THE ULTIMATE STRENGTH OF PRE-STRESSED AND CONVENTIONALLY REINFORCED CONCRETE UNDER THE COMBINED ACTION OF MOMENT AND SHEAR

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LEHIGH UNIVERSITY
Fritz Laboratory Report 223.17
October 1957
ACKNOWLEDGEMENTS

This work has been carried out at

LEHIGH UNIVERSITY
FRITZ ENGINEERING LABORATORY
DEPARTMENT OF CIVIL ENGINEERING

Director
Professor W.J. Eney

as a part of an investigation sponsored by

PENNSYLVANIA STATE HIGHWAY DEPARTMENT
U.S. BUREAU OF PUBLIC ROADS
REINFORCED CONCRETE RESEARCH COUNCIL
CONCRETE PRODUCTS DIVISION, AMERICAN-MARIETTA COMPANY
AMERICAN STEEL AND WIRE DIVISION, U.S. STEEL CORPORATION
JOHN A. ROEBLING'S SONS CORPORATION
LEHIGH UNIVERSITY

The author is especially indebted to

Professor CARL E. EKBERG, JR.
(Chairman of the Concrete Division)
Professor BRUNO THURLIMANN

for their valuable advice

Active assistance in the preparation of this study was given by: Mssrs. Mario J. Danieri, Charles R. Wilson, Jose L. Montemayor, Research Assistants, and Mr. Alex W. Adler. The manuscript was typed by Miss Elizabeth R. Arbogast.

The author wishes to express his sincere thanks and appreciation for all the help and advice he enjoyed in the course of this investigation.
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SYNOPSIS

The prediction of the ultimate strength of prestressed and conventionally reinforced concrete beams under the combined action of moment and shear (abbreviated called shear strength) thus far had to be based on empirical formulas. A theoretical solution was primarily hampered because no failure mechanism pertaining to the region of moment and shear was established.

A rational approach to the problem is sought by stating a hypothesis concerning the failure mechanism for the region of diagonal cracks. The deformations of this region are considered to result from a shear rotation about the neutral axis of the prospective failure cross-section. By observing consecutively the stresses caused by these deformations, the equilibrium conditions and a suitable failure criterion for plain concrete (e.g. Mohr's failure criterion), the condition of failure can be obtained.

The theory is first presented for simply reinforced concrete beams (simply supported or restrained) and later generalized for prestressing, web and compression reinforcement.

Theoretical estimates are compared with test results. Reasonable agreement is obtained.
1. INTRODUCTION

1.1 GENERAL

In the past half-century of concrete research, the question of shear has incessantly occupied a vast interest. A great number of experimental investigations, covering more than a thousand beam tests, have been reported in the literature. In spite of this wide experience, it was not possible to realize a rational approach for the determination of the ultimate strength of reinforced concrete beams under the combined action of moment and shear, abbreviately, but somewhat unprecisely, called "shear strength". The various formulas, suggested in the literature, are all of an empirical nature and bear, therefore the disadvantage that their validity is to some extent restricted to the particular conditions of the tests from which they are derived, especially since most of the test beams can only be considered as reduced models of true structures. It would certainly be preferable to find first a rationally founded theory and use tests later for its verification. In spite of the odds, which have to be expected, such a rational approach is attempted in this investigation.

The scope of this investigation was originally confined to prestressed concrete. The approach made here
permitted, however, the consideration of the problem as a whole, including conventional reinforcement, prestressing, as well as end restraint and the shape of the cross-section. The finding that the ultimate shear strength of prestressed concrete members can be derived from the strength of the similar non-prestressed member is of great practical significance, because the multitude of test results available for conventionally reinforced beams may compensate somewhat for the lack of information concerning the shear strength of prestressed concrete.
1.2 STATEMENT OF THE PROBLEM

The cognition that elastic stresses or strains under working load very often do not represent a satisfactory criterion for the estimation of the safety of a structure, has in the past decade greatly influenced the research on the "problem of shear" in that the interest has more and more been concentrated toward the prediction of the ultimate strength. Especially for a material like prestressed concrete, the determination of the ultimate strength has always been felt to be imperative. While this did not cause any special difficulties for members failing in pure bending, it was not possible, as mentioned before, to find a rational solution for members failing in the region subjected to moment and shear. There exist various empirical formulas for conventionally reinforced beams, but very little information is available for prestressed concrete.

The specific problem of this paper is therefore to determine the ultimate strength of prestressed and conventionally reinforced beams, failing in connection with the development of diagonal cracks in the region subjected to moment and shear (Fig. 1.2/1).

The designation used for such failures is not uniform. Terms like
Crack Pattern of a Beam Subjected to Moment and Shear
Shear Failure (S)
Diagonal Tension Failure (DT)
Shear Compression Failure (SC)
Bond Failure (B)

can be found in the literature. Even though the term "Shear Failure" is the most unprecise of all, it is the one most often used and serves somewhat as an overall expression for others. Especially in the earlier stage of the research, failures due to slippage of the longitudinal reinforcement, i.e. "Bond Failures," were often erroneously interpreted as shear failures because of their similar appearance to the latter.

"Diagonal Tension Failures" are characterized by a sudden development of one or more diagonal cracks immediately followed by failure. If failure occurs by crushing of the concrete compressive zone considerably after the formation of diagonal cracks, it is usually called "Shear Compression Failure." As for the theory stated in this paper, it will not be necessary to distinguish between these different types of failure, because the derivation of the formulas accounts for all the mentioned phenomena, excluding bond.

The ultimate strength of a beam is given by some authors as the shear force $V_u$ acting at the failure cross-section (Fig. 1.2/2) and by others as the corresponding moment, $M_{su}$.
Fig. 1.2/2
Forces at Failure

Fig. 1.2/3
Stresses at Failure
As will be shown, both of these quantities taken separately, fail to present a theoretically satisfactory expression for the ultimate strength of a cross-section subjected to moment and shear. The inseparable concatenation of moment and shear requires clearly that the combination of both must be considered. Since it would be rather inconvenient, however, to specify the strength always by two quantities, it has been chosen in this paper, for reasons explained later, to express the strength by the moment capacity of the prospective failure cross-section as a function of the relative magnitude of the shear force, i.e.

\[ M_{su} = M_{su} \left( \frac{M}{Vh} \right) \]  

(1/2/1)

Instead of carrying on the correct, but somewhat lengthy term, "ultimate moment of a cross-section under the combined action of moment and shear", this quantity \( M_{su} \) will henceforth be referred to as "ultimate shear-moment".

In order to determine this moment capacity, it is necessary to obtain information concerning the concrete and steel stresses produced by the moment and shear. The distribution and extension of concrete stresses is of primary importance, since it is always by exhausting the capacity of the concrete compression zone that beams finally
fail. (Failures by rupture of the longitudinal reinforcing occur only for such low percentages of reinforcement as to be outside the range of practicability). The generally accepted simplifying assumptions that

1. The contribution of the longitudinal reinforcement to the shear transfer is negligible

2. The shear transfer along the diagonal crack is negligible

together with the knowledge of the stress-strain relationship of steel and concrete, and the statement of equilibrium conditions, are not sufficient to determine the shear strength, as will be explained.

For a beam without web reinforcement, the above assumptions mean that the tension force $T$ (see Fig.1.2/2) acts in the direction of the longitudinal reinforcement and that the lines of action of $T$, $C'$ (inclined compression force) and $R$ (resultant of the external forces to the left of the considered cross-section) intersect in one point.

To satisfy the equilibrium condition, the horizontal component $(C)$ of the force $C'$ has to be equal to $T$. The couple $C$ and $T$ constitute the ultimate shear moment
The vertical component ($V_c$) of $C'$ in turn has to equal the ultimate shear force $V_u$

$$V_c = V_u \quad (1.2/3)$$

Expressed in terms of the stresses (see Fig. 1.2/3) these equations become

$$C = \int_0^{h_1} \sigma_c \, dA_c$$

$$T = \int_{A_{st}} \sigma_{st} \, dA_{st} \quad (1.2/4)$$

$$V_c = \int_0^{h_1} \tau \, dA_c$$

A great number of investigators have immortalized their names by assuming various distributions of the concrete normal stresses $\sigma_c$, but numerical comparisons show that the deviation of the ultimate moment resulting from various reasonable choices is rather small. What really matters, is the depth of the neutral axis ($h_1$). If this depth were known, the ultimate shear moment could be determined fairly accurately.

Unfortunately, this quantity $h_1$, has not, as yet, been derived by rational approaches.
As is sufficiently known, the depth of the neutral axis (h₁) for pure bending, is found with the aid of Navier-Bernoulli's hypothesis that plane cross-sections remain plane after deformation; a hypothesis which is sometimes referred to as deformation condition, strain condition, or compatibility condition, the latter term being used in this paper. This hypothesis does not hold true for regions subjected to moment and shear, especially not after formation of diagonal cracks. Thus it can be said, that the particular difficulty of the problem of shear is caused by the inability to formulate an appropriate compatibility condition.

Before discussing how this difficulty may be overcome, it shall be clarified why cracks in the bending zone usually do not, and why cracks in the shear zone do, devaluate Navier-Bernoulli's hypothesis.
1.3 COMPARISON WITH PURE BENDING

Navier-Bernoulli's hypothesis that plain cross-sections remain plain after deformation was adopted from the very beginning to reinforced concrete design, and it is surprising to note how little concern arose about its adaptability after the formation of cracks.

According to the mathematical theory of elasticity, the hypothesis holds true only for pure bending or bending with axial load, but not if shear is present. Furthermore, it is originally stated in terms of displacements and only on account of continuity is it possible to conclude that the strain distribution over a cross-section must be linear. Since cracks sharply interrupt the continuity, there is no direct reason why this hypothesis should still hold true. In fact, the relative displacement between concrete and steel contradicts such an assumption. Experimental evidence shows (Ref. 1.3/1; Fig. 1.3/1), that not even the steel strains of different layers of reinforcement follow a straight line over a vertical cross-section. Long experience has taught, however, that the assumption of a linear distribution of concrete and steel strains yields, in general, satisfactory results for the prediction of both stresses and strength. The reason can be found by considering first the
Fig. 1.3/1

Strain Distribution Over a Cross-Section of a Conventionally Reinforced Beam (Tests see Ref. 1.3/1)

Note: all dimensions in inches
actual strain distribution along the whole beam and second, the sensitivity of the ultimate strength to deviations from the straight line assumption.

An example of the distribution of concrete and steel strains for a beam with only vertical cracks in the region of pure bending is shown in Fig. 1.3/2. The relative strain alternations depend mainly on the quality of bond. Since for comparatively good bond the end sections of the portion of pure bending remain plane, one may assume without great error, that the average steel strains, together with the average concrete compressive strains, form a straight line over a vertical cross-section. Such an averaging process is obviously not possible for poor bond (see Fig. 1.3/2). Thus one can conclude that Navier-Bernoulli's hypothesis, in terms of strains, applies only if cracks do not result in a strain concentration.

Considerations of the theory of errors reveal, furthermore, the fact that the ultimate moment is relatively insensitive to deviations from a straight line assumption. A deviation on the part of the steel, as shown in Fig. 1.3/1 will therefore have a rather small effect on the ultimate moment, whereas a strain concentration, which can surpass the surrounding strains by several hundred per cent, can yield results, radically different from those obtained with a straight line assumption.
Ect (Concrete top strains)

$\varepsilon_{ct}$ (Concrete top strains)

$\varepsilon_{ct\ av}$

Fig.

$\varepsilon_{st\ av}$

$\varepsilon_{st}$ (steel strains)

$\varepsilon_{cb}$ (Concrete bottom strains)

good bond

poor bond

Strain Distribution of Conventionally Reinforced Beam
Aside from the fact that Navier-Bernoulli's hypothesis is not correct in the region of shear, failures due to diagonal cracks are always associated with a pronounced strain concentration (Fig. 1.3/3). The situation is worsened because the bond stresses not only have to level out the increase of the steel stresses due to cracking, but they have also to build up the tension force in the reinforcement according to the moment gradient. Thus a strain condition of the prospective failure cross-section cannot possibly be stated without considering the surrounding deformations. This is the fundamental difference between vertical and diagonal cracks: the first permit and the latter exclude, in general, a compatibility condition confined to cross-sections only.

It can furthermore be seen that the quality of bond, so far never taken into account quantitatively, is a determining variable of the problem of shear.
Fig. 1.3/3

Strain Distribution for a Beam With Diagonal Cracks
1.4 POSSIBILITIES OF APPROACH

As implied in the foregoing, there are in principle, two possibilities of attacking the problem, i.e., either to find or to evade a formulation of a compatibility condition. This distinction is chosen to concisely introduce the ideas of some investigators, however, without giving reference to all of them, a task which was already undertaken by the thorough literature research published in Ref. 1.4/1; 1.4/2.

Morsch (1.4/3), one of the first and most eminent investigators in this field, and Ritter (1.4/4), evaded a compatibility condition by stating the famous "truss-analogy", i.e., by considering the beam as a truss with the longitudinal reinforcement and the concrete top zone as tension and compression chords respectively, the truss bracing being formed by the web reinforcing (tension bars) and by the web concrete acting under 45° in compression. This constitutes clearly a rational approach. The depth of the concrete compression chord remains, however, unknown, thus the truss analogy can only furnish approximate information about the ultimate shear-moment if the failure is due to yielding of the steel. The development of modern reinforcing steels with high distinct yield points or with none at all, has shifted the failure cause from the steel
to the concrete. The truss-analogy must therefore be abandoned for the prediction of the ultimate shear moment. Mörsch's ideas prevail, however, in all specifications, as far as the computation of allowable stresses in the web reinforcement is concerned, by means of the well-known "shear diagram". This was, after all, the ultimate goal of Mörsch's research, i.e., to dimension the web reinforcement such that failure occurs in bending rather than in shear. This school of thought was further developed and refined, notably by Dischinger (1.4/5) and Rausch (1.4/6).

Several investigators, e.g., Richart (1.4/7), Morretto (1.4/8), and Clark (1.4/9), tried to derive the shear strength by formulating the nominal average shearing stress ($\nu_u$) at failure as a function of various parameters such as the concrete strength ($f'_c$), the ratio of longitudinal and web reinforcement ($p$ and $r$), and the ratio of the shear span* to the effective depth ($a/h$) of the beam,

$$\nu_u = \frac{v_u}{bh} = F (f'_c; p; r; a/h) \quad (1.4/1)$$

Since this nominal shearing stress has no actual significance in the configuration of the stresses at failure, it is not surprising that this approach failed to give satisfactory agreement outside the range of the tests set up for the derivation of the particular formulas.

* The shear span is usually understood to be the horizontal distance between the support and the first load
Most of the extensive research performed in the last decade at the University of Illinois belongs in the category of avoiding the formulation of a compatibility condition. The approach suggested in the publication "Strength in Shear of Reinforced Concrete Beams" by Laupa, Siess and Newmark (1.4/2) is characterized by its striking simplicity which marks its special value but also its limitation. Their basic idea was to derive an empirical expression for the quantity

\[
\frac{M_{su}}{bh^2f'_c} \left( k + np' \right)
\]

as a function of whatever variable might be of concern. For the interpretation of test results, it proved advantageous to introduce the term \( k \), i.e. the relative depth of the neutral axis as ordinarily determined for transformed areas (straight line theory). The quantity "n" refers to the ratio of the modulus of elasticity of steel to that of concrete and \( p' \) is the ratio of the compressive reinforcement. The investigators suggested for beams without web reinforcement

\[
\frac{M_{su}}{bh^2f'_c} \left( k + np' \right) = 0.57 - \frac{4.5 f'_c}{10^4} \quad (1.4/2)
\]

This empirical expression was obtained by fitting a straight line through a plot of a great number of test results. Some
limitation lies in the lack of revealing a rational failure mechanism. It was therefore necessary to derive the influence of stirrups again purely empirically, which resulted in a much poorer agreement with test results. For the same reason, no rational modification is possible for prestressed concrete.

Moody, Viest, Elstner, and Hognestad (1.4/10) have chosen to give empirical information about the shear strength essentially in terms of the steel stress at failure.

\[
f_s = 0.729 \left[ 6.9 \times 10^{-4} E_s \left( -1 + \sqrt{1 + \frac{1450}{pE_s/k_1k_2f_c^2}} \right) \right]
\]

(1.4/3)

With this information, the ultimate moment can be calculated by observing the equilibrium conditions alone, which the authors have chosen to formulate very meticulously.

Only one attempt to state an outright rational strain or compatibility condition can be found in the literature. In his dissertation, "Shear Strength Of Simply Supported Prestressed Concrete Beams", Zwoyer (1.4/11) suggested that the average strains of the whole shear span remain in a linear relationship over the depth of the beam. It is hard, however, to conceive what bearing this average strain should have on the stress concentration at the failure cross-section.

As for other methods of attack, one might be inclined to rely on some kind of a lower bound theorem as successfully
used in the plastic analysis of steel structures. It has to be said, however, that due to the development of cracks, the exact dimensions of the load carrying parts of the structure are essentially unknown, thus excluding the application of such methods.

The approach proposed in this paper is an endeavor to obtain a rationally founded compatibility condition. In addition to the variables taken into account in most investigations, the influence of the bond and the influence of the state of stress in the concrete, are considered. This does not make the theory more complicated; on the contrary, they permit a rational numerical explanation of phenomena which so far had to be derived empirically.
2. THE INFLUENCE OF THE STATE OF STRESS ON SHEAR FAILURES

2.1 GENERAL

As mentioned in the introduction, a theoretically satisfactory failure condition can only be expressed by a combination of both moment and shear force. Since the final failure occurs always by destruction of the concrete compressive zone, it will logically depend on the state of stress prevailing in this zone, and not only on the normal stress component as generally presumed. Consequently it will be necessary first to determine the stress configuration produced by the combination of moment and shear and second to compare this configuration with a suitable failure criterion for plain concrete.

The first task will, admittedly, involve some guessing; the influence of its arbitrariness can, however, be estimated.

Concerning the second point, it has to be emphasized that, contrary to some statements in the literature, reliable failure criteria for plain concrete are firmly established. The one stated by Mohr (2.1/1) some 50 years ago, for example, has thoroughly been checked by many tests and many investigators.
2.2 THE STATE OF STRESS OF BEAMS SUBJECTED TO MOMENT AND SHEAR

Prior to the formation of diagonal cracks, the approximate stress distribution can easily be determined, if one retains the assumption that plain cross-section remain plain after deformation of the beam — an assumption which, due to the presence of shear, is theoretically not quite correct, but which seems permissible in view of the far graver idealizations necessary because of the materials employed.

With this assumption the formulas for the stresses over a given cross-section become

\[ \sigma_x = \frac{F}{bh} - \frac{Fey}{I} + \frac{M}{I} \psi \]

\[ \tau_{xy} = \frac{VQ}{bl} \]

(2.2/1)

where \( Q = \int_{\bar{y}}^{y_c} y \, dA \)

An example of this stress distribution is shown in Fig. 2.2/1 for a rectangular non-prestressed cross-section.

After the formation of diagonal cracks the stress configuration changes abruptly because the internal forces previously carried by the concrete tension zone have to be redistributed (Fig. 2.2/2). This information, however, does
Fig. 2.2/1
Stress Distribution of an Uncracked Rectangular Cross Section

Fig. 2.2/2
Redistribution of Internal Forces After Formation of the Diagonal Crack
not suffice to determine the state of stress prevailing in an element of length \( dx \) of the concrete compressive zone (Fig. 2.2/2). This requires the additional, and to some degree arbitrary, assumption that

the distribution curves of the normal stresses on both sides of \( dx \) are second order parabolas with the vertex at the concrete top fiber (Fig. 2.2/3).

This assumption is not so plausible as it might appear at first glance. Firstly, the parabolic shape is not rationally rooted because the vertical sections above the crack must not necessarily remain plane, and secondly, if they do remain plane and if the stress strain relationship of concrete is parabolic, the curve to the left of the element \( dx \) could not possibly have its vertex at the same level as the curve to the right. It will therefore be necessary to estimate the influence of the arbitrariness of this assumption. With the notations given in Fig. 2.2/3, the normal stresses become

\[
\sigma = \sigma_0 \left[ 1 - \left( \frac{y}{h_L} \right)^2 \right] \quad (2.2/2) \\
\sigma' = \sigma'_0 \left[ 1 - \left( \frac{y}{h_L + \tan \gamma \cdot dx} \right)^2 \right] \quad (2.2/3)
\]

where the unprimed quantities refer to the right, and the primed to the left side of the element \( dx \). The maximum stresses at \( y = 0 \) are denoted by the subscript \( o \).
Normal Stress Distribution

Shearing Stress Distribution

\[
\sigma_{\text{max}} = \sigma = 1.5 \sigma_{\text{av}}
\]

\[
\tau_{\text{max}} = 1.54 \tau_{\text{av}}
\]

Fig. 2.2/3

Stress Distribution at the Concrete Compressive Zone
For a state of equilibrium, the resultant forces of the two stress blocks have to be equal, thus

\[
\sigma_o' = \frac{h_1}{h_1 + \tan \gamma} \sigma_o \tag{2.2/4}
\]

This configuration of normal stresses implicitly determines the magnitude and distribution of the shearing stresses which can be found by considering the equilibrium conditions of an element \(dx \cdot y\) (shaded in Fig. 2.2/3).

\[
\bar{T}(y) = \frac{\int_0^y \sigma \, dy - \int_0^y \sigma' \, dy}{dx} \tag{2.2/5}
\]

By substituting 2.2/2, 2.2/3 and 2.2/4 in 2.2/5, one obtains

\[
\bar{T}(y) = \frac{\sigma_o}{dx} \left[ y - \frac{y^3}{3h_1^2} - \frac{h_1 y}{h_1 + \tan \gamma \cdot dx} + \frac{h_1 y^2}{(h_1 + \tan \gamma \cdot dx)^3} \right] dy
\]

\[
= \frac{\sigma_o}{dx} \left[ y \cdot \tan \gamma \frac{y}{h_1 + \tan \gamma \cdot dx} - \frac{y^3}{3h_1^2} \left( \frac{3h_1^2 \tan \gamma + 3h_1 \tan^2 \gamma \cdot dx + \tan^3 \gamma \cdot dx^2}{(h_1 + \tan \gamma \cdot dx)^3} \right) \right]
\]

Consequently,

\[
\bar{T}(y) = \lim_{dx \to 0} \bar{T}(y) = \sigma_o \left[ \frac{y \cdot \tan \gamma}{h_1} - \frac{y^3 \tan \gamma}{h_1^3} \right] \tag{2.2/6}
\]
or finally,

$$\tau (y) = \sigma_0 \cdot \tan \gamma \left[ \frac{y}{h_1} - \frac{y^3}{h_1^3} \right]$$  \hspace{1cm} (2.2/6)$$

The integral of these shearing stresses over the whole compressive zone must equal the shear force $V$:

$$V = \int_0^{h_1} b \tau dy = b \sigma_0 \tan \gamma \left[ \frac{h_1}{2} - \frac{h_1}{4} \right]$$

or

$$\sigma_0 = \frac{4V}{\tan \gamma \cdot bh_1}$$  \hspace{1cm} (2.2/7)$$

and by substituting in 2.2/6

$$\tau (y) = \frac{4V}{bh_1} \left[ \frac{y}{h_1} - \frac{y^3}{h_1^3} \right]$$  \hspace{1cm} (2.2/8)$$

According to the last equation the distribution curve of the shearing stresses is a parabola of the third order (Fig. 2.2/4), with

$$\tau_{\text{max}} \sim 1.54 \tau_{\text{av}}$$

at

$$y = \frac{h_1}{\sqrt{3}}$$

The combination of $\sigma$ and $\tau$ as shown in Fig. 2.2/4 determines the state of stress at any point of the prospective failure cross-section if one neglects local vertical normal stresses.
produced by concentrated loads. This simplification is maintained also in the vicinity of the loads, a measure which is conservative in the light of failure criteria.

In order to estimate the accuracy of this result, it is compared in Fig. 2.2/5 with two limiting cases, one by assuming a triangular, and one by assuming a rectangular normal-stress block. This comparison permits the conclusion that a failure criterion applied to the parabolic assumption determines the strength of the concrete compressive zone fairly accurately.
Distribution of Shearing Stresses Resulting From Various Normal Stress Assumptions

Fig. 2.2/5
2.3 MOHR'S FAILURE CRITERION

The failure criterion for plain concrete, introduced by Mohr (2.1/1) does not actually constitute a failure hypothesis, in that no failure cause is presupposed as, for example, in hypotheses of maximum shear or maximum normal stress. It resulted essentially from an apt method of presenting test data, which revealed the fact that the entirety of Mohr's circles representing the states of stress at failure has one common envelope which can be approximated by a parabola of the second order (2.3/1). This experimental root marks the special practicability of Mohr's failure criterion, but also implies its shortcomings. Among the latter counts the fact that it does not reveal any failure mechanism. Consequently, a prediction of the failure plane is, in general, not possible.

Many textbooks give misleading information in this respect by presuming that the angle between the failure plane and the direction of the minor principal stress is equal to one-half of the angle formed in Mohr's presentation by the horizontal axis and the radius of the circle in question, pointing to the contact of circle and envelope. This misinterpretation stems probably from the entirely different concept of Coulomb's failure hypothesis.
for soil, which presupposes the failure mechanism by stating that a given plane slides when

\[ \tau > \tau_0 + k \sigma \]

where \( \tau \) and \( \sigma \) are the stress components of the plane in question, and \( \tau_0 \) and \( k \) are material constants. In Mohr's presentation, this would mean that the failure envelope is a straight line, and that the inclination of the failure plane is indeed determined as explained above. Since this concept does not apply to concrete, there is no reason why the rules concerning the inclination of cracks should still be valid. On the contrary, experimental evidence contradicts such rules. In general, cracks form perpendicular to the direction of the minor principal stress. The crack pattern is therefore assumed to follow the stress trajectories.

Mohr's failure criterion is often criticized for another shortcoming: due to experimental difficulties and due to the two-dimensional form of presentation, the influence of the intermediate principal stress \( \sigma_3 \) (\( \sigma_1 < \sigma_3 < \sigma_2 \)) is neglected. This fact is, however, of no concern for this consideration, since the problem dealt with herein involves anyhow, only a plane state of stress.

As known, a plane state of stress can be characterized at any point by the so-called Mohr's stress-circle (Fig. 2.3/1), which can mathematically be expressed as
Mohr's Circle

Fig. 2.3/1

Mohr's Failure Criterion

\( \sigma_c \) = Concrete compressive strength

\( \sigma_t \) = Concrete tensile strength

Fig. 2.3/2
\( \sigma^2 - (\sigma_1 + \sigma_2) \sigma + \tau^2 + \sigma_1 \sigma_2 = 0 \)  
\hspace{1cm} (2.3/1)

where \( \sigma_1 \) and \( \sigma_2 \) are the principal stresses, and \( \sigma \) and \( \tau \) are the stress variables convention: \((\sigma_1, \sigma_2); \text{tension} (-); \text{compression} (+)\).

In accordance with a great number of tests, (Ref. 2.2/1), the failure envelope (Fig. 2.3/2) for a given concrete can be written as

\[ \left( \frac{\tau}{\tau_s} \right)^2 + \frac{\sigma}{\sigma_T} = 1 \hspace{1cm} (2.3/2) \]

where \( \sigma_T \) (negative) is the linear tensile strength, and \( \tau_s \) is the shearing stress at \( \sigma = 0 \). Since \( \tau_s \) is difficult to obtain experimentally, it is convenient to express the failure parabola in terms of the concrete compressive strength \( \sigma_c \).

\[ \tau^2 = \left[ \sigma_c - 2\sigma_T - 2 \sqrt{(\sigma_T - \sigma_c) \sigma_T} \right] (\sigma - \sigma_T) \hspace{1cm} (2.3/3) \]

or with the abbreviation

\[ k = \left| \frac{\sigma_T}{\sigma_c} \right| \]

\[ \tau^2 = \sigma_c \left[ 1 + 2k - 2 \sqrt{(k + 1)k} \right] (\sigma + k \sigma_c) \hspace{1cm} (2.3/4) \]
These equations do not yet constitute a failure criterion for $\sigma$ and $\tau$, they specify merely the shape of the failure envelope. The failure criterion can be found by observing that a particular stress circle, given by $\sigma_x$, $\sigma_y$, and $\tau_{xy}$, has to contact the envelope. A general stress circle is given by

$$\sigma^2 + \tau^2 - (\sigma_x + \sigma_y) \sigma + \sigma_y \sigma_x - \tau_{xy}^2 = 0 \quad (2.3/5)$$

where $\sigma$ and $\tau$ are the coordinates of any point of the circle, and $\sigma_x$, $\sigma_y$, and $\tau_{xy}$ are the stress components of an element $d\mathbf{x} d\mathbf{y}$ of a body in the $x$-$y$ coordinate system (Fig. 2.3/3).

The points of intersection of any circle with the parabola can be found by substituting Eq. 2.3/4 in Eq. 2.3/5. Solved for the $\sigma$ coordinate of the points of intersection ($\sigma_1$ and $\sigma_{II}$), this yields

$$\sigma_{I,II} = \frac{-A\sigma_c + \sigma_x + \sigma_y \pm \sqrt{(A\sigma_c - \sigma_y - \sigma_x)^2 - 4(Ak\sigma_c^2 + \sigma_x \sigma_y - \tau_{xy}^2)}}{2} \quad (2.3/6)$$

where

$$A = 1 + 2k - 2 \sqrt{(k + 1) k}$$

If now the circle shall be tangential to the parabola, then $\sigma_{I,II}$ has to be single-valued, i.e., the square root in Eq. 2.3/6 has to vanish. Consequently,
Fig. 2.3/3

Body and Stress Coordinates

Fig. 2.3/4

Mohr's Failure Criterion

Adapted for plane state of stress with $\sigma_y = 0$ ($\sigma_c = 8\sigma_T$). Any stress combination $\sigma_x, \tau_{xy}, \sigma_y = 0$ outside the shaded area will cause failure.
This is now the mathematical form of Mohr's failure criterion. In the course of substitution, however, some imaginary solutions were incorporated. If, namely,

$$\sigma_x + \sigma_y - A \sigma_C < -2k \sigma_C$$

then the contact point would lie outside the real region of the parabola. Therefore, Equation 2.3/7 applies only if

$$\sigma_x + \sigma_y - A \sigma_C > -2k \sigma_C$$  \hspace{1cm} (2.3/8)

The possibility of a failure configuration exists, however, also in region excluded by Eq. 2.3/8, namely, if the stress circle has the contact point at the vertex of the parabola, i.e.,

$$\left( k\sigma_C \right)^2 + (\sigma_x + \sigma_y) k\sigma_C + \sigma_y \sigma_x - \tau_{xy}^2 = 0$$  \hspace{1cm} (2.3/9)

Consequently, the failure criterion has to be expressed separately for the two regions, which are divided by Eq. 2.3/8.

As for the application to failure conditions of beams, use is made of the previous assumption that $\sigma_y = 0$. A further simplification is possible by relying on experimental findings concerning the ratio of tensile to compressive strength. Although this ratio is not constant, it
would, in view of the complexity of the problem, lead too far to carry this parameter on to the general theory of shear failures. Ros (Ref. 2.3/1), has found that this ratio varies from about one-sixth to one-tenth. Thus an average value of

$$k = \left| \frac{\sigma_T}{\sigma_c} \right| = \frac{1}{8}$$

is chosen hereafter, reducing the failure criterion to

\[
\left( \frac{\sigma_x}{\sigma_c} \right)^2 - \frac{\sigma_x}{\sigma_c} + \left( \frac{2\tau_{xy}}{\sigma_c} \right)^2 = 0
\]

for \( \frac{\sigma_x}{\sigma_c} > \frac{1}{4} \) \hspace{1cm} (2.3/7b)

and

\[
\left( \frac{8\tau_{xy}}{\sigma_c} \right)^2 - \left( \frac{8\sigma_x}{\sigma_c} \right) - 1 = 0
\]

for \( \frac{\sigma_x}{\sigma_c} < \frac{1}{4} \) \hspace{1cm} (2.3/9b)

These findings can geometrically be interpreted in the \( \sigma - \tau \) diagram as an affine transformation of the original failure envelope (parabolic part) and the circle of the compressive strength (elliptical part), the ratio of affinity being one-half (Fig. 2.3/4).
2.4 APPLICATION OF FAILURE CRITERION TO BEAMS

Approach by Bresler and Pister. An attempt to apply a failure criterion to determine the shearing strength of reinforced concrete beams has already been made by Bresler and Pister (Ref. 2.4/1, pp. 21 to 31). Instead of only dealing with normal stresses, the authors rightly pointed out the importance of considering the state of stress which causes failure.

Based on the investigations of the ultimate strength of reinforced concrete beams in bending by Hognestad, Hanson, and McHenry (2.4/2), it was assumed that the general normal stress at failure be

\[ \sigma_c = \frac{3900 + 0.35 f'_c f'_y}{3200 + f'_c} \]  

(2.4/1)

With a further assumption that the steel stress of the longitudinal reinforcing approximately equals the yield stress \( f_y \) (an assumption of very limited accuracy), the depth of the neutral axis was calculated to be

\[ \frac{h_1}{h} = \frac{p f_y}{\sigma_c} \]  

(2.4/2)
The shear force capacity \( V \) of the concrete compressive zone was then obtained by

\[
V = \tau \cdot bh_1 \quad (2.4/3)
\]

where \( \tau \) is a function of \( \sigma \), according to the failure criterion stated by the authors.

This concept implies that for a given concrete cylinder-strength \( f'_c \) the failure is determined by the shearing force \( V \) only, and that \( \sigma \) is constant. This would, in effect, mean that a shear force smaller than indicated by Formula 2.4/3 could never cause failure. Thus one could obtain a moment capacity of any desired magnitude by simply shifting the shearing force \( V \) far enough from the support.

A misconception lies in the assumption that for a given \( f'_c \), only \( \tau \), but not \( \sigma \) is a variable, or in other terms, only one point of the failure criterion is being considered. In reality, the number of combinations of \( \sigma \) and \( \tau \) which determine the failure is infinite. For a particular beam, the actual combination causing failure depends on the load configuration.

**New Approach.** With the aid of the shear and normal stress distribution (established in Section 2.2) and a failure criterion (given in Section 2.3), the
implements are ready for a determination of the manner in which the bending moment and the shear force affect the shear failure.

There is, however, one important question as to where and how a stress configuration has to be critical. One could think of several possibilities, i.e., failure occurs if:

1. The stress combination at one point is critical
2. The failure criterion is fulfilled at all the points of the compression zone
3. The average stresses over the whole compression zone satisfy the failure condition
4. The average stresses over a certain portion of this zone are critical

From experience, it is known that the first assumption only applies in cases where a sudden failure propagation can develop, such as in a rupture modulus test or in notched bar tests. Due to the small yet present compressive ductility of concrete, such an assumption is unsuitable. So is the second, because the stresses never become critical in the vicinity of the neutral axis. The third and the fourth assumptions both seem reasonable. By considering the shear and the normal
stress diagrams in Fig. 2.2/4, however, it is evident that with Assumption (3) the influence of the normal stresses is somewhat underestimated. In this paper preference is therefore given to Assumption (4) by considering the critical portion to extend from the vertex of the shear distribution curve to the top of the beam.

Thus the average stresses which serve to formulate the failure condition are defined as

\[
\sigma_{av}^* = \frac{\int_0^{h_1/\sqrt{3}} \sigma \, dy}{h_1/\sqrt{3}} = \frac{4 \, C}{3 \, bh_1}
\]

and

\[
\tau_{av}^* = \frac{\int_0^{h_1/\sqrt{3}} \tau \, dy}{h_1/\sqrt{3}} \sim \frac{V}{bh_1}
\]

(See Fig. 2.2/4)

Hence,

\[
\tau_{av}^* = \frac{3 \, V}{4 \, C} \sigma_{av}^* .
\]

(2.4/5)

By substituting this expression in the equations of the failure criterion (Eqs. 2.3/7b and 2.3/9b) one obtains:

for \( \frac{\sigma_{av}^*}{\sigma_c} > \frac{1}{4} \)

\[
\left(\frac{\sigma_{av}^*}{\sigma_c}\right)^2 - \frac{\sigma_{av}^*}{\sigma_c} + \left(\frac{3 \, V}{2 \, C} \frac{\sigma_{av}^*}{\sigma_c}\right)^2 = 0
\]
or
\[
\frac{\sigma_{av}^*}{\sigma_c} = \frac{1}{1 + \left(\frac{3}{2}\frac{V}{C}\right)^2}
\]  \hspace{1cm} (2.4/6a)

and for
\[
\frac{\sigma_{av}^*}{\sigma_c} < \frac{1}{4}
\]

\[
\left(\frac{6 \sigma_{av}^*}{\sigma_c} V\right)^2 - \left(\frac{8 \sigma_x}{\sigma_c}\right) - 1 = 0
\]

or
\[
\frac{\sigma_{av}^*}{\sigma_c} = 1 + \sqrt{1 + \frac{1}{4} \left(\frac{3}{2}\frac{V}{C}\right)^2}
\]  \hspace{1cm} (2.4/6b)

These formulas express the influence of the shear force on the problem of shear failure. They show that the average normal stress \(\sigma_{av}^*\), at which the beam fails, is by no means a function only of the concrete strength, but depends very significantly on the ratio of the shear force to the horizontal compressive force. The larger this ratio gets, the smaller becomes the normal stress capacity of the concrete compressive zone. One can, in the same way also express the shear stress capacity,

\[
\frac{\tau_{av}^*}{\sigma_c} = \frac{\frac{3}{4} \frac{V}{C}}{1 + \left(\frac{3}{2}\frac{V}{C}\right)^2}
\]  \hspace{1cm} (2.4/7a)
and

\[
\frac{\tau_{av}^*}{\sigma_c} = \frac{1 + \sqrt{1 + \frac{1}{4}(\frac{3V}{C})^2}}{\gamma(\frac{3V}{C})} \tag{2.4/7b}
\]

A direct application of these relations to beam problems is obviously made difficult, since C, in contrast to V, is not given by the loading condition, but depends on the depth of the compressive zone (h₁). If one would like to convert these last formulas to a direct determination of the shear strength of beams, one would have to find accurate expressions for h₁ and C, which would necessarily involve functions of all the parameters known to influence the shear strength. This procedure would be rather inaccurate. One can expect greater accuracy by working with the variation of the normal stresses (Eq. 2.4/6 and 2.4/6b; Fig. 2.4/1) as follows: The shear strength is considered to be a problem of moment capacity, where the presence of the shear force results in a reduction of the normal stress capacity of the concrete compressive zone (according to Eq. 2.4/6 and 2.4/6b) and naturally, in the formation of diagonal cracks.

On the basis of this view, it seems permissible to introduce practical simplifications, i.e., firstly in
\[
\frac{\sigma_{av}}{\sigma_c} = \frac{1}{1 + \left(\frac{3V}{2C}\right)^2} \quad \text{(Eq. 2.4/6)}
\]

\[
\frac{\tau_{av}}{\sigma_c} = \frac{3V/4C}{1 + \left(\frac{3V}{2C}\right)^2} \quad \text{(Eq. 2.4/7)}
\]
the region

\[ 1 < V/C < 2 \]

or

\[ \frac{\sigma_{av}^*}{\sigma_c} < \frac{1}{4} \]

Equations 2.4/6a and 2.4/7a are practically identical to Eq. 2.4/6b and 2.4/7b, respectively, thus sparing the need of the latter. Secondly, no grave error results by assuming that

\[ C = \frac{M}{0.9h} \]

This last assumption is naturally only used to determine the ratio \( \frac{\sigma_{av}^*}{\sigma_c} \), but it constitutes by no means an expression for the direct computation of the ultimate shear moment. Thus, the normal stress capacity of the concrete compressive zone at the end of a diagonal crack becomes

\[ \frac{\sigma_{av}^*}{\sigma_c} = \frac{1}{1 + 1.8 \left( \frac{Vh}{M} \right)^2} \]

or, with the terms used in the next chapter

\[ \sigma_0 = \frac{f'_c}{1 + 1.8 \left( \frac{Vh}{M} \right)^2} \]  

(2.4/8)

It will be noted that \( \sigma_c \) is taken to be equal to \( f'_c \), a transition which does not hold true for pure bending,
where the equation \( \sigma_c = 0.85 f'_c \) or more elaborate formulas are used. Such a reduction was found to be unnecessary for shear failures because the critical stresses are, in contrast to the ones of flexural failure, confined to one cross-section. This rational approach is in good agreement with experimental findings as will be seen in Chapter 6. The influence of the \( \frac{M}{Vh} \) ratio on \( \sigma_0 \) (see Fig. 2.4/2) is somewhat over-accentuated, especially for small values of \( \frac{M}{Vh} \), because, contrary to the simplification assumed, the diagonal crack and the longitudinal reinforcement does contribute to the shear transfer. It is, however, conservative to neglect this contribution.
\[ \frac{\sigma_{av}}{\ell'_{c}} = \frac{1}{1 + (1.8) \left( \frac{Vh}{M} \right)^2} \]
3. THEORY OF SHEAR FAILURES FOR CONVENTIONALLY
REINFORCED BEAMS WITHOUT WEB AND COMPRESSION REINFORCEMENT

3.1 BASIC CONCEPT

The aim of this chapter is to postulate an idealized deformation mechanism which furnishes the missing compatibility condition. A new approach is first outlined for the simplest case of conventional reinforcement and generalized in the next chapter for prestressing, web and compression reinforcement.

Since Navier-Bernoulli's hypothesis was found to hold approximately, except in the region crossed by a diagonal crack (see Introduction), it follows that the deformations of a beam outside this region result from rotations of vertical cross-sections about the neutral axis. By analogy it is arbitrarily stipulated that the deformation of the diagonally cracked portion stems also from a rotation, i.e. a rotation about the as yet unspecified end of the diagonal crack (see Fig. 3.1/1).

This basic assumption permits the formulation of a compatibility condition by observing the deformations caused
Fig. 3.1/1

Idealized Mechanism of the Relative Displacements Prior to a Shear Failure (Shear Rotation)
by this shear rotation. From Fig. 3.1/1 it can be seen that the shortening of the concrete top zone has to be

$$\Delta l_{\text{top}} = \phi h_1$$  \hspace{1cm} (3.1/1)

and the elongation of the bottom zone, attributed to a pull out effect of the reinforcement relative to the concrete becomes

$$\Delta l_{\text{bottom}} = \phi (h - h_1) \frac{1}{\sin \alpha}$$  \hspace{1cm} (3.1/2)

(For notations see Fig. 3.1/1)

Hence

$$\frac{\Delta l_{\text{top}}}{\Delta l_{\text{bottom}}} = \frac{h_1}{h - h_1} \sin \alpha$$  \hspace{1cm} (3.1/3)

This last equation can now be considered as a compatibility condition. Notably, it is stated in terms of finite deformations and not, as usual, in terms of strains. This measure is necessary because the strain singularity associated with shear failure requires a consideration of the deformations in the whole region of the prospective shear failure.

Otherwise the structure of the formula is similar to the one normally used

$$\frac{\varepsilon}{\varepsilon_{\text{bottom}}} = \frac{h_1}{h - h_1}$$  \hspace{1cm} (3.1/4)
The sin-term enters because the considered cross-section is partially inclined under an angle $\alpha$.

The next question is now, obviously, to determine the deformations $\Delta l_{\text{top}}$ and $\Delta l_{\text{bottom}}$. For these considerations only the portion to the left of the crack is observed.
3.2 THE DEFORMATION OF THE CONCRETE TOP FIBER

By using the notation given in Fig. 3.2/1, the contraction of the concrete top fiber can be expressed as

$$\Delta l_{\text{top}} = \int \varepsilon (x; y = 0) \, dx$$  \hspace{1cm} (3.2/1)

Only one point of the curve \( \varepsilon (x; y = 0) \) is known beforehand, i.e. the ultimate compressive strain \( \varepsilon_{\text{cu}} \) will certainly occur at the location of crushing, thus

$$\varepsilon (x = 0; y = 0) = \varepsilon_{\text{cu}}$$

The remaining portion of the curve has first to be established with the aid of some simplifying assumption with respect to the stress distribution in the deformed portion of the concrete. Since these assumptions will obviously be quite arbitrary, an attempt is made to bound the probable value of \( \Delta l_{\text{top}} \) by varying the stipulations concerning the flow of the internal forces. In order to lend the problem to a mathematical treatment, a uniform stress distribution in each vertical cross-section had to be assumed. The variation of flow of internal forces is achieved by choosing different shapes for the lower boundaries of the stress blocks. A plausible choice of such a boundary would be a curve as shown in Fig. 3.2/1, starting tangentially with an angle \( \gamma \) at \( x = 0 \) \( y = h_1 \) and proceeding
Fig. 3.2/1

Strain Distribution at the Concrete Top Fiber
concavely downwards to the origin of the crack at \( x = \frac{h - h_1}{\tan \alpha} \), \( y = h_1 \). Unfortunately such a curve would lead to an elliptical integral for \( \Delta l_{\text{top}} \), and thus would make a solution in general terms impossible. Therefore, two limiting boundary curves were considered.

(1) A parabola of the second order with a horizontal tangent at \( x = 0; y = h_1 \)

(2) A straight line from \( x = 0; y = h_1 \) to the origin of the diagonal crack.

The stress-strain relationship, which will serve to convert the assumed stresses into strains is, in agreement with a great number of tests, (Ref. 2.31), chosen to be

\[
\eta = 2 \xi - \xi^2 \quad (3.2/2)
\]

or \( \xi = 1 - \sqrt{1 - \eta} \)

where \( \eta = \frac{\sigma_x}{\sigma_o} \)

\( \xi = \frac{\varepsilon_x}{\varepsilon_{\text{cu}}} \)

\( \sigma_x = \) stress component in x-direction (variable)

\( \sigma_o = \) maximum stress component in x-direction (usually smaller than the ultimate normal stress according to Formula 2.4/8)

\( \varepsilon_x = \) strain component in x-direction

\( \varepsilon_{\text{cu}} = \) ultimate concrete strain
Parabolic Boundary. The equation of a parabolic boundary (Fig. 3.2/2) of the sort outlined before is

\[ y = h_1 + \frac{\tan^2 \alpha}{h-h_1} x^2 \]  
(3.2/3)

On account of the uniform stress distribution assumed, and by considering the equilibrium condition of different stress blocks

\[ G = \sigma_0 bh_1 = \sigma_x \]  
by

one gets

\[ \sigma_x = \sigma_0 \frac{h_1}{y} = \sigma_0 \frac{1}{1 + \frac{\tan^2 \alpha}{h_1(h-h_1)} x^2} \]  
(3.2/4)

The strain distribution of the concrete top zone (Fig. 3.2/2) can thus be obtained by substituting Equation 3.2/4 in 3.2/2,

\[ \varepsilon_x = \varepsilon_{cu} \left[ 1 - \sqrt{1 - \frac{1}{1 + \frac{\tan^2 \alpha}{h_1(h-h_1)} x^2}} \right] \]  
(3.2/5)

hence

\[ \triangle l_{\text{top}} = \varepsilon_{cu} \int_0^{h_1-h_1/h \tan \alpha} \left[ 1 - \sqrt{1 - \frac{1}{1 + \frac{\tan^2 \alpha}{h_1(h-h_1)} x^2}} \right] dx \]

the solution of which is
Strain Distribution at the Concrete Top Fiber for the Parabolic 1 and Straight Line Boundary 2 Assumptions

Fig. 3.2/2
\[ \Delta l_{\text{top}} = \frac{\varepsilon_{\text{cu}} h}{\tan \alpha} \left[ 1 - \frac{h_1}{h} - \sqrt{1 - \frac{h_1}{h}} + \frac{h_1}{h} \left(1 - \frac{h_1}{h}\right) \right] \]

(3.2/6)

**Straight Line Boundary.** The procedure for the straight line assumption follows the same pattern as before with the exception that the origin of the coordinate axis was shifted to the intersection of the straight line with the top fiber (Fig. 3.2/3). Hence

\[ y = x \cdot \tan \alpha \quad (3.2/7) \]

and

\[ \Delta l_{\text{top}} = \frac{\varepsilon_{\text{cu}} h}{\tan \alpha} \int \left[ 1 - \sqrt{1 - \frac{h_1}{x \cdot \tan \alpha}} \right] dx \]

\[ \frac{h_1}{\tan \alpha} \]

(3.2/8)

which after a transformation of the variables becomes

\[ \Delta l_{\text{top}} = \frac{\varepsilon_{\text{cu}} h_1}{\tan \alpha} \int \frac{1}{\frac{h_1}{h}} \left[ \frac{1}{\xi^2} - \frac{1}{\xi^2} \right] d\xi \]

\[ \frac{h_1}{h} \]

(3.2/8a)

The solution of this integral is

\[ \Delta l_{\text{top}} = \frac{\varepsilon_{\text{cu}} h}{\tan \alpha} \left[ 1 - \frac{h_1}{h} - \sqrt{1 - \frac{h_1}{h}} + \frac{h_1}{2h \ln} \frac{1 + \sqrt{1 - \frac{h_1}{h}}}{1 - \sqrt{1 - \frac{h_1}{h}}} \right] \]

(3.2/9)
Fig. 3.2/3

Stresses for the Straight Line Assumption
The two functions for $\Delta l_{\text{top}}$ given by Equation 3.2/6 and Eq. 3.2/9 deviate considerably from each other as can be seen in Fig. 3.2/4. It seems plausible, although it cannot be proved, that for any given ratio $\frac{h_1}{h}$ the parabolic assumption overestimates and the straight line assumption underestimates the value of $\Delta l_{\text{top}}$. It will be seen later, however, that upon combining each of the two curves with the equilibrium conditions, they yield very similar functions for the sought unknown $\frac{h_1}{h}$ or $M_{su}$. It shall also be mentioned that several other assumptions for the boundary of the stress blocks were considered by choosing parabolas ending not at the bottom of the beam, but somewhere between mid-depth and bottom. Again the difference in $\Delta l_{\text{top}}$ did not greatly affect the outcome of the final result.
Fig. 3.2/4

$\Delta l_{\text{top}}$ as a Function of $\frac{h_t}{h}$

- Parabolic assumption
- Straight line assumption
- Formula 3.4/12
3.3 THE DEFORMATION OF THE BOTTOM ZONE

As explained before, the elongation of the bottom zone is attributed to a pull-out effect of the longitudinal reinforcement with respect to the surrounding concrete. Since this pull-out effect is obviously a function of the bond characteristic, it is apparent that the influence of bond will enter the considerations. On the other hand, a new variable is added to the already complicated problem — a variable which from experience is known to be accompanied by considerable scatter.

Observations of actual pull-out tests suggest that (5.1/2)

$$\Delta l_{\text{bottom}} = \lambda \varepsilon_{\text{st}}^2$$  \hspace{2cm} (3.3/1)

where $\lambda$ depends on the bond characteristic. The reciprocal of $\lambda$, i.e. $\frac{1}{\lambda}$, will hereafter be called "bond coefficient."

As discussed in Chapter 5, conventional pull-out tests do not directly represent the magnitude of $\Delta l_{\text{bottom}}$, since due to the rotation $\phi$ the resulting tension force does not act in the direction of the tendons. The coefficient $\lambda$ will therefore take on a greater value than for axial pull-out.
3.4 SOLUTION

By assuming a rectangular stress block of the magnitude \( \sigma_0 \) (see Eq. 2.4/8) over the concrete compressive zone\(^*\) the ultimate shear moment becomes

\[
M_{su} = \sigma_0 h_1 b \left( h - \frac{h_1}{2} \right)
\]

(3.4/1)

This expression contains only one unknown, i.e. \( h_1 \). An equation for \( \frac{h_1}{h} \) is therefore regarded as the solution of the problem. Similarly to pure bending, this solution is obtained by combining the compatibility and equilibrium conditions.

The Compatibility Condition, derived in Section 3.1 to 3.3 can now be transformed to the two alternative expressions in terms of strains, namely

\[
\varepsilon_{st} = \sqrt{\frac{\varepsilon_{cu} h}{\lambda \tan \alpha \sin \alpha}} \left[ \left( \frac{h}{h_1} - 1 \right) \left( 1 - \frac{h_1}{h} \right) \sqrt{1 - \frac{h_1}{h}} \sqrt{\frac{h_1}{h} \left( 1 - \frac{h_1}{h} \right)} \right]
\]

(3.4/2)

\(^*\) This simplifying assumption, though differing from the one made in Chapter 2, seems permissible, since the shape of the stress block does not greatly affect the ultimate moment.
obtained by substituting Equation 3.2/6 and 3.3/1 in 3.1/3; (parabolic assumption) and

\[
\ell_{st} = \sqrt{\frac{\varepsilon_{cu} h}{\lambda \tan \alpha \sin \alpha}} \left[ (\frac{h}{h_1} - 1) \left( 1 - \frac{h_1}{h} \right) + \frac{h_1}{2h} \ln \left( \frac{1 - \frac{h_1}{h}}{1 - \frac{h_1}{h_1}} \right) \right]
\]

(3.4/3)

obtained by substituting Equation 3.2/9 and 3.3/1 in 3.1/3 (straight line assumption).

The Equilibrium Condition requires that the compressive force \( C \) must be equal and opposite to the tension force \( T \); i.e. with the assumption and notation used before.

\[
\frac{\sigma_0}{b h_1} = A_{st} E_{st} \ell_{st} \leq \ell_{st}
\]

(3.4/4)

or

\[
\frac{\sigma_0}{p E_{st}} = \frac{h}{h_1} \leq \ell_{st}
\]

where

\[
A_{st} = \text{Area of longitudinal tension reinforcement.}
\]

\[
E_{st} = \text{Modulus of elasticity or secant modulus of steel, depending on whether or not the steel is still in the elastic range.}
\]

\[
\frac{A_{st}}{bh} = p = \text{Ratio of reinforcing steel}
\]

The Implicit Solution for \( \frac{h_1}{h} \) is found, as mentioned before, by combining Equations 3.4/2 or 3.4/3 with Eq. 3.4/4.
\[
\frac{\sigma_o}{p_{Es}} \sqrt{\tan \alpha \sin \alpha} \cdot \frac{\epsilon_{cu}}{h} = \frac{h}{h_1} \sqrt{\frac{h}{h_1} - 1} \left( 1 - \frac{h_1}{h} - \sqrt{1 - \frac{h_1}{h}} \right) + \frac{h_1}{2h} \ln \left( 1 + \sqrt{1 - \frac{h_1}{h}} \right)
\]
(parabolic assumption) \hspace{1cm} (3.4/5)

\[
\frac{\sigma_o}{p_{Es}} \sqrt{\tan \alpha \sin \alpha} \cdot \frac{\epsilon_{cu}}{h} = \frac{h}{h_1} \sqrt{\frac{h}{h_1} - 1} \left( 1 - \frac{h_1}{h} - \sqrt{1 - \frac{h_1}{h}} \right)
\]
(straight line assumption) \hspace{1cm} (3.4/6)

These alternative expressions constitute the general solution of the problem treated in this chapter. The righthand side contains only the unknown \( \frac{h_1}{h} \); the quantities to the left of the equal sign are given parameters.

The graphical presentation of these solutions (Fig. 3.4/1) permits now the drawing of conclusions as to what boundary assumption is preferable. Since the ultimate shear moment increases with increasing \( \frac{h_1}{h} \), it is evident that the straight line assumption yields more conservative results and will therefore be chosen for further applications. The comparatively small difference between the two curves, in spite of the much greater deviation of \( \Delta l_{top} \), as seen in Fig. 3.2/4, seems to justify the hope that this theory is not sensitive to inaccuracies in the basic assumptions, especially since the rather uncertain parameters, i.e., \( \alpha, \lambda, \) and \( \epsilon_{cu} \), occur within the square root.
Fig. 3.4/1 Solution for $h_1/h$ as a function of $\sigma_0$, $E_s$, $\alpha$, $\epsilon_{cu}$ and $h$

$$q = \frac{\sigma_0}{pE} \sqrt{\frac{\lambda \tan \alpha}{\epsilon_{cu} h}}$$

Parabolic Assumption

Straight Line Assumption

Explicit Approximation

$$\frac{h_1}{h} = \frac{1}{0.75q + 1.65}$$
The Explicit Solution. In order to lend such a solution to practical applications, it should be expressed in an explicit form. It is therefore sought to approximate the original curve by means of the method of least squares, considering also the weight of any portion of curve according to the relative probability of occurrence.

The shape of the curve, given by Equation 3.4/6, suggests an approximation of the following kind

$$
\left(\frac{h_1}{h}\right)_{\text{app.}} = \frac{1}{k_1 q + k_2} \tag{3.4/8}
$$

where $k_1$ and $k_2$ are the constants to be found by the method of least squares and $q$ stands for

$$
q = \frac{\sigma_0}{p_{\text{Est}}} \sqrt{\lambda \tan \alpha \sin \alpha \frac{\varepsilon_{\text{cu}}}{h}}
$$

Unfortunately the method becomes very laborious for this particular case and no explicit expression for $k_1$ and $k_2$ can be found, because by choosing, for example, $n$-reference points, one ends up with two polynomials in $k_1$ and $k_2$ of the $(3n + 1)^{th}$ order, the solution of which could only be found by trial and error. Under these circumstances one might as well start out to find $k_1$ and $k_2$ directly by trial and error. It was preferred, however, to choose a different approach. If the sought approximation can be expected to coincide closely with the original curve, then one can apply the method of least squares to the reciprocal function.
and still obtain practically the same k-values.

Consequently it is required that

\[
\sum_{1}^{n} w_n \left( \left( \frac{h}{h_1} \right)_n - k_1 q_n - k_2 \right)^2 = \min
\]

which means: the sum of the squares of the errors over \(n\)-reference points of weight \(w_n\) be a minimum.

Hence the partial derivatives with respect to \(k_1\) and \(k_2\) have to be zero.

\[
\frac{\partial}{\partial k_1} \sum_{1}^{n} w_n \left( \left( \frac{h}{h_1} \right)_n - k_1 q_n - k_2 \right)^2 = 0
\]

and

\[
\frac{\partial}{\partial k_2} \sum_{1}^{n} w_n \left( \left( \frac{h}{h_1} \right)_n - k_1 q_n - k_2 \right)^2 = 0
\]

or

\[
\sum_{1}^{n} w_n \left( \left( \frac{h}{h_1} \right)_n - k_1 q_n - k_2 \right) q_n = 0
\]

and

\[
\sum_{1}^{n} w_n \left( \left( \frac{h}{h_1} \right)_n - k_2 q_n - k_2 \right) q_n = 0
\]

The two last equations determine the values of the constants \(k_1\) and \(k_2\). The weight of the reference points for the particular curves in question was derived from a
statistical survey of the frequency of test results in specified ranges of \( \frac{h_1}{h} \). This frequency curve is shown in Figure 3.4/1. The computations for the \( k \)-values is presented in Table 3.4/1. By comparing the first and the last column of this table, it can be seen that the approximation is so accurate as to lie within the thickness of the line for the greatest portion of the curve in Fig. 3.4/1.

Thus the explicit solution for our problem can be written as

\[
\frac{h_1}{h} = \frac{1}{0.75 \left( \frac{\sigma_0}{p E_{st}} \sqrt{\frac{\lambda \tan \alpha \sin \alpha}{\varepsilon_{cu} h}} \right) + 1.65} \quad (3.4/11)
\]

where

\[
\sigma_0 = \frac{f_c'}{1 + 1.8 \left( \frac{V_h}{M} \right)^2} \quad (2.4/8)
\]

With these two formulas the ultimate shear moment

\[
M_{su} = \sigma_o b h_1 (h - \frac{h_1}{2}) \quad (3.4/1)
\]

can be calculated.

It was thus possible to determine the ultimate shear moment by a rational approach, the applicability of which has, of course, first to be proven by comparison with test results. Anticipating a thorough comparison, it can be seen that the order
### TABLE 3.4/I

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<th>$h_1/h$</th>
<th>$q$</th>
<th>$w$</th>
<th>$h_1/h$</th>
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<th>$wq$</th>
<th>$q^2$</th>
<th>$wq^2$</th>
<th>$q \cdot h_1$</th>
<th>$h_1/h$</th>
<th>$\frac{1}{k_1 q + k_2}$</th>
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<td>714.68</td>
<td>1440.29</td>
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</table>

\[
\begin{align*}
1440.29 - 1567.33 k_1 - 163.91 k_2 &= 0 \\
169.82 - 163.91 k_1 - 28.75 k_2 &= 0
\end{align*}
\]

\[k_1 = 0.746 = 0.75\]

\[k_2 = 1.654 = 1.65\]

**Derivation of the k-values for the approximation of the expression**

\[q = \frac{h}{h_1} \sqrt{\frac{(h/h_1 - 1)(1 - h_1/h)}{h_1}} \sqrt{1 - \frac{h_1}{h}} + \frac{h_1}{2h} \ln \frac{1+ |1-h_1/h|}{1- |1-h_1/h|} \]

by:

\[\frac{h_1}{h} = \frac{1}{k_1 q + k_2}\]
of the variable is in accordance with experimental evidence
in that the formulas indicate an increase of the ultimate
moment with

(1) Increasing concrete strength
(2) Increasing ratio of reinforcement
(3) Improvement of bond characteristic
  (i.e. decreasing $\lambda$-values)
(4) Increasing deformability of the concrete
  compressive zone $\left( \frac{\varepsilon_{\text{cu}}}{\tan \alpha \sin \alpha} \right)$
(5) Increasing ratio of $M/Vh$ (for symmetrical two point loading, with
  increasing shear span)

For later use it is convenient to trace this solution
once again back to the compatibility condition. An exactly
identical solution to 3.4/11 could be obtained by taking

$$
\Delta_{1\text{top}} = \frac{\varepsilon_{\text{cu}} h}{\tan \alpha} \cdot \frac{\left( \frac{1 - k_2}{k_1} \frac{h_1}{h} \right)^2}{\left( \frac{h}{h_1} - 1 \right)}
$$

(3.4/12)

(given as a dashed line in Fig. 3.2/4).

This brings the compatibility condition to the simple
form

$$
\varepsilon_{st} = \left( \frac{1 - k_2}{k_1} \frac{h_1}{h} \right) \frac{\varepsilon_{\text{cu}}}{h \sin \alpha \tan \alpha}
$$
or by substituting the $k$-values

$$
\varepsilon_{st} = 1 - 1.65 \frac{h_1}{h} \sqrt{\frac{\varepsilon_{cu}}{\lambda \sin \alpha \tan \alpha}} \quad (3.4/13)
$$

This expression is used later on to derive the formulas for more complicated cases.
4. GENERALIZATION FOR PRESTRESSING
WEB AND COMPRESSION REINFORCEMENT

4.1 PRESTRESSING

The approximate knowledge of the failure mechanism, gained by the rational approach, permits a direct determination of additional variables, such as prestressing, web and compression reinforcement. This feature is especially favorable for prestressed members because their shear strength can reliably be related to those of non-prestressed beams, thus sparing the need of experimentally redetermining the influence of all the variables already aggravating the research of conventionally reinforced concrete.

The initial state of stress, produced by the prestressing, forbids a direct application of the compatibility condition stated in the last chapter. Rather than change the compatibility condition itself, one can introduce fictitious forces which compensate the initial state of stress. The following notations will be used for these considerations:

\[ T_i, \sigma_i, \varepsilon_i: \] Initial pretensioning force
(stress; strain) before release

\[ T_i', \sigma_i', \varepsilon_i': \] Initial post-tensioning force
(stress; strain) before creep and shrinkage

take place
\( \Delta T_{ie}; \Delta \sigma_{ie}; \Delta \varepsilon_{ie} \): Losses due to inelastic deformation of concrete (creep and shrinkage) and steel (creep of the tendons)

\( \Delta T_{el}; \Delta \sigma_{el}; \Delta \varepsilon_{el} \): Losses due to the elastic shortening of the concrete

\( T_0 \): Fictitious force which, applied at the centroid of the tendons, would relieve the concrete stresses

\( T_{eff}; \sigma_{eff}; \varepsilon_{eff} \): Effective prestressing force (stress; strain) after deduction of all the losses

Since the initial state of stress is caused exclusively by the prestressing (the stresses due to dead load are not considered as initial stresses, even though they have to be taken into account for the calculation of the prestressing losses), it must be possible to relieve these stresses by applying a fictitious force \( (T_0) \) at the centroid of the prestressing tendons. This force has to be larger than the effective prestressing force, since the elastic losses have to be overcome (but not the inelastic losses which remain as residual strains without causing stresses). Thus the fictitious force is:
for pretensioning,

\[ T_o = T_i - \Delta T_{ie} = T_{eff} + \Delta T_{el} \quad (4.1/1) \]

or, expressed by the steel strains,

\[ \epsilon_o = \epsilon_i - \Delta \epsilon_{ie} = \epsilon_{eff} + \Delta \epsilon_{el} \quad (4.1/1a) \]

and for post-tensioning,

\[ T_o = T_{i'} - \Delta T_{ie} + \Delta T_{el} = T_{eff} + \Delta T_{el} \quad (4.1/2) \]

or

\[ \epsilon_o = \epsilon_{i'} - \Delta \epsilon_{ie} + \Delta \epsilon_{el} = \epsilon_{eff} + \Delta \epsilon_{el} \quad (4.1/2a) \]

Considering now a free body diagram at a prospective failure cross section (Fig. 4.1/1), one can always split the external moment \( M \) into a couple

\[ T = C = \frac{M}{(h - h_1)} \quad (4.1/3) \]

The tensile force acting at the centroid of pretensioning can be imagined to consist of two parts

\[ T = T_o + T_+ \quad (4.1/4) \]

* For post-tensioning, \( \Delta T_{el} \) cannot be considered as a real loss; it is better defined by the strain \( \Delta \epsilon_{el} \), i.e. the elastic shortening of the concrete at the centroid of tendons due to post-tensioning. Thus,

\[ \Delta T_{el} = EA_{st} \Delta \epsilon_{el} \]
Fig. 4.1/1

Free Body Diagram at a Prospective Failure Cross-Section

\[ M = C \left( h - \frac{h_1}{2} \right) = T \left( h - \frac{h_1}{2} \right) \]
i.e., of the force $T_o$ and whatever force $T_+$ is required to add up to $T$. From the above discussion, it is seen that $T_o$ acting alone relieves all the initial stresses. From Equation 4.1/4, it follows that

$$\varepsilon_{st} = \varepsilon_o + \varepsilon_+ \quad (4.1/4a)$$

The compatibility condition, as derived in the last chapter, can therefore be applied to the configuration of the remaining forces, stresses, or strains. Thus, according to Equation 3.4/13,

$$\varepsilon_+ = \frac{1 - 1.65}{0.75} \frac{h_1}{h} \sqrt{\frac{\varepsilon_{cu} h}{\lambda \sin \alpha \tan \alpha}} \quad (3.4/13)$$

The equilibrium condition, on the other hand, becomes

$$T = E_{st} A_{st} (\varepsilon_o + \varepsilon_+) = C = \sigma_o b h_1 \quad (4.1/5)$$

which is identical to Equation 3.4/4 except for the replacing of $\varepsilon_{st}$ by $\varepsilon_o + \varepsilon_+$.

The solution $h_1/h$ for prestressed concrete can therefore be obtained by combining Equation 3.4/13 and Eq.4.1/5.

$$\frac{h_1}{h} = \frac{1 + 0.75 \varepsilon_o \sqrt{\frac{\lambda \sin \alpha \tan \alpha}{\varepsilon_{cu} h}}}{0.75 \frac{\sigma_o}{PE_{st}} \sqrt{\frac{\lambda \sin \alpha \tan \alpha}{\varepsilon_{cu} h}} + 1.65} \quad (4.1/6)$$
This solution accounts numerically for the empirically known fact that prestressing increases the shear strength. This becomes clearly evident by transforming Equation 4.1/6 to

\[
\left( \frac{h_1}{h} \right)_{\text{with prestressing}} = \left( \frac{h_1}{h} \right)_{\text{without prestressing}} \left(1 + 0.75\varepsilon_0 \sqrt{\frac{\lambda \sin \alpha \tan \alpha}{E_o E_{cu} h}} \right)
\]

Since \( \varepsilon_0 \) is a direct measure of the degree of prestressing, Equation 4.1/6a suggests that the ratio of depth of the neutral axis increases linearly with the degree of prestressing. Thus also, the ultimate shear moment increases, according to

\[
M_{Su} = \sigma_o b h_1 \left( h - \frac{h_1}{2} \right)
\]

Furthermore, it can be presumed that the effect of an error in the somewhat uncertain estimation of the coefficient \( \lambda \) is lessened, since its influence on the two factors on the right hand side of Eq. 4.1/6a is of opposite rate.

There might arise some concern that the percent-wise increase of the ratio \( h_1/h \) increases also with \( \lambda \), i.e. with deterioration bond conditions. The absolute value of \( h_1/h \) however, increases with improved bond characteristic. What Eq. 4.1/6a expresses, in effect, is that the percent-wise improvement which can be gained by prestressing becomes greater with poorer bond conditions, a statement which seems conceivable. It is naturally implied that no bond failure develops.
4.2 WEB REINFORCEMENT

The remarks made in the beginning of the last chapter concerning the advantage of a rational approach prevail also for the determination of the influence of the web reinforcement: it is possible to trace the solution back to the case without web reinforcement.

The beneficial influence of stirrups is rooted in the following four points:

(1) The stirrup forces contribute directly to the ultimate shear moment.

(2) A portion of the shear force \( V \) is carried by the stirrups, thus increasing the normal stress capacity of the concrete compressive zone.*

(3) The ductility of the concrete compressive zone, i.e. \( \varepsilon_{cu} \), is increased by the presence of stirrups.

(4) The quality of bond is improved when the longitudinal reinforcement is laced by stirrups.

* This is the reason why beams with stirrups show a smaller influence of the shear span than beams without web reinforcement.
For the time being, only the first two points are considered numerically, since first, $\varepsilon_{cu}$ does not affect the ultimate shear moment significantly, and second, only qualitative information concerning the improvement of the bond due to stirrups is available at present.

Actual measurements of the stirrup forces by means of strain gages (Ref. 1.2/1 and 4.2/1) permit the conclusion that the stirrups crossed by the diagonal failure crack reach or surpass the yield point of the steel, thus sparing the need for a consideration of the shear rotation $\phi$.*

In order to avoid the carrying on of insignificant details, it is furthermore assumed that the diagonal crack subtends an angle of $45^\circ$ and extends over the whole depth of the beam, yielding the following formulas (see Fig. 4.2/1).

\[
V_{\text{stirrup}} = rbh f_y \; (4.2/1)
\]

\[
(V_{\text{stirrup}} \leq V)
\]

and

\[
M_{\text{stirrup}} = \frac{rbh^2}{2} f_y \; (4.2/2)
\]

* Such a consideration showed that only if $h_1/h$ is larger than 0.3 is it theoretically possible that the stirrups in the immediate proximity of the concrete crushing zone may be stressed below the yield point.
Simplified Assumption for the Computation of the Stirrup Moment \( (M_{\text{stirrup}}) \)
where

\[ r = \frac{A_w}{bs} = \text{ratio of stirrup reinforcement} \]

\[ A_w = \text{area of two legs of one stirrup} \]

\[ s = \text{spacing of the stirrups} \]

\[ f_y' = \text{yield strength of the stirrups} \]

As for Equation 4.2/11, it has naturally to be observed that \( V_{st} \) cannot exceed the total shear force \( V \).

On account of the contribution of the stirrups to the shear transfer, Equation 2.4/8 has to be modified to

\[
\sigma_o = \frac{f_c'}{1.8 \left( \frac{(V-V_{st})h}{M} \right)^2 + 1} \quad (4.2/3)
\]

Since only the ratio of \( M/V \), but not the forces themselves, are determined beforehand by the loading conditions, it would be necessary to solve Eq. 4.2/3 in connection with Eq. 3.4/1 and 3.4/11 by iteration. For the practice, such a procedure would certainly be too involved. It can safely be omitted, since this results only in a slight underestimation of the ultimate shear strength. The reason for introducing Equation 4.2/3 is mainly for the purpose of checking tests results where \( M_{su}/V_u \) is known.
4.3 COMPRESSION REINFORCEMENT

In order to avoid unnecessary repetition of similar derivations, the influence of the compression reinforcement is given directly for prestressed concrete, keeping in mind that for non-prestressed members $\varepsilon_0$ is zero and $\varepsilon_+$ has to be replaced by $\varepsilon_{st}$.

The compatibility condition suffers no change by the introduction of compression reinforcing, thus

$$
\varepsilon_+ = \frac{1 - 1.65 \frac{h_1}{h}}{0.75} \sqrt{\frac{\varepsilon_{cu} h}{\lambda \tan \alpha \sin \alpha}}
$$

(3.4/13)

As for the equilibrium condition, the contribution of the compression reinforcement to the total force $C$ has to be taken into account, using the following notations:

$$
A'_{st} = \text{area of compression reinforcement}
$$

$$
f_y'; (\varepsilon_y') = \text{yield stress (strain) of the compression reinforcement}
$$

and for later use,

$$
p' = \frac{A'_{st}}{bh} = \text{ratio of compression reinforcement}
$$

$$
f'; (\varepsilon') = \text{steel stress (strain) of compression reinforcement at failure}
$$
Since the concrete strains of the prospective failure zone are considerably larger than the yield strain of steel (\(\varepsilon_{cu} \sim 0.0036\), as compared to \(\varepsilon_{y'} \sim 0.0015\)), one could be inclined to presume that the compressive reinforcement always reaches its yield strength. Experimental evidence contradicts, however, such a presumption. If, for example, \(p = p'\), then the tensile reinforcement would have to exceed the yield strength (equilibrium), which is incompatible with test results.

Thus it has to be concluded that the steel strains do not follow the concentration of the concrete strains. For reasons of simplicity, the strain of the compression reinforcement at failure is arbitrarily assumed as

\[ \varepsilon' \approx 0.5 \varepsilon_{y'} \quad (4.3/1) \]

and

\[ f' \approx 0.5 f_y' \quad (4.3/2) \]

Thus the equilibrium condition becomes

\[ \sigma_0 h_1 + 0.5 A_{st} f_{y'} = E(\varepsilon_0 + \varepsilon_+) A_{st} \quad (4.3/3) \]

Consequently the solution for \(h_1/h\) is obtained by substituting Equation 4.3/3 in Eq. 3.4/13;
This is now the most general solution for the ratio \( \frac{h_1}{h} \).

The compression reinforcement has, naturally, also to be taken into account for the formulation of the ultimate shear moment, which, by including the contribution of vertical stirrups, becomes

\[
M_{su} = bh^2 \left[ \left( \frac{\sigma_0 h_{1}}{h} \right) + \frac{p'_f y'_f}{2h} \right] \left( 1 - \frac{h_1}{2h} \right) + \frac{rf_y}{2}
\]

\[\text{contribution of the beam}\]
\[\text{cont. of compressive reinforcement}\]
\[\text{contribution of stirrups}\]

\[\text{(4.3/5)}\]

For the interpretation of test results of beams with unnaturally high percentage of compression reinforcement, it is stipulated that the compression force carried by the reinforcement cannot exceed the force carried by the concrete compression zone, i.e.

\[
0.5 bh f'_y \leq \sigma_0 bh_1
\]

\[\text{(4.3/6)}\]

This condition will scarcely ever be critical for the practice.
5 DERIVATION OF PARAMETERS

5.1 THE BOND COEFFICIENT $1/\lambda$

The derivation of the bond coefficient $1/\lambda$, defined by the equation

$$\Delta l_{\text{bottom}} = \lambda \epsilon_{st}^2 \quad (3.3/1)$$

constitutes one of the major difficulties of the theory because it is hardly accessible to direct tests. Axial pull-out tests are not representative in magnitude, since due to the shear rotation $\phi$, the pull-out force in the beam does not act in the direction of the tensile reinforcement, but under some angle, depending on the actual rotation and on the local split-away of concrete. A duplication of these complex conditions, by means of pull-out tests with lateral forces, seems hopeless because an application of the latter -- complicated and undetermined in itself -- would make reliable measurements practically impossible. An attempt is therefore made to compose the derivation of $\lambda$ in part from findings of various investigators.

Most of these findings are given by the authors in the form of diagrams, and scarcely any explicit formulations, even for linear pull-out tests, can be found, probably because
of the large scatter always associated with such tests. For the application to the theory presented herein, this scatter is not alarming, because the final solution (Eq. 4.3/4) is relatively insensitive to normal errors in the estimation of $\lambda$. As will be seen, the range of variation of $\lambda$ (from about 1000 to 100,000 inches) far exceeds the possible errors of a particular $\lambda$ -value, thus justifying a consideration of the bond characteristic.

The general characteristic of pull-out tests can, for example, be observed in the thorough bond investigations reported by Clark (5.1/1), which covered pull-out tests on standard specimens as well as on beams. For the latter, the longitudinal reinforcement was exposed at a certain distance from the supports (see Fig. 5.1/1) in order to permit measurements of the relative displacement of steel and concrete by means of dial gages. The results of these beam tests coincided closely with those of the standard pull-out tests with the same anchorage length. The magnitude of the relative displacement ($\Delta 1$) as obtained from these tests is, however, not representative for the purpose of our theory because the obliquity of the pull-out force due to the shear rotation $\phi$ is, compared to the one in a normal beam, significantly decreased by the exposure of the bar over a considerable
Fig. 5.1/1

Test Set-Up Used by Clark
length. Nevertheless, these tests may certainly serve to indicate the general trend of the relation between \( \Delta l \) and the pull-out force \( (P) \).

Figure 5.1/2 shows one of the diagrams from Clark's tests which is also representative for the pull-out data of many other authors.

The factors which will have to be considered for an empirical formulation of the \( \lambda \)-values are in accordance with experience,*

1. The concrete strength
2. The diameter of the reinforcing bars
3. The prestressing
4. The roughness of the reinforcing bars

The influence of the concrete strength was directly derived from beam tests. In Series B of the shear investigation of Moody, Viest, Elstner, and Hognestad (1.4/10), 16 beams were tested, the only variable of which was the concrete strength. Their ultimate shear moment vs. concrete strength is plotted in Fig. 5.1/3. The corresponding \( \lambda \)-values (Fig. 5.1/4) were calculated backwards with the aid of the anchorage length was found to be of negligible influence as long as no slip at the non-loaded end of the bar occurs.

---

* The anchorage length was found to be of negligible influence as long as no slip at the non-loaded end of the bar occurs.
Fig. 5.1/2
Pull-Out Characteristic

\[ \Delta I = \lambda \varepsilon^2 \]
Fig. 5.1/3 \( M_{su} \) as a function of \( f'_c \) from the series B of the tests by Moody, Viest, Elstner and Hognestad.

Fig. 5.1/4 Bond coefficient \( \lambda \) as a function of \( f'_c \) (derived from the tests above).
theory outlined in this paper, taking \( \varepsilon_{cu} = 0.0036 \) and \( \alpha = 45^\circ \). The large scatter of the \( \lambda \)-diagram is a logical consequence of the favorable feature that the ultimate shear moment has a relatively small response to variations in \( \lambda \).

In order to determine the influence of the concrete strength on \( \lambda \), it is therefore necessary to select an idealized conservative curve in the moment-strength diagram (Fig. 5.1/3) and trace this curve forth into the \( \lambda \)-diagram (Fig. 5.1/4). This permits an estimation of the sought relationship with

\[
\lambda = \lambda_5 (0.5 + \frac{2500 \text{ psi}}{f_c})
\]  

(5.1/1)

where \( \lambda_5 \) is the coefficient at the arbitrary chosen reference strength of \( f_c = 5000 \text{ psi} \). Obviously \( \lambda_5 \) still depends on the other influencing factors mentioned before.

Equation 5.1/1 is in qualitative agreement with findings of other authors, notably with those of Clark (5.1/1).

The influence of the bar-diameter \( d \), was, for example, studied by Djabry (5.1/2) by means of pull-out tests with plain round bars, the results of which are summarized in Figure 5.1/5. Three sets of \( \lambda \)-values were derived from this data, such that the assumption

\[
\Delta l = \lambda \varepsilon_{st}^2
\]

coincided with the experimental values at \( \Delta l = 0.01 \), 0.02 and 0.03 cm respectively. Plotting these \( \lambda \)-values as a function of the bar diameter \( d \) (Figure 5.1/6) suggests
Fig. 5.1/5  Tests by Djabry. Pull-Out Displacement vs. Steel Stress. (Plain Round Bars)

Fig. 5.1/6  Coefficient $\lambda$ vs. Bar Diameter
Derived from Djabry's Tests.
a straight line assumption

\[ \lambda = k \cdot \kappa \left( 0.5 + \frac{2500 \text{ psi}}{f'_c} \right) \cdot d \quad (5.1/2) \]

where \( \kappa \) is a coefficient which may still be dependent on the prestressing and the bar roughness and \( k \) is a factor for the transition from straight pull-out tests to the actual pull-out effect in beams.

If a cross-section contains bars of different diameters, the question arises, what value of \( d \) should be used in Equation 5.1/2. It is believed that a plausible assumption should be based on the arithmetic mean value of the bar cross-areas

\[ A_{\text{equ}} = \frac{1}{n} \sum_{n} A_n \]

where \( n \) is the number of bars of one cross-section. Thus

\[ d_{\text{equ}} = 2\sqrt{\frac{A_{\text{equ}}}{\pi}} \]

or

\[ d_{\text{equ}} = \sqrt{\frac{n}{\sum_{n} d_n^2}} \quad (5.1/3) \]

The influence of prestressing was studied in the course of an investigation at Lehigh University (Ref. 5.1/3) which had as its primary object the anchorage characteristic of prestressing strands. A great number of pull-out specimens
of various lengths were cast around 7/16 inch pretensioned strands. The prestressing was only released at the free end whereas the pull-out end remained under tension, thus simulating the prestressing condition occurring at a crack. It was shown in this investigation that the distribution of the bond stresses and, consequently, the pull-out characteristic is not appreciably altered by varying the prestressing. Thus \( \lambda \) can be considered as being independent of the degree of prestressing.

The Influence of the Roughness. The term roughness, as interpreted in this paper, does not only refer to the surface of the bar, it includes also other means of improving the bond characteristic, i.e. notches, ridges (deformed bars) and distortion of bars or wires (strands).

Since no comprehensive treatment of this subject including strands could be found, a series of pull-out tests was performed at the Fritz Laboratory, the results of which are given in the appendix.

The tentative values for the coefficient of roughness obtained from these tests are given in Table 5.1/I.
<table>
<thead>
<tr>
<th></th>
<th>Not Rusted</th>
<th>Rusted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plain Round Bars</td>
<td>18500</td>
<td>13500</td>
</tr>
<tr>
<td>Deformed Bars</td>
<td>9500</td>
<td>6500</td>
</tr>
<tr>
<td>Strands</td>
<td>6500</td>
<td>4500</td>
</tr>
</tbody>
</table>

**TABLE 5.1/1**

Coefficients of Roughness (χ) for Standard Pull-out Tests

With the aid of these χ-values, it is now possible to compare numerically the difference between axial pull-out and pull-out in beams. One could obtain, for example, the same idealized curve of the λ-values as given in Figure 5.1/4 by choosing the transition factor k to be three. In other words, due to the obliquity of the pull-out force in beams, the displacement Δl is approximately three times larger than for axial pull-out.

Consequently the coefficient λ can be calculated as

\[ \lambda = 3(0.5 + \frac{2500 \text{ psi}}{f_c}) \chi d \]  

(5.1/4)

In spite of the somewhat vague basis of these derivations, this empirical formula for λ substituted in the solution for the ultimate shear moment showed satisfactory agreement with test results (see Chapter 6).
5.2/1 THE INCLINATION \( \alpha \) OF DIAGONAL CRACKS

Since it was found that under given load conditions diagonal cracks more or less follow the stress trajectories, the angle \( \alpha \) could always be determined accordingly. In experimental investigations there occur, however, always cases where the crack pattern differs considerably from the stress-trajectories without significantly affecting the ultimate shear moment. By comparing the outlined theory with a great number of test results, it was furthermore found that the best agreement was achieved by assuming an angle \( \alpha \) of 45°. Thus it seems permissible to idealize the real conditions by assuming always

\[
\alpha = 45°
\]

more so, since this assumption yields theoretically conservative results.
5.3 THE ULTIMATE CONCRETE STRAIN $\varepsilon_{cu}$

The uncertainty about the magnitude of the ultimate concrete strain, reflected in many investigations, is to a great degree caused by its sensitivity to the speed of loading and by the difficult techniques of measurement. It seems conceivable that the loading speed cannot be a determining factor for the ultimate strength, a contention which is substantiated by considering the small influence of $\varepsilon_{cu}$ on the final solution of the theory presented herein. Thus an average value of

$$\varepsilon_{cu} = 0.0036$$

is assumed for later computations and recommendations.

Some qualitative reflections on the influence of the state of strain on $\varepsilon_{cu}$ may, however, serve to explain an experimentally observed phenomenon. If, namely, a simply supported beam is unsymmetrically loaded with one concentrated load, it can always be observed that the opening of the diagonal cracks in the short shear span is larger than in the longer shear span, although the maximum moment is the same. This can only be attributed to the different state of strain due to the different magnitude of the shear force on either side of the applied load. One can thus conclude
that the deformation and consequently $\varepsilon_{cu}$, is greater the larger the magnitude of the shear force becomes.
The theory of shear failures stated herein is compared in this chapter with the results of 278 beam tests which could be found in the more recent literature. According to the general validity of the theory, a broad field of variables could be considered in this comparison, including:

- Cross-Section
- Shear Span
- Prestressing
- Web Reinforcement
- Compression Reinforcement
- End Restraint

The following tables give the properties of the test beams and the major steps of computation required for the theoretical solution, the latter involving always:

1. \( \sigma_0 \rightarrow \text{Eq. 2.4/8} \)
2. \( \lambda \rightarrow \text{Eq. 5.1/4 and Table 5.1/I} \)
3. \( h_1/h \rightarrow \text{Eq. 4.3/4} \)
4. \( M_{su} \rightarrow \text{Eq. 4.3/5} \)

For T-beams it has naturally to be observed that the depth of the neutral axis should not be taken larger than the depth of the flange.
For the computation of the ultimate strength of restrained beams the contributions of the compression reinforcement and the shear transfer \((V_{st})\) of the stirrups were neglected. This conservative measure seems justifiable on account of rapid changes of the free body forces within the relatively short shear span: firstly, the change from negative moments over the support to the positive moment at the end of the shear span makes a full development of the steel compression force according to the concrete compression strains unlikely; and secondly, the stirrups close to the concrete crushing zone probably do not reach their yield strength. Thus a reduction of the shear force carried by the concrete should not be considered. This presumption has on the other hand little effect on the moment contribution of the stirrups.

The ratios \(M_{\text{calculated}}/M_{\text{test}}\) are given in the last column of the tables including those results where large deviations could be assigned to definitely known reasons, for example, very short shear spans (theory underestimates the shear strength) or erratic results within a test series. The frequency curves of the ratio \(M_{\text{calc}}/M_{\text{test}}\) are given for all the tests in Fig. 6/1 and 6/2. It should be noted that the theory resulted from rationally founded assumptions which had
at times been chosen conservatively. The peaks of the frequency curves lie therefore in general on the conservative side.

As for the prestressed beams (with the frequency peak at 0.8) no empirical modification was sought for the time being, since only one test series with a limited number of variables was available. As mentioned in the introduction, it is planned to study the particular conditions of prestressed concrete in the light of this theory by a series of tests which might eventually suggest an empirical or theoretical modification.
\[ \frac{M_{\text{calc}}}{M_{\text{test}}} \]

<table>
<thead>
<tr>
<th>Description</th>
<th>N</th>
<th>( m )</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simply supported; no web or comp. reinf.</td>
<td>52</td>
<td>0.96</td>
<td>0.15</td>
</tr>
<tr>
<td>Simply supported; with comp. reinf.</td>
<td>30</td>
<td>1.04</td>
<td>0.15</td>
</tr>
<tr>
<td>Simply supported; with web reinf.</td>
<td>59</td>
<td>0.90</td>
<td>0.12</td>
</tr>
<tr>
<td>T Section</td>
<td>20</td>
<td>1.04</td>
<td>0.18</td>
</tr>
<tr>
<td>Prestressed</td>
<td>32</td>
<td>0.80</td>
<td>0.10</td>
</tr>
<tr>
<td>Restrained</td>
<td>85</td>
<td>0.94</td>
<td>0.17</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>278</strong></td>
<td><strong>0.94</strong></td>
<td><strong>0.16</strong></td>
</tr>
</tbody>
</table>

**Note:**
- \( N \) = number of tests
- \( m \) = mean value of \( \frac{M_{\text{calc}}}{M_{\text{test}}} \)
- \( \sigma \) = standard deviation of \( \frac{M_{\text{calc}}}{M_{\text{test}}} \)

**Fig. 6/1**

Frequency Curve of the Ratio \( \frac{M_{\text{calc}}}{M_{\text{test}}} \)
Fig. 6/2  Composite Frequency Curve of the Ratio \( \frac{M_{\text{calc}}}{M_{\text{test}}} \)

(Number of tests = 278)
<table>
<thead>
<tr>
<th>Beam</th>
<th>$M_{Vn}$</th>
<th>$f'_c \text{ psi}$</th>
<th>$p%$</th>
<th>$\sigma_0 \text{ ksi}$</th>
<th>$\lambda$</th>
<th>$h_1/h$</th>
<th>$M_{calc}$</th>
<th>$M_{test}$</th>
<th>$M_{calc}/M_{test}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.41</td>
<td>5320</td>
<td>1.90</td>
<td>4.607</td>
<td>23763</td>
<td>0.176</td>
<td>495.0</td>
<td>468</td>
<td>1.06</td>
</tr>
<tr>
<td>2</td>
<td>&quot;</td>
<td>2420</td>
<td>&quot;</td>
<td>2.095</td>
<td>37560</td>
<td>0.253</td>
<td>309.6</td>
<td>288</td>
<td>1.08</td>
</tr>
<tr>
<td>3</td>
<td>&quot;</td>
<td>3735</td>
<td>&quot;</td>
<td>3.234</td>
<td>28649</td>
<td>0.210</td>
<td>407.2</td>
<td>423</td>
<td>0.96</td>
</tr>
<tr>
<td>4</td>
<td>&quot;</td>
<td>2230</td>
<td>&quot;</td>
<td>1.931</td>
<td>39717</td>
<td>0.261</td>
<td>293.2</td>
<td>327.6</td>
<td>0.89</td>
</tr>
<tr>
<td>5</td>
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<td>4450</td>
<td>1.90</td>
<td>3.853</td>
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<td>0.193</td>
<td>450.0</td>
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<td>414</td>
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<td>&quot;</td>
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<td>0.283</td>
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<td>252</td>
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</tr>
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<td>1.90</td>
<td>5.169</td>
<td>22509</td>
<td>0.165</td>
<td>524.4</td>
<td>432</td>
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<td>&quot;</td>
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<td>29901</td>
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</tr>
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<td>5530</td>
<td>&quot;</td>
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<td>23326</td>
<td>0.172</td>
<td>504.7</td>
<td>486</td>
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<td>&quot;</td>
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<td>0.234</td>
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<td>381.6</td>
<td>0.92</td>
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<td>13</td>
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<td>1.90</td>
<td>4.745</td>
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<td>502.5</td>
<td>450</td>
<td>1.21</td>
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<tr>
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<td>&quot;</td>
<td>2.827</td>
<td>31010</td>
<td>0.224</td>
<td>375.5</td>
<td>349.2</td>
<td>1.08</td>
</tr>
<tr>
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<td>&quot;</td>
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<td>&quot;</td>
<td>4.693</td>
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</tr>
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<td>&quot;</td>
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<td>305.5</td>
<td>306</td>
<td>1.00</td>
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</table>

Reference: 1.4/10
Reinforcement: 2 No. 7 Deformed bars
Dimensions: 6" x 12" Beams. $a = 36\"$, $h = 10.56\$, $L = 108\$" Total length = 120"
<table>
<thead>
<tr>
<th>Beam</th>
<th>h</th>
<th>$M_{Vh}$</th>
<th>$f'c$</th>
<th>p</th>
<th>Reinf Bars</th>
<th>$c_{ho}$</th>
<th>$\lambda$</th>
<th>$h_{1/h}$</th>
<th>$M_{calc}$</th>
<th>$M_{test}$</th>
<th>$M_{calc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>in.</td>
<td></td>
<td>psi</td>
<td>%</td>
<td>No.-Size</td>
<td>ksi</td>
<td>in.</td>
<td>k-in</td>
<td>k-in</td>
<td>k-in</td>
<td>k-in</td>
</tr>
<tr>
<td>A1</td>
<td>10.30</td>
<td>3.06</td>
<td>4400</td>
<td>2.17</td>
<td>1-11</td>
<td>3.689</td>
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<td>455,9</td>
<td>425</td>
<td>1.07</td>
</tr>
<tr>
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<td>3.00</td>
<td>4500</td>
<td>2.15</td>
<td>2-8</td>
<td>3.749</td>
<td>29555</td>
<td>0.205</td>
<td>531.1</td>
<td>254</td>
<td>1.13</td>
</tr>
<tr>
<td>3</td>
<td>10.55</td>
<td>2.99</td>
<td>4500</td>
<td>2.22</td>
<td>2-7; 1-6</td>
<td>3.744</td>
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<td>0.222</td>
<td>575.4</td>
<td>535</td>
<td>1.08</td>
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<td>4570</td>
<td>2.37</td>
<td>4-6</td>
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<td>21988</td>
<td>0.238</td>
<td>629.1</td>
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<td>1.25</td>
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<td>B1</td>
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<td>3.00</td>
<td>3065</td>
<td>1.62</td>
<td>1-8; 2-4</td>
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<td>0.227</td>
<td>397.0</td>
<td>398</td>
<td>1.00</td>
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<td>1.60</td>
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<td>27824</td>
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<td>0.97</td>
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<td>2430</td>
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<td>0.265</td>
<td>351.4</td>
<td>394</td>
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<td>C1</td>
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<td>2.99</td>
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<td>0.81</td>
<td>1-7</td>
<td>0.766</td>
<td>78826</td>
<td>0.222</td>
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<td>880</td>
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<td>2-5</td>
<td>0.728</td>
<td>58466</td>
<td>0.255</td>
<td>129.7</td>
<td>173</td>
<td>0.75</td>
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<td>3</td>
<td>10.75</td>
<td>2.93</td>
<td>1000</td>
<td>0.80</td>
<td>3-4</td>
<td>0.827</td>
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<td>149.1</td>
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<td>4</td>
<td>10.80</td>
<td>2.92</td>
<td>980</td>
<td>0.82</td>
<td>2-4; 2-3</td>
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<td>37759</td>
<td>0.271</td>
<td>154.5</td>
<td>178</td>
<td>0.87</td>
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</table>

Reference: 1.4/10
Reinforcement: Deformed Bars
Dimensions: 7" x 12" Beams. a = 31.5", h = Variable, 10.30 — 10.80. L = 63".
Total Length = 75".

TABLE 6/2 - Tests by Moody (Series A), 1953.
Simple-Span Rectangular Beams Without Web Reinforcement
<table>
<thead>
<tr>
<th>Beam</th>
<th>a</th>
<th>h</th>
<th>$\frac{M}{Vh}$</th>
<th>$f'c$</th>
<th>p</th>
<th>Reinf Bars</th>
<th>$\sigma_0$</th>
<th>$\lambda$</th>
<th>$h_1/h$</th>
<th>$M_{calc}$</th>
<th>$M_{test}$</th>
<th>$\frac{M_{calc}}{M_{test}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-2</td>
<td>48</td>
<td>10.58</td>
<td>4.54</td>
<td>3900</td>
<td>2.08</td>
<td>3-No.6</td>
<td>3.587</td>
<td>23962</td>
<td>0.221</td>
<td>473.4</td>
<td>458.4</td>
<td>1.03</td>
</tr>
<tr>
<td>S-3</td>
<td>10.31</td>
<td>4.57</td>
<td>2140</td>
<td>1.90</td>
<td>2-No.7</td>
<td>1.969</td>
<td>40871</td>
<td>0.255</td>
<td>290.7</td>
<td>364.8</td>
<td>0.80</td>
<td></td>
</tr>
<tr>
<td>S-4</td>
<td>10.31</td>
<td>4.66</td>
<td>3800</td>
<td>4.11</td>
<td>2-No.10</td>
<td>3.508</td>
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<td>537.6</td>
<td>1.01</td>
<td></td>
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<td>S-5</td>
<td>48</td>
<td>10.31</td>
<td>4.66</td>
<td>4330</td>
<td>4.11</td>
<td>2-No.10</td>
<td>3.998</td>
<td>38311</td>
<td>0.269</td>
<td>592.7</td>
<td>537.6</td>
<td>1.10</td>
</tr>
<tr>
<td>S-7</td>
<td>10.72</td>
<td>4.48</td>
<td>2140</td>
<td>0.93</td>
<td>3-No.4</td>
<td>1.964</td>
<td>23355</td>
<td>0.196</td>
<td>239.4</td>
<td>276.0</td>
<td>0.87</td>
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<tr>
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<td>10.58</td>
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<td>2.096</td>
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<td>0.216</td>
<td>271.4</td>
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<td>0.73</td>
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Reference: 1.4/2
Reinforcement: Deformed Bars
Dimensions: 6" x 12" Beams, $h = 10.31 \rightarrow 10.72$, $L = 108"$, Total Length = 120".

TABLE 6/3 - Tests by Laupa, 1953.
Simple-Span Rectangular Beams Without Web Reinforcement
**Reference:** 6.1/1  
**Reinforcement:** Deformed Bars  
**Dimensions:** 6” x 12” Beams, a = 36”, h = 10.58”, L = 108”,  
Total length = 120”.

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<th>$p$</th>
<th>Reinf. Bars</th>
<th>$\sigma_0$</th>
<th>$\lambda$</th>
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<th>$M_{calc}$</th>
<th>$M_{test}$</th>
<th>$\frac{M_{calc}}{M_{test}}$</th>
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<td>1.38</td>
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<th>p</th>
<th>$\sigma_o$</th>
<th>$\lambda$</th>
<th>$h_1/h$</th>
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Reference: 4.2/1
Reinforcement: 2 No. 7 Deformed bars.
Dimensions: 8" x 18" Beams. $a =$ Variable, 18" to 36". $h = 15.37"$. $L = 72"$.

TABLE 6/5 - Tests by Clark, 1951.
Simple-Span Rectangular Beams Without Web Reinforcement
Compression Reinforcement: $f_y = 45$ ksi
Reinforcement: Deformed bars
Reference: 1.4/10
Dimensions: 7" x 24" Beams. $a = 32"$, $h = 21"$, $L = 96"$, Total length = 120"

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<th>$p'$</th>
<th>$\sigma_o$</th>
<th>$\lambda$</th>
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TABLE 6/6 - Tests by Moody (Series III), 1953. Simple Span Rectangular Beams Without Web Reinforcement, With Compression Reinforcement
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<th>p</th>
<th>$p'$</th>
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<th>$\lambda$</th>
<th>$h_1/h$</th>
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<th>$M_{test}$</th>
<th>$M_{calc}$/$M_{test}$</th>
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</table>

Dimensions: 4 to 6.25 x 8 Beams. h = 7", L = 60", total length 65".
Compression Reinforcement: $f_y \approx 48$ ksi.
Reference: 6.1/2

**TABLE 6/7 - Tests at M. I. T., 1951. Simple Span Rectangular Beams Without Web Reinforcement, With Compression Reinforcement**
TABLE 6/8 - Tests By Moretto, 1945. Simple-Span Rectangular Beams Without Web Reinforcement, With Compression Reinforcement

<table>
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<th>p'</th>
<th>σ₀</th>
<th>λ</th>
<th>h₁/h</th>
<th>Mᵦcalc</th>
<th>Mᵦtest</th>
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Reference: 1.4/8
Compression Reinforcement: fy ≈ 48 ksi
Reinforcement: Deformed bars
Dimensions: 5.5" x 21" Beams, h = 18.25", a = 32", L = 96", total length = 120".
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Tension reinforcement: Deformed bars
Web reinforcement: Deformed bars, 3/8-in; Vertical stirrups, $fy = 48$ ksi

**TABLE 6/9 - TESTS BY CLARK, 1951**

Simple-Span Rectangular Beams With Web Reinforcement (Continued on next page)
<table>
<thead>
<tr>
<th>Beam</th>
<th>a</th>
<th>( \frac{M}{Vh} )</th>
<th>( f_c' )</th>
<th>( p )</th>
<th>( r )</th>
<th>( \sigma_0 )</th>
<th>( \lambda )</th>
<th>( h_1/h )</th>
<th>( M_{bend} )</th>
<th>( M_{stir} )</th>
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\( h = 15.37'' \quad b = 8'' \quad L = 72'' \quad \gamma = 48.63 \text{ksi} \)

| D1-1 | 18 | 1.95 | 3800  | 1.63 | .46 | 2.58 | 36571 | .229   | 991   | 208   | 1200  | 1218  | .99          |
|      | 2  | 1.77 | 3790  | 1.63 | .46 | 2.40 | 36626 | .239   | 959   | 208   | 1167  | 1443  | .81          |
|      | 3  | 2.21 | 3560  | 1.63 | .46 | 2.60 | 37972 | .225   | 984   | 208   | 1192  | 1038  | 1.15         |

\( h = 12.37'' \quad b = 6'' \quad L = 96'' \quad \gamma = 46.5 \text{ksi} \)

| D2-1 | 18 | 2.62 | 3480  | 1.63 | .61 | 2.75 | 38482 | .216   | 1006  | 276   | 1283  | 1173  | 1.09         |
|      | 2  | 2.41 | 3755  | 1.63 | .61 | 2.86 | 36821 | .214   | .036  | 276   | 1313  | 1263  | 1.04         |
|      | 3  | 2.25 | 3595  | 1.63 | .61 | 2.65 | 37756 | .223   | 994   | 276   | 1270  | 1353  | .94          |
|      | 4  | 2.24 | 3550  | 1.63 | .61 | 2.61 | 38035 | .224   | 986   | 276   | 1262  | 1355  | .93          |

| D3-1 | 18 | 3.02 | 4090  | 2.44 | .92 | 3.41 | 35098 | .250   | 1413  | 154   | 1598  | 1.15        |
|      | 2  | 3.02 | 4090  | 2.44 | .92 | 3.41 | 35098 | .250   | 1413  | 154   | 1598  | 1.15        |
|      | 3  | 3.27 | 4060  | 3.42 | .46 | 3.47 | 39676 | .272   | 751   | 108   | 870   | 966   | .84          |

| D4-1 | 18 | 3.02 | 4090  | 2.44 | .92 | 3.41 | 35098 | .250   | 1413  | 154   | 1598  | 1.15        |
|      | 2  | 3.02 | 4090  | 2.44 | .92 | 3.41 | 35098 | .250   | 1413  | 154   | 1598  | 1.15        |
|      | 3  | 3.27 | 4060  | 3.42 | .46 | 3.47 | 39676 | .272   | 751   | 108   | 870   | 966   | .84          |

* \( M_{stir} = \frac{V}{2} \times \frac{h}{2} \)  \( \text{(Vst)} \) \( V \)
VI

\[ M = \frac{V}{\lambda} \]

\[ \sigma_o = f'c, \quad \frac{M}{(V-V_{st})h} = \infty \]

<table>
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<tr>
<th>Beam</th>
<th>h</th>
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<th>p</th>
<th>r</th>
<th>( \lambda )</th>
<th>( h_L/h )</th>
<th>( M_{bend} )</th>
<th>( M_{st} )</th>
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* \( V_{st} > V \) therefore: \( M_{st} = V \cdot \frac{h}{2} \), \( \sigma_o = f'_c \), \( \frac{M}{(V-V_{st})h} = \infty \)

Reference: 6.1/1

Tension Reinforcement: Deformed bars
Web Reinforcement: Vertical deformed stirrups, \( f_y \approx 45 \text{ ksi} \)
Dimensions: 6" x 12" beams. \( a = 36"\), \( h = 9.23"\) — 10.72".
  Total Length = 120". \( L = 108" \)

**Table 6/10 - Tests by Gaston, 1952.**

Simple-Span Rectangular Beams With Web Reinforcement
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<th>( f'_c ) (ksi)</th>
<th>p (%)</th>
<th>( \sigma_o ) (ksi)</th>
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Reference: 6.1/3
Reinforcement: Deformed Bars
Dimensions: \( b' = 3" \). \( b = 13" \). \( h = 5" \). \( a = 10" - 37" \). \( t = 1.1/4" \).
Total length = 42" - 92".

Simple-Span T-Beams Without Web Reinforcement
| Beam | a  | h   | $\frac{M}{Vh}$ | $f'_c$ | p   | $\sigma_0$ | $\lambda$ | $\epsilon_0$ | $h_1/h$ | $M_{calc}$ | $M_{test}$ | $\frac{M_{calc}}{M_{test}}$ |
|------|----|-----|----------------|-------|-----|------------|------|----------|--------|-----------|-------------|---------------|------------------|
| S-1  | 36 | 9.16 | 3.93 | 3620 | 0.320 | 3.24 | 9144 | 0.35 | 0.08 | 135 | 224 | 0.60 |
| S-2  | 36 | 9.28 | 3.88 | 3660 | 0.316 | 3.26 | 9086 | 1.75 | .12 | 192 | 312 | 0.62 |
| S-3  | 36 | 9.38 | 3.84 | 4290 | 0.312 | 3.82 | 8316 | 0.84 | .08 | 168 | 252 | 0.67 |
| S-4  | 36 | 8.00 | 4.50 | 3085 | 0.916 | 2.83 | 10063 | 4.06 | .45 | 383 | 387 | 0.99 |
| S-5  | 36 | 8.30 | 4.34 | 6260 | .883 | 5.71 | 6907 | 2.62 | .25 | 527 | 558 | 0.94 |
| S-6  | 36 | 8.20 | 4.39 | 7990 | .893 | 7.30 | 6243 | 3.85 | .22 | 580 | 593 | 0.98 |
| S-7  | 36 | 8.44 | 4.27 | 3550 | .868 | 3.23 | 9249 | 3.66 | .38 | 426 | 504 | 0.84 |
| S-8  | 36 | 8.20 | 4.34 | 6120 | .758 | 5.59 | 6977 | 0.18 | .11 | 244 | 315 | 0.77 |
| S-9  | 36 | 8.20 | 4.34 | 4760 | .774 | 4.35 | 7874 | 1.99 | .21 | 333 | 412 | 0.81 |
| S-10 | 36 | 8.20 | 4.34 | 5790 | .774 | 5.29 | 7156 | 2.52 | .20 | 391 | 475 | 0.82 |
| S-11 | 36 | 8.80 | 4.09 | 2580 | .333 | 2.32 | 11282 | 1.26 | .14 | 142 | 192 | 0.74 |
| S-12 | 36 | 8.20 | 4.34 | 4760 | .774 | 4.35 | 7874 | 1.06 | .17 | 278 | 376 | 0.74 |
| S-13 | 36 | 8.20 | 4.34 | 4840 | .774 | 4.42 | 7807 | 4.90 | .33 | 492 | 539 | 0.91 |
| S-14 | 36 | 8.20 | 4.34 | 4660 | .715 | 4.26 | 7960 | 4.60 | .30 | 447 | 502 | 0.89 |
| S-15 | 36 | 8.35 | 4.31 | 2896 | .351 | 2.63 | 10484 | 3.08 | .19 | 194 | 264 | 0.74 |
| S-16 | 36 | 8.06 | 4.47 | 3530 | .364 | 3.23 | 9279 | 3.13 | .17 | 199 | 268 | 0.74 |

Reference: 1.4/11
Reinforcement: Rusted Wires
Dimensions: 6" x 12" Beams.  a = Variable 24—54.  h = Variable 8.00—9.38

TABLE 6/12 - Tests By Zwoyer, 1953
Simple-Span Prestressed Concrete Rectangular Beams Without Web Reinforcement
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<th>$p$</th>
<th>$\sigma_0$</th>
<th>$\lambda$</th>
<th>$\epsilon_0$</th>
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* $M_{st} \leq V \cdot \frac{h}{2}$

Ref: 1.4/10

Longitudinal Reinforcement: Deformed Bars

Web Reinforcement: Stirrups (deformed)

Length: 15' 4"

Span: 8'

TABLE 6/13 - Tests By Elstner, 1954

Restrained Rectangular Beams With Web Reinforcement
| Beam | $M/Vh$ | $f'_c$ | $p$ | $r$ | $f_y$ | $\sigma_0$ | $\lambda$ | $h_1/h$ | $M_{berd}$ | $M_{st}$ | $M_{calc}$ | $M_{test}$ | $M_{calc}/M_{test}$ |
|------|--------|------|----|----|------|--------|------|--------|-----------|---------|-----------|------------|----------------|----------------------|
|      | psi    | %    | %  | ksi | ksi | 10^2  | k-in. | k-in.  | k-in.    | k-in.   | k-in.     | k-in.      |                       |
|      |        |      |    |     |      |        |       |       |          |         |           |            |                       |
| Series II. 7" x 24" Beams. $a = 32"$. $h = 21"$ |        |      |    |     |      |        |       |       |          |         |           |            |                       |
| II-21a | 0.76   | 3580 | 2.72 | 0.52 | 47   | 0.87  | 378  | 0.470  | 969    | 379  | 1348  | 1653    | 0.82                       |
| b     | 0.76   | 3640 | 2.72 | 0.52 | 47   | 0.88  | 374  | 0.468  | 982    | 379  | 1362  | 1509    | 0.90                       |
| 22a   | 0.76   | 3000 | 2.72 | 0.95 | 44   | 0.73  | 421  | 0.482  | 826    | 652  | 1478  | 1600    | 0.92                       |
| b     | 0.76   | 2710 | 2.72 | 0.95 | 44   | 0.66  | 449  | 0.489  | 754    | 652  | 1406  | 1546    | 0.91                       |
| II-23a | 0.76   | 3230 | 2.72 | 1.47 | 41   | 0.78  | 402  | 0.477  | 883    | 934  | 1818  | 1600    | 1.14                       |
| b     | 0.76   | 3160 | 2.72 | 1.47 | 41   | 0.77  | 407  | 0.479  | 866    | 934  | 1801  | 1866    | 0.96                       |
| II-e  | 0.76   | 3420 | 2.72 | 1.72 | 43   | 0.83  | 388  | 0.473  | 930    | 1153 | 2085  | 2080    | 1.00                       |
| f     | 0.76   | 3330 | 2.72 | 2.14 | 43   | 0.81  | 395  | 0.475  | 908    | 1189 | 2097  | 1813    | 1.16                       |

| Series IV. 7" x 15" Beams. $a = 48"$. $h = 12"$ |        |      |    |     |      |        |       |       |          |         |           |            |                       |
| IV-m  | 2.00   | 2860 | 4.76 | 0.52 | 53   | 1.97  | 434  | .396   | 631    | 141  | 773    | 1001     | 0.77                       |
| n     | 2.00   | 3710 | 4.76 | 0.95 | 45   | 2.55  | 370  | .370   | 779    | 219  | 998    | 1357     | 0.74                       |
| o     | 2.00   | 3420 | 4.76 | 1.47 | 47   | 2.35  | 3888 | .378   | 730    | 352  | 1083   | 1494     | 0.72                       |

TABLE 6/13 (Continued)
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Tension Reinforcement: Deformed Bars

Continued on Span = 8 ft.

Total Length = 15' 4"

Reference: 1.4/10

TABLE 6/14 - Tests By Moody, 1954

Restrained Rectangular Beams Without Web Reinforcement
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<th>Beam</th>
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TABLE 6/14 (Continued)
7 SUMMARY AND CONCLUSIONS

(1) The prediction of the ultimate strength of prestressed and conventionally reinforced concrete beams under the combined action of moment and shear was approached by stating a hypothesis concerning the failure mechanism in the region of diagonal cracks.

(2) The failure mechanism is considered to consist of a "shear-rotation" about the neutral axis of the broken failure cross-section. The depth of the neutral axis at failure could then be obtained by observing:
   (a) the deformations caused by the shear rotation;
   (b) the critical stresses resulting from these deformations; and
   (c) the equilibrium conditions of the internal forces.

(3) The failure is ultimately attributed to the destruction of the concrete compressive zone. The strength of this zone, depending on the prevailing state of stress, was derived according to Mohr's failure criterion for plain concrete.

(4) The deformation of the tension zone was attributed to a pull-out effect of the longitudinal reinforcement relative to the surrounding concrete. This measure
permitted a numerical consideration of the influence of the bond characteristic on the shear strength.

(5) The information concerning the ultimate strength of a beam is expressed in terms of moment capacity (ultimate shear moment), while considering, however, the influence of the magnitude of the shear force \( \frac{M}{Vh} \) ratio).

(6) The comparison with 278 test results confirmed the satisfactory validity of the theory for:

(a) Simply supported beams with and without web or compression reinforcement

(b) Prestressed beams

(c) Restrained beams

(d) Beams of various cross-sections

(7) According to the theory and in agreement with experimental findings, the ultimate shear moment increases with

(a) concrete strength;

(b) ratio of tension reinforcement;

(c) ratio of web reinforcement;

(d) ratio of compression reinforcement;

(e) prestressing;

(f) bond coefficient \( \frac{1}{\lambda} \); and

(g) ratio \( \frac{M}{Vh} \) (for symmetrical loading: shear span).

The rate of moment increase with these parameters is shown in an example in Fig. 7/1.
(8) The mode of failure of a given beam depends on the load configuration. It is therefore impossible to provide against shear failure for the beam as such without due regard to the particular load conditions.

(9) With the theory outlined in this paper it is easily possible to specify an adequate factor of safety ($s$) against shear failures in the form

$$\frac{M_{su/calc}}{M_{design/min}} \geq s$$
Fig. 7.1
Rate of Influence of Various Parameters

1.4/1 Hognestad, E. What Do We Know About Diagonal Tension and Web Reinforcement in Concrete? University of Illinois Bulletin, Vol. 49, No. 50. March, 1952.


2.1/1 Mohr, O. "Die Schubfestigkeit des Betons." *Armierter Beton*, 1911.


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<td>$A'_{st}$</td>
<td>Area of longitudinal compression reinforcement</td>
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<td>$A_w$</td>
<td>Area of two legs of one stirrup</td>
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<td>Inclined compressive force</td>
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<td>$E'_{st}$</td>
<td>Eccentricity of prestressing force</td>
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<td>Effective prestressing force</td>
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<td>$f'$</td>
<td>Steel stress of compression reinforcement at failure</td>
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<td>Moment of inertia of the uncracked section</td>
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<td>jh</td>
<td>Lever arm of the internal forces</td>
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<td>k</td>
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<td>$T_i'$</td>
<td>Initial post-tensioning force before creep and shrinkage take place</td>
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<tr>
<td>V</td>
<td>Shear force</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$V_c$</td>
<td>Shear force carried by the concrete compressive zone</td>
</tr>
<tr>
<td>$V_{st}$</td>
<td>Shear force carried by the stirrups</td>
</tr>
<tr>
<td>$V_u$</td>
<td>Ultimate shear force</td>
</tr>
<tr>
<td>$W_n$</td>
<td>Weight of a reference point &quot;n&quot; of the curve given by Eq. 3.4/6</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Angle defined in Fig. 3.2/1</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Angle defined in Fig. 3.2/1</td>
</tr>
<tr>
<td>$\Delta l_{bot}$</td>
<td>Deformation of the bottom zone (see Fig. 3.1/1)</td>
</tr>
<tr>
<td>$\Delta l_{top}$</td>
<td>Deformation of the top zone (see Fig. 3.1/1)</td>
</tr>
<tr>
<td>$\Delta T_{el}$</td>
<td>Loss of prestress due to elastic shortening of concrete</td>
</tr>
<tr>
<td>$\Delta T_{ie}$</td>
<td>Loss of prestress due to inelastic deformations of concrete and steel</td>
</tr>
<tr>
<td>$\Delta \varepsilon_{el}$</td>
<td>Elastic change of strain due to prestressing</td>
</tr>
<tr>
<td>$\Delta \varepsilon_{ie}$</td>
<td>Inelastic change of strain due to prestressing and sustained load</td>
</tr>
<tr>
<td>$\Delta \sigma_{el}$</td>
<td>Elastic change of stress due to prestressing</td>
</tr>
<tr>
<td>$\Delta \sigma_{ie}$</td>
<td>Inelastic change of stress due to prestressing and sustained load</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Strain</td>
</tr>
<tr>
<td>$\varepsilon'$</td>
<td>Steel strain of compression reinforcement at failure</td>
</tr>
<tr>
<td>$\varepsilon_{ct}$</td>
<td>Steel strain in excess of prestressing strain at failure</td>
</tr>
<tr>
<td>$\varepsilon_{cb}$</td>
<td>Concrete bottom strains</td>
</tr>
<tr>
<td>$\varepsilon_{ct}$</td>
<td>Concrete top strains</td>
</tr>
<tr>
<td>$\varepsilon_{cu}$</td>
<td>Ultimate concrete strain</td>
</tr>
</tbody>
</table>
\[ \varepsilon_{\text{eff}} \] Effective steel strain in prestressing tendons
\[ \varepsilon_i \] Initial pretensioning strain before release
\[ \varepsilon_i' \] Initial post-tensioning strain before creep and shrinkage take place
\[ \varepsilon_{\text{st}} \] Steel strain
\[ \varepsilon_x \] Strain in x direction (3.2/1)
\[ \varepsilon_y' \] Yield strain of compression or web reinforcement
\[ \eta = \frac{\sigma_x}{\sigma_0} \]
\[ \kappa \] Coefficient of roughness
\[ \frac{1}{\lambda} \] Bond coefficient
\[ \xi = \frac{\varepsilon_x}{\varepsilon_{\text{cu}}} \]
\[ \sigma \] Normal stress
\[ \sigma' \] Concrete stress defined in Fig. 2.2/3
\[ \sigma_{\text{av}} \] Average stress in the concrete compressive zone
\[ \sigma_{\text{av}}^* \] Average stress of a portion of the concrete compression zone (Eq. 2.4/4)
\[ \sigma_c \] Compressive strength of concrete
\[ \sigma_0 \] Concrete normal stress at failure (Eq. 2.4/8)
\[ \sigma_0' \] Concrete stress defined in Fig. 2.2/3
\[ \sigma_{\text{st}} \] Steel stress
\[ \sigma_t \] Tensile strength of concrete
\[ \sigma_x, \sigma_y \] Stress components (Fig. 2.3/3)
\( \sigma_1 \)  
\( \sigma_2 \)  
\( \sigma_3 \)  
\( \tau \)  
\( \tau_{av} \)  
\( \tau_{av}^* \)  
\( \tau_s \)  
\( \tau_{xy} \)  
\( \tau_{yx} \)  
\( \phi \)  

Principle stresses  
Shear stress  
Average shear stress of the concrete compressive zone  
Average shear stress of a portion of the concrete compressive zone (Eq. 2.4/4)  
Shear stress (Fig. 2.3/2)  
Shear stresses (Fig. 2.3/1 and 2.3/3)  
Shear rotation
APPENDIX

Pull-out Tests to Derive the Coefficient of Roughness $\chi$.

Pull-out specimens:

Cylinders 6" x 12" vertically cast
Concrete strength $f_{c'} = 4660$ psi
Steel (two specimens of each)
1/2" plain bar
7/8" plain bar
1/2" deformed bar
7/8" deformed bar
1/2" strand
7/16" strand

The average pull-out displacements $\Delta l$ as a function of the steel strain are plotted in the following figures, together with the theoretically assumed curves:

$$
\Delta l = \chi \left( 0.5 + \frac{2500 \text{ psi}}{f_{c'}} \right) d \leq 2
$$
\( \epsilon \times 10^{-3} \) 

\( \Delta l \) (inches)

\[ \begin{align*} 
\frac{1}{2}'' \text{ Plain Bar} \\
\text{(not rusted)} \\
\kappa = 18,500 \\
\end{align*} \]

\[ \begin{align*} 
\frac{7}{8}'' \text{ Plain Bar} \\
\text{(not rusted)} \\
\kappa = 18,500 \\
\end{align*} \]
Deformed Bar (not rusted)
\( \kappa = 9500 \)

deformed bar theoretical curve

testing

Deformed Bar (rusted)
\( \kappa = 6500 \)

deformed bar theoretical curve

testing
\( \frac{1}{2}'' \) Strand
\( \kappa = 6500 \)

- Pullout Displacement \( \Delta l \) (inches)
- \( \epsilon \) (\( x 10^{-3} \))

\( \frac{7}{16}'' \) Strand
(not rusted)
\( \kappa = 6500 \)

- Pullout Displacement \( \Delta l \) (inches)
- \( \epsilon \) (\( x 10^{-3} \))