surface - Partial Sectioning 220A
+ Inner Surface - Partial Sectioning T-12
—— Estimated Mean Curve - Partial Sectioning

FIGURE 18
Figure 19

Comp. FLANGE DISTRIBUTION

-20 -10 0 10 10 0 -10 ksi

WEB DISTRIBUTION

20 ksi

10

10 ksi

Outer surface

Inner surface

Complete Sectioning

Mean curve

RESIDUAL STRESS DISTRIBUTION

6M 15'2"
Figure 20
DIFFERENCE BETWEEN COMPLETE SECTIONING AND PARTIAL SECTIONING

Figure 21
Figure 22
DISTRIBUTION OF RESIDUAL STRESS AT THREE LOCATIONS IN A SINGLE BEAM (T-6)

Figure 23
Partial Sectioning

Figure 24
COMP. FLANGE PATTERN

COMP. = 5 ksi

WEB PATTERN

RESIDUAL STRESS DISTRIBUTION
(Complete Sectioning)

FIGURE 25
Outside Surface

Inner Surface

Partial Sectioning

Mean Curve

Figure 26
Figure 27
DIFFERENCE BETWEEN RESULTS OBTAINED BY TWO INVESTIGATORS

Figure 28
Figure 29 - RESIDUAL STRESS DISTRIBUTION
Figure 30

RESIDUAL STRESS DISTRIBUTION
(Partial Sectioning)
FLANGE DISTRIBUTION

FLANGE TIP FACE DISTRIBUTION

WEB DISTRIBUTION

RESIDUAL STRESS DISTRIBUTION (Complete Sectioning)

FIGURE 31
FLANGE DISTRIBUTION

WEB DISTRIBUTION

RESIDUAL STRESS DISTRIBUTION
(Complete Sectioning)

FIGURE 32
COLD BENDING RESIDUAL STRESSES IN 8WF31

Figure 33
Partial Sectioning

Outside

Inside

Mean Curve

Surface Partial Sectioning

Figure 34
Figure 35

Partial Sectioning

12WF50
FIGURE 36

Estimated Mean Curve

Outer Surface Complete Sectioning

Inner Residual stresses before cold-heading test
Figure 37 (a) - COLD BENDING OF 5WF18-1/2-COLUMN AND DEFLECTIONS
FIG. 38: (a) VARIATION OF RESIDUAL STRESS (KSI) ALONG A 8WF31 BEAM

(b) THE VARIATION OF RESIDUAL STRESS (KSI) AT SECTION i-j AS A FUNCTION OF BEAM LENGTH
CROSS-SECTION STRESS-STRAIN CURVE FOR AS-DELIVERED MATERIAL

Figure (a)

CROSS-SECTION STRESS-STRAIN CURVE FOR ANNEALED MATERIAL

Figure (b)

Figure 39
THE INFLUENCE OF RESIDUAL STRESS ON COLUMN STRENGTH AS INDICATED BY THEORY AND TESTS

Figure 40
Figure 41

Figure 42