Prestressed Concrete Bridge Members
Progress Report 11

STATIC TESTS ON PRESTRESSED CONCRETE BEAMS
USING 7/16" STRANDS

by
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(Not for Publication)

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Mr. A.E. Cummings (Chairman) Reinf. Concrete Research Council
Mr. L.A. Porter, Pennsylvania Department of Highways
Mr. J.L. Stinson, U.S. Bureau of Public Roads
Mr. N. VanEenam, U.S. Bureau of Public Roads
Mr. S.L. Selvaggio, Concrete Products Co. of America
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Mr. H.K. Preston, John A. Roebling's Sons Corporation
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Prof. W.J. Eney, Lehigh University

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GENERAL

The pilot tests described herein constitute the first series of a program aimed to determine the bond characteristics of 7/16" strands. At the last meeting of the LPCC it was decided to pour and test fourteen beams to study the behavior of the above mentioned strands. However, during the conduct of these tests it was decided to change the type of loading and include centerline as well as third point loading. To this effect two more beams were added to the program. The program was further subdivided by the inclusion of strands made by two different manufacturers, hereafter referred to as A and B. Although nominally designated as 7/16" strands the areas of these strands are different, hence the different percentages of steel. The material that follows includes the third point loading series only.

A. DESCRIPTION OF BEAMS:

The cross-section of the beams is 6"x12", the longer dimension being horizontal. Twelve feet long, they were tested on 11'-6" on centers. Two beams were poured at a time, each containing two strands placed 8 inches apart and 2 inches from the bottom thus reducing the eccentricity to 1 inch. The percentages of steel used in these tests were .467 (A) and .454 (B). The prestressing of the strands was performed by means of two 35 ton mechanical jacks and the jacking frame previously used on similar tests. One of the manufacturers in his catalogue makes definite recommendations as to the load to be used in prestressing its strand and the other does not specify any particular stress. The recommended load of 18,900 lbs. was used for both types of strands. The unit stress in case A was 168,900 psi and in case B 173,500 psi. The load was applied to the strands through calibrated dynamometers of a type similar to the ones described in Progress...
The strands were stress-relieved and their ultimate strengths varied from 253,000 psi for strand A to 266,000 for strand B. It was noticed that although the two reels were kept under identical storage conditions that the strand designated as B started to rust lightly shortly after reaching the laboratory. No rusting was observed on strand A. The only cleaning performed on the strands consisted of a pass or two with a cloth impregnated with carbon tetrachloride. This cleaning did not remove the light rust on strand B.

**B. CONCRETE PROPERTIES:**

The strengths of the concrete used in these tests were selected to be 3500, 5500, and 7500 psi at release. The aggregates were from the source used before for other concrete work poured in the laboratory. The fineness modulus of the sand was found to be 2.32 and that of the crushed stone (maximum size 1/2") 5.43. The mix proportions for the four different strengths used are shown in Table I below:

<table>
<thead>
<tr>
<th></th>
<th>Mix 0 5500 at 28 Days</th>
<th>Mix I 3500 psi At Release</th>
<th>Mix II 5500 psi At Release</th>
<th>Mix III 7500 psi At Release</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cement, Hi Early:</td>
<td>610</td>
<td>536</td>
<td>714</td>
<td>940</td>
</tr>
<tr>
<td>Water</td>
<td>325</td>
<td>334</td>
<td>334</td>
<td>334</td>
</tr>
<tr>
<td>Sand</td>
<td>1170</td>
<td>1190</td>
<td>1120</td>
<td>1110</td>
</tr>
<tr>
<td>Stone</td>
<td>1910</td>
<td>1940</td>
<td>1820</td>
<td>1670</td>
</tr>
<tr>
<td>Adm.: Darex AEA</td>
<td>--</td>
<td>120 cc</td>
<td>225 cc</td>
<td>300 cc</td>
</tr>
<tr>
<td>Slump</td>
<td>1/2&quot;</td>
<td>3&quot;</td>
<td>1 1/2&quot;</td>
<td>1/2&quot;</td>
</tr>
</tbody>
</table>

**NOTE:** Darex AEA was added to insure the workability which otherwise would have had to be sacrificed in the 7500 psi mix; hence all the prestressed concrete mixes contained Darex AEA with average air content of around 4%.

Control cylinders to determine the strengths at release, at test time, and for modulus of elasticity were poured with each pair of beams. In addition to the cylinders, 6x6x36" specimens were cast to determine
the modulus of rupture of the concrete at the time of test. All the beams were moist cured for at least a week prior to the releasing of the strands. Depending on the testing schedule the curing was either continued in the moist room or the beams were removed to the laboratory for testing.

C. TEST PROCEDURE:

1. At Release

(a) Slip Measurements

A slip gage composed of two Ames dials mounted on a bracket was used for each strand at both ends of the beam. The plungers of the Ames dials rested on smooth metal plates applied to the concrete by means of regular sealing wax. The rotation of each strand was determined through the use of metallic arms attached to the brackets supporting the Ames dials of the slip gage.

(b) Strains in Concrete

The strains in the concrete were first measured by Huggenberger gages placed side by side and forming short continuous lines at both ends of a beam, however the system proved to be quite unreliable and was later substituted by continuous lines of Whittemore gages on both sides of each beam.

2. Static Tests

(a) Loading Conditions

Third point loading was used in the testing of the first eight beams. The general procedure consisted in loading each beam in three cycles. The first cycle beyond the cracking load, the second cycle 125% to 150% of the observed cracking load, and the third cycle to collapse.
(b) **Instrumentation**

I. **Deflection gage:**

An Ames dial mounted on an angle iron seating on short steel pins over the supports provided the means for measuring deflections. To avoid the inconvenience of resetting the gage too often, in the third cycle the deflections were measured with a scale.

II. **Whittemore gages:**

Whittemore gages were used to measure the strain in the concrete up till cracking load. Placed at the center of the span were two lines of plugs spaced at 2 inches.

III. **Slip gages:**

The slip gages used were reinstalled on the strands and used in a manner identical to that at release.

(c) **Crack Marking & Photography**

To make the detection of cracks easier all the beams were whitewashed prior to the static tests. Cracks were marked as to their height at different load levels and were later retouched for photography.

D. **TEST RESULTS AND OBSERVATIONS:**

1. **At Release**

   In all cases the prestress was transferred by releasing the jack gradually.

   (a) **Whittemore Readings**

   The Whittemore gage plugs were placed in continuous lines at the level of the strands for all but two of the prestressed beams. Readings were taken with the specimens resting on the forms and also on 11'-6" centers. The time elapsed between these two positions varied between an hour for series II to two hours for series I. The curves drawn from these readings are shown in Figures 1 and 2.
The plot for beams I-AI and I-BI shows that the maximum strain is attained within 27 inches or 62 diameters from the ends of the beams. A study of the curves obtained for beams I-AII and I-BII show a similar pattern although the trend is much less apparent. However, one can venture to say that the maximum strain is reached within 32 inches or 73 diameters. These results compare remarkably well with the results reported by Mr. J.R. Janney for differing sizes of wires and strand and varying strength of concrete in his paper entitled: "The Nature of Bond in Pretensioned Prestressed Concrete". The shape of the stress transfer curve at either end of the beams is not defined and the curves for the different faces of the same beam are in some instances quite different, nonetheless they are given to illustrate the results obtained at release. No apparent relationship can be established between strength of concrete and length of anchorage. This observation is further sustained by the seeming lack of relationship between slip at release and the strength of concrete as can be seen from Table II. Thus it appears that the length of anchorage is not a function of adhesion but rather a function of Poisson's Ratio and consequently of the wedge effect at the free end of the strand.

<table>
<thead>
<tr>
<th>Age days</th>
<th>Beam</th>
<th>Strand No.</th>
<th>Slip At Release**</th>
<th>Strength of Concrete (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Jacking End 0.001&quot;</td>
<td>Anchor End 0.001&quot;</td>
</tr>
<tr>
<td>9</td>
<td>1-AI</td>
<td>1</td>
<td>83</td>
<td>63</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>83</td>
<td>70</td>
</tr>
<tr>
<td>9</td>
<td>1-BI</td>
<td>1</td>
<td>106</td>
<td>71</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>79</td>
<td>64</td>
</tr>
<tr>
<td>7</td>
<td>1-AII</td>
<td>1</td>
<td>105</td>
<td>102</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>56</td>
<td>58</td>
</tr>
<tr>
<td>7</td>
<td>1-BII</td>
<td>1</td>
<td>66</td>
<td>131</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>88</td>
<td>75</td>
</tr>
<tr>
<td>28</td>
<td>1-AIII</td>
<td>1</td>
<td>126</td>
<td>48</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>74</td>
<td>36</td>
</tr>
<tr>
<td>28</td>
<td>1-BIII</td>
<td>1</td>
<td>149</td>
<td>62</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>119</td>
<td>70</td>
</tr>
</tbody>
</table>

*Only 2 cylinders tested.

**It is of interest to note that slip at release or near the ultimate load was accompanied by rotation of the strands. No attempt has been made to correlate the two phenomena.
The total elastic deformations as measured with the Whittemore gages for the different beams are given in Table III.

TABLE III

<table>
<thead>
<tr>
<th>Beam</th>
<th>Deformations (in x 10^{-4})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>North Face</td>
</tr>
<tr>
<td>1-AI</td>
<td>214</td>
</tr>
<tr>
<td>1-BI</td>
<td>281</td>
</tr>
<tr>
<td>1-AII</td>
<td>169</td>
</tr>
<tr>
<td>1-BII</td>
<td>218</td>
</tr>
</tbody>
</table>

Since no determination of the modulus of elasticity was done at the time of release the values in Table III represent only total strain over the entire lengths of the beams. One must also remember that in some of the cases the readings were taken an hour and a half to two hours after releasing the prestress which would necessitate the inclusion of the effect of plastic flow into the computations.

2. Static Tests

For summary of results see Table IV.

(a) O-Series

Designed for a concrete strength of 5500 psi at 28 days beam 1-A0 was cracked before the test, therefore its behavior did not approach the calculated values. The low cracking load was caused by the existing crack and it is believed that the slipping which occurred at 68.6% of ultimate load was probably a direct consequence of the cracking of the beam prior to the test. In spite of this fact however, and as seen in Fig. 4, the crack pattern is very satisfactory. There are five cracks on either side of the centerline spaced at about one foot apart. This crack distribution is evidence of satisfactory bonding qualities of the non-stressed strands. Beam 1-B0 behaved better than its equivalent as the calculated cracking load of 1740 lbs. is only within 340 lbs. of the observed one; and even though slip was
recorded at 94% of ultimate, the value of it does not exceed 5/1000ths of an inch. The calculated stress of the steel is 241,000 psi, a little over 90% of the ultimate value. It is interesting to note that the number of cracks in I-BO matches that of beam 1-AO and the spacing is the same. Beam I-BO failed by crushing of concrete in the center whereas beam 1-AO crushed under one of the loading points. Both beams deflected appreciably before final collapse as can be seen from Table IV.
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1-AO</td>
<td>29</td>
<td>5540</td>
<td>23(N) 55(S)</td>
<td>15</td>
<td>7280</td>
<td>5000</td>
<td>68.6%</td>
<td>8720</td>
<td>142,500</td>
<td>1750</td>
</tr>
<tr>
<td>1-BO</td>
<td>28</td>
<td>5561</td>
<td>5(N) 1(S)</td>
<td>4</td>
<td>8500</td>
<td>8000</td>
<td>94.0%</td>
<td>8820</td>
<td>241,000</td>
<td>1740</td>
</tr>
<tr>
<td>1-AI</td>
<td>26</td>
<td>4020</td>
<td>70(N) 56(S)</td>
<td>16</td>
<td>6000</td>
<td>5900</td>
<td>98.4%</td>
<td>8200</td>
<td>181,000</td>
<td>4090</td>
</tr>
<tr>
<td>1-BI</td>
<td>27</td>
<td>4050</td>
<td>75(N) 56(S)</td>
<td>0</td>
<td>7210</td>
<td>7000</td>
<td>97.1%</td>
<td>8380</td>
<td>222,000</td>
<td>4090</td>
</tr>
<tr>
<td>1-AII</td>
<td>26</td>
<td>7040</td>
<td>2(N) 0(S)</td>
<td>0</td>
<td>7740</td>
<td>--</td>
<td>100%</td>
<td>8900</td>
<td>253,000</td>
<td>4520</td>
</tr>
<tr>
<td>1-BII</td>
<td>27</td>
<td>7000</td>
<td>68(N) 73(S)</td>
<td>5.5</td>
<td>8410</td>
<td>6300</td>
<td>75%</td>
<td>9100</td>
<td>184,000</td>
<td>4520</td>
</tr>
<tr>
<td>1-AIII</td>
<td>32</td>
<td>8267</td>
<td>8(N) 3(S)</td>
<td>2</td>
<td>7920</td>
<td>7500</td>
<td>94.6%</td>
<td>9060</td>
<td>210,000</td>
<td>4800</td>
</tr>
<tr>
<td>1-BIII</td>
<td>31</td>
<td>6879</td>
<td>11(N) 11(S)</td>
<td>0</td>
<td>8580</td>
<td>8400</td>
<td>98.0%</td>
<td>9080</td>
<td>246,000</td>
<td>4350</td>
</tr>
</tbody>
</table>

*Fell and cracked while transporting
(b) **I-Series**

Designed for a strength of 3500 at release the cylinders for both beams tested at 4020 for I-AI and 4050 for I-BI. Slip occurred in both beams at over 95% of ultimate load. The one significant difference between the behaviors of the two beams is that only one strand slipped in beam I-BI and it occurred at the end nearest to the failure plane. The crack pattern in both beams is regular and confined within the loading points. It was observed that upon removal of the load the cracks closed and the beams recovered practically all of their deflection. The calculated cracking loads for both beams agreed very closely with the actual ones, (Table IV). Collapse occurred in beam I-BI by crushing of concrete, similarly the concrete crushed at the top of beam I-AI under one of the load points.

(c) **II-Series**

The concrete in these two beams was designed for a strength of 5500 psi at release. At test time cylinders poured with the beams tested at 7000 psi. Beam I-AII is the only beam among the eight discussed that did not exhibit any slip at or near ultimate load of 7740 lbs. Upon removal of the second cycle maximum load of 6500 lbs, the cracks closed up. Failure occurred at the moment the concrete crushed in the center of the span because of excessive yielding of the steel. Cracking was distributed uniformly within the middle third of the span with an approximate spacing of eight inches.

Beam I-BII although exhibiting a similar crack distribution as its companion beam failed suddenly and as can be judged from Fig.5 in a diagonal tension crack. The only significant slip in the strands occurred at the end nearest to the failure plane. The load at which slip occurred is only 75% of the actual ultimate load. The calculated stress in the steel at that load is 184,000 psi a little over the
initial prestressing stress. Although the beam failed suddenly an appreciable deflection (±4.25") presaged the failure which even though sudden was not unexpected.

(d) **III- Series**

This pair of beams contained the strongest concrete of all the beams tested under third point loading. The calculated cracking load for beam I-AIII compares very favorably with the observed load. Slip did not occur until 94.6% of ultimate and its value in all cases was less than 10/1000th of an inch. The cracks as evidenced from Fig. 4 are confined within the constant moment region with an average spacing of approximately 6". The beam failed when concrete under one of the load points crushed at a load of 7920 lbs. Beam I-BIII behaved in a fashion much like its equivalent beam in that the crack distribution, although slightly different, is very similar. The slip occurred at 98% of the ultimate load of 8580 lbs. It is interesting to note that this beam cracked at a load appreciably higher than the rest of the beams. With the exception of the un-prestressed beams, the cracking loads of the remaining five beams varied within 800 lbs. of each other. The maximum recorded deflection for beam I-AIII, and I-BIII was 5 7/8" and 5 in., respectively.
E. CONCLUSIONS

Although much more satisfactory than the tests conducted on 7/16" strands last summer, the reported tests are not very conclusive. However, they demonstrated one important fact which should be kept in mind in the search for the maximum size of strands to be used in pretensioned construction. The uniform distribution of the prestressing over the concrete section is as important as the percentage of steel. In the tests performed last summer the percentage of steel was 0.151% whereas in the present tests the percentages are .467 and .454. Although the steel percentage was increased three times in the herein reported beams, nonetheless one must consider the manner in which the prestressing force is transferred to the concrete. In one case the force is concentrated within one strand, thus highly stressing the concrete surrounding it. This results in what probably may be termed as a highly yielded concrete around the strand hence only with a small amount of stress being distributed over the remainder of the section. Compared to the foregoing case, the present beams contain two strands which even though spaced at 8 inches apart distribute the prestressing force over a larger area of concrete thus resulting in a uniformly stressed section with increased efficiency.

The performance of the beams has been very satisfactory especially if one considers the fact that the equivalent design load for the prestressed beams is 2210 lbs. (See appendix). The design load is defined as the load which produces 0 stress at the bottom fibers of the beams. Such computations presuppose the knowledge of the prestressing force which has been computed according to the expression recommended in "Criteria for Prestressed Concrete Bridges". The value calculated for the prestressing force at the time of the tests was 16,400 per strand or a total force of 32,800 lbs. per beam.
Thus the resulting losses can be determined as being 13%. This calculated value of the prestressing force was also the basis for the determination of the cracking loads. The close relation between the observed and the calculated cracking loads show that the calculated prestressing force at the time of release was almost the same in all the beams and more important still that the basis of computations was sound. The question then arises as to what will one call a safe maximum size of strand. Is it a strand that does not slip at ultimate load or is it a strand that slips at a considered safe percentage of the ultimate? In the tests reported here even the lowest of the observed slip load (6300 lbs) is equivalent to 2.85 (Design + Dead Load). Nonetheless the undisputed fact remains that designers and constructors would be more willing to use a strand if they know that it withstood the ultimate load of a beam which it was prestressing without slip.

As mentioned at the beginning of this report the first eight beams constitute only one part of the whole program, the second part, although essentially similar to the one just reported on, differs in that the beams are to be tested with centerpoint loading. Therefore, it would be premature to make any recommendations for any future testing. However, tentatively here are some of the aspects of the problem that could be investigated:

1. Distribution of the prestress throughout the section and its effect on bond of strands.
2. Minimum percentage of steel and its relation to the size of strands.
3. Minimum spacing between strands.
4. Behavior under dynamic loading.
5. Minimum cover of concrete.
(a) Beam 1-AI at Release. Resting on Forms

(b) Beam 1-AI Two Hours after Release. Supported 11'-6" cc

(c) Beam 1-BI at Release. Resting on Forms.

(d) Beam 1-BI Two Hours After Release. Supported 11'-6" cc

FIG. 1 CONCRETE STRAIN DISTRIBUTION
(a) Beam 1-AII At Release. Resting on Forms

(b) Beam 1-AII 1/2 Hour After Release. Supported 11'–6" cc

(c) Beam 1-BII At Release. Resting on Forms.

(d) Beam 1-BII 1/2 Hour After Release. Supported 11'–6" cc

FIG. 2. CONCRETE STRAIN DISTRIBUTION
FIG. 3 LOAD DEFLECTIONS FOR BEAMS 1-BII AND 1-AII
Fig. 4  Cracking Pattern in Beams After Ultimate Load Test

Fig. 5  Cracking Pattern in Beams After Ultimate Load Test
Appendix I

Equivalent Design Load Computations

Design Load is defined as the load producing 0 stress at the bottom. Losses due to Creep, Shrinkage and Elastic Deformation according to the criteria published by BPR are expressed as follows:

\[ 6000 + 16 f_{cs} + 0.04 f_{si} \]

where \( f_{cs} \) can be computed from the design stress in the concrete. It is interpreted as being the stress due to prestressing and dead load.

\[ \frac{4}{6} \times 1050 = 700 \text{ psi} \] at centroid of steel due to prestressing

D.L. Stresses: \( M = \frac{75 \times 11.5 \times 12}{8} = 14,850 \text{ in.lbs.} \)

\[ f_c = \frac{14850 \times 1}{216} = 69 \text{ psi} \]

\[ f_{cs} = 700 - 69 = 631 \text{ psi} \]

Losses:

\[ 6000 + 631 \times 16 + 0.04 \times 174,000 \]

\[ = 6,000 + 10,100 + 6900 = 23,060 \text{ psi} \]

Net Prestressing Stress:

\[ 174,000 - 23,060 = 150,940 \text{ psi} \] Force: 16,400 lbs.

For Strands B

\[ P = \frac{152,250}{69} = 2210 \text{ lbs.} \]

Cracking Load Computations:

Assuming a Modulus of Rupture Value of \( 0.15f_c \) for low strengths and \( 0.10f_c \) for the higher strengths, let us compute the cracking loads:

\[ \begin{align*}
1-AI & : M.R.: 4020x.15 = 605 \text{ psi} \\
1-BI & : M.R.: 4050x.15 = 608 \text{ psi} \\
1-AII & : M.R.: 7040x.10 = 704 \text{ psi} \\
1-BII & : M.R.: 7000x.10 = 700 \text{ psi}
\end{align*} \]

\( \{ \text{Actual M.R. 610 psi, 740 psi} \} \)
1-AIII : 8267 x .10 = 827 psi
1-BIII : 6879 x .10 = 688 psi

A-I: -610 = 455+455-206-0.32P
B-I: -610 = 455+455-206-0.32P
A-II: -740 = 704-0.32P
B-II: -740 = 704-0.32
A-III: -827 = 704-0.32
B-III: -688 = 704-0.32

P_{cracking} = 4090 \text{ lbs.}
P_{cracking} = 4090 \text{ lbs.}
P_{cracking} = 4520 \text{ lbs.}
P_{cracking} = 4520 \text{ lbs.}
P_{cracking} = 4800 \text{ lbs.}
P_{cracking} = 4350 \text{ lbs.}
Computations for Ultimate Loads

Based on assumptions established in Progress Report No. 5.

1-B0: $f'_0 = 5561$ psi  
\[ x = \frac{3}{2} \cdot \frac{266,000}{7000} \cdot (0.0454) \cdot x = 1.305 \text{ in.} \]
\[ z = 4 - 1.305 \cdot 0.375 = 3.512 \]
\[ M_u = 266,000 \cdot 3.512 \cdot 0.2178 = 203,000 \text{ in/lbs.} \]
\[ \frac{P}{2} = 4410 \Rightarrow P = 8820 \text{ lbs.} \]

Stress at Slip: \[ 266 \times \frac{8000}{8820} = 241,000 \text{ psi} \]

1-BI: $f'_c = 4050$  
\[ x = 1.79 \text{ in.} \]
\[ z = 4 - 1.375 \cdot 1.79 = 3.329 \]
\[ M_u = 266,000 \cdot 3.329 \cdot 0.2178 = 193,000 \text{ in/lbs.} \]
\[ \frac{P}{2} = \frac{123,000}{46} = 4190 \Rightarrow P = 8380 \text{ lbs.} \]

Stress at Slip: \[ 266 \times \frac{7000}{8380} = 222,000 \text{ psi} \]

1-BII: $f'_c = 7000$  
\[ x = 1.035 \text{ in.} \]
\[ z = d - \frac{3}{8} \cdot x = 4 - 0.375 \cdot 1.035 = 3.613 \]
\[ M_u = 266,000 \cdot 3.612 \cdot 0.2178 = 209,000 \text{ in/lbs.} \]
\[ \frac{P}{2} = \frac{219,000}{46} = 4550 \Rightarrow P = 9100 \text{ lbs.} \]

Stress at Slip: \[ 266 \times \frac{6300}{9100} = 184,000 \text{ psi} \]

1-BIII: $f'_c = 6879$  
\[ x = 1.055 \text{ in.} \]
\[ z = 4 - 0.375 \cdot 1.055 = 3.604 \]
\[ M_u = 266,000 \cdot 3.604 \cdot 0.2178 = 208,500 \text{ in/lbs.} \]
\[ \frac{P}{2} = \frac{4540}{9080} \Rightarrow P = 9080 \text{ lbs.} \]

Stress at Slip: \[ 266 \times \frac{8400}{9080} = 246,000 \text{ psi} \]
Computations for Ultimate Load

Based on the assumptions established in Progress Report No.5.

1-A0: \( f'_c = \frac{5640}{2} \cdot \frac{253,000}{5640} \cdot (0.000467) \cdot 4 = 1.25 \text{ in.} \)

\[ z = 4 - 0.375 \cdot 1.25 = 3.546 \text{ in.} \]

\[ M_u = 252,000 \cdot 3.546 \cdot 0.224 = 200,000 \text{ in/lbs.} \]

\[ P = \frac{4360}{2} = 8720 \text{ lbs.} \]

Stress at Slip: \( 253,000 \cdot \frac{5000}{8720} = 142,500 \text{ psi} \)

1-A1: \( f'_c = 4020 \) \( x = 1.76 \text{ in.} \)

\[ z = 4 - 0.375 \cdot 1.76 = 3.34 \text{ in.} \]

\[ M_u = 252,000 \cdot 3.34 \cdot 0.224 = 188,500 \text{ in/lbs.} \]

\[ P = \frac{188,500}{2} = 4100 \cdot \frac{8200}{46} = 8200 \text{ lbs.} \]

Stress at Slip: \( 253,000 \cdot \frac{5900}{8200} = 181,000 \text{ psi} \)

1-AII: \( f'_c = 7040 \) \( x = 1.00 \text{ in.} \)

\[ z = d - 3 \cdot \frac{3}{8} \cdot x = 4 - 0.375 \cdot 1.00 = 3.625 \text{ in.} \]

\[ M_u = 252,000 \cdot 3.625 \cdot 0.224 = 205,000 \text{ in/lbs.} \]

\[ P = \frac{205,000}{2} = 4450 \cdot \frac{8900}{46} = 8900 \text{ lbs.} \]

Stress at Slip: \( 253,000 \cdot \frac{8900}{8900} = 253,000 \text{ psi} \)

1-AIII: \( f'_c = 8524 \) \( x = 0.83 \text{ in.} \)

\[ z = 4 - 0.375 \cdot 0.83 = 3.689 \text{ in.} \]

\[ M_u = 253,000 \cdot 3.689 \cdot 0.224 = 208,000 \text{ in/lbs.} \]

\[ P = \frac{208,000}{2} = 4530 \cdot \frac{9060}{46} = 9060 \text{ lbs.} \]

Stress at Slip: \( 253,000 \cdot \frac{7500}{9060} = 210,000 \text{ psi} \)
APPENDIX III

Moduli of Elasticity

Values of Moduli of Elasticity.

The following values were obtained, at the time of testing, by the following three methods:

1. From the deflection of the beams.
2. The Secant Modulus
3. Slope of the Last Cycle of Loading (3 cycles total)

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<th>Beam Test</th>
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