This paper has been entitled "Tensile Stress-Strain Characteristics of Structural Steel". A more appropriate title perhaps, would be "The Uncertainty of the Stress-Strain Characteristics of Structural Steel".

In much of the literature in the field of plasticity theory attention is directed toward determining two or three-dimensional stress distributions for an arbitrary law of strain-hardening and plastic flow. Much attention has been given to distinctions between "flow" theories and "deformation" theories. Deformations are considered far into the strain-hardening region and it becomes important to differentiate between true stress or strain and nominal stress or strain in order to obtain from a simple tension test applicable stress-flow information that is applicable to more complicated states of stress.

Such theoretical work in the field of plasticity is of great importance and it would be easy to cite many important practical applications. But when the structural engineer thinks of "plasticity" he visualizes the sort of work that Professor Baker and his associates
are doing in Great Britain. Their results are placing within the
gasp of the structural engineer usable methods for the analysis
of the real strength of structural frames. From the conventional
but indirect design procedures based upon elastic stress analyses
and a permissible working stress, they are leading us to more direct
and realistic procedures based upon calculation of actual usable
strength as a certain percentage of the collapse or limit load.

To the ultimate user of a bridge or building the load-carrying
structural framework will be acceptable if it is:

1. Strong enough throughout its lifetime to carry all loads
   that may be applied (expected or accidental)

2. Rigid and unyielding enough not to vibrate, buckle, or
deflect to an extent that would reduce the usefulness
   for the intended purpose,

3. Enduring enough to withstand any corrosive action and not
to fail by fatigue,

4. Economical in choice and use of material.

Although items 3 and 4 are of great importance, they will not
be considered further at the present time. The first two requisites,
strength and rigidity, in a structure made of steel, are satisfied in conventional design practice by making each separate member of such size and shape as will transmit the calculated forces without exceeding in any of its parts a "unit stress" or "intensity of force transmission" greater than a specified "working stress", which is less than the stress at which marked inelastic action starts for the particular material in question. Elasticity and proportionality between stress and strain are assumed to exist and the stresses are calculated by "elastic theory".

It is important to keep in mind the fact that a structure may have much more strength than can be utilized if the deflections cause unsightly distortion, lead to failure of malfunctioning of non-load carrying parts or machinery, or if the accompanying flexibility results in psychologically uncomfortable vibration, or malfunctioning under load. In the case of the Tacoma Narrows Suspension Bridge ample strength under static load was available but the structure failed because its flexibility permitted destructive aerodynamically induced vibrations.

Unfortunately, in plastic-range design, the overall deflection of the structure cannot be predicted with anything like the accuracy
that may be expected within the elastic range. This statement is made in spite of the acknowledged fact that there are many definite solutions in plastic theory for assumed stress-strain relationships. Nevertheless, an uncertainty of plastic range deflection must be expected in supposedly identical structures because of the variation that does exist in the stress-strain properties of the material in the inelastic range. In the elastic range, on the other hand, the stress-strain properties are relatively invariant for a given material and this fact permits accurate prediction of actual deflections.

The main purpose of this paper will be to study the variations in the inelastic stress-strain properties in the simplest of stress states (uniaxial tension or compression) and for the commonest of structural materials (structural steel). Most of these variations in stress-strain characteristics and the conditions that cause them have been discovered in the materials testing laboratories many years ago. It is important now that we give this subject renewed consideration because of its importance to the ultimate acceptance of plastic range design.

The following subdivisions outline the remainder of the paper:
(1) The conditions causing variations in stress-strain characteristics.

(2) The relationship between uniaxial stress-strain properties and the deflections of a structural section.

(3) Variation in stress-strain properties in rolled structural sections.

(4) Variation in stress-strain properties of rolled plate material.

(5) A summary of factors leading uncertainty in plastic range deflection characteristics in the finished structure.

In both the elastic and plastic stages the strength and deformations of a structural frame may be calculated on the basis of a knowledge of the tensile and compressive stress-strain properties of the steel. As long as the structure is in the elastic range the calculation of its deformation depends on the elastic constants. Results of many of tests of common structural steels have shown that these constants are not changed appreciably by variations in chemical composition, prior plastic straining, and prior thermal history. On the other hand, variation of any one or more of these same three items may have a very marked effect on effect on the inelastic part
of the stress-strain relationship. Therefore, in elastic design, within relatively narrow limits, the engineer can predict with certainty the deflections of a structure. In plastic design, on the other hand, because of the great variance of the inelastic part of the stress-strain diagram, unpredictable in many instances, it is not possible to calculate with much accuracy the deflection of a structure. This is not due to inadequacy of plastic theory but is a result of the intrinsic nature of the material we are dealing with, i.e., structural steel. Inelastic deflection predictions could be made accurately only if the inelastic part of the stress-strain curve of the material could be specified to vary within narrow limits when purchased to a given specification. The elastic constants, on the other hand, do remain within such narrow limits of variation that they are no longer even specified.

It should be mentioned, in passing, that the inelastic part of the stress-strain curve is of recognized importance, even in elastic design. In the first place, the lower limit of initial inelastic behavior, divided by a so-called "factor of safety", determines for a particular steel the "allowable working stress". More important, in every structure, there are countless locations where local stress-
concentrations exist. If fatigue is not the criterion of design these local stresses are not calculated at all. Nevertheless, the inherent plasticity of the material is called upon in these locations and some localized plastic flow must take place without fracture. So long as the primary load carrying stresses are below the "allowable working stress" the local plastic deformations do not contribute appreciably to the overall deflection of the structure, hence, if not dangerous, they are usually of relative unimportance on a large scale and do not cause a preciable errors in deflection calculation based on an assumed elastic behavior.

It is the primary intent in this paper to discuss the circumstances and give some idea as to the extent of the variance that may be expected in the inelastic stress-strain properties of structural steel. In developing the greater use of plastic theory as applied to actual design it is obviously important to recognize the existence of this variance and meet the resulting problems either by close control of materials or by acceptance of the variation within a probable range that may be determined, perhaps by statistical studies of a large number of samplings.

For the purposes of elastic design there are accepted specifications
values for the elastic modulus and the minimum yield point. What is now needed is an accepted "standard" for the stress-strain curve beyond the elastic range and the assurance that by use of this "standard" curve one may predict with sufficient accuracy the corresponding inelastic behavior of structural members by use of existing analytical procedures. That this may be a very elusive quest is evidenced by the fact that no such standard curves are now available.
of structural members by use of existing analytical procedures.

From an educational point of view, if the "theory of plasticity" is to find widespread acceptance in structural design, it would be desirable to present correct information in student textbooks regarding the stress-strain curve of the most widely used material, structural steel. Figure 1 is taken from the Sixth Edition of "The Elasticity and Resistance of Materials" by Wm. H. Burr, prepared in 1903. These stress-strain curves were made in 1896 and 1899 and clearly show the usual relationship between the elastic and plastic deformations just beyond the yield point. It is of special note that the "Rock Island Bridge Steel", tested in 1896, meets fully the A.S.T.M. Yield Point, Ultimate Strength, and dutility Specifications for Bridge Steel currently in use more than fifty years later. Unfortunately, some very modern and highly respected text books in strength of materials have depicted the stress-strain curve of structural steel as shown in Figure 2, wherein the total strain in the lower yield point region has about the same magnitude as the elastic strain at the yield point. Such a stress-strain curves may lead to misconceptions that have sometimes been made in formulating theories of inelastic analysis, namely, that the dotted or dashed lines are satisfactory approximation if the upper yield point is ignored. If is not impossible, with proper
prior strain-history, to obtain a stress-strain curve similar to Figure 2 in the initial range. Such a curve would be anything but typical and certainly not a plausible basis for any theory of plasticity relating to a steel structure.

As to the basic phenomena of yielding of structural steel, there is a voluminous literature on the subject and no attempt will be made herein to discuss the fundamental problem as to how yield is initiated within a metallic crystal or aggregate of metallic grains. We will commence rather with the overall behavior of a standard tensile or compression test specimen, strained slowly through the elastic and plastic ranges, during which time frequent observations are made of the load and deformation. If precise determination of $E$ and the proportional limit is not attempted, an automatic stress-strain recorder may be used. Figures 3 and 4 show typical stress-strain curves of a miscellaneous assortment of structural carbon and alloy steels. These curves have been traced directly from actual automatic recordings of load and deformation with the scale of the ordinate and abscissae adjusted to indicate stress and strain. Figure 3 is to a scale that gives the entire nominal stress strain diagram and Figure 4 shows only the elastic and early plastic strain up to two or three percent. It is to be noted that the flat portion of the curve extend
over a strain of from 0.005 to 0.025. Even in high yield point alloy steels the same phenomenon is noted.

Stress-strain curves of individual samples are of little use, however, in establishing a basic stress-strain curve that might be used in structural analysis in the plastic range. There is needed a systematic statistical study over a wide range of samples to include all of the variables that affect the shape of a stress-strain curve. To facilitate such a study, the stress-strain curve might be catalogued as shown in Figure 5, wherein seven items are shown, the numerical values of which would permit the replotting of a sufficiently accurate stress-strain curve in any given case. In a large mass of data each of these seven items could be studied statistically to determine its range of variation within defined limits of probability and from such a statistical study a basic minimum curve might be arrived at.

The cataloguing of stress-strain curves might be justifiably simplified by ignoring the upper yield point entirely. The upper yield point is a condition of instability, is sensitive to surface roughness, rate of strain, and other variables. Furthermore, the contribution of the upper yield point to the strength of a member loaded into the plastic range is rather negligible if it exists at all, and
certainly disappears entirely in a bent beam, for example, as the limit of complete plasticity is approached.

The upper yield point, nevertheless, is the important strength criterion now used in elastic design. The upper yield point is important, therefore, in that our present notions the yield strength of structural steel are conditioned by the presently as to available accumulation of data, most of which reports only the upper yield point. It is additionally unfortunate that the acceptance of structural steel is based on mill test reports that often give a misleading estimate of steel strength even when tests are made strictly according to A.S.T.M. Standards (E8-46) for "Tension Testing of Metallic Materials". This is particularly the case in using the older beam and balance type testing machines for which the specification reads "When the yield point of the material is reached, the increase of load stops, but the operator runs the poise a trifle beyond the balance position......the corresponding stress is taken as the yield point". The italics have been inserted by the author. Furthermore, since there is no rate of strain specified the mill tests are made at a speed that raises the upper yield considerably. As an extreme example, we have on record a mill test report for a silicon structural steel quoting a yield point of 60,300 psi. and an ultimate of 82,200 psi. Tested at a show rate in
the laboratory a well-defined upper yield point of 47,150 psi and an ultimate of 84,250 psi were determined. A great deal of similar information has been recorded and the practices are defended on the basis that all mills use a similar practice and that the test is therefore satisfactory for comparative purposes. Steel mills are changing to modern hydraulic testing machines in which case the maximum load at the "bolt in the gage" is accepted as the yield point. This is an improvement but a specification limiting the strain rate to a reasonably low value is needed together with a determination of the lower yield point. It is obvious from the foregoing that most mill test records, while possibly suitable for comparative purposes, are not of much use in defining the yield point that might be the basis for plastic design.

Turning again to Fig. 5, $P_p$ has been noted as the stress at an offset strain of 0.0001. This is an arbitrary selection of strain, chosen to determine the general shape of the curve and the true proportional limit, if it exists at all, will be a much lower stress if the strain measuring apparatus is sufficiently sensitive.

To predict the overall plastic strength and deformation of a structural member it is most important to know $E_{LY}$ and $P_{LY}$ the strain and strengthening the lower yield point. In many cases a structural
Member will deflect far beyond the limit of structural usefulness without entering the general strain-hardening region. However, the initial rate of strain-hardening will provide the additional information necessary in those cases where plastic strains are this large. Test records usually report none of the three important plasticity stress and strain measures, $P_{LY}'$, $e_{LY}'$, and $\frac{dp}{de}$.

The simplification of the stress-strain curve as shown in Fig. 6, ignoring $P_{p}$ and $P_{VY}$ as relatively unimportant will be considered by many as providing enough information in the case of structural steel. In the case of stainless steel and nonferrous metals, stress-strain diagrams similar in shape to that shown in Figure 7 could be cataloged for purposes of statistical analysis by recording the stress at several arbitrary offsets of strain.

As is well known, the lower yield point region represents a very inhomogeneous state of strain, during which initial plastic regions develop and spread until finally their general distribution permits strain-hardening to be evidence by the upward slope of the stress-strain curve. Figures 7, 8, and 9 illustrate the various stages in the lower yield point region, the test specimen having been whitewashed to accentuate the surface evidence of yielding that causes disruption of the mill scale along slip regions. In Fig. 7, the lower yield
point region has just been entered. In Fig. 8, the strain \_
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\_, and in Fig. 9, the lower yield point strain has been
exhausted and general strain-hardening has just commenced.

As an example of the variation within a particular sample of steel
tensile test data giving the sample size and average values of \( P_{UY} \),
\( P_{LY} \), and \( \frac{dP}{d\varepsilon} \) are presented in Table 1, along with the estimated
standard deviation, a useful index of variation. The material

\[ \text{refer also to Figure 10.} \]

\[ \text{Also report on variation between different heats as indicated by \( \text{Chang's tests} \).} \]

Unfortunately, although a statistical study might be made of the
variation in properties within one particular heat, the further
variations between heats and between the different mills, makes the
problem of actually determining an acceptable minimum, or standard,
stress-strain curve difficult if not impossible.

The preceding paragraphs have discussed the usual shape of the initial part of the tensile (or compressive) stress-strain diagram of the as-received material. The shape of the stress-strain diagram and other mechanical properties of the as-received material when tested under exactly the same conditions are determined principally by these factors:

(1) The chemical composition.

(2) The amount, sense of prior plastic strain or "cold work".

(3) Prior heat effects, including the magnitude, duration, and rate of change of temperature. The resultant properties are also dependent on the sequence of effects (2) and (3).

Limiting ourselves to standard tensile and compression tests, the mechanical properties are further determined by the conditions of the test itself,

(4) The type of test specimen, including size, shape, surface finish.

(5) Temperature during test

(6) Speed of deformation during test

The acceptance of the material and the permissible working stresses usually are determined by the aforementioned standard tensile tests.
These are the properties we might therefore presume to exist in the structure—but such is hardly the case. The processes of fabrication and erection introduce a further series of heat and plastic strain effects that result in additional variation in properties of the material and leave residual stresses within the member.

Referring to factors (2) and (3) of the preceding paragraph, brief review of the influence of prior plastic overstrain and heat effects on the initial stress-strain curve will be presented. Some years ago tests made on a wide variety of structural carbon, silicon, and low-alloy steel plates by F. Opila and the writer showed that the tensile and compressive stress-strain curves were initially very nearly identical, tested either in or transverse to the direction of rolling. Figure 11, taken from that report is typical.