Built-up Members in Plastic Design

ULTIMATE STRENGTH DESIGN CURVES FOR LONGITUDINALLY STIFFENED PLATE PANELS WITH LARGE b/t

by

Joseph F. Vojta

and

Alexis Ostapenko

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Fritz Engineering Laboratory
Department of Civil Engineering
Lehigh University
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Fritz Engineering Laboratory Report No. 248.19
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1. INTRODUCTION

The purpose of the ultimate strength design curves presented here is to rapidly determine the dimensions of a longitudinally stiffened plate panel that will just sustain a simultaneous axial and lateral loading. Given a set of loads and overall dimensions and assuming some relative proportions of the cross section, the dimensions of the cross section can be found. A series of different sets of relative proportions can be assumed and the most advantageous section selected from these.

The dimensions and range of loads for which these curves apply are those used in ship bottom plating.

The theory and the computer program used in obtaining the data for these design curves are described in ULTIMATE STRENGTH DESIGN OF LONGITUDINALLY STIFFENED PLATE PANELS WITH LARGE b/t*.

* Vojta, J. F. and Ostapenko, A. ULTIMATE STRENGTH DESIGN OF LONGITUDINALLY STIFFENED PLATE PANELS WITH LARGE b/t, Fritz Laboratory Report No. 248.18, Lehigh University, August, 1967.
2. PRELIMINARY CONCEPTS

The type of cross section analyzed consists of a plate stiffened by a series of tee-stiffeners, (Fig. 1). The loads considered are a compressive axial load, \( P' \), acting in the direction of the stiffeners and a uniformly distributed lateral load, \( q \), on the plate side of the cross section causing compression in the plate at the middle of the span during bending (Fig. 2). The ends A and B can either be both fixed or both simply-supported. There are two sets of curves, one for each end condition; however, their use follows identical procedures.

An idealized cross section shown in Fig. 3 is used in the design curves and for the calculations of the required cross section. In non-dimensional terms it can be described by the parameters \( A_s/A_p \), \( A_f/A_s \), \( b/t \), \( d/t \).

The design curves apply when \( b/t \) is large enough (\( b/t \geq 45 \)) for the plate to buckle before the ultimate load is reached.

The following ranges of values were used in the development of the design curves:

\[
\frac{A_s}{A_p} = 0.20 \text{ to } 0.48 \\
\frac{A_f}{A_s} = 0.35 \text{ to } 0.60 \\
b/t = 60 \text{ to } 110
\]
\[ Q = q(T) = 40 \text{ to } 480 \]
\[ \sigma_r / \sigma_{yp} = 0.00 \text{ to } 0.15 \]

However, extrapolations outside these ranges can be used in most cases.

The following values are assumed as fixed in the development of the curves:

Yield stress of plate and stiffener, \( \sigma_{ys} = \sigma_{yp} = 47.0 \text{ ksi} \)
Modulus of elasticity, \( E = 29,600 \text{ ksi} \)
Poisson's Ratio, \( \mu = 0.30 \).
3. Method of Using the Ultimate Strength Design Curves

The use of the design curves will now be illustrated schematically. This is followed by the procedure outlined in steps and by a numerical example.

3.1 Schematic Example

Known from the loading, geometry and material conditions are the parameters:

- \( B \) = Total panel width (in.)
- \( L \) = Length of section (in.)
- \( P' \) = Total panel axial load (kips)
- \( \sigma_r \) = Plate compressive residual stress (ksi)
- \( \sigma_{yp} \) = Plate yield stress (ksi)
- \( \sigma_{ys} \) = Stiffener yield stress (ksi)
- \( E \) = Modulus of elasticity (ksi)

The geometric parameters \( A_s/A_p, A_f/A_s \), \( b/t \) and \( d/t \) are all assumed. Dimensions of the cross section can be obtained through the use of the design curves which find the plate width, \( b \). This can be repeated many times with different assumed parameters and the most advantageous section can be selected depending upon the criteria required.
Figure 4 shows a schematic sketch of the design curves. Beginning in Box "A", compute S for the chosen values of b/t and d/t. For b/t < 80 use S_2 on the bottom scale as a starting point; for b/t \geq 80 use S_1 on the top scale. Assuming d/t, compute Q

\[ Q = q_{2}(\frac{d}{t}) \]  

Project a vertical line to the curve with the correct residual stress ratio \( \frac{\sigma_f}{\sigma_{yp}} \) and the correct Q.* A horizontal line is then to be drawn to the right for b/t > 80, following dotted lines, (or to the left for b/t \geq 80, solid lines).

The intercept on the N axis (N' value in Fig. 4) is a value based on \( A_s/A_p = 0.60 \). If the designer chooses these values for the cross section, he can stop here and compute b as follows:

\[ b \text{ (inches)} = 0.0003895 \left( \frac{P'}{B} \left( \frac{b}{t} \right)^2 \right) \text{ for s. s. ends} \]  

\[ b \text{ (inches)} = 0.001287 \left( \frac{P'}{B} \left( \frac{b}{t} \right)^{1.65} \right) \text{ for fixed ends} \]

If different values of \( A_s/A_p \) and \( A_f/A_s \) are chosen, the cross section must be modified in Boxes "B" and "C".

For \( A_s/A_p \neq 0.34 \) and \( A_f/A_s \neq 0.60 \), the designer continues the horizontal line through the N axis to the curve corresponding to the correct Q and \( A_s/A_p \) in Box "B". A vertical line from here will intercept the R axis at a value pertaining to \( A_f/A_s = 0.60 \), with \( A_s/A_p \), b/t,

* Interpolation between curves for Q varies approximately with the logarithm of Q. All other interpolations are approximately linear.
and d/t as previously chosen. However, since R is a complicated term, not readily useful for computing b, one should continue on to Box "C" in all cases.

After locating the correct curve in Box "C", the vertical line through the R axis should be extended until it intersects this curve. A horizontal line will now give a value on the N axis. The plate width b can then be found from Eqs. 4 to 5:

\[
b \text{ (inches)} = 0.000223 \left( \frac{P'}{B} \right)^2 \text{ for s. s. ends} \quad (4)
\]

\[
b \text{ (inches)} = 0.0017246 \left( \frac{P'}{B} \right)^{1.65} \text{ for fixed ends} \quad (5)
\]

Having obtained b from one of the equations of Eqs. 2 to 5, the area values can now be computed for the idealized cross section as follows:

Plate area, \( A_p = \frac{b^2}{(b/t)} \) \quad (6)

Stiffener area, \( A_s = \left( \frac{A_s}{A_p} \right) A_p = \frac{A_s}{A_p} \frac{b^2}{(b/t)} \) \quad (7)

Stiffener flange area, \( A_f = \left( \frac{A_f}{A_s} \right) A_s = \left( \frac{A_f}{A_s} \right) \left( \frac{A_s}{A_p} \right) \left[ \frac{b^2}{(b/t)} \right] \) \quad (8)

Web area, \( A_w = A_s - A_f = \left( \frac{A_s}{A_p} \right) \left[ 1 - \left( \frac{A_f}{A_s} \right) \right] \left[ \frac{b^2}{(b/t)} \right] \) \quad (9)

Total area, \( A = A_s + A_p = \left[ 1 + \left( \frac{A_s}{A_p} \right) \right] \left[ \frac{b^2}{(b/t)} \right] \) \quad (10)

The dimensions of the idealized cross section are:

Plate width (center to center of stiffeners), b(in.), from Eqs. 2 to 5.
Plate thickness (in.), $t = \frac{b}{(b/t)}$  \hspace{0.5cm} (11)

Stiffener depth (in.), $d = \frac{(d/t) \cdot b}{(b/t)}$ \hspace{0.5cm} (12)

Web plate thickness, (in.), $w = \frac{A_w}{d} = \frac{A_s}{A_p} \left[1 - \frac{A_f}{A_s}\right] \frac{b}{(d/t)}$ \hspace{0.5cm} (13)

The stiffener flange can be proportioned as desired as long as its area is as given by Eq. 8.

Figure 5 presents the design curves for panels with simply-supported ends; Fig. 6 presents the design curves for fixed ends.

3.2 Detailed Outline of Steps to Follow

1. Assume $d/t$, $b/t$, $A_s/A_p$, and $A_f/A_s$ and compute $S$, $Q$, $\sigma_t/\sigma_{yp}$. The formula for $S$ is given on the design charts.

   \[ Q = q \frac{(d/t)}{\text{(q in psi)}} \]  \hspace{0.5cm} \text{From Eq. (1)}

   \[ \frac{\sigma_t}{\sigma_{yp}} = \frac{\sigma_t}{47.0} \]  \hspace{0.5cm} 47.0 is the yield stress for which these curves are valid.

2. Pick the correct scale for the starting point $S$ depending upon $b/t < 80$ or $b/t \geq 80$. Draw a vertical line from $S$ to the correct curve.

3. Draw a horizontal to the N axis; to the right for $b/t < 80$ or to the left for $b/t \geq 80$.

4. If $A_s/A_p = 0.34$ and $A_f/A_s = 0.60$, compute $b$ from Eq. 2 or 3 then go to Step 8.
If either $A_s/A_p \neq 0.34$ or $A_f/A_s \neq 0.60$ project a horizontal line to the curve corresponding to the chosen $A_s/A_p$.

5. Draw a vertical line to the curve corresponding to the chosen $A_f/A_s$.

6. Draw a horizontal line to the N axis.

7. Compute $b$ from Eq. 4 to 5.

8. Compute areas (Eq. 6 to 10) and cross-sectional dimensions (Eqs. 11 to 13).

3.3 Numerical Example

Given: Panel with fixed ends

- $Q = 11.25$ psi
- $P' = 12,800$ kips
- $B = 41'-8" = 500$ in.
- $\ell = 22'-8" = 272$ in.
- $\sigma_r = 0.000$ psi
- $\sigma_{yp} = \sigma_{ys} = 47.0$

Solution:

Assume: $b/t = 60$, $d/t = 16.0$, $A_f/A_s = 0.35$, $A_s/A_p = 0.48$

1. Compute $S$, $Q$, $\frac{\sigma_r}{\sigma_{yp}}$, and use the design curves for fixed ends, Fig. 6.

\[
S = 45.0 \sqrt{\frac{B \ell}{P' (d/t) \left( \frac{b}{l} \right) 0.65}}
\]

\[
= 45.0 \sqrt{\frac{500 (272)}{12800 (16)(60) 0.65}} = 9.675
\]
\[ Q = q \left( \frac{d}{t} \right) = 11.25 \times 16 = 180.0 \]

\[ \frac{\sigma_r}{\sigma_{yp}} = \frac{0.000}{47.0} = 0.000 \]

2. Since \( b/t < 80 \), the lower scale is used. A line is drawn upward to the curve for \( Q = 180 \), \( \frac{\sigma_r}{\sigma_{yp}} = 0.000 \) (solid line).

3. Draw a horizontal line to the right (\( b/t < 80 \)) which gives \( N = 0.676 \).

4. Both \( A_s/A_p \) and \( A_f/A_s \) were chosen not to be the standard values, thus a horizontal line is drawn to the right to the curve for \( \frac{A_s}{A_p} = 0.48 \), \( Q = 180 \).

5. A vertical line is drawn down to the curve for \( \frac{A_f}{A_s} = 0.35 \). (This intercepts the R axis at \( R = 32.05 \)).

6. Draw a horizontal line to the left giving \( N = 0.653 \).

7. From Eq. 5

\[ b = 0.0017246 \left[ \frac{p'(\frac{b}{t}) 1.650}{B N \left[ 1 + \left( \frac{A_s}{A_p} \right) \right]} \right] \]

\[ 0.0017246 \left[ \frac{(12800)(60) 1.650}{500 (0.653)(1.48)} \right] = 39.0 \text{ in.} \]

8. Compute the values of the areas of the cross section

\[ A_p = \frac{b^2}{(b/t)} = \frac{(39.0 \text{ in})^2}{60} = 25.35 \text{ in}^2 \quad \text{from Eq. 6} \]

\[ A_s = \left( \frac{A_s}{A_p} \right) A_p = 0.48 (25.35 \text{ in}^2) = 12.17 \text{ in}^2 \quad \text{from Eq. 7} \]
\[ A_f = \left( \frac{A_f}{A_s} \right) A_s = 0.35 \times (12.17 \text{ in}^2) = 4.26 \text{ in}^2 \quad \text{from Eq. 8} \]

\[ A_w = A_s - A_f = 12.17 \text{ in}^2 - 4.26 \text{ in}^2 = 7.91 \text{ in}^2 \quad \text{from Eq. 9} \]

Total area = \( A_s + A_p = 12.17 \text{ in}^2 + 25.35 \text{ in}^2 = 37.52 \text{ in}^2 \quad \text{from Eq. 10} \)

Note: These values are for the idealized cross section. For the total panel the values are multiplied by \( \frac{500}{39.0} = 12.82 \approx 13 \).

Computation of the idealized cross section:

\[ b = 39.0 \text{ in. as computed in Step 7.} \]

\[ t = \frac{b}{(b/t)} = \frac{39.0 \text{ in}}{60} = 0.65 \text{ in} \quad \text{from Eq. 11} \]

\[ d = t(d/t) = 0.65 \text{ in (16.)} = 10.4 \text{ in} \quad \text{from Eq. 12} \]

\[ w = \frac{\left( \frac{A_s}{A_p} \right) \left[ 1 - \left( \frac{A_f}{A_s} \right) \right]}{(d/t)} b = \frac{0.48 \times (1 - 0.35)}{16.0} 39.0 = 0.761 \text{ in} \quad \text{from Eq. 13} \]

 Arbitrarily let the stiffener flange be 3/4 in. thick (that is, \( t_f = 0.75 \text{ in} \)). Then \( b_f = \frac{A_f}{t_f} = \frac{4.62 \text{ in}^2}{0.75 \text{ in}} = 5.68 \text{ in} \).

The idealized cross section for this example is shown in Fig. 7. The total panel will then consist of a plate 500 inches wide with stiffeners spaced 39 in. center to center.
4. SYMBOLS

A Total area of idealized cross section

\( A_f \) Area of flange, idealized cross section

\( A_p \) Area of plate, idealized cross section

\( A_s \) Area of stiffener, idealized cross section

\( A_w \) Area of web, idealized cross section

B Total panel width

b Width of idealized cross section, i.e. center to center of stiffeners

\( b_f \) Width of stiffener flange

d Depth of cross section

E Modulus of elasticity

\( \ell \) Length of panel

\( N,R,S \) Values from design charts

\( m' \) Bending moment acting on total panel cross section

\( P' \) Axial load (kips) on total panel cross section

Q \( q (d/t) \)

q Uniformly distributed lateral load (ksi)

t Thickness of plate

\( t_f \) Thickness of stiffener flange

w Thickness of stiffener web

\( \mu \) Poisson's Ratio

\( \sigma_r \) Compressive residual stress in plate (ksi)
4. SYMBOLS (continued)

\[ \sigma_{yp} \quad \text{Yield stress of plate} \]
\[ \sigma_{ys} \quad \text{Yield stress of stiffener} \]
5. FIGURES
Fig. 1 General Cross Section

Fig. 2 Loading Condition

Fig. 3 Idealized Cross Section
Fig. 4. Schematic Plot of Design Curves
DESIGN CURVES FOR SIMPLY-SUPPORTED ENDS

1. Assume $b/t$, $d/t$, $A_s/A_p$, $A_f/A_s$
2. Compute $S$, $Q$, $o/R_o$, $o/R_p$
3. Start in plot above:
   Use lower scale for $b/t < 80$.
   Use upper scale for $b/t \geq 80$.
4. Use curves on right for $b/t < 80$.
   Use curves on left for $b/t \geq 80$.
5. Find final $N$ value
6. Compute $b$:
   \[
   b = \frac{P'(b/t)}{BN(1 + A_s/A_p)}
   \]

$P'$ = Axial force on panel (kips)  \( q \) = Lateral load (psi)
$B$ = Total panel width (in.)  $L$ = Panel length (in.)
$Q = q(d/t)$, psi

Fig. 5 Design Curves for Simply-Supported Ends
DESIGN CURVES FOR FIXED ENDS

1. Assume $b/t, d/t, A_f/A_p, A_s/A_p$
2. Compute $S, Q, \sigma_f/\sigma_y, \sigma_s/\sigma_y$
3. Start in plot above:
   - Use lower scale for $b/t < 80$.
   - Use upper scale for $b/t \geq 80$.
4. Use curves on right for $b/t < 80$.
   - Use curves on left for $b/t \geq 80$.
5. Find final $N$ value
6. Compute $b$:
   \[
b = 0.001724 \left( \frac{P(t/f) \sigma_y}{B N (1 + A_f/A_p)} \right)^{1/650}
\]

- $P'$ = Axial force on panel (kips)
- $q$ = Lateral load (psi)
- $B$ = Total panel width (in.)
- $L$ = Panel length (in.)
- $Q = q(d/t)$, psi

Fig. 6 Design Curves for Fixed Ends
Fig. 7 Cross Section - Numerical Example
6. ACKNOWLEDGMENTS

This report is part of a research project on Built-Up Members in Plastic Design carried out at Fritz Engineering Laboratory, Lehigh University, Bethlehem, Pennsylvania. Dr. A. Ostapenko is Director of the Project. Dr. L. S. Beedle is Director of the Laboratory and Acting Head of the Department of Civil Engineering. The project sponsor is the Naval Ship Engineering Center, Department of the Navy. The authors are thankful for their support, and in particular to Mr. John Vasta who initiated this project.

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This report presents ultimate strength design curves for longitudinally stiffened plate panels as used in ship bottom plating. The curves determine the dimensions of a plate panel that will just sustain a given simultaneous axial and lateral loading condition. Repeated use of the curves will give the optimum cross section.

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combined loads, design curves, longitudinal stiffeners, ship plates, ultimate strength
Fritz Engineering Laboratory Report No. 248.19
ULTIMATE STRENGTH DESIGN CURVES FOR LONGITUDINALLY STIFFENED PLATE PANELS WITH LARGE b/t by Joseph F. Vojta and Alexis Ostapenko. Aug. 1967, v, 19p., illus., design curves.

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CORRECTION SHEET

for

FRITZ LABORATORY REPORT 248.19

"ULTIMATE STRENGTH DESIGN CURVES FOR LONGITUDINALLY
STIFFENED PLATE PANELS WITH LARGE \( e/b \)

by
Joseph F. Vojta
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August, 1967

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| 3    | Step 3 should have \( N = 0.652 \)  
|      | Step 5 should have \( R = 33.85 \)  
|      | Step 7 should have 0.627 instead of 0.653 and 10.6 instead of 39.0  
|      | Step 8 should have: \( A_p = \frac{R^2}{(\frac{b}{e})} = \frac{(50.6)^2}{80} = 27.52 \text{ in}^2 \)  
|      | \( A_p = (\frac{A_2}{A}) A_p = 0.48(27.52 \text{ in}^2) = 13.21 \text{ in}^2 \)  
|      | \( A_f = (\frac{A_2}{A}) A_f = 0.35(18.27 \text{ in}^2) = 6.63 \text{ in}^2 \)  
|      | \( A_w = A_3 - A_f = 13.27 \text{ in}^2 - 6.63 \text{ in}^2 = 6.64 \text{ in}^2 \)  
|      | Total corr = \( A_3 + A_p = 13.21 \text{ in}^2 + 27.21 \text{ in}^2 = 40.73 \text{ in}^2 \) |
\[ \frac{b}{4} = \frac{50}{10.52} = 12.3 \times 13. \]

\[ b = 40.6 \text{ in. as computed in step 7.} \]

\[ x = \frac{b}{(4a)} = \frac{40.6}{60} = 0.68 \text{ in.} \]

\[ d = x(4a) = 0.68 \times 16 = 10.8 \text{ in.} \]

\[ \omega = \frac{a + b (1 - 0.35)}{16} = \frac{40.6}{16} = 2.53 \text{ in.} \]

\[ \beta_1 = \frac{463}{0.75} = 618 \text{ in.} \]

Lost line should read: spaced 40.6 in. center to center.

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**Fig. 7**

Changes must be made to correspond to the changes made on p. 61 to 62.

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C. F. Larson

Re: Fritz Laboratory Reports No. 248.18 "Ultimate Strength Design of Longitudinally Stiffened Plate Panels With Large b/t" by J. F. Vojta and A. Ostapenko, and No. 248.19 "Ultimate Strength Design Curves for Longitudinally Stiffened Plate Panels With Large b/t" by J. F. Vojta and A. Ostapenko

Gentlemen:

Copies of the above listed reports have been sent to you under separate cover. These reports present results of an analytical study on the ultimate strength of longitudinally stiffened plate panels subjected to the combined action of lateral (normal) and axial loads and having large b/t ratios.

Design curves are given as a practical means for a rapid design of stiffened panels for a prescribed set of loads and overall dimensions. For the convenience of the user the design curves and pertinent instructions are summarized in a separate report (No. 248.19).

We anticipate submitting report 248.18 for publication and would appreciate your comments on the material as well as suggestions for the publication medium.

Sincerely yours,

Alexis Ostapenko
Professor of Civil Engineering

AO:mlc