Welded Rigid Frames and Their Components

PRELIMINARY DESIGN OF UNBRACED MULTI-STORY FRAMES

by

W. Hansell

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Approved and recommended for acceptance as a dissertation in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

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A B S T R A C T

This study describes the initial stage of a plastic design method for unbraced multi-story frames. The initial design data includes the number of stories, bays, and dimensions of the frame; and the working loads which the frame must carry together with appropriate load factors. The objective is to find a suitable bending moment, shear force and axial force distribution throughout the frame for three load combinations, without prior knowledge of the frame member sizes. By "suitable" is meant that members selected to resist the preliminary design forces and moments will be reasonably close to those found adequate after necessary design checks. References are cited to suggest appropriate design checks.

The basic method of structural analysis in this study is termed plastic moment balancing. This method is based on a modified version of the lower bound theorem of simple plastic theory. The modifications concern in-plane frame instability. Relative horizontal deflection or sway between the floors of an unbraced frame is considered by formulating equilibrium in an initially assumed deflected state. An approximate account of frame instability effects is thus included in the preliminary design. Subsequent inelastic sway deflection checks are needed to verify the initial sway assumptions. Methods for establishing the initial sway assumptions remain for future investigation.
The three basic equilibrium requirements in plastic moment balancing include story, girder, and joint equilibrium. These equilibrium requirements are considered separately in a definite sequence which eliminates any moment carry-over between joints but which permits considerable versatility in seeking desirable distributions of moment. In fact the relative distribution of moments is assigned in a meaningful parametric form at the beginning of the preliminary design.

The moment balancing parameters must be statically admissible but may be assigned to accomplish certain preliminary design objectives. The design objectives considered in this study include restricted hinge patterns in girders, double curvature bending in columns, and girders of constant depth on one level. The moment balancing parameters used in the preliminary design may also be assigned on the basis of inelastic sway investigations of similar frames. If this is done, the results of the plastic moment balance provide a reasonable and rapid prediction of inelastic frame behavior. From this viewpoint, plastic moment balancing is at least as rational a preliminary design approach as most available allowable stress or plastic design methods.

The term restricted hinge pattern means that a girder does not develop the two hinges required for a girder sway mechanism at ultimate load. A simple method for specifying restricted girder hinge patterns in the plastic moment balance is included for the purpose of story sway stiffness control under factored combined load or gravity load.

When a restricted hinge pattern is specified in the girder plastic moment balance, an increased portion of the sway response of
the girder is in the elastic range. A method is proposed for estimating
the relative amounts of elastic and elastic-plastic sway response of
girders at the beginning of the preliminary design. This method uses the
portal distribution of sway moments and the girder sway mechanism as
limiting conditions.

The main result of this study is to provide a "bridge" between
structural research and design practice, by restating a design philosop­
hy with origins more than half a century old. A design example for
a 24 story 3 bay frame is included in the Appendix.
1. INTRODUCTION

A desirable method of analysis has been described as one which: (1) is easy to learn and remember, (2) introduces few new technical terms or radically new concepts, (3) enables us to see the approximate answer from the beginning, and (4) is flexible.¹ There is no claim to having accomplished these objectives in this preliminary design study, but they indicate the philosophy which has guided this work. Items (3) and (4) are especially germane to the design of highly indeterminate structures like an unbraced multi-story frame.

The characteristics of a desirable design method have been influenced by the digital computer. The age of electronic computation has extended the scope but not the basis for design or the need for adaptable design methods.

1.1 PROBLEM STATEMENT

The problem considered in this study is the following: given the number of stories, bays, and dimensions of an unbraced planar multi-story frame, and the working loads which the frame must carry together with appropriate load factors; find suitable distributions of bending moment, shear force and axial force throughout the frame for appropriate

¹References are indicated thus (1)
load combinations, without prior knowledge of the frame member sizes. By "suitable" we mean that members selected to carry the preliminary design forces and moments will be reasonably close to those found acceptable after necessary design checks.

The term preliminary analysis might be used to describe this study but analysis usually implies predicting the response of a frame with known members, or relative member properties. The term preliminary design is preferred because this study intends to estimate the moments and forces in the frame as the first step in the design process.

We may choose between two apparently different design methods for indeterminate frames: the allowable stress method, and the plastic design method. The differences are more apparent than real in many respects because both methods find their justification in the behavior of rigid frames and their components in the inelastic range (Chapter 1, Ref. 2). This study uses a modified plastic design approach with appropriate checks on elastic behavior under working loads. Two load factors are applied to the working load to obtain required ultimate load capacities. These are the load factors $F_{1R}$ for gravity loading and $F_{2R}$ for combined (gravity plus wind) loading.

The statical method is the fundamental approach to the plastic analysis of structures used in this investigation. The statical method satisfies both the equilibrium and plastic-moment conditions, but not necessarily the mechanism condition, of the simple plastic theory and is based on the lower-bound theorem (Art. 2.2, Ref. 3). Although this theorem does not intend to consider the significant changes in
geometry which may occur in an unbraced frame, these geometry effects are fully accounted for by modifying the equilibrium condition (Art. 1.3). A rigorous theoretical investigation of this equilibrium modification is a fundamental and unsolved problem in the field of inelastic instability. Nevertheless, the design checks which follow the preliminary design phase serve to bridge this gap in theory and allow us to proceed with confidence.

It will be assumed in this study that all horizontal forces in the plane of a frame are resisted by the rigidly connected girders and columns without assistance from diagonal bracing, walls, cladding, or shear transfer between adjacent frames through the floor system. These qualifications serve to define the term unbraced. They also suggest that a completely unbraced frame is rare in practice. However the trend in multi-story building architecture is away from the granite exterior of the Empire State Building with its substantial interaction between cladding and bare frame in resisting sway due to wind. Modern architectural components and fireproofing methods place a gradually increasing reliance on the steel skeleton to resist wind, with the bare unbraced frame as a limiting and conservative condition.

A prime concern in multi-story frame design is the sway (or the physiological response to sway acceleration and vibration) induced by wind at the working load level. The influence of sway on the strength and stability of the frame in the inelastic range of (overload) response is an equally vital concern in providing a margin of safety. The following articles describe the behavior of unbraced multi-story frames and their components with particular reference to
sway effects in the inelastic range. The purpose of the discussion is to suggest what factors should be considered in preliminary design.

1.2 THE \( \Delta P \) EFFECT IN COLUMNS

The total wind shear in a story is the sum of the wind forces acting on the frame above that story. The wind shear causes shear forces, bending moments, and axial loads in the columns. The column moments are balanced by moments in the girders immediately above and below the story. In addition to these internal forces and moments, wind also causes relative horizontal deflection or sway of an unbraced frame between the levels above and below the story. The product of total gravity load above the story and relative sway deflection gives an additional moment, termed the \( \Delta P \) moment. The \( \Delta P \) moment adds to the wind moments in the columns and girders but does not change the total wind shear in the story.

The \( \Delta P \) effect is one factor influencing the equilibrium of an unbraced multi-story frame which is frequently insignificant for single story or braced multi-story frames. It is useful to investigate the relative reduction in shear capacity of a column due to the \( \Delta P \) effect. The columns in Fig. 1.1 will assist in this investigation. The column in Fig. 1.1(a) carries an axial load \( P \) and end-moments \( M_{pc} \) and \( qM_{pc} \) at its upper and lower ends. The plastic moment capacity of the column is reduced from \( M_p \) to \( M_{pc} \) by the axial load. The reduced plastic moment is conveniently approximated by (3)

\[
M_{pc} = 1.18 \left(1 - \frac{P}{P_y}\right) M_p \quad \frac{P}{P_y} \geq 0.15
\]
for a \( W \) shape bent about its strong axis, where \( P_y \) is the plastic load (area of column times yield stress). The end-moment ratio \( q \) determines the moment at one end of the column as a multiple of the column moment at the opposite end. A positive end-moment ratio causes double curvature bending.

The column in Fig. 1.1(a) is in equilibrium in the unswayed position and resists a horizontal shear

\[
Q_0 = \frac{(1 + q) M_{pc}}{h}
\]  

(b)

where the subscript \( o \) identifies the unswayed position and \( h \) is the column height. The column in Fig. 1.1(b) carries the same axial load and end-moments, but it is in equilibrium in the sway position. The sway deflection index, or chord rotation, \( \Delta/h \), is a convenient non-dimensional measure of the sway, \( \Delta \). The sway reduces the shear capacity of the column from \( Q_0 \) to

\[
Q_\Delta = Q_0 - P \frac{\Delta}{h}
\]  

(c)

where the subscript \( \Delta \) is used to identify the sway position. The relative reduction in shear capacity

\[
\frac{Q_o - Q_\Delta}{Q_o} = \frac{P h}{(1 + q) M_{pc}} \left( \frac{\Delta}{h} \right)
\]  

(d)

is a linear function of the sway deflection index. To indicate the influence of other structural parameters on the relative \( P \Delta \) reduction in shear capacity, Eq. (d) may be transformed to
where

\[ \frac{Q_o - Q_\Delta}{Q_o} = \left( \frac{P/P_y}{M_{pc}/M_p} \right) \left( \frac{d_c/r_c}{2f} \right) \left( \frac{1}{1 + q} \right) \left( \frac{h}{r_c} \right) \left( \frac{\Delta}{h} \right) \]  

and \((d_c/2fr_c)\) has a value close to unity (approximately 1.06) for most W or H column sections in Ref. 5.

Equations (a and e) indicate that the relative \(P\Delta\) reduction in shear capacity depends on the \(P/P_y\) ratio, the end-moment ratio \(q\), and the slenderness ratio \(h/r_c\) in the plane of bending, in addition to the sway deflection index \(\Delta/h\). Figure 1.2 graphically indicates the predictions of Eq. (e) for two \(P/P_y\) and two \(q\) values. The vertical axis in this graph is the relative reduction in shear capacity and the horizontal axis is the product \((h/r)(\Delta/h)\). Notice that values of \((Q_o - Q_\Delta)/Q_o\) greater than unity correspond to negative values of \(Q_\Delta\). In terms of structural behavior, this means that the swayed column does not help to resist wind shear but instead, the column applies an additional horizontal shear to the remaining elements of the frame.

If \(\Delta/h = 0.002\) is accepted as a nominal working load sway deflection index, we may conservatively estimate the sway deflection index at ultimate load to be on the order of 10 times this value, although considerably smaller values of \(\Delta/h\) at ultimate load are more likely (see Art. 1.6). For a column with a slenderness ratio of 25, this gives the product \((h/r)(\Delta/h) = 0.5\). Then Fig. 1.2 indicates
PA shear reductions of from 10 to 20 percent for P/P_y = 0.3. With P/P_y = 0.6 and the same value of (h/r)(Δ/h) the shear reductions increase to the range from 35 to 70 percent.

Several conclusions may be drawn from this brief study:

(1) The PA shear reduction is significant only when both the axial load and the sway deflection are appreciable. This explains why PA effects are usually ignored for braced frames where the sway is limited and for single-story unbraced frames where P/P_y is frequently less than 0.15. Except in the top several stories of an unbraced multi-story frame, both of the elements which promote the PA effect are present and cannot be neglected in design.

(2) Engineering judgment suggests that some tentative limitation be placed on the sway deflection index at ultimate load, primarily to avoid the uneconomical reduction in shear capacity which results when the PA effect is not controlled. This limitation is a useful parameter in preliminary design calculations but it is not a parameter which cannot be exceeded in final design checks. That is, the computed ultimate load deflection may have any reasonable value, consistent with the requirement that the frame be able to carry the ultimate load in the deflected state.

(3) Of the four parameters which determine the PA effect, the P/P_y ratio is the most important. If P/P_y exceeds 0.3 and the product (h/r)(Δ/h) approaches 0.5, the PA shear reduction is significant (in excess of 10 to 20 percent). This is regarded as a tentative and possibly unconservative rule, but it gives an approximate estimate.
One tacit assumption in this study of relative $PA$ effects is that one of the column end-moments is equal to the reduced plastic moment $M_{pc}$. This condition may not occur for several reasons:

(1) The same column size is used in two or three stories of a frame to simplify fabrication and erection. The axial load in the upper story of a two tier column is less than that in the lower story. This increases the value of $M_{pc}$ in the upper story which also must resist a smaller wind shear, and therefore smaller wind moments in the column. In addition, column splices are frequently located from 1 to 3 feet above the floors. The moment capacity at the splice may influence the column end-moments.

(2) Lateral-torsional buckling of a laterally unsupported column may limit major axis end-moments to a value less than $M_{pc}$. This limitation is most likely to occur for columns bent in single curvature ($q = -1$). However, single curvature column bending occurs infrequently in multi-story frames except occasionally in bottom story windward columns or when extreme checkerboard live load patterns are considered.

(3) A column with a relatively large slenderness ratio (say $h/r_c > 40$) in the plane of bending and high axial load (say $P/P_y > 0.8$) may be unable to resist $M_{pc}$, even in double curvature bending, ($q = +1$) because of excessive deflection between the ends of the column in the plane of bending (Fig. 11.10, Ref. 6). (It is important to distinguish between the $PA$ effect and the amplification of moment due to deflection between the ends of a column. Moment amplification is usually of secondary importance in comparison with the $PA$ effect for
unbraced frame columns bent in double curvature.) Columns with a slenderness ratio of 40 or more in the plane of bending occur infrequently in multi-story frames unless they are bent about their weak axis or they extend through two or more stories without intermediate support in the plane of the frame. Columns fabricated from ASTM A512 steel (yield strength on the order of 100 ksi) will also tend to have relatively larger slenderness ratios than columns using lower strength steels.

(4) The column end-moments must be balanced by, and frequently are limited by, the girder end-moments. (The example discussed in Art. 1.6 and Fig. 1.6 involves a $P/A$ shear reduction of about 23 percent at ultimate load with $P/P_y$ values for the four columns ranging from 0.43 to 0.87 and $(h/r)(\Delta/h) \approx 0.1$ at ultimate load. The column moments are limited by the girder plastic moment capacities.)

Thus the moment assumptions in Fig. 1.1(b) may not be realized in practice. Nevertheless, our study of relative $P/A$ effects serves to identify the contributing parameters, and to give a numerical evaluation (independent of refined inelastic column theory) of their potential contribution.

1.3 PRELIMINARY DESIGN FOR $P/A$ EFFECTS

A basic assumption in the simple plastic theory of structures is that equilibrium can be formulated in the undeflected state of the structure. Our study of $P/A$ effects suggests that this assumption must be modified in an attempt to apply plastic concepts to the design of a tall unbraced multi-story frame particularly in the middle and lower
stories. The preliminary design method in this study deals with the sway problem in several steps:

**Step (1)** The ultimate load sway deflection in each story is assumed at the beginning of the preliminary design. The ideas involved in making this assumption remain for future study. Equilibrium is then formulated in this initially assumed deflected state.

**Step (2)** After tentative member sizes have been selected to carry the moments determined in Step (1), the ultimate load sway deflections corresponding to these members are estimated using the moment diagrams from Step (1).

If the sway deflection check in Step (2) indicates sways which are less, in every story, than the initial sway assumptions, we may conclude that the frame design is conservative with regard to PA effects. If the sway deflection check indicates sways larger than the initial sway assumptions we may follow one of two alternatives.

**Step (3) - Alternative (1)** Attempt to show that the tentative member sizes are adequate to carry the increased PA moments. At this stage the deflection check is more accurately described as an iterative stability check. The basic ideas described by Vianello and expanded by Newmark may be applied with modifications to this frame stability check. The calculations can be protracted and may result in no convergence, particularly if the initial sway assumptions
are inappropriate. Then an alternative course is attractive, although not necessarily as economical.

**Step (3) - Alternative (2)** Use the sways obtained from the sway deflection check in Step (2) in place of the initial sway assumptions, and repeat the preliminary design, starting with Step (1). This may also be described as an iterative process but each iteration gives new member sizes in addition to deflections.

**Step (4) - A sway deflection check at working load** completes the consideration of sway effects. If working load sway limitations are quite conservative, the working load sway stiffness requirement may control the design and should be checked following Step (1). This will insure serviceability under working load conditions. The sway deflection check in Step (2) should also be used to insure an adequate margin of safety against inelastic frame instability under factored loading.

The inelastic sway analysis method in Ref. 10 may be used in Step (2) to obtain a complete story shear versus sway curve. This curve provides the information needed to determine the adequacy of the preliminary design for sway effects at ultimate load and at working load.

This outline of the steps involved in dealing with the sway problem is purposely brief and incomplete in essential details. Its
purpose here is to indicate the general nature of the method used to circumvent the deflection limitation of simple plastic theory.

The efficiency of this approach stems from the direct consideration of sway effects right from the beginning of the preliminary design and depends, in part, on the ability to make reliable initial ultimate load sway estimates. Step (3) of the approach introduces an element of iteration which is not especially attractive from the standpoint of efficient design practice but this step is required only to correct an inappropriate initial sway estimate. Since the second cycle of preliminary design will tend to result in a frame with increased sway stiffness, Step (3) could use sway deflections somewhat smaller than those found in the preceding Step (2) with some increase in economy.

The design example in Appendix 1 tentatively suggests that the member sizes obtained in the preliminary design may not be sensitive to substantial changes in the initial sway assumption.

1.4 OTHER INELASTIC INSTABILITY EFFECTS

A second basic assumption in the simple plastic theory of structures is that instability of the frame or any part of the frame does not occur prior to ultimate load. Concern for stability problems in tall inelastic frames has prompted one British investigator to write in 1961:(11)

"It must be stated quite clearly that there is no theoretical justification for applying plastic theory to the design of tall buildings, and by the nature of the problem, there can be no theoretical justification. There can only be a
practical justification; that the calculations lead to a reasonable design, and that with that reasonable design, when erected, the structure does not fall down."

This is a refreshingly frank and realistic assessment. However, reservations concerning the theoretical justification for design methods certainly are not confined to the plastic theory. In fact, a second British investigator has observed: (12)

"The plastic method of design owes its conception in England to the fundamental shortcomings of elastic design. It is only when these shortcomings are understood that the logic and charm of the plastic method can be appreciated to the full."

The investigation of stability problems in multi-story frames has recently (since 1961) lead to a better understanding of the inelastic interaction between rigidly connected columns and girders. Both analytical and experimental studies (10, 13, 14, 15, 90) of the inelastic response of beam-and-column subassemblages have helped to clarify many of the stability problems which have impeded the application of plastic concepts to the design of multi-story frames. Many of these developments are summarized and extended in Ref. 6. The methods for dealing with inelastic restrained columns free to sway in Chapters 14 to 19 of Ref. 6 and the more refined analysis in Ref. 10 are of particular interest in suggesting design checks for unbraced frames, once tentative member sizes have been selected. It is the purpose of this preliminary design study to suggest the basis for this initial selection of members.
1.5 THE ROLE OF GIRDLERS IN RESISTING SWAY

One of the important conclusions to stem from inelastic sway subassemblage studies is the dominant role which the girders play in resisting sway deflection in a story. This role is illustrated in an approximate manner in Fig. 1.3. Figure 1.3(a) shows the deflected shape of one story of an unbraced frame caused by wind shear. The total sway $\Delta$ may be estimated$^{(16,19)}$ as the sum of the sway $\Delta_c$ due to column bending with no joint rotation (Fig. 1.3b) plus the sway $\Delta_g$ due to bending of the girders with rigid body rotation of the columns (Fig. 1.3c). The girders thus help to prevent sway by restraining end-rotation of the columns.

The working load sway deflection estimates in Examples 21.16 of Ref. 2 and D11 of Ref. 17 substantiate the idea that girder bending frequently is the major contribution (say 80 percent) to the total sway deflection at working load in unbraced frames of practical proportions. This explains why sway deflection control is more effectively achieved by increasing girder sizes than by increasing the stiffness of columns. The result is a frame with relatively "strong" girders and "weak" columns (with emphasis on "relative" - the columns are not weak in an absolute sense.)

Since girder flexure dominates the sway in a story, it follows that any factor which changes the stiffness of the girders will have a significant influence on the sway in adjacent stories. For the purpose of this discussion, girder stiffness may be defined as the ratio $\delta M/\delta \theta$ where $\delta M$ is a change in girder end-moment and $\delta \theta$ is the
corresponding change in end rotation. The formation of a plastic hinge in a girder will reduce the girder stiffness by a factor of about 2. A second plastic hinge (which does not unload) will reduce the girder stiffness to zero, if strain-hardening effects are neglected.

The successive stages in the formation of plastic hinges due to increasing wind moments in a girder with constant plastic moment capacity, \( M_p \), are illustrated in Fig. 1.4. If \( F_{2R} \) is the load factor for combined (gravity plus wind) loading, the total gravity load on the girder is \( F_{2R} w L_g \) where \( L_g \) is the clear girder span and \( w \) is the uniformly distributed (dead plus live) working load. Figures 1.4(a) and (b) show the gravity loads and the moment diagram for the girder before any wind moments are applied. The girder end-moments are \( M_A \) and \( M_B \) where subscripts A and B identify the windward and leeward ends of the girder and wind is acting from the left. The end-moments are considered positive when clockwise as shown in Fig. 1.4(a).

In this study, all girder moment diagrams are drawn in two steps. First, the statically determinate moment diagram for the gravity load is drawn assuming the girder rests on simple supports. The maximum ordinate of this diagram is

\[
M_S = \frac{1}{8} F_{2R} w L_g^2
\]

in Fig. 1.4(b). Second, the linear moment diagram for the end-moments \( M_A \) and \( M_B \) is superimposed. This linear diagram will be termed the fixing line for convenient reference, because it serves to "fix" the moment at any girder section. Moments which cause tension on the bottom
of the girder between its ends are considered positive and are measured downward from the fixing line to the statically determinate moment diagram.

The girder end-moments due to gravity load are assumed to be the fixed-end moments

\[ M_B = -M_A = \frac{2}{3} M_S \]  

in Fig. 1.4(b). This is a reasonable assumption if the girder span is one of several nearly equal interior spans. It is further assumed that the girder plastic moment capacity, \( M_p \), exceeds the fixed-end moments.

When wind moments caused by wind from the left are applied to the girder, the fixing line rotates clockwise as indicated in Fig. 1.4(c). In the portal and cantilever methods of wind analysis, a point of inflection is assumed at the center of each girder. This convenient assumption will be followed in this study to approximate the elastic range of response to wind. The change in end moments, \( \delta M \), due to wind in the elastic range is then as indicated in Fig. 1.4(d) where

\[ \delta M_A = \delta M_B \]

On the upper levels of a multi-story frame, the difference between the plastic moment and the fixed-end moment is frequently small and, in fact, may vanish, because the girder sizes in this part of the frame are governed by the factored gravity load with load factor \( F_{1R} > F_{2R} \). The result is that the moment

\[ M_B = \frac{2}{3} M_S + \delta M_B \]
at the leeward end of the girder soon reaches $M_p$ under increasing wind moments. This marks the end of the elastic range of response to wind. The first plastic hinge in a girder may be expected to form at the leeward end as indicated in Fig. 1.4(e), (unless the end-moments due to gravity load are considerably smaller than the fixed-end moments). The girder wind moment diagram when this leeward hinge forms is shown in Fig. 1.4(f) with a point of inflection assumed at midspan. The wind moment at the leeward end is limited to

$$\delta M_B = M_p - \frac{2}{3} M_S$$

in this figure.

A further increase in girder wind moments can be absorbed by increasing the windward wind moment $\delta M_A$ with $\delta M_B$ held constant at the value from Eq. (d). The fixing line rotates about the leeward plastic hinge as in Fig. 1.4(g). The girder wind moment diagram in Fig. 1.4(h) indicates that the point of inflection shifts toward the leeward end. The result of the leeward plastic hinge is to redistribute the wind moments. The redistributed wind moments permit the girder to carry larger total wind moments than the assumptions of the cantilever or portal methods would indicate. This is one example of the "logic and charm" and of the economy inherent in the plastic method.

The moment diagram in Fig. 1.4(g) may be expected to occur for the girders on the upper levels, at and near the roof, of an unbraced multi-story frame. As we proceed down from the roof, the girder wind moments increase in a manner analogous to a cantilever. The ability of
the leeward plastic hinge to redistribute the increasing wind moments
is limited by the formation of a second plastic hinge in the girder as in
Fig. 1.4(j).

The second plastic hinge is aptly described as a sagging hinge when
it occurs between midspan and the windward end of the girder. The com­
bination of leeward and sagging hinges transforms the girder into a sway
mechanism (Fig. 1.5(a)) with vertical deflection determined by the
rotation of the joint at the windward end of the girder. The girder can
continue to "ride" with the swaying frame with no contribution to sway
stiffness of the adjacent stories while increasing wind moments are redis­
tributed to other members.

The girder wind moment diagram in Fig. 1.4(k) indicated the
substantial leeward shift of the point of inflection when the girder sway
mechanism forms. It is obvious from a comparison of Figs. 1.4 (f) and
(k) that the portal and cantilever wind moment distributions are not
optimum.

The moment diagrams in Fig. 1.4 (g) and (j) may occur in the
middle levels of an unbraced multi-story frame, several stories below
the roof. In this region, at least some of the girder sizes will be
controlled by combined load rather than by gravity load. As we proceed
downward in this region, the girder plastic moment capacities must
increase, and the sagging hinge in Fig. 1.4(j) shifts toward the wind­
ward end of the girder.

In the bottom levels of a tall unbraced frame, the sagging
hinge in some girders may reach the windward end of the girder as
indicated in Fig. 1.4(m). This produces a girder panel mechanism illustrated in Fig. 1.5(b). Again the girder can "ride" with the swaying frame with large but not excessive vertical deflection, but with no sway stiffness contribution.

In summary, the girder moment diagrams involve a gradual transition from Fig. 1.4(e) at the roof to Fig. 1.4(j) at the bottom level of a tall unbraced frame. The behavior illustrated in this figure forms the basis for the plastic moment balancing theory\(^{(6,20)}\) for girders presented in Chapter 5 of this study.

Although the girder wind moments change gradually with increasing distance from the roof, this does not suggest that the moment diagram for every girder on a given level will have the same form, at ultimate combined load. In fact, frame stability and associated sway stiffness considerations may result in different girder moment diagrams and hinge patterns on the same level in the middle and lower story zones of a tall unbraced frame, even when all girder spans and loads are identical. This point appears to have been absent in some previous plastic design studies.

1.6 AN EXAMPLE OF INELASTIC SWAY BEHAVIOR

The incremental sway stiffness of a story, \(K_s = \frac{\delta H_T}{\delta \Delta}\) (where \(\delta H_T\) is a change in wind shear and \(\delta \Delta\) is the corresponding increment of sway deflection), which results from the successive formation of plastic hinges in girders is aptly illustrated in the design example in Ref. 10. This example investigates the inelastic sway behavior at the 20th level below the roof of a 24 story, 3 bay, unbraced frame designed by the
plastic method. The data in Fig. 1.6 and Table 1.1 was abstracted from this design example. The frame elevation, the wind shear versus sway deflection behavior, and the sequence of hinge formation are indicated in Fig. 1.6. Incremental sway stiffness calculations for the story are given in Table 1.1. Row 6 in this table gives the story sway stiffness, $K_s$, after each hinge forms and row 7 indicates the ratio of $K_s$ to the elastic sway stiffness at working load.

The main points of interest in Fig. 1.6 are the reductions in sway stiffness following the formation of plastic hinges, and the hinge patterns at ultimate load and at the mechanism load. According to simple plastic theory, a frame sway mechanism could occur with 8 column hinges in the story below level 20 (a strong-beam, weak-column design), or with 6 girder hinges on level 20 (a strong-column, weak-beam design), or in a more likely mixed mode involving at least 6 column and girder hinges at level 20.

The first 3 hinges form at the leeward end of the 3 girders on level 20, as expected. The sway stiffness of the story decreases rapidly to 68, 34, and 15 percent of the elastic sway stiffness after these hinges form. The story shear when the third girder hinge forms is 98 percent of its ultimate value. At this point the story is in stable equilibrium (positive sway stiffness) at a sway deflection index, $\Delta/h = 0.0045$, which is only 1.67 times the working load value.

The fourth hinge occurs at the windward end of the center (most stiff) girder. This girder is then reduced to a panel mechanism and its contribution to the story sway stiffness is cancelled. The ultimate wind
Table 1.1

SWAY STIFFNESS CALCULATIONS AT LEVEL 20 BELOW ROOF
(Fig. 1.6) from Ref. 10

<table>
<thead>
<tr>
<th>Row</th>
<th>Condition</th>
<th>Working Load</th>
<th>Hinge 1</th>
<th>Hinge 2</th>
<th>Hinge 3</th>
<th>Hinge 4</th>
<th>Hinge 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Story shear $H_T$</td>
<td>114</td>
<td>150</td>
<td>159</td>
<td>162</td>
<td>146</td>
<td>136</td>
</tr>
<tr>
<td>2</td>
<td>(kips)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Deflection Index</td>
<td>0.0027</td>
<td>0.0039</td>
<td>0.0045</td>
<td>0.0050</td>
<td>0.0090</td>
<td>0.01</td>
</tr>
<tr>
<td>3</td>
<td>$\delta h$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Sway $\Delta$ (in.)</td>
<td>0.39</td>
<td>0.46</td>
<td>0.56</td>
<td>0.65</td>
<td>0.72</td>
<td>1.30</td>
</tr>
<tr>
<td>4</td>
<td>$\delta Q$ after hinge (kips)</td>
<td>20</td>
<td>9</td>
<td>3</td>
<td>-16</td>
<td>-10</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>$\delta \Delta$ after hinge (in.)</td>
<td>0.10</td>
<td>0.09</td>
<td>0.07</td>
<td>0.58</td>
<td>0.14</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Incremental Sway Stiffness $K_s = \delta Q/\delta \Delta$ (kips/in.)</td>
<td>293</td>
<td>200</td>
<td>100</td>
<td>43</td>
<td>-29</td>
<td>-71</td>
</tr>
<tr>
<td>7</td>
<td>$K_s/K_{s, WL}$</td>
<td>1.00</td>
<td>0.68</td>
<td>0.34</td>
<td>0.15</td>
<td>-0.10</td>
<td>-0.24</td>
</tr>
</tbody>
</table>
shear is reached when this fourth hinge forms at $\Delta/h = 0.005$, or 1.85 times the working load sway.

Fig. 1.6 indicates that only one (leeward end) hinge develops in the exterior girders at ultimate load and that no column hinges are formed at this load. The ultimate load hinge pattern at level 20 is restricted relative to any possible hinge pattern for a frame sway mechanism.

Beyond $\Delta/h = 0.005$ the sway stiffness is negative and the story is in a state of unstable equilibrium. However, the story shear decreases slowly with increasing sway and the fifth girder hinge does not form until $\Delta/h$ reaches nearly 0.009. At this point the moments in column C below level 20 have nearly reached $M_{pc}$ and the story shear has decreased to less than 90 percent of its ultimate value. Further sway of the frame occurs under rapidly decreasing wind shear and is in the nature of a plastic mechanism.

The combination of the PA effect and the incremental story sway stiffness reduction due to plastic hinges has two significant results in the example in Fig. 1.6.

(1) The ultimate story shear capacity is reach WITHOUT FORMING A FRAME SWAY MECHANISM.

(2) The frame sway mechanism forms at a LOWER STORY SHEAR and a LARGER SWAY DEFLECTION than at ultimate load.

These results reflect the principle difference between the in-plane behavior in the lower stories of an unbraced multi-story frame and the assumptions of the simple plastic theory.
Note that these results do not stem from $P_A$ effects alone or from plastic-hinge–induced sway stiffness reduction alone, but from the combination of these factors. This explains why $P_A$ effects are nil at working load. Note also the rapid change in the incremental story sway stiffness caused by individual girder plastic hinges. A change in the flexural rigidity $(EI)$ of one girder will change the story sway stiffness but the formation of a leeward plastic hinge in the girder appears to have a more pronounced influence on the story sway stiffness. This suggests that if reasonable preliminary design assumptions are made for $P_A$ effects and the ultimate load hinge pattern, the preliminary members will tend to provide adequate sway stiffness and story shear capacity.

Strain hardening effects were neglected in this sway study, in the interest of simplicity, but their probable influence on the story shear versus sway deflection behavior in Fig. 1.6 is qualitatively evident. Once the fourth girder hinge has formed, strain hardening at some of the earlier formed hinges would serve to increase the story sway stiffness and to raise the shear versus sway curve. This would tend to offset the negative values of $K_s$ for $\Delta/h$ considerably larger than 0.005. Even a minimal amount of ductile cladding would have similar results. However, secondary factors such as differential settlement, initial crookedness of columns, shear distortion of joints, local buckling, and column shortening together with axial loads in the girders might also be active. Furthermore, many of these secondary factors would tend to exert a greater influence in the range of larger deformation on the descending branch than on the ascending branch of the shear versus sway curve. The net result is that the shear versus sway behavior beyond the ultimate load can be considered as indefinite and unessential for many practical purposes.
The sway behavior beyond ultimate load does have significance in two respects. It serves to measure the ductility (capacity for deformation) and the reserve energy absorbing capacity of the frame. These factors are related to seismic and blast behavior.

1.7 INELASTIC COLUMNS

The behavior of column C in this example (Fig. 1.6) is especially interesting because this column exhibits substantial inelastic behavior at ultimate load but does not reach a hinge condition. Note that wind causes a considerable increase in the axial load on the leeward columns C and D (35 percent for column C) and a corresponding decrease in axial load in the windward columns A and B. The behavior of the leeward column D would have been similar to column C, except that the plastic moment capacity of girder CD on level 20 limits the moment in column D. Note also that the windward and leeward roles of columns A to D are reversed when wind acts from the right.

Column D illustrates the behavior of leeward columns in a "strong-column, weak-beam" design in which the column moments are limited to much less than $M_{pc}$ by the plastic moment capacity of the adjacent girders. The behavior of column C is closer to that for the leeward columns of a "strong-beam, weak-column" design in which the plastic moment capacity of the girders does not limit the column moments.

The reduction in axial load on the windward columns forces these columns into the strong-column, weak-beam category. It is evident that the strong-column, weak-beam category could be used for both windward
and leeward columns by increasing column sizes. When the column sizes are decreased, it is unlikely that the windward columns could be designed to fall within the weak-column, strong-beam category (unless the wind loads in the columns are small relative to the gravity loads) since these same columns are leeward columns for wind in the opposite direction. The column size is controlled by the leeward condition under combined load, or by gravity load.

Thus the behavior of a frame in which all columns are in the weak-column, strong-beam category is of little practical interest. In addition, there appears to be little reason to demand that all leeward columns should be designed to remain elastic in the strong-column, weak-beam category if rotation capacity and lateral bracing requirements for these columns are not excessive. The recent evidence reported in Ref. 21 and Chapter 7 of Ref. 22 suggests that the rotation capacity provided by inelastic columns with a positive end-moment ratio (bent in double curvature, \( q > 0 \)) may be more than adequate to justify their use in plastic design with reasonable limitations on the strong axis slenderness ratio and required rotation capacity.

An equilibrium calculation in the deflected state shows that column C below level 20 in Fig. 1.6 carries a moment of \( 0.96 \text{ M}_{pc} \) at ultimate load. This forces the ultimate load moment into the knee of the moment versus end-rotation curve for this column and requires very little rotation capacity while permitting substantial inelastic behavior. It is pertinent to comment that column C resists a moment of \( 0.94 \text{ M}_{pc} \) when the first girder hinge forms at 80 percent of the ultimate story shear. This first hinge occurs in the girder on the windward side of
column C. The change in moment in this column due to increasing story shear is therefore limited in part by the adjacent girder hinge. Thus, although the response of column C is definitely inelastic at ultimate combined load, this column is on the fringe of the strong-column, weak-beam category. It is reasonable to expect a larger rotation capacity requirement in column C if girder BC is increased in size.

This study does not intend to investigate the behavior of columns in the weak-column, strong-beam category nor to advocate their extensive use in design practice. However the practice of controlling sway deflection by increasing girder sizes suggests that this category of column and girder designs is of practical interest.

1.8 REMARKS ON PRELIMINARY DESIGN CRITERIA

Several ideas germane to a preliminary design method are evident from the behavior described in Fig. 1.6. We may choose between two basically different combined load criteria in preliminary design calculations:

1) The deflected state at the ultimate combined load, i.e., the peak of the sway versus story shear curve in Fig. 1.6 or,

2) The deflected state when a plastic mechanism forms under combined load.

It is clear from Fig. 1.6 that the second condition should use substantially larger sway deflections than the first but that the story shear when a mechanism forms is definitely less than the ultimate
story shear. However, the story shear reduction between the ultimate
and mechanism loads can be only roughly estimated in the preliminary
design stage.

It has been suggested (Art. 13.2 of Ref. 6) that "a structure
should be analyzed (and designed) for a series of possible loading paths
or for a condition which is independent of the loading paths. Formation
of the mechanism gives such a condition, although it may be rather con-
servative for some structures." The "conservative" nature of the
mechanism condition stems from the shear reduction after ultimate load.

The mechanism condition has been used in many previous frame
design studies (discussed in Art. 2.2), in part because there is an
element of analytical definiteness and simplicity associated with a simple
plastic theory mechanism, and secondly, because simple methods dealing
with the ultimate load condition have not been available. However, the
indefiniteness of the shear versus sway behavior of a real frame beyond
the ultimate load condition does not lend support to the mechanism
condition as a design criterion. The ultimate load condition is sug-
gested as a reasonable alternative preliminary design criterion with the
provision that methods using this criterion should remain nearly as
simple as those which deal with the mechanism condition. The plastic
moment balancing procedure for girders in Chapter 5 of this study may
be applied to both the ultimate load and the mechanism load criteria
with only minor variations in the procedure.
1.9 RESTRICTED HINGE PATTERNS

The design example reviewed in Fig. 1.6 clearly illustrates that sway mechanisms do not occur in every girder on a level at ultimate combined load. Only four of the six or more hinges required for a frame mechanism at level 20 actually form before or at ultimate load. Furthermore, only one of the three girders reaches a sway or panel mechanism in the process. If we select the ultimate load as our preliminary design criterion, it is necessary to consider restricted hinge patterns in the girders. By restricted hinge pattern we mean that one or more girders on a level does not form a sway or panel mechanism at ultimate combined load, although a single hinge may occur at the leeward end of these girders.

A restricted girder hinge pattern requires that the girder have a larger plastic moment capacity than that needed for a girder mechanism. Figure 1.4(g) illustrates the moment diagram for a girder with a restricted hinge pattern. Note that the maximum positive moment $M_C$ is less than the plastic moment $M_p$. The same is true of the maximum positive moments at ultimate load in the exterior girders AB and CD in Fig. 1.6. The ultimate load hinge patterns for these girders are restricted.

The behavior illustrated in Fig. 1.6 suggests that when hinges have formed at the leeward end of each girder on one level, most of the wind shear capacity in the adjacent stories has been exhausted. This assumed restricted hinge pattern (or another appropriate pattern) may be coupled with assumptions concerning the ratio $C = M_C / M_p$ for each girder, where $M_C$ is the maximum positive (sagging) moment (Fig. 1.4(g))
and $M_p$ is the positive plastic moment capacity for a girder. These assumptions together with others concerning the deflected state of the frame and the distribution of wind moments to the girders at ultimate combined load provide the information needed to estimate the required plastic moment capacities for the girders and columns. This is the essence of the plastic moment balancing approach to preliminary design proposed in this study.

At this point some insight from the pen of Hardy Cross is appropriate: (1)

"One of the most powerful methods of analysis is, in a sense, no method at all - namely, to guess at a solution and then see if it satisfies statics and geometry. The facility of guessing at solutions is capable of great development. One of the chief values of formal analyses is to aid in its development."

This quotation is cited in response to reservations concerning the compounding of assumptions. It also points to the need for more "formal analyses" like that in Ref. 10 to aid in formulating complete and substantiated design recommendations for restricted hinge patterns and related assumptions.

Restricted hinge patterns may be considered as an approximate device for sway deflection control at ultimate load. They provide a means for considering the interrelated requirements of strength and stiffness. Such requirements are not limited to the ultimate combined (gravity plus wind) load condition.

Adequate strength and sway stiffness must be provided to carry the factored gravity loads. Girders are frequently proportioned to
form three-hinges plastic mechanisms under the ultimate gravity load. It is evident that the girder hinges serve to reduce the sway stiffness of an unbraced frame. If this reduction in sway stiffness is too severe, an unbraced and unclad frame may collapse in an inelastic sidesway buckling mode.

Sidesway buckling appears to be unlikely in the lower stories of an unbraced, unclad frame because of the more demanding sway stiffness requirements in this zone for combined loading. However, these sway stiffness requirements are present in a much reduced degree in the upper stories where wind load effects are smaller and sidesway frame buckling seems to be more likely in this height zone. When substantial live load reductions, various code specified wind load patterns, higher strength steel columns in lower stories, and the different load factors for gravity and combined loads are considered, the zone where sidesway frame buckling under gravity loading is most likely, becomes less clear cut.

Unbraced multi-story single bay frames are intuitively more prone to sidesway buckling than multi-bay frames under similar gravity loads. It has been suggested in a recent paper that intuition may be misleading on this question if the increments of sway stiffness with additional bays are not proportional to the additional gravity loads carried. Although approximate elastic energy methods were used to estimate critical sidesway buckling loads in this paper the conclusions are at least qualitatively valid for inelastic sidesway frame buckling. This is particularly true if all of the girders in the additional bays form three-hinged mechanisms under ultimate gravity
loads because these girders can contribute little increase to the sway stiffness in adjacent stories.

Regardless of where it may occur, sidesway frame buckling may be controlled by using appropriate restricted hinge patterns in preliminary design for gravity loads. For example, if the maximum positive (sagging) moments in one or more girders on a level, due to gravity loads, are limited to say 0.9 times the positive plastic moment capacity of the girders, most of the sway stiffness contribution of these girders is preserved at ultimate gravity load. It is felt that current inelastic sidesway frame buckling studies will help to establish the frame geometry and load parameters which may combine to make sidesway buckling the critical failure mode under gravity loading. These results should be of value in indicating when restricted hinge patterns should be considered in preliminary design for gravity loads.

1.10 SUMMARY

This chapter describes the results of the $P\Delta$ effect on the ultimate strength under combined load of unbraced multi-story frames and indicates a method for including the $P\Delta$ effect in preliminary design. The most effective device for controlling $P\Delta$ effects is to increase the size of girders because girder flexure is the major factor which governs sway deflection.

The wind moments in girders which form plastic hinges are compared with the girder moments according to the portal method of wind analysis. A leeward girder hinge produces a more favorable
distribution of wind moments than the portal method would indicate but reduces the girder contribution to story sway stiffness. The redistribution of wind moments is limited by the formation of a second plastic hinge between midspan and the windward end of the girder. This produces a girder mechanism and eliminates the girder contribution to incremental story sway stiffness.

An example illustrates the facts that: (1) the ultimate story shear is reached without forming a frame sway mechanism and (2) the frame sway mechanism forms at a lower story shear and a larger sway deflection than are present at ultimate load. The factors which combine to determine the ultimate load are the $P_A$ effect and plastic-hinge-induced sway stiffness reduction. Each girder hinge causes a significant reduction of incremental sway stiffness. The ultimate story shear is reached when the incremental sway stiffness changes from positive to negative.

To consider the interrelated requirements of strength and stiffness at ultimate load it is suggested to use restricted girder hinge patterns in the preliminary design. The maximum positive moment is less than $M_p$ in a girder with a restricted hinge pattern. The combination of restricted girder hinge patterns and estimated $P_A$ effects is proposed as a modification of the simple plastic theory to approximately account for the in-plane frame instability phenomenon at ultimate load.
The essence of this discussion is that we should make reasonable provisions for restricted girder hinge patterns and \( P\Delta \) effects in the preliminary design of the lower stories of an unbraced multi-story frame. Members selected in the preliminary design are then closer to those found adequate in later design checks than would be the case if frame stability effects are neglected.
2. PREVIOUS RESEARCH

2.1 EARLY CONTRIBUTIONS

It is revealing to note that plasticity concepts were tested and applied to the design of beams in several apartment type buildings over 50 years ago, \(^{(24)}\) and that the tangent modulus theory of inelastic column buckling has been available, but not fully accepted or interpreted, for three-quarters of a century. (See page 463 of Ref. 25.) Information on the inelastic flexural stress distribution in a mild steel beam was obtained in 1899 (Art. 3.2, Ref. 26). Recognition of ductility as a vital and simplifying factor in the design of steel structures has been repeatedly advocated under many titles (limit, ultimate strength, plastic design) for more than four decades (Refs. 27 to 36, also see Ref. 3 and Appendix I of Ref. 25). At the beginning of this period Kist stated that

"In the design of a redundant structure, it is not necessary to use the equations of elasticity to determine the redundants; it is only necessary to assume values for them, any assumptions at all but preferably the most advantageous ones, provided such assumptions are compatible with the conditions of equilibrium." \(^{(27)}\)

One might well append - "in a conservatively assumed deflected state" - to this quotation in recognition of inelastic instability effects. Nor did instability inhibit the early application of limit design concepts
to such a slender structure as the transmission tower - tested under a 50 percent overload - in 1910! (Ref. 32 and Ref. 34, p. 63)

Although much of the effort to extend plastic design concepts to multi-story frames has been compressed into the last two decades, progress in this field seems overdue and certainly not hasty or lacking in precedent.

2.2 RECENT STUDIES

The potential advantages and logic of plastic design for tier buildings were compared with conventional practice in a forward looking paper at the beginning of this decade. (37) This paper provides a vital perspective concerning design assumptions in practice which, in some respects, "theoretically seem quite indefensible" but which "have given entirely satisfactory service." The point is that theory may serve to guide and improve design practice but should not, and does not, limit design practice in matters of conflict between theory and experience.

Comprehensive surveys of the experimental basis for plastic design and of frame stability investigations are given in Refs. 38 and 39. A concise review of the numerous assumptions which have been made in the analysis of unbraced multi-story frames is included in Chapter 13 of Ref. 6.

The term plastic moment balancing is used to describe the approach to plastic theory used in this study. In the literature the method is referred to as plastic moment distribution (40) (because it is superficially similar to elastic moment distribution) or relaxation
of yield hinges. \(^{(41)}\) The versatility of the approach, particularly for multi-story frames, is evident from several texts. \(^{(18,26,42,43)}\)

Chapter 9 compares plastic moment balancing and plastic moment distribution. Both of these methods employ equilibrium conditions, but in a different manner.

Interest in a plastic approach to the design of unbraced multi-story frames and in related stability problems is evident from numerous contributions in the literature. Many of these are listed and compared in Table 2.1 which includes contributions from four continents. This table does not intend to be complete for this would require a separate paper. Instead, the table groups similar contributions and indicates how each group deals with selected facets of the unbraced frame design problem. A blank entry in this table indicates a topic not considered in the reference.

There is some risk of incomplete description and interpretation in Table 2.1. For example, Ref. 46 considers plastic design concepts for combined loads on unbraced frames, but is primarily concerned with a subassemblage type design for gravity loads. The original papers should be consulted before drawing conclusions. Nevertheless, the comparisons in Table 2.1 indicate some of the trends apparent in previous studies of plastic design methods for unbraced frames.

Table 2.1 compares various design methods on the basis of:
(1) method of analysis, and consideration given to; (2) PΔ effects, (3) inelastic columns, (4) restricted hinge patterns, and (5) the criterion for combined load capacity (Art. 1.8). A variety of analysis
### TABLE 2.1
COMPARISON OF PLASTIC DESIGN AND ANALYSIS METHODS
FOR UNBRACED MULTI-STORY FRAMES

<table>
<thead>
<tr>
<th>Item No. (year)</th>
<th>Ref. Method of Analysis</th>
<th>PA Effects Considered</th>
<th>Inelastic Hinge Columns Permitted</th>
<th>Restricted Girder Patterns Considered</th>
<th>Criterion for Combined Load Capacity</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 45 (1952) *</td>
<td>Combination of Plastic Mechanisms</td>
<td>Yes Point hinges, constant M_p</td>
<td>Mechanism</td>
<td>Frame stability not within scope of paper</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 40 41 (1954) *</td>
<td>Plastic Moment Distribution</td>
<td>Mentioned</td>
<td>With reservations</td>
<td>Equilibrium and plastic moment conditions</td>
<td>Discussion refers to a variation of this method in France &quot;40 years ago&quot;</td>
<td></td>
</tr>
<tr>
<td>3 46 26 (1956)</td>
<td>Plastic Mechanism and Subassemblage</td>
<td>Mentioned</td>
<td>For weak axis bending only</td>
<td>Elastic girders</td>
<td>Strong-beam, Weak-column</td>
<td></td>
</tr>
<tr>
<td>4 47 (1956)</td>
<td>First order elastic-plastic analysis</td>
<td>Yes Point hinges, constant M_p</td>
<td>Mechanism</td>
<td>Frame Stability not within scope of paper</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 48 (1956)</td>
<td>Incremental second order elastic-plastic analysis</td>
<td>Directly</td>
<td>Yes Point hinges, constant M_p</td>
<td>Ultimate load</td>
<td>Computer analysis. Includes column shortening effects</td>
<td></td>
</tr>
</tbody>
</table>

*Also described in several texts (Refs. 18, 26, 42, 43, 44)*
<table>
<thead>
<tr>
<th>Item (and year)</th>
<th>Ref. No.</th>
<th>Basic Method of Analysis</th>
<th>P/A Effects Considered</th>
<th>Inelastic Columns Permitted</th>
<th>Restricted Girder Hinge Patterns Considered</th>
<th>Criterion for Combined Load Capacity</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>49 (1958)</td>
<td>Incremental Dynamic Elastic- Plastic Analysis</td>
<td>Yes</td>
<td></td>
<td></td>
<td>Dynamic behavior</td>
<td>Matrix analysis of computer program</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>51 (1960)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>52 (1964)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>53 (1958)</td>
<td>Several inelastic analyses</td>
<td>Yes</td>
<td>With strong reservations</td>
<td>Yes</td>
<td>Ultimate load</td>
<td>Inelastic frame study of stability</td>
</tr>
<tr>
<td>8</td>
<td>54 (1959)</td>
<td>Model tests</td>
<td>Yes</td>
<td>Yes</td>
<td>Partial failure observed</td>
<td>Maximum test load</td>
<td>34 models failed in sway mode</td>
</tr>
<tr>
<td>9</td>
<td>11 (1960)</td>
<td>Plastic Mechanism</td>
<td>Mentioned</td>
<td>No, until ultimate load</td>
<td>Mentioned in discussion</td>
<td>Mechanism</td>
<td>Relies on cladding for stability</td>
</tr>
<tr>
<td>10</td>
<td>9 (1961)</td>
<td>Virtual work deflection analysis</td>
<td>Mentioned</td>
<td>Yes, Point hinges, constant $M_{pc}$</td>
<td></td>
<td>Mechanism</td>
<td>Avoids incremental analysis</td>
</tr>
<tr>
<td>11</td>
<td>55 (1961)</td>
<td>Plastic moment distribution</td>
<td>Yes</td>
<td>Yes, Point hinges, constant $M_{pc}$</td>
<td></td>
<td>Mechanism</td>
<td>AISC Spec. Formula (20) controlled columns</td>
</tr>
<tr>
<td>12</td>
<td>20 (1961)</td>
<td>Plastic moment balancing</td>
<td>Directly, using initial sway estimate</td>
<td>Yes</td>
<td></td>
<td>Mechanism</td>
<td>Suitable for manual or computer calculations</td>
</tr>
<tr>
<td>Item</td>
<td>Ref. No. and Year</td>
<td>Basic Method of Analysis</td>
<td>PA Effects Considered</td>
<td>Inelastic Girder Capacity Considered</td>
<td>Restricted Load Capacity Considered</td>
<td>Criterion for Combined Load Capacity</td>
<td>Remarks</td>
</tr>
<tr>
<td>------</td>
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</tr>
<tr>
<td>13</td>
<td>56 (1962)</td>
<td>Several elastic-plastic analyses</td>
<td>Directly</td>
<td>Yes, Point hinges, constant Mpc</td>
<td></td>
<td>Ultimate load</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>57 (1962)</td>
<td>An elastic strength-stiffness approach based on assumed deflection</td>
<td>Directly with initial sway estimate</td>
<td>No</td>
<td>Elastic stiffness and strength</td>
<td>Considers stiffness in plane and out of plane of frame</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>58 (1963)</td>
<td>Incremental first order elastic-plastic analysis</td>
<td>Yes</td>
<td></td>
<td>Mechanism</td>
<td>Matrix analysis, computer program</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>59 (1964)</td>
<td>Plastic mechanism followed by elastic inverse iteration</td>
<td>Using an inverse iterative routine</td>
<td>Yes, Point hinges, constant Mpc</td>
<td>Mechanism</td>
<td>Axial load stiffness reductions nil in example</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>60 (1964)</td>
<td>Plastic mechanism and Plastic moment distribution</td>
<td>Mentioned</td>
<td>Yes, Point hinges, constant Mpc</td>
<td>Minimum Weight Mechanism</td>
<td>Weight insensitive close to minimum weight solution</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>8 (1964)</td>
<td>Plastic virtual work formulated for swayed frame</td>
<td>Directly, with initial sway estimate</td>
<td>Within limits</td>
<td>Mechanism including PA effect</td>
<td>Relates mechanism sway to working load away limit</td>
<td></td>
</tr>
<tr>
<td>Item No. and (year)</td>
<td>Ref.</td>
<td>Basic Method of Analysis</td>
<td>PA Effects Considered</td>
<td>Inelastic Columns Permitted</td>
<td>Restricted Girder Hinge Patterns Considered</td>
<td>Criterion for Combined Load Capacity</td>
<td>Remarks</td>
</tr>
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</tr>
<tr>
<td>19</td>
<td>61 (1965)</td>
<td>Plastic mechanism with elastic amplification factors for sway</td>
<td>Using an iterative routine</td>
<td>No, Until ultimate load</td>
<td></td>
<td>Mechanism</td>
<td>Computer program used</td>
</tr>
<tr>
<td>20</td>
<td>62 (1965)</td>
<td>Iterative instantaneous stiffness incremental analysis</td>
<td>Directly</td>
<td>Yes, With spreading inelastic zones</td>
<td>Nonlinear moment-curvature relation enforced</td>
<td>Ultimate load</td>
<td>Frame test reproduced by computer</td>
</tr>
<tr>
<td>21</td>
<td>63 (1965)</td>
<td>Several elastic-plastic and inelastic analyses</td>
<td>Directly</td>
<td>Yes, Using equivalent stiffness for inelastic range</td>
<td></td>
<td>Ultimate load</td>
<td>Loads applied at joints. Strain-hardening considered</td>
</tr>
<tr>
<td>22</td>
<td>64 (1965)</td>
<td>Incremental first order elastic-plastic analysis</td>
<td></td>
<td>Yes, Point hinges, constant $M_p$</td>
<td></td>
<td>Mechanism</td>
<td>Matrix formulation, computer program</td>
</tr>
<tr>
<td>23</td>
<td>65 (1965)</td>
<td>Rigid plastic</td>
<td></td>
<td>Yes</td>
<td></td>
<td>Mechanism</td>
<td>Matrix formulation</td>
</tr>
<tr>
<td>24</td>
<td>6 (1965)</td>
<td>Inelastic sway subassemblage analysis</td>
<td>Directly</td>
<td>Yes, Spreading plastic zones</td>
<td>Yes</td>
<td>Ultimate load</td>
<td>Semi-graphical method of analysis</td>
</tr>
<tr>
<td>25</td>
<td>66 (1965)</td>
<td>Incremental second order interactive elastic-plastic analysis</td>
<td>Directly</td>
<td>Yes, Point hinges, constant $M_p$</td>
<td></td>
<td>Negative sway stiffness</td>
<td>Matrix formulation, computer program</td>
</tr>
<tr>
<td>Item</td>
<td>Ref. No. and (year)</td>
<td>Method of Analysis</td>
<td>( PA ) Effects Considered</td>
<td>Inelastic Columns Permitted</td>
<td>Restricted Girder Hinge Patterns Considered</td>
<td>Criterion for Combined Load Capacity</td>
<td>Remarks</td>
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</tr>
<tr>
<td>26</td>
<td>67 (1965)</td>
<td>Model tests</td>
<td>Yes, interaction formula proposed</td>
<td>Yes</td>
<td>Partial failure mechanism observed</td>
<td>Maximum test load</td>
<td>31 models failed in sway mode</td>
</tr>
<tr>
<td>27</td>
<td>23 (1965)</td>
<td>Elastic approximations</td>
<td></td>
<td></td>
<td></td>
<td>Frame stability study - elastic energy approximations</td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>68 (1965)</td>
<td>Model tests</td>
<td>Yes</td>
<td>Yes</td>
<td>Maximum test load</td>
<td>Earthquake study. Repeated load tests</td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>69 (1966)</td>
<td>Incremental first order elastic-plastic analysis</td>
<td>Yes, Continuous reduction of ( M_{pc} )</td>
<td></td>
<td>Uncontrolled sway</td>
<td>Column loads decrease frame capacity</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>70 (1966)</td>
<td>Plastic moment distribution</td>
<td>By increasing wind shear</td>
<td>Yes, Point hinges, constant ( M_{pc} )</td>
<td>Minimum weight mechanism</td>
<td>Iterative computer solution for minimum weight</td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>71 (1966)</td>
<td>Iterative second-order elastic-plastic modified slope deflection analysis</td>
<td>Directly</td>
<td>Yes, Slope-deflection equations modified for column hinges, residual stress</td>
<td>Diverging iteration</td>
<td>Iterative computer program, includes column shortening effects</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>72 (1966)</td>
<td>Second order elastic-plastic slope deflection analysis</td>
<td>Directly</td>
<td>Yes, Spreading plastic zones considered approximately</td>
<td>Negative sway stiffness</td>
<td>Experimental verification. Large ( PA ) effects observed in tests</td>
<td></td>
</tr>
</tbody>
</table>
methods are evident with some preference given to the plastic mechanism approach. Several papers combine the plastic mechanism method with an assumption of elastic column behavior up to ultimate load (based in part on concern for inelastic lateral-torsional column buckling). This provides a solution to the design problem, but it is difficult to evaluate the results because little evidence is presented to verify the elastic column assumption.

There is nearly unanimous agreement on the need for considering $\mathcal{E}$ effects indicated in Table 2.1. However, well defined methods for dealing with $\mathcal{E}$ effects in preliminary design are not plentiful.

There seems to be no consensus of opinion on the use of inelastic columns in Table 2.1 although the work on inelastic columns mentioned in Art. 1.4 is of sufficiently recent origin to restrict its fullest evaluation by the structural engineering profession. Concern for lateral-torsional instability and associated reductions in rotation capacity has tended to discourage the use of inelastic unbraced columns in plastically designed multi-story frames (8,11,26) unless the unattractive conditions of elastic girders or weak axis column bending are also retained. Article 1.7 suggests that a somewhat more liberal use of inelastic columns in double curvature may be justified, pending further study.

The value of restricted hinge patterns to control inelastic frame stability has been stressed in several investigations, particularly in discussion, as indicated in Table 2.1. Partial mechanisms have been considered in many analyses, although not for the purpose of controlling frame stability.
Many investigations in Table 2.1 have used a plastic mechanism as the design criterion for combined load. Only the more recent of these studies have attempted to include PΔ effects along with the mechanism criterion; using an iterative technique based on elastic column behavior in several studies (but omitting verification of the elastic column assumption), and a sway deflection analysis to check initial sway estimates in other studies. None of the investigations which use the mechanism criterion for combined load have attempted to consider the shear reduction between ultimate load and the mechanism load (Art. 1.8, Fig. 1.6). Those studies which use the ultimate load as the design criterion for combined loading are analysis methods in which the response of a frame with known member sizes is investigated. No method of considering the ultimate load criterion in the initial selection of members is included in Table 2.1.

2.3 FRAME STABILITY INVESTIGATIONS

A comprehensive review of inelastic frame instability under combined loads is given in Ref. 53, including the discussion of this paper. Three examples are used to illustrate the concept of deteriorated critical loads. Under the heading "Practical considerations and recommendations" it is suggested that a "generalized Rankine" formula may be used to account for the sway stiffness deterioration caused by plastic hinges. This frame stability design equation relates the ultimate load capacity \( W_u \) under gravity or combined loading to two different upper bound estimates of load capacity for an unbraced frame. The upper bound load parameters are the simple plastic theory failure
load \( W_p \) for gravity or combined loading and the theoretical elastic sidesway frame buckling (bifurcation) load \( W_{cr} \). The Rankine-Merchant equation empirically relates these frame load parameters in the form

\[
\frac{1}{W_u} = \frac{1}{W_p} + \frac{1}{W_{cr}}
\]  

The usefulness of this equation in preliminary design depends on the ability to estimate \( W_{cr} \) before selecting members for the frame. Simple approximate methods for estimating \( W_{cr} \) from known frame geometry and load parameters are proposed in Ref. 8 (paragraph 93) and Ref. 67 (Part 2). These estimates of \( W_{cr} \) provide an indication of the possible reduction in ultimate load capacity \( W_u \) due to inelastic frame instability at the beginning of the preliminary design. The simple plastic theory failure load \( W_p \) can then be adjusted accordingly. This recent preliminary design approach to the frame stability design problem deserves further study.

The data in Figs. 31 and 32 in the discussion of Ref. 53 suggests that the Rankine-Merchant formula (as interpreted in this reference) is a conservative lower bound for inelastic frame instability effects but the scatter of data in these figures is revealing. The comments in Ref. 53, including discussion, concerning the contribution of strain hardening and shear resisting walls in the plane of an unbraced frame, to the stability of the frame are worthy of careful study. The paper also calls attention to "the importance of putting limits on the use of comprehensive mechanisms of collapse in tall buildings" (the restricted hinge pattern idea).
A pronounced scatter of data on a Rankine-Merchant frame stability plot is reported in a recent refined inelastic stability investigation (Fig. 8, Ref. 76). However, most of the frames in this investigation use rectangular rather than \( W \) shapes. It is interesting to note that the best agreement between the theoretical ultimate loads in this study and the Rankine-Merchant approximation is obtained for those frames with larger horizontal loads.

An improved design approximation for sidesway buckling of unbraced frames under gravity loading is proposed in Ref. 77. The choice of nondimensional parameters in this reference differs slightly from those of the Rankine-Merchant formula and results in a much reduced scatter of frame buckling data (Fig. 22, Ref. 77). Rectangular portal frames using identical \( W \) shapes for beams and columns are considered in this reference, but the proposed design approximation is considered to have wider application (Chapter 15, Ref. 6). The domain in which frame instability causes no reduction in ultimate gravity load capacity is clearly defined in Ref. 77. This is a particularly valuable feature of this contribution.

2.4 MULTI-STORY FRAME STABILITY MODEL STUDIES

Three testing programs which investigated the multi-story frame stability problem using small steel models are included in Table 2.1. A total of 66 unbraced multi-story model tests involving both gravity and combined loading are reported in Refs. 54 and 67. All but 10 of these models were predicted to fail by forming girder gravity load mechanisms according to simple plastic theory. However 65 models
collapsed under reduced loads with the formation of frame sway mechanisms. Usually the frame instability failure mechanism was isolated in the first one or two stories at the bottom of the models. These model tests demonstrate that inelastic frame instability is a definite possibility for very slender unbraced and unclad multi-story frames. The columns and girders used in the models had length-to-depth ratios on the order of 30 and 60 respectively. These values are about 3 times the length-to-depth ratios in multi-story frames of practical proportions. The models were purposely made slender to promote frame instability failures.

The extrapolation of model stability test results to a prototype frame of practical proportions is not an obvious or trivial matter. A simple prediction formula relating the frame instability failure loads of a model and a prototype frame is proposed in Ref. 67. The prediction relation is based on Eq. 2.1 and the assumption that plastic hinges occur in the same sequence and the same locations in model and prototype. A dimensional analysis of the area, plastic moment, and flexural rigidity (EI) ratios between prototype and model is included in Ref. 67. Several factors such as residual prototype column stresses, out-of-plane deformation, and the stiffening contribution of model joints, need careful consideration in interpreting frame stability model test results. The demonstration that a small metal model can be proportioned and tested to reasonably simulate the observed ultimate load behavior of a full scale prototype frame would be particularly valuable.

Frame stability design recommendations are proposed in Part 2 of Ref. 67. These design recommendations involve redefining $W_{cr}$ in
Eq. (2.1) using the modified Euler formula

\[ W_{cr} = \frac{\pi^2 EI_c}{h_{cr}^2} \]  \hspace{1cm} (2.2)

The proposed empirical expression for \( h_{cr} \) is a function of frame geometry and load parameters and the column base restraint condition. All of the data needed to estimate \( h_{cr} \) is available at the beginning of the preliminary design. This leaves the column rigidity \( EI_c \) in Eq. (2.2) and the simple plastic theory failure load \( W_p \) in Eq. (2.1) as open preliminary design parameters to be assigned.

2.5 DETERIORATED CRITICAL LOAD STUDIES

Many discussions of the deteriorated critical load concept for dealing with elastic-plastic frame stability problems are available in the British literature (Refs. 53, 73-75, 8, 11, 16). This concept can be carried to extremes (for example, Fig. 10 in the discussion of Refs. 8 and 60) in an effort to dramatically demonstrate the results of extensive hinge patterns on frame stability. While such demonstrations serve a useful (but easily misunderstood) purpose, it is vital to keep the inelastic frame stability problem in proper perspective by noting that "any analyses or tests carried out in this subject must relate to representative designs; unless this is observed it is easy to produce evidence for or against catastrophic collapse merely by tinkering about with stiffness ratios" (paragraph 34, Ref. 53).
Description of Deteriorated Critical Load Concept

The deteriorated critical load concept intends to give a quantitative measure to the tendency for frame instability as plastic hinges form under increasing gravity or combined loads. The plastic hinges are replaced by true hinges which, neglecting strain hardening effects, cause a deterioration of sway stiffness. To measure this changing stiffness, the elastic sidesway buckling loads or deteriorated critical loads are evaluated for several similar hypothetical frames with plastic hinges replaced by true hinges (M = 0). Hypothetical proportional vertical loads are applied at the joints to replace the proportional gravity loads on the real, partially plastic frame. Wind loads are of course omitted in the buckling (bifurcation) analyses of the hypothetical frames.

The purpose of the remarks which follow is to describe the second order elastic-plastic combined load versus sway behavior of the real frame and to relate this behavior to the deteriorated critical loads for the hypothetical frames. Based on these remarks, the deteriorated critical load concept is then reviewed from the viewpoint of load versus sway behavior under combined loading and under gravity loading.

Elastic-Plastic Combined Load Versus Sway Behavior

It can be shown that the deteriorated critical loads are horizontal asymptotes for the second order, elastic-plastic, load versus sway curve in the range of small deflections for proportional combined loading, because these critical loads correspond to a singular stiffness matrix for the hypothetical frames (Ref. 78). This is illustrated
in Fig. 2.1 where $W_{cr}(n)$ indicates the deteriorated critical load (or proportional load parameter) for an unbraced frame with n hypothetical hinges. While this figure does not pertain to a "representative design" it does qualitatively represent the behavior of an idealized elastic-plastic unbraced frame under proportional gravity-plus-wind loading, without indulging in an intricate, second order, mathematical analysis. (67) The details of the frame are immaterial in this conceptual discussion. The similarity between the sway behavior in Figs. 1.6 and 2.1 is evident, although different vertical axes are used in these figures. To keep Fig. 2.1 simple, it is assumed that 3 hinges are required to reach the ultimate combined load and that a fourth hinge produces a rigid-plastic sway mechanism at a load which cannot exceed (but which is frequently less than) the ultimate load. This assumed behavior is quite similar to that described in Art. 1.6 for a "representative design."

The load versus sway curve for the real frame (Fig. 1.7) is built up from segments of the second order, elastic, load versus sway curves for the hypothetical frames. (67) Each of the elastic sway curves corresponds to a different hypothetical frame similar to the real frame, but with plastic hinges replaced by true hinges. This reduces the sway stiffness of the hypothetical frames (Art. 1.6). Constant moments are applied at the true hinges to represent the effect of the constant moments in the real frame. The only elastic sway curve which passes through the origin is the one for the real frame with no hinges. The remaining elastic sway curves are shifted because of the sway produced by the constant moments applied at the true hinges.
Each sway curve for one of the hypothetical frames under combined load may be compared with the load-deflection curve for an elastic flagpole with proportional vertical and horizontal loads plus a constant moment applied at the top of the flagpole. The constant moment causes an initial deflection of the flagpole which is similar to the sway of the hypothetical frame due to the constant moments applied at the true hinges. A series of flagpoles with different flexural rigidities are needed to simulate the reduced sway stiffness of the hypothetical frames as hinges form under increasing loads. Each flagpole has a critical vertical elastic buckling load (Euler load). As the proportional vertical load at the top of the flagpole approaches the Euler load, the horizontal deflection at the top increases rapidly. The Euler loads for the flagpoles are analogous to the deteriorated critical loads for the hypothetical frames.

In the elastic range of Fig. 2.1, before any hinges have formed, the critical load asymptote is usually an order of magnitude larger than the load on the real frame. As plastic hinges form under increasing loads, the critical load asymptotes are depressed; each asymptote corresponding to a particular hinge pattern with decreasing sway stiffness.

The critical load asymptotes are approached from below (segments 0-1, 1-2, 2-3 in Fig. 2.1) on the ascending branch of the second order load versus sway curve, and from above (segment 3-J) on the descending branch of this curve. Although the load versus sway curves are described as asymptotic (inferring large sway) the sway deflections of the frame remain finite and within the limitations of the small
deflection theory because new plastic hinges form with increasing sway before the deflections become large. Finally, a sufficient number of hinges form to produce a plastic sway mechanism (point J, Fig. 2.1). The load versus sway curve then follows the second order rigid-plastic mechanism curve which is asymptotic to the zero load axis under proportional combined loads.

The comments in Art. 1.8 concerning the descending branch of the load versus sway curve apply to Fig. 2.1 beyond the ultimate load. The upward concavity of segment 3-J is evident in an analysis based on idealized (point) plastic hinges. If the analysis is refined to consider finite, spreading, inelastic zones this upward concavity may be masked, but then $W_{cr}$ decreases as the inelastic zones spread.

**Evaluation of Deteriorated Critical Load Concept**

Guides for interpreting the results of deteriorated critical load calculations are considered next, first for the combined load case, and second for the symmetrical gravity load case. These cases are then compared to indicate their fundamentally different influence on frame stability.

We may now ask - what information can be inferred from knowledge of the deteriorated critical loads, $W_{cr}(n)$ alone, in isolation from sway data? First, if the hinge pattern used to compute $W_{cr}(n)$ is not the hinge pattern which actually occurs at a given stage of combined loading, or something close to the actual hinge pattern, very little in the way of valid conclusions can be obtained from the value of $W_{cr}(n)$ alone. We can only guess at the possible behavior which might follow if the
assumed hinge pattern could be realized. (This is exactly what is done, for example, in the discussion of Refs. 8 and 60, and in the comments concerning Fig. 19 in Ref. 53). Therefore we will assume that, in addition to values of $W_{cr}(n)$, the correct hinge locations for a stated combined loading history are known or can be validly assumed.

**Evaluation for Combined Loading**

The following conclusions can then be based on $W_{cr}(n)$ alone for the case of combined loading:

1. If $W_{cr}(n)$ exceeds the gravity loads on the frame
   a. the sway stiffness is positive,
   b. the frame can support larger gravity and wind loads,
   c. the gravity loads cannot exceed $W_{cr}(n)$, if no consideration is given to strain-hardening-induced stiffness contributions.

The load-sway coordinates for condition (1) are represented by points on the curve 0-1-2-3 in Fig. 2.1.

2. If $W_{cr}(n)$ is less than the gravity loads on the unbraced and unclad frame:
   a. the sway stiffness is negative.

But here we must stop! Nothing definite can be said about whether the frame can or cannot support the loads. In Fig. 2.1, the load-sway coordinates for condition (2) are represented by points on the elastic sway curve K-3-J. We cannot tell from $W_{cr}(n)$ alone whether the load-sway coordinates are above or below the ultimate load at point 3. If
the coordinates are below point 3 it is obvious that the frame can support the loads, with a different hinge pattern, on the ascending branch of the load versus sway curve.

In other works, very little in the way of useful conclusions can be inferred from knowledge of the deteriorated critical loads alone, unless these loads exceed the required ultimate combined load. If however we supplement the deteriorated critical load data with a step-by-step, second order, elastic-plastic load versus sway investigation, the values of $W_{cr}(n)$ becomes more meaningful as asymptotes. The deteriorated critical loads for the combined loading condition, considered in isolation from sway data, are of limited value.

Note that when a kinematically admissible (complete or partial) plastic sway mechanism forms under combined loading the corresponding deteriorated critical load is always zero and the sway stiffness is always negative. This is also true in the case where the rigid-plastic mechanism curve intersects the rising load versus sway curve (that is, point J coincides with point 3 in Fig. 2.1). The fact that the deteriorated critical load $W_{cr}$ (mechanism) in Fig. 2.1 is zero, most certainly does not mean that the frame cannot carry the combined loads which cause the sway mechanism. There may be other reasons, such as lateral-torsional instability or inadequate rotation capacity, which may limit the frame loads to something less than the mechanism loads. If these localized instability effects are provided for, the frame can carry the combined loads which cause the sway mechanism although it may do so on the ascending (stable) branch of the load versus sway curve, rather than the descending (unstable) branch of this curve.
Thus, design methods which use a mechanism as the failure criterion for combined load need not be rejected for reasons of frame instability if complete recognition of PA effects is included. The literature on deteriorated critical loads would lead one to conclude otherwise. Note carefully that we have not said the mechanism condition will be reached, but rather that the combined loads corresponding to the sway mechanism condition can be supported. It is also pertinent to comment that the mechanism sway deflection (point J in Fig. 2.1) and the shear reduction from the ultimate to the mechanism conditions may be substantial. Of course, the shear reduction is an analytical rather than a real phenomenon under true gravity loading.

Evaluation for Gravity Loading

Consider now the case of gravity loading only. The deteriorated critical loads may be interpreted in a somewhat different manner. Assume, for simplcity, that the frame and gravity loads are symmetrical so that, at least initially, the gravity loads cause no sway. As the gravity loads are increased, and plastic hinges begin to form, the sway stiffness and the deteriorated critical loads decrease. Unloading of the hinge at one end of a girder under incipient sway conditions should be considered in estimating \( W_{cr}(n) \).

If we again assume that the hinge pattern used to compute \( W_{cr}(n) \) is at least close to the pattern which actually occurs at a given stage of gravity loading, the following conclusions may be inferred from \( W_{cr}(n) \) alone:

1. If \( W_{cr}(n) \) exceeds the gravity loads on the frame:
   (a) the sway stiffness is positive,
(b) the frame can support larger gravity loads,
(c) the gravity loads cannot exceed \( W_{cr}(n) \) if strain hardening effects are ignored.

(2) If \( W_{cr}(n) \) is less than the gravity loads:
   (a) sidesway frame buckling may be expected under gravity loads not less than \( W_{cr}(n) \),
   (b) the frame buckling loads may exceed \( W_{cr}(n) \) if more hinges than are necessary for sidesway buckling are used to estimate \( W_{cr}(n) \) or if strain hardening contributes to the sway stiffness of the frame.

Comparison of Gravity and Combined Loading

The deteriorated critical loads for the gravity load condition, together with the known (zero) sway, do provide a useful indication of the tendency for sidesway frame buckling. This contrasts with the interpretation of deteriorated critical loads for the combined load condition. The reasons for these contrasting interpretations are:

(1) The deflections caused by symmetrical gravity loads do not contribute to the critical sway buckling modes which correspond to the deteriorated critical loads, while the converse is true for the combined load condition.

(2) Frame instability under combined loading can be described as amplification of wind induced sway, due to the PΔ effect and to plastic hinge sway stiffness.
reductions. But sidesway frame buckling under gravity loads is a bifurcation process which is fundamentally identical in nature to the behavior considered in the calculation of the deteriorated critical loads. Once these differences are clearly understood, the interpretation of deteriorated critical loads can proceed on a sound basis.

Conclusions

This review of the deteriorated critical load concept from a graphical load-versus-sway viewpoint may be compared with the evaluation using minimum potential energy principles in Ref. 74. This reference concludes that,

"The concept of the deteriorated critical load clarifies thinking on the subject of elastic-plastic instability of structures. It is however only of very limited assistance in the actual calculation of failure loads, since the deteriorated structure cannot be obtained unless a full elastic-plastic analysis has in any case been derived. Moreover, if deteriorated critical loads are considered in isolation from a complete analysis, misleading results may be obtained."

These conclusions are in complete agreement with the preceding discussion. The comments in paragraphs 77 to 85 of Ref. 8 lend further support to these conclusions. It is felt that these conclusions deserve renewed emphasis, based on what are considered to be overly conservative reservations in the literature for frame stability problems in plastically designed unbraced multi-story frames. While these problems
cannot be ignored, neither should they be overemphasized to the detriment of progress.

It is hoped that the evaluation of the deteriorated critical load concept from a load-versus-sway viewpoint (which is somewhat more tangible and familiar to the engineering profession than minimum potential energy) will help to further clarify the significance of this concept. The reservations expressed here concerning the limitations of the deteriorated critical load idea apply with equal force to the alternative "degree of restraint" approach described in Ref. 53 and previous papers. The degree of restraint approach to stability problems for combined loads again involves neglect of load-versus-sway behavior and the assumption of hinge patterns with no demonstration that these patterns are statically admissible or a valid approximation of actual frame behavior. We cannot object to assumptions per se but we do hold reservations when the assumptions are unverified either by design checks or by previous experience with well defined limitations.

The inelastic unbraced sidesway subassemblage analysis described in Ref. 10 provides a much-needed method for checking assumptions made in preliminary design. After some experience with the restricted hinge pattern idea (Art. 1.9) and its limitations, it should be possible to simplify the design check for sway to chart form. This will provide a rapid and convenient method for handling the frame stability problem in plastically designed, unbraced, multi-story frames. It is this need for rapid and convenient methods which has promoted interest in the degree of restraint idea and the deteriorated critical load concept. While alternative methods of similar convenience
remain to be presented, progress in the field of inelastic instability suggests that such methods will soon be available.

2.6 LOAD FACTORS

The load factors used to obtain required ultimate load capacities from the working loads have a direct bearing on the economy and the overload performance of a multi-story building. Reference 53 comments that, "The presence or absence of warning of collapse must decide the load factor to some extent." This comment would appear to penalize a frame which exhibits linear overload behavior followed by a sharp knee in its load versus sway curve (Fig. 13, Ref. 53) simply because the first hinges form at a load close to the ultimate load capacity.

If the load factor is related (in a somewhat arbitrarily defined but very real fashion) to the probability of failure, varying the load factor according to "the presence or absence of warning of collapse" is a curious interpretation of this relationship. Some precedent for this interpretation is found in Ref. 79 which indicates that a "structure statically determinate and likely to give little warning if collapse is approached" represents an adverse condition which should be considered in assessing load factor requirements. Note that a rigid frame is not statically determinate and that changing load factors will change the ultimate load capacity but not necessarily the nature of warning of collapse. The collapse condition may still occur with little change in the degree of warning. Incidentally, the overload serviceability of a structure which gives "little warning" of collapse
may be superior to a companion structure with similar ultimate load capacity which exhibits markedly nonlinear behavior and signs of distress under small overloads. Consideration of a "limiting load factor" to provide an adequate margin of overload serviceability is also emphasized (in a subordinate role) in Refs. 79 and 8. The inelastic shear versus sway analysis method proposed in Ref. 10 provides information for assessing the overload serviceability of an unbraced frame.

This load factor issue is economically significant because several later papers have suggested that substantially higher load factors (up to 3.0) should be used for buildings with little or no cladding, because of concern for frame stability, (8,16) and because in some cases "the collapse load would then be astonishingly sudden, and the load factor would have to be raised on that account as indeed it was in the case of brickwork" (paragraph 92 of Ref. 16).

Larger load factors for materials with more variable properties, like brickwork, are certainly appropriate. But the extension of larger load factors to a ductile and less variable steel rigid frame, designed to carry loads without assistance from cladding, and with full account taken of instability (PA) effects, is considered in the light of recent progress, to be an overly conservative reaction to the frame stability problem.

The subassemblage approach to the inelastic wind shear versus sway problem (Ref. 10) certainly removes much of the doubt concerning frame stability under combined load conditions. It is anticipated that
current studies of inelastic sidesway frame buckling under gravity loads (a limiting case of the combined load stability problem in which the lateral loads vanish) will result in a similar degree of confidence. If warning of collapse is considered important, then simple non-structural devices to provide this warning could be developed and installed during construction, without sacrificing the economy of the steel frame by using increased load factors. Incidentally, such devices could provide valuable information on the working load behavior of buildings if they are designed to measure wind induced sway acceleration and vibration data.

An increase in load factors (ranging from 2.0 to 8.0) is recommended for structures in which the composite action of the steel skeleton and more variable cladding material (which may be interrupted by doors and windows or which may be removed in renovation) are counted upon to preserve the stability of the structure (paragraph 136, Ref. 53). These load factors gradually increase with the cladding contribution but seem quite conservative in the lower range. A downward trend in these composite load factor recommendations should be anticipated as further information on the interaction between frame and cladding, and methods for predicting this interaction, become available. (80,81)

Progress in the study of structural safety, (79,82,83) and the trend toward changing load factors suggest that load factors should be considered as open parameters in formulating a preliminary design method. This practice is followed in this study. The load factors assigned in a particular design problem should obviously depend in part
on the design criterion (ultimate load or mechanism load, Art. 1.8). The minimum required load factors \( F_{IR} = 1.70 \) for gravity load and \( F_{2R} = 1.30 \) for combined load are suggested in Ref. 6 with the understanding that the refined methods of inelastic analysis described in this reference (or other methods of similar rationality) should be used with these suggested load factors.

Part 1 of the American Institute of Steel Construction Specification (Allowable Stress Design, Ref. 5) permits a 1/3 increase in allowable stresses for combined (gravity plus wind) loads. This empirical allowable stress provision is carried over to plastic design by using \( F_{2R} = F_{IR}/1.33 \). This load factor relationship is used in quoting numerical values of preliminary design parameters throughout this study.

2.7 SCOPE OF THIS STUDY

This study describes the basis for the initial stage of a plastic design method for unbraced multi-story frames. The objective is a preliminary design including a complete bending moment, axial force, and shear distribution for gravity loading and for combined (gravity plus wind) loading. Member sizes are included in some of the numerical examples, but the main emphasis is on obtaining the preliminary moment distribution from initial frame geometry and load data. The design checks and secondary design considerations which complete the total structural design process are not included in this study, except by reference.
The preliminary design begins with the definition of initial data in Chapter 3. The principal parts of the plastic moment balance are concerned with frame statics, the girder plastic moment balance, and the joint balance. These parts are described in Chapters 4 to 6. The influence of base restraint on the plastic moment balance for bottom story columns is considered in Chapter 7. Chapter 8 is concerned with plastic moment balancing parameters and design objectives for girders. A numerical example is included in Appendix 1 to illustrate methods described in the text.
3. PRELIMINARY DESIGN CONSIDERATIONS

This study is limited to one regular type of frame in the interest of simplicity and emphasis on fundamentals. The principles can be extended to other types of frames with appropriate modifications.

In this chapter the frame layout and loading assumptions are described. Brief comments on load history and live load reductions are included. Then the load combinations and preliminary design criteria for an unbraced multi-story frame are given. The chapter concludes with a statement of sign conventions and notation.

3.1 FRAME LAYOUT

The frames considered in this preliminary design study are described as follows:

1. The frames have a rectangular profile. Columns extend without offset from roof to foundation and girders extend across the full width of the frame. Frames with "missing" girders or columns or with column footings at different elevations are excluded.

2. The girder elevations and depths on each level are reasonably constant so that the girders may be assumed to have a common horizontal centerline for the purpose of structural analysis. The story height between
levels is the distance between the common horizontal centerlines of the girders. In the bottom story, the story height is measured from the constant elevation at top of footings. The story heights and girder depths in adjacent stories may vary as required.

3. The distance between column centerlines in the plane of the frame will be termed the bay width. The width of adjacent bays may vary as required.

4. The distance between adjacent frames, perpendicular to the plane of the frames will be termed the frame spacing, which may vary in any desired manner.

5. The frames and loads are considered to act in a single plane. Biaxial bending of columns is not considered. Lateral support for girders and columns is provided by bracing or cladding to prevent out-of-plane deformation.

6. The girder-to-column connections are rigid in the sense that they will transmit the full plastic moment capacity (with appropriate axial load reduction) of each member without local buckling or excessive distortion.

7. No bracing or cladding is used in the plane of the frame to resist horizontal sway in a story.

8. Individual girders have a uniform section.

9. The column bases may be pinned, fixed, or elastically restrained in the plane of the frame.
The term frame geometry will be used to describe the following data: number of stories and height of each story; number of bays and width of each bay; frame spacing; and base restraint conditions. It is assumed that the frame geometry is completely specified at the beginning of the preliminary design.

The frame geometry considered in this study is indicated in Fig. 3.1. Sketch (a) shows a regular frame with unequal bays and stories, which conforms to the frame geometry limitations stated above. Sketch (b) indicates ten irregular framing features which are not considered in this study. Most of these features could be considered with appropriate modifications in the statics conditions. Only the dimensions indicated in Fig. 3.1(a) plus the distance between adjacent frames are known when the preliminary design commences.

3.2 LOADS AND LOADING CONDITIONS

The loads considered in this preliminary design study include the following items:

1. The girders carry uniformly distributed dead and live loads.

2. Concentrated vertical dead loads may be applied at the center of each joint from members which are perpendicular to the plane of the frame. These joint loads are used to account for walls and parapets. In addition, column dead loads, including fireproofing, are applied as joint loads at the top of each column.
3. Uniformly distributed wind loads are applied as concentrated loads at floor levels.

4. Live loads originating from floors (but not the roof) may be reduced as permitted by the live load reduction provisions of the governing building code.

5. All loads are assumed to be statically applied without impact, repetition, or time variation.

6. The internal forces and moments in the frame are assumed to be independent of load history for the purpose of preliminary design.

The influence of load history on the load-deflection behavior and ultimate load capacity of an unbraced frame is qualitatively described in Chapter 13 of Ref. 6. This reference indicates that the application of ultimate vertical loads, followed by increasing wind loads, may be expected to produce larger ultimate wind load capacities than other loading sequences. Other investigators have suggested that a loading sequence involving full vertical loading, followed by increasing wind loading, should be considered to be more severe than a proportional loading sequence. (61)

No quantitative demonstration of potential differences in ultimate load capacity resulting from different load histories is available in the literature. However, this problem is under current study. It is known that the deflections of an inelastic structure do depend on the load history. Since deflections also influence the equilibrium of
an unbraced multi-story frame, it is logical to expect some variation in ultimate load capacity with different load histories. Unless significant changes in ultimate load capacity can be shown to result from different load histories, item 6 in the above list may be accepted as a reasonable design assumption.

The principles involved in dealing with uniformly distributed girder loads may be applied to any type of load distribution (Art. 1.5).

3.3 LOAD COMBINATIONS

The first step in the design is to establish or estimate the dead, live, and wind loads on the frame. The ideas involved in this load definition step are adequately described in several texts\(^{2,17,96}\) and need no further elaboration here except to comment that load requirements for a plastically designed frame are virtually the same as those for a frame proportioned by allowable stress methods. In particular, the application of live load reduction factors is equally appropriate for each of these design methods. The fundamental basis for live load reduction is the reduced probability of the simultaneous application of full (code specified) live loads on large tributary floor areas.\(^{84}\) This basis is quite independent of the design method used to proportion the frame. It should be noted however that some authors have suggested various interpretations of how live load reductions should be applied in a plastic design method.\(^{8,16}\) The live load reduction provisions in Sect. 3.5 of Ref. 85 are used in the design example in Appendix 1 of this study.
For the purpose of preliminary design, the gravity load component of the column loads may be computed on the basis of tributary floor areas. The wind load component of the column loads will be discussed in Chapter 4. The sum of the dead load and the reduced live load on a girder or column will be termed the working load.

The next step in the design is to consider the possible loading conditions and design criteria which may control the required member sizes, and to assign appropriate load factors to each loading condition. The discussion in Chapter 1 suggests the preliminary design criteria in Table 3.1. The load factors in this table are taken from Ref. 6. The plastic design examples in that reference provide some guidance in selecting controlling design criteria. For instance, criterion A1 governs the size of girders and columns in the top 3 or 4 stories of the plastically designed unbraced frames in Ref. 6. The remarks in Art. 1.8 apply to criterion B1.

Frame geometry parameters such as the number of stories, number of bays, and the ratio of total height above a level to total width of the frame at that level, will obviously have some bearing on the domains for controlling design conditions. Frame load parameters such as the ratio of total gravity load to total wind shear in a story and the load factor ratio \( F_{1R}/F_{2R} \) will also influence the domains for controlling design conditions. Domain equations which combine frame geometry and load parameters are given in Refs. 16 and 61. Amplification factors for the \( P_0 \) effect are also considered in the domain equations of paragraph 17, Ref. 61. Note that these
TABLE 3.1
LOADING CONDITIONS AND PRELIMINARY DESIGN CRITERIA
FOR UNBRACED FRAMES

A. For gravity loads (including dead plus reduced live load)

Required Load Factor \( F_{1R} = 1.70 \)

**Design Criteria:**

1. Adequate strength to carry the factored gravity loads with the formation of plastic mechanisms.

2. Adequate stiffness in each story to avoid sway instability under factored gravity loads.

B. For combined loads (including dead plus reduced live loads and wind loads from either direction in the plane of the frame)

Required Load Factor \( F_{2R} = 1.30 \)

**Design Criteria:**

1. Adequate strength and stiffness to carry the factored combined loads including \( PA \) effects.

2. Adequate stiffness to control sway deflection at the (unfactored) working load level.
equations are derived for a frame with equal bays and for a constant load factor ratio $F_1/F_2 = 1.25$, neglecting live load reduction.

Domain equations or charts which identify controlling design criteria are helpful when a preliminary design is worked out manually. However, when the preliminary design is done by a computer, the more direct approach of considering each design criterion in turn and selecting the controlling criterion for each member can be used to advantage. The direct approach is also more generally valid when frames with unequal bays are considered or when restricted hinge patterns and live load reductions are applied.

A primary concern in the design of unbraced frames is the ability of the frame to resist horizontal (sway) deflection. Resistance to sway in a story is conveniently measured by the sway stiffness of the story as defined in Chapter 1. The columns in the story and the girders on the levels above and below the story contribute to the story sway stiffness. The preliminary design method considered in this study does not involve direct consideration of sway stiffness parameters. Nevertheless, sway stiffness for criteria A2 and B1 in Table 3.1 may be anticipated by specifying appropriate restricted hinge patterns at the beginning of the preliminary design process.

An example of the frame geometry and load data needed to begin a preliminary design is given in Table A1 in Appendix 1.
3.4 NOTATION AND SIGN CONVENTIONS

A consistent and easily applied sign convention is used in this study as follows:

End-moments, end-rotations, column chord rotations, and column shear couples are positive when clockwise.

This sign convention requires that positive moments on a joint must act in the counterclockwise sense. Compressive axial forces in columns are considered positive. These sign conventions are illustrated in Fig. 3.2 where all forces, moments, and rotations are shown in their positive directions. It is helpful to note that the column shear couple Qh and the column end-moments cannot all be positive for a column in equilibrium. Horizontal wind forces are taken as positive when acting from left to right. Then a positive wind generally causes a positive shear couple in most of the columns in one story.

The subscript notation for moments in Fig. 3.2 is as follows:

A and B denote the left and right ends of a girder.
U and L denote the upper and lower ends of a column.

The prefix subscript j identifies a moment acting on a joint with moment center at the center of the joint. Chapter 4 indicates the difference between the girder end-moments \( M_A \) and \( M_B \) and the joint moments \( M_{JA} \) and \( M_{JB} \).
To identify the joints and bays on one level of a frame, the joints and bays are numbered from left to right. The levels and stories are numbered from roof to foundation. Columns are identified by the joint number above and girders by the joint number at the left end. This numbering system is indicated in Fig. 3.1(a). Numerical subscripts are enclosed in parentheses and separated by commas, with joint or bay numbers given first in all double subscripts. The capital letters, I, J, K, and N are used to identify numerical subscripts.

This subscript system provides a simple transition from text notation to computer compiler notation, and completely identifies the members or portion of the frame considered in a formula. In essence, the subscripts are a convenient and concise bookkeeping device. The subscripts help to identify the many similar quantities which describe the statics of a multi-story frame.

For example \( M_A(J,I) \) and \( M_B(J,I) \) are the end-moments on the girder in bay \((J)\) on level \((I)\), and

\[
\sum_{J} M_{G(I)} = \sum_{J} (M_{JA(J,I)} + M_{JB(J,I)})
\]

is the sum of the joint moments from the girders on level \((I)\). It is understood that \((J)\) goes from 1 to the number of bays in this summation. Similarly,

\[
\sum_{J} M_{C(I)} = \sum_{J} (M_{JU(J,I)} + M_{JL(J,I)})
\]

is the sum of the joint moments for the columns in the story below level \((I)\) where \((J)\) is understood to range from 1 to the number of joints on level \((I)\).
4. FRAME STATICS

This chapter considers equilibrium requirements for the joints of a rigid frame and for the columns in each story of a frame. First, the effect on joint equilibrium of members with finite depth is investigated. Then, equilibrium between the column moments in a story and the forces causing sway of the story is formulated. The story and joint equilibrium conditions are next combined to establish girder moment requirements. At this stage the moment balancing concept is introduced to obtain sway moments for the girders. These sway moments provide the starting point for the plastic moment balancing process described in subsequent chapters. Finally, this chapter considers methods for estimating the axial load in columns due to sway effects.

An important feature of equilibrium requirements for unbraced frames is the influence of sway deflection on equilibrium. For this reason, all equilibrium requirements are formulated in the deflected state of the frame. It is evident that the frame deflections are not known in the preliminary design stage. Nevertheless it is prudent to include estimated sway deflection effects in the preliminary design process. If reasonable sway deflection assumptions are used, the members selected to carry the preliminary design moments will be better able to provide for sway deflection effects than would be the case if deflection effects are ignored in the preliminary design.
Many tests of rigid frames and their components have demonstrated that plastic hinges form outside of properly proportioned and braced joints connecting beams and columns.\(^{(86-91)}\) This increases the strength and stiffness of the frame above that predicted by an analysis based on centerline dimensions and members of zero depth. From the design viewpoint, this finite joint size effect indicates that the frame members may be proportioned for plastic moments less than the peak moment values at the center of a joint. This does not mean that the forces within the joint can be neglected in design.

To consider the beneficial effects of finite joint size in a design method, it is necessary to locate the critical sections which define where controlling moments act in the connected members around the joint. It is suggested that plastic hinges in welded rigid frame girders be assumed to occur at a distance of the girder depth from the column flange.\(^{(90)}\) A somewhat more conservative procedure is to assume that plastic hinges form in girders at the column flanges and in columns at the girder flanges. Then both girders and columns are designed to resist the moments which develop over their clear spans. This clear span assumption will be used to locate the critical sections in girders and columns in this preliminary design study.

The strengthening and stiffening effects of finite joint size are an inherent advantage of rigid frame construction which contributes nothing to material or labor costs. Nor is the potential reduction in member size a trivial matter as indicated in Fig. 6.7 of Ref. 6. The use of clear girder spans has adequate precedent in the structural
literature. Clear heights are advocated in the design of columns by several authors. (6,17,92-97) Conservative estimates of girder depth can be based on span-to-depth limitations for deflection or on previous experience. Girder depths of 1/20 to 1/10 times the span are typical. Column depths are normally confined to a narrow range from say 10 inches to 18 inches for W shapes. An assumed column depth of one foot is frequently appropriate.

Fortunately, consideration of finite joint size in the preliminary design process using the plastic moment balancing method can be accomplished with little increase in design labor. In the interest of simplicity, the "depth" of a joint with different girder depths framed into opposite flanges of a column will be assumed equal to the depth of the smaller girder.

Consider now the equilibrium conditions for joints of finite size. The forces acting on an interior girder-to-column flange joint in a rigid frame are displayed in Fig. 4.1(a). The joint is shown in its deflected state consisting of a rotation \( \theta_j \), but shear and flexural distortions are omitted. Axial forces from the girders are also neglected on the assumption that these forces are small.

The size of the joint is established by the smaller depth \( d_b \) of the girders and the depth \( d_c \) of the column. The resultant of the forces on each face of the joint includes a moment and a horizontal or vertical shear force. These will be termed the end-moment and end-shear for the connected members. The axial column loads above and below the joint complete the freebody diagram in Fig. 4.1(a). The forces from
members which frame into the joint in the plane perpendicular to the diagram are omitted for clarity.

All forces and moments in Fig. 4.1 are shown in their positive sense, according to the sign conventions and notation established in Chapter 3. If the joint rotation $\theta_j$ is small, equilibrium of moments about the center of the joint gives

$$ M_A - \frac{1}{2} \frac{d}{c} V_A + M_B + \frac{1}{2} \frac{d}{c} V_B $$

$$ + M_L - \frac{1}{2} \frac{d}{b} Q_L - \frac{1}{2} \frac{d}{b} \theta_j P_L $$

$$ + M_U - \frac{1}{2} \frac{d}{b} Q_U - \frac{1}{2} \frac{d}{b} \theta_j P_U = 0 \quad (4.1) $$

The terms $P \theta_j \frac{d}{b}/2$ include the moment on the rotated joint due to the column loads. Example 1 in Appendix 2 illustrates that these terms are rarely significant when compared with the moment capacity of the columns.

In view of the small contribution of the $P \theta_j \frac{d}{b}/2$ terms in Eq. (4.1) these terms will be dropped. This amounts to stating that joint rotation may be neglected in formulating equilibrium for multi-story frame joints of finite size. Although this is an accepted practice, no justification of this practice or indication of its limitations is available in the literature.

To further simplify the joint equilibrium condition, it is convenient to group the end-moment and end-shear contributions from each connected member in Fig. 4.1 in the form
These moments will be termed the joint-moments for the connected members, identified by the prefix subscript $j$. The joint-moments are the algebraic sum of a member end-moment and the member end-shear times the half-depth of the joint. The moment center for the joint-moments is understood to be the center of the joint.

The moment equilibrium condition for the finite joint can now be written in the conventional form

$$M_{jA} + M_{jB} + M_{jU} + M_{jL} = 0$$

(4.3)

This conventional representation of moment equilibrium for a joint is illustrated in Fig. 4.1(b). The preceding discussion shows that this conventional form of joint equilibrium is valid for joints of finite size, with the assumption that axial load from the girders and columns have a negligible influence on moment equilibrium. It is further assumed that no eccentric reactions are applied to the joint by members which connect adjacent frames. Obvious modifications of Eq. (4.3) are required for joints with eccentrically framed spandrel beams.
4.2 COLUMN EQUILIBRIUM

Equilibrium conditions for a column will be studied with the aid of Fig. 4.2. Figure 4.2(a) shows the web view of the column with girders of depth \(d_{bU}\) and \(d_{bL}\) at its upper and lower ends. The clear column height \(h_c = h - (d_{bU} + d_{bL})/2\) is conservatively determined by the girders with least depth in the plane of the frame. Figure 4.2(b) shows the forces and moments acting on the deflected column between joint centers and Fig. 4.2(c) gives similar information for the clear column height. Free body diagrams of the column forces and moments on the upper and lower joints are indicated in Fig. 4.2(d). It is assumed that the rigid girder-to-column-flange connections prevent bending of the column within the joints in Fig. 4.2. All quantities in this illustration are shown positive, as defined in Art. 3.4.

We are interested in equilibrium expressions for the horizontal shear force \(Q\) and the end-moments \(M_U\) and \(M_L\) on the column in the de­flected state. Consider first the shear force. From Fig. 4.2(b) the horizontal shear force is

\[
Q = -\frac{1}{h} (M_{jU} + M_{jL} + PA) \tag{4.4}
\]

The shear couple \(Q \frac{d_b}{2}\) on the joint freebody diagrams (Fig. 4.2(d)) can be expressed as

\[
Q \frac{d_b}{2} = -\frac{1}{2} \frac{d_b}{h} (M_{jU} + M_{jL} + PA) \tag{a}
\]
while moment equilibrium for these joint diagrams gives

\[ M_U = M_{jU} + Q \frac{d_{bU}}{2} + P_y U \]

\[ M_L = M_{jL} + Q \frac{d_{bL}}{2} + P_y L \]

Here \( y_U = \theta_{jU} \frac{d_{bU}}{2} \) and \( y_L = \theta_{jL} \frac{d_{bL}}{2} \) are the sway within the joints. Article 4.1 indicates that terms of the form \( Py = P \theta \frac{d_b}{2} \) in Eq. (b) are small relative to the column moment capacity. However, it does not follow that the Py terms are small in comparison with the \( Q \frac{d_b}{2} \) terms, which in fact may vanish.

The expressions (b) for the end-moments can be simplified if we assume

\[ \theta_{jU} = \theta_{jL} = \Delta/h \]  

in evaluating the Py terms. Then combination of Eqs. (a) to (c) yields

\[ M_U = M_{jU} - \frac{1}{2} \frac{d_{bU}}{h} (M_{jU} + M_{jL}) \]

\[ M_L = M_{jL} - \frac{1}{2} \frac{d_{bL}}{h} (M_{jU} + M_{jL}) \]

(4.5)

since the PA and Py terms cancel. These results may also be obtained by assuming that the column moments vary linearly between the joint centerlines.

The assumption used in Eq. (c) permits us to estimate the influence of joint rotation on the column end-moments, \( M_U \) and \( M_L \). This
assumption may or may not lead to conservative column end-moments, but in any practical situation the differences are trivial. Example 2 in Appendix 2 illustrates this fact. It is of interest to note that by considering the finite joint size in Example 2, the required column moment capacity is reduced from M at centerline of joint to (7/8) M at face of joint. The resulting 12.5 percent moment reduction may permit a reduction in the column size.

Again we have arrived at results (Eqs. 4.4 and 4.5) which follow the accepted practice of neglecting joint rotation. The literature rarely treats the degree of approximation or the limitations of these results. This leads to the valid but curious mixture of clear span assumptions for proportioning girders coupled with total story height assumptions for designing columns in some recent structural publications.

4.3 STORY EQUILIBRIUM

There are three equilibrium conditions for each story of a multi-story frame. The first step in formulating equilibrium in a story is to determine the horizontal shear force due to wind loads above the story. If \( H_{(I)} \) is the factored wind load applied at level \( (I) \), the total factored wind shear in the story below level \( (I) \) is

\[
H_T(I) = \sum_{(I)} H_{(I)} \quad (4.6)
\]

where \( \sum_{(I)} \) indicates summation for all levels above and including level \( (I) \).
If we sum the shears from Eq. (4.4) for each column below level (I), then shear equilibrium for the story requires that

\[
H_T(I) = \sum_{(J)} Q(I, J) = \frac{1}{h(I)} \sum_{(J)} (M_{jU}(J, I) + M_{jL}(J, I)) - \left(\frac{A}{h(I)}\right) \sum_{(J)} P(J, I)
\]

where \(P(J, I)\) is the factored gravity-plus-sway load in the column below joint (J) and \(\sum_{(J)}\) indicates summation over all columns in the story. The subscript \((J, I)\) denotes the column below joint (J) on level (I). A second condition of vertical equilibrium in the story gives

\[
P_T(I) = \sum_{(J)} P(J, I)
\]

(4.7)

where \(P_T(I)\) is the total factored gravity load in the columns below level (I). The resultant of the column sway loads is a moment couple so the column sway loads cancel in Eq. (4.7). The third story equilibrium condition, involving the overturning moment due to wind loads and the resisting moment due to column sway loads and moments, is considered in Art. 4.5.

It is convenient to introduce the symbol \(\sum M_C(I)\) for the column moment sum in the story below level (I), so that

\[
\sum M_C(I) = \sum (M_{jU}(J, I) + M_{jL}(J, I))
\]

(4.8)

Then from Eqs. (a) and (4.7), the column moment sum is

\[
\sum M_C(I) = - (H_T h(I)) - (P_T A(I))
\]

(4.9)
Although this equation involves moments, it is based on the horizontal and vertical equilibrium conditions for forces in the story. Note that the total shear in the story is $H_T$ and not $H_T + P_T \Delta/h$. This indicates that the $PA$ effect does not change the story shear but it does influence the column (and girder) moments. The column moment sum computed from Eq. (4.9) will be referred to as the story sway moment in the story below level (I). The sway moment is the sum of the story shear times the story height plus the total gravity load above the story times the story sway.

The story sway moments are negative when the wind shear and sway are caused by wind acting from left to right. Live load reductions for the columns in the story are used to determine the total gravity load, $P_T$, in Eq. (4.9). An alternative procedure is to use the live load reductions for the girders on levels above the story in estimating $P_T$. However, the girder live load reductions do not reflect the reduced probability of simultaneous live load on the tributary floor area for a column.

A portion of the story sway moment is resisted at the upper end and the remainder at the lower end of the columns in a story. This represents a vertical distribution of the story sway moment. If $D_{V(I)}$ is the factor defining the portion of the story sway moment assigned to the upper end of the columns, then the column moment sums in the story below level (I) are

$$\sum_{J}^{M_{jU(I,I)}} = D_{V(I)} \sum_{C(I)}^{M_{C(I)}}$$

(4.10)
at the upper level (I), and

\[
\sum_{(J)} M_{jL(J,I)} = (1 - D_V(I)) \sum M_C(I) \tag{4.11}
\]

at the lower level (I + 1). In these equations, \( D_V \) is aptly termed the vertical distribution factor for the story sway moments. This vertical distribution factor must be assigned in the range \( 0 < D_V < 1 \) for each story. The value \( D_V = 0.5 \) is frequently used to assign half of the story sway moment to each end of the columns. Once the vertical distribution of sway moments is decided, the column moments sums in Eqs. (4.10) and (4.11) are conserved in the plastic moment balancing process.

The vertical distribution of sway moments is illustrated in Fig. 4.3 which shows the positive joint moments in two stories of a frame. All of the moments in this sketch carry numerical subscripts (omitted for clarity) and may have different values from member to member. The story sway moments \( \sum M_C \) are shown at the left center of each story. These sway moments are distributed, using the vertical distribution factors in the triangles above and below, to the upper and lower ends of the columns. This distribution gives the column moment sums, \( \sum M_{jU} \) and \( \sum M_{jL} \), on the left in Fig. 4.3. It is obvious that some of the joint moments in this figure must be negative to conserve equilibrium. Based on the deflected state shown in Fig. 1.3(a) we may anticipate negative joint moments for most of the columns, except the windward column, when wind acts from left to right.
Chapter 6 discusses vertical distribution factors in more depth. At this stage it suffices to comment that the portal and cantilever methods of wind analysis use constant vertical distribution factors of 0.5. Other values of $D_V$ may be both possible and useful.

4.4 GIRDER MOMENT SUMS

We can now combine the joint and story equilibrium conditions to establish moment requirements for the girders. First we add the moment equilibrium condition, Eq. (4.4), for each joint on a level to obtain

$$\sum_{G(I)} = \sum_{L} (M_{jA(J,I)} + M_{jB(J,I)}) = -\sum_{L} M_{jL(J,I-1)} - \sum_{A} M_{jU(J,I)} \quad (a)$$

where $\sum_{G(I)}$ is the sum of the girder sway moments on level (I). Then Eqs. (4.10) and (4.11) give

$$\sum_{G(I)} = -(1 - D_V(I-1)) \sum_{C(I-1)} - D_V(I) \sum_{C(I)} \quad (4.12)$$

The girder sway moment sum is obtained by adding the sway moments which are distributed to level (I) from the adjacent stories. This girder sway moment sum is conserved in the plastic moment balancing process. It is helpful to note that if the sway moments in the stories above and below a level are zero, under gravity loading, the girder sway moment sum on that level must vanish.
Some portion of the girder sway moment sum must be resisted by each girder on the level. This implies a distribution of moments to the girders. If \( M_G(J,I) \) is the girder sway moment and \( D_G(J,I) \) is the girder distribution factor, for the girder in span \( J \) on level \( I \), then

\[
M_G(J,I) = M_A(J,I) + M_B(J,I) = D_G(J,I) \sum M_G(I)
\]

(4.13)

When this equation is summed for each girder on level \( I \), it is apparent that

\[
\sum D_G(J,I) = 1
\]

(4.14)

Any set of girder distribution factors which sums to unity on a level is statically admissible.

The horizontal distribution of sway moments to the girders is illustrated in Fig. 4.3. The column moment sums at level \( I \) are combined to give the girder sway moment sum, \( \sum M_G \), shown to the left at level \( I \) in this figure. Then the girder distribution factors, \( D_G \), inside the triangles below the girders, are used to obtain the sway moment, Eq. (4.13), for each girder. This sway moment provides the starting point for the plastic moment balance for the girders.

The girder distribution factor for each girder on a level must be assigned subject to the sum in Eq. (4.14). Chapter 8 discusses girder distribution factors in more depth. Here we may briefly comment that the portal method of wind analysis used girder distribution factors, \( D_G \), proportional to the bay width. Other values of \( D_G \) are both
possible and useful. The vertical and girder distribution factors provide a simple preliminary design device for specifying how the frame components are to share in carrying the frame loads.

4.5 COLUMN SWAY LOADS

In addition to the horizontal and vertical equilibrium conditions for a story, formulated in Art. 4.3, there remains the story moment equilibrium condition. This condition considers equilibrium between the overturning moment caused by wind and by sway-induced eccentricity of the gravity loads above a level, and the resisting moment provided by column moments and axial loads.

The axial load in columns under combined loading may be considered as the sum of two components. The first or gravity load component may be estimated based on tributary floor areas and live load reductions for a column, together with any dead loads applied at joints. The second or sway component has no vertical resultant when summed over the columns in one story. The sway components combine to give a resultant moment couple in the story which is the major contribution to resisting the overturning moment in the story.

Derivation: Story Moment Equilibrium Condition

The story moment equilibrium condition will be described with reference to Fig. 4.4 which shows the positive factored wind forces \( H(I) \) and resultant factored gravity loads \( W(I) \) on four levels of an unbraced frame. Also shown are the positive column moments \( M_{JU}(J,4) \).
the sway load components \( P_{S(J,4)} \), and the shears in the columns below level 4. The resultant shear \( H_T(4) \) below level 4 and the overturning moment

\[
M_{OT}(4) = H(1) \left[ h(1) + h(2) + h(3) \right] + H(2) \left[ h(2) + h(3) \right] + H(3) \left[ h(3) \right] \\
+ W(1) \left[ \Delta(1) + \Delta(2) + \Delta(3) \right] + W(2) \left[ \Delta(2) + \Delta(3) \right] + W(3) \left[ \Delta(3) \right]
\]

at level 4 are indicated in the bottom left corner of Fig. 4.4. The second line in this expression is the overturning moment due to sway above level 4. The total overturning moment must be balanced by the sum of the column moments \( M_{jU(J,4)} \) plus the resultant moment couple due to the column load sway components, \( P_{S(J,4)} \). Using the symbol \( M_{CC(4)} \) for this resultant moment couple, (subscript CC for column couple) the story moment equilibrium condition can be written in the form

\[
M_{OT}(4) = M_{CC(4)} + \sum_{(J)} M_{jU(J,4)} \tag{b}
\]

at level 4 of the frame in Fig. 4.4, or in the alternative form

\[
M_{OT}(4) = M_{CC(3)} + \sum_{(J)} M_{jL(J,3)}
\]

This alternative form eliminates the girder wind shears on level 4 and indicates how different freebody diagrams may be used to formulate the story moment equilibrium condition.

**Derivation: Column Sway Loads**

Consider now the column loads \( P_{S} \) due to wind induced sway. The column sway loads are similar to the flexural stresses in a cantilever
beam. The girders of the frame act like the web of a cantilever in that the girder shears due to wind cause axial forces in the columns. The column sway loads are the algebraic sum of the girder wind shears. Thus, \( P_{s}(2,4) \) is the sum of the shears from the 8 girders in Fig. 4.4 which frame to column (2) on and above level (4).

The column sway loads \( P_{s} \) on the windward side of the frame are shown as tensile loads in Fig. 4.4 in accordance with elementary cantilever flexural behavior. However, if the exterior girders in this figure are relatively stiff in comparison with the interior girders, then \( P_{s}(2,4) \) and \( P_{s}(3,4) \) may be reversed, and the frame behavior may approach to that of two single bay frames which are tied together by the interior girders. Thus the cantilever method of wind analysis should (but in its conventional form does not) consider the variations from elementary cantilever flexural behavior which can result from various distributions of girder wind shears in the bays of an unbraced frame.

The plastic moment balancing approach does provide a method for considering the distribution of girder shears in estimating the column sway loads. Figure 4.5 indicates how the shears in the girders framed to joint (J) on level (I) may be determined from the girder distribution factors \( D_{G} \) and the girder sway moment sum \( \sum M_{G(I)} \) for level (I). The girder sway moments for bays (J-1) and (J) are indicated above the girders, using Eq. (4.13). The girder wind shears are obtained by dividing the sway moments by the corresponding spans between column centerlines as shown below the girders in Fig. 4.5. If \( \delta P_{s}(J,I) \) is the compressive increment of the column sway load in column (J) due to
girder shears on level (I), then equilibrium in the vertical direction for joint (J) gives

$$
\delta \mathbf{P}_S(J,I) = V(J-1,I) - V(J,I) = \left[ \frac{D_G}{L} (J-1,I) - \frac{D_G}{L} (J,I) \right] \sum M_G(I)
$$

(4.15)

using the information in Fig. 4.5. The bracketed term in this equation is a statically derived dimensional distribution factor which may be positive, negative, or zero. This bracketed term will be referred to as the sway load distribution factor for column (J), below level (I). The girder wind shears in Eq. (4.15) are considered positive when they cause a counterclockwise couple on the girders as shown in Figs. 4.4 and 4.5. Then positive girder sway moments cause positive wind shears in the girders.

We can determine \( \mathbf{P}_S(J,I) \) by summing the increments from Eq. (4.15). To simplify the result, we will assume that the sway load distribution factors for column (J) are constant from the roof to level (I). Then the column sway loads below level (I) take the form

$$
\mathbf{P}_S(J,I) = \left[ \frac{D_G}{L} (J-1) - \frac{D_G}{L} (J) \right] \sum M_G(I)
$$

(4.16)

for each column (J). The bracketed sway load distribution factor in Eq. (4.16) may also be considered as the weighted average of the corresponding terms in Eq. (4.15). If more convenient estimates are
acceptable, we may assume a value for the sway load distribution factor in Eq. (4.16) based on the range of the girder distribution factors $D_G$ for the girders on levels above the column under investigation. (See Chapter 8.) It is helpful to mention that for the left hand column, $(J) = 1$, and the first term in the brackets of Eqs. (4.15) and (4.16) is absent. The last term in these brackets drops out for the right hand column. If we sum Eq. (4.16) over the columns in one story, the result is zero, indicating that the column sway loads have the required zero vertical resultant.

**Evaluation**

Now we want to demonstrate that the values of $P_{S(J,I)}$ from Eq. (4.16) satisfy the story moment equilibrium condition. First we can use Eq. (4.12) to expand the sum in Eq. (4.16). For the frame in Fig. 4.4, at level 4, we obtain

$$
\sum_{(I)=1}^{4} \left( \sum_{(I)} M_G(1) \right) = - \left(1 - D_V(1-1)\right) \sum_{(I)} M_C(1-1) - D_V(1) \sum_{(I)} M_C(1)
- \left(1 - D_V(2-1)\right) \sum_{(I)} M_C(2-1) - D_V(2) \sum_{(I)} M_C(2)
- \left(1 - D_V(3-1)\right) \sum_{(I)} M_C(3-1) - D_V(3) \sum_{(I)} M_C(3)
- \left(1 - D_V(4-1)\right) \sum_{(I)} M_C(4-1) - D_V(4) \sum_{(I)} M_C(4)
$$

Notice that the last term in each line of this equation cancels with the $D_V$ part of the first term in the next line, so that Eq. (c) reduces to

$$
\sum_{(I)=1}^{4} \left( \sum_{(I)} M_G(1) \right) = - \sum_{(I)=1}^{3} \sum_{(I)} M_C(I) - D_V(4) \sum_{(I)=1} M_C(4)
$$

(d)
From Eqs. (4.16) and (d) we conclude that the column sway loads are proportional to the sum of the sway moments in the stories above the story under consideration.

Next we use Eq. (4.9) to evaluate the first term in Eq. (d), and also substitute the column moment sum from Eq. (4.10) for the last term. Thus we obtain

\[
\sum_{(I)=1}^{4} \left( \sum_{(I)=1}^{3} (H_I h)(I) \right) + \sum_{(I)=1}^{3} (P_T \Delta)(I) - \sum_{(J)} M_{jU(J,4)} (e)
\]

Consider now the first two sums on the right hand side of Eq. (e). The term \(\sum (H_I h)(I)\) is the first line of the overturning moment, Eq. (a).

We can manipulate the second sum as follows for the frame in Fig. 4.4:

\[
\sum_{(I)=1}^{3} (P_T \Delta)(I) = \left[ w(1) \Delta(1) + \left[ w(1) + w(2) \right] \Delta(2) + \left[ w(1) + w(2) + w(3) \right] \Delta(3) \right]
\]

Then the term \(\sum (P_T \Delta)(I)\) in Eq. (e) is the same as the second line in Eq. (a), so we obtain from Eq. (4.16)

\[
P_{S(J,4)} = \left[ \left( \frac{D_G}{L} \right)_{(J-1)} - \left( \frac{D_G}{L} \right)_{(J)} \right] \left\{ \begin{array}{c} M_{OJ(4)} - \sum_{(J)} M_{jU(J,4)} \end{array} \right\} (f)
\]

for each column \((J)\) below level 4.

Finally we may compute the resisting moment due to the column sway loads by using any convenient moment center. If we take moments about the left hand column at level 4 in Fig. 4.4, the column loads, \(P_{S(J,4)}\), give a moment of

\[
M_{CC(4)} = P_{S(2,4)} L(1) + P_{S(3,4)} (L(1) + L(2)) + P_{S(4,4)} (L(1) + L(2) + L(3))
\]
where \( P_s \) is positive when it causes compression, as in Eq. (4.15).

Using Eq. (f) this reduces to

\[
M_{CC}(4) = (D_G(1) + D_G(2) + D_G(3)) \left\{ O_T(4) - \sum_{(J)} M_{jU}(J,4) \right\}
\]

Since the girder distribution factors must sum to unity from Eq. (4.14), we have verified that \( M_{CC}(4) \) satisfies the story moment equilibrium condition, Eq. (b), when the column load sway components are evaluated from Eq. (4.16).

We have seen that the \( \Delta \)A effect changes the column and girder moments but not the shear in a story. An additional result of the \( \Delta \)A effect is now evident. From Eqs. (4.16) and (e) it is apparent that the column sway loads in each story include a cumulative contribution from the \( \Delta \)A effect in every story above the one under consideration. While this is intuitively evident, no demonstration of the cumulative axial load contribution for an unbraced frame is available in the literature.

4.6 SUMMARY

This chapter explains how the girder sway moments

\( M_G = M_{jA} + M_{jB} \)

may be determined from known frame geometry and load data plus three additional groups of assigned parameters: (1) the sway deflection index \( \Delta/h \); (2) the vertical distribution factors \( D_V \); and (3) the girder distribution factors \( D_G \). This completes the first major part of the plastic moment balancing process. An example of tabular calculations for the vertical and horizontal distribution of sway moments
is included in Appendix 1, Tables A3 and A4. The next step is to determine girder plastic moments and joint moments so as to conserve the value of $M_G$ for each girder. This is the topic of Chapter 5.
5. GIRDERS EQUILIBRIUM

The fundamentals of plastic moment balancing for girders are explained in this chapter. The discussion begins with consideration of girder statics. Plastic moment conditions are purposely avoided in this statics discussion. Then the conditions which form the basis for the plastic moment balance are stated and described in graphical and algebraic form. Consideration is given to girder mechanisms and to restricted girder hinge patterns. The chapter concludes with a discussion of girder moment diagrams for gravity and for combined loading.

5.1 STATICAL RELATIONS

Our purpose in this article is to obtain non-dimensional statical relations between the end-moments and the maximum positive (sagging) moment for a girder. We also will find an expression for the girder sway moment in terms of the end-moments. Statical relations for a girder carrying a uniformly distributed load are obtained from Fig. 5.1, which shows the distance \( L \) between column centers, the depth \( d_c \) of the columns, and the clear span

\[
L_g = L - d_c
\]  

(5.1)

of the girder. If the columns at the left end A and the right end B are of different depth, it is reasonable to take \( d_c \) as the average column depth. All moments and forces in Fig. 5.1 are shown in their positive
sense and wind is assumed to act from left to right. This figure indicates the total factored gravity load $F_w L$, the positive girder end-moments $M_A$ and $M_B$, the end-shears $V_A$ and $V_B$, and the girder joint moments $M_{JA}$ and $M_{JB}$. Article 4.1 indicates that the joint moments

$$M_{JA} = M_A - V_A \frac{d}{c}/2$$

$$M_{JB} = M_B + V_B \frac{d}{c}/2$$

are simply a replacement for the combined effect of the end-moments and shears on joint equilibrium. The end-shears

$$V_A = \frac{F_w L}{2} \frac{2}{g} + \frac{M_A + M_B}{L_g}$$

$$V_B = \frac{F_w L}{2} \frac{2}{g} + \frac{M_A + M_B}{L_g}$$

may be replaced in Eq. (5.2) to give the joint moments in the non-dimensional form

$$\frac{M_{JA}}{M_{pm}} = \frac{M_A}{M_{pm}} - \left( 4 - \frac{M_A + M_B}{2 M_{pm}} \right) \frac{d_c/L}{1 - d_c/L}$$

$$\frac{M_{JB}}{M_{pm}} = \frac{M_B}{M_{pm}} + \left( 4 + \frac{M_A + M_B}{2 M_{pm}} \right) \frac{d_c/L}{1 - d_c/L}$$
The non-dimensionalizing term

\[ M_{pm} = F \frac{w L^2}{g} \quad (5.5) \]

is simply a convenient substitution for the known values of the gravity load, span, and load factor. Equation (5.5) may also be regarded as the minimum plastic moment capacity which is needed to carry only the factored gravity load. The maximum ordinate of the simple beam moment diagram in Fig. 5.1(d) is \( 2 M_{pm} \). The required girder plastic moment capacity always exceeds \( M_{pm} \) unless \( F = F_1R \) for the gravity loading condition.

The girder sway moment (Art. 4.4)

\[ M_G = M_{jA} + M_{jB} \quad (5.6) \]

follows directly from Eqs. (5.1 and 5.4) in the non-dimensional form

\[ G = \frac{M_G}{M_{pm}} \left( 1 - \frac{dC}{L} \right) = \frac{M_A + M_B}{M_{pm}} \quad (5.7) \]

The horizontal distribution of sway moments in Eq. (4.13) gives the girder sway moment \( M_G(J,I) \) for span \( (J) \) on level \( (I) \). Once \( M_G \) is known, girder equilibrium is considered in isolation from the frame so numerical subscripts are not needed. The moment ratio \( G \) is a function of the sway moment distributed to the girder from adjacent stories, and the factored

\#The load factor \( F \) in Eqs. (5.5 and 5.9) is to be interpreted as an open parameter in Art. 5.1 only. All other references to \( M_{pm} \) defined by Eq. (5.5) and \( R \) defined by Eq. (5.9) will assume that \( F = F_{2R} \) from combined load.
gravity load on the girder. For convenient reference, \( G \) is termed the 
**sway moment coefficient** for the girder. Note that Eq. (5.7) is 
statistically derived and independent of plastic moment conditions.

Consider the statical relationship between the end-moments \( M_A \)
and \( M_B \), and the maximum positive (sagging) moment \( M_C \). The results are 
simplified by using the dimensionless moment factors

\[
\begin{align*}
A &= \frac{M_A}{M_p} \\
B &= \frac{M_B}{M_p} \\
C &= \frac{M_C}{M_p}
\end{align*}
\]

(5.8)

in place of the moments. The non-dimensionalizing factor \( M_p \) may be 
regarded as any convenient moment value because we are concerned only 
with statics, exclusive of plastic moment conditions, in this article. 
(Later we will consider \( M_p \) as a required plastic moment but we want to 
avoid this connotation in the context of statical considerations, for 
the reasons indicated at the end of this article.) The statical 
equations which follow are further simplified by the dimensionless 
factor

\[
R = \frac{M_p}{M_{pm}} = \frac{16 M_p}{F w L^2 g}
\]

(5.9)*

which is a convenient substitution for the gravity load, span, and load 
factor. The maximum ordinate of the simple beam moment diagram in 
Fig. 5.1(d) is \((2/R) M_p\). 

*See footnote on Eq. (5.5)
To find an expression for $M_C$ it is expedient to first determine the distance $X$ from the right end of the girder cross-section where the maximum positive moment acts. Using the right hand freebody diagram in Fig. 5.1(e) and noting that $V_C = 0$, we have $V_B = F w X$. But $V_B$ is also given by Eq. (5.3). Hence, by equating these two expressions for the right end-shear and using Eqs. (5.8 and 5.9) we obtain

$$\frac{X}{L_g} = \frac{1}{2} + (A + B) \frac{R}{16} \quad (5.10)$$

This result is valid for any values of $A$, $B$, and positive $R$. Notice that if $X < 0$ the maximum positive moment $M_C = -M_B$ occurs at the right end of the girder. The negative sign is needed here because a moment causing tension on the bottom of the girder at the right end is a negative end-moment according to our clockwise positive end-moment convention. On the other hand, if $X > L_g$ then the maximum positive moment $M_C = +M_A$ occurs at the left end of the girder as in Fig. 1.4(m).

If the maximum positive moment lies within the span $(0 < X < L_g)$, we may again refer to the right hand freebody diagram in Fig. 4.1(e) to find $M_C$, using the known value of $X$. After non-dimensionalizing we obtain the moment factor

$$C = -B + \frac{2}{R} \left(1 + \frac{R}{8} (A + B)\right)^2 \quad (5.11a)$$

$$C = +A + \frac{2}{R} \left(1 - \frac{R}{8} (A + B)\right)^2 \quad (5.11b)$$

for $-8 \leq (A + B) R \leq 8$
which gives \( M_c \) from Eq. (5.8). The bounds on \((A + B) R\) follow from the bounds on \(X\). The left hand freebody diagram in Fig. 5.1(e) gives Eq. (5.11b).

The product \((A + B) R\) in Eqs. (5.10 and 5.11) can be evaluated from Eqs. (5.7 and 5.9) with the result

\[
(A + B) R = G
\]  

(5.12)

Then, Eqs. (5.10 and 11) can be written, including the girder sway moment coefficient \(G\), in the form

\[
\frac{X}{L_g} = \frac{1}{2} + \frac{1}{16} G
\]  

(5.13)

\[
C = - B + \frac{2}{R} \left( 1 + \frac{G}{8} \right)^2
\]  

(5.14a)

\[
C = + A + \frac{2}{R} \left( 1 - \frac{G}{8} \right)^2
\]  

(5.14b)

for \(-8 \leq G \leq 8\)

Equations (5.14) are valid if the maximum positive moment occurs between the ends of the girder. Once we have distributed the sway moments to the girders, Eq. (5.12 to 5.14) can be used to estimate the girder moments as a function of one of the end-moment factors, \(A\) or \(B\).

Equations (5.10 and 5.11) represent statical relations between the five parameters \(A, B, C, R,\) and \(X/L_g\). Given three of these parameters we can find the remaining two from Eq. (5.10 and 5.11) or the
alternatives:

For \( 0 \leq (C + B)R \leq 8 \)

\[
A = -B + \frac{8}{R} \left( \sqrt{\frac{1}{2} (C + B)R} - 1 \right)
\]

\[
\frac{X}{L_g} = \sqrt{\frac{1}{8} (C + B)R}
\]

For \( 0 \leq (C - A)R \leq 8 \)

\[
B = -A - \frac{8}{R} \left( \sqrt{\frac{1}{2} (C - A)R} - 1 \right)
\]

\[
\frac{X}{L_g} = 1 - \sqrt{\frac{1}{8} (C - A)R}
\]

These statically derived expressions involve \( R \) and pairs of values of \( A \), \( B \), and \( C \) on the right hand side. If \( A \), \( B \), and \( C \) are known and \( 0 < X < L_g \), we can find \( R \) from

\[
R = \frac{16}{(A + B)^2} \left( C - \frac{1}{2} (A - B) - \sqrt{(C - A)(C + B)} \right)
\]

if \( (A + B) \neq 0 \)

These equations are recorded here for convenient reference. The plastic moment balancing process uses Eqs. (5.10 to 5.17) but in a simplified form.

Note that we have not enforced any plastic moment conditions on the girder moments in this statical discussion. (Plastic moment
conditions are applied in the next article.) This means that all of Eqs. (5.1 to 5.17) may be applied at any load level, including working load.

For example, if estimates of the girder sway moment coefficient G, and one end-moment factor A or B, are available at working load (F = 1 in Eqs. (5.5 and 5.9)), we can use Eq. (5.14) to obtain the statically defined maximum positive moment factor C. Then Eqs. (5.15 or 5.16) completely define the girder moment diagram at working load. We may use this diagram for deflection calculations or for secondary design checks.

For a second example, we may need the length of the negative moment region, \( x_{pi} \), in Fig. 5.1(d), to check lateral bracing requirements for the bottom flange of a girder. The right hand freebody diagram in Fig. 5.1(e) may be used to establish that

\[
\frac{x_{pi}}{L_g} = \frac{x}{L_g} \left(1 - \sqrt{\frac{C}{C + B}}\right)
\]

for \( 0 \leq x \leq L_g \). The value of X at any load level may be obtained from the statical equations in this article by properly choosing the load factor F.

Table 3.1 indicates that either gravity loading or combined loading may control the size of a particular girder. In addition to the girder moments for the governing loading condition we also need the girder moments for the other noncontrolling load because these latter moments may be critical in the design of girder cutouts and splices or
of adjacent columns. This may happen for instance when the noncontrolling girder moments result in an unfavorable column moment gradient. The statics relations in this article are of value in estimating girder moments for the noncontrolling loading condition. This is the most cogent reason for considering the girder statics without reference to plastic moment conditions. Girder moments for noncontrolling loading conditions are considered in Art. 5.4.

5.2 PLASTIC MOMENT BALANCE FOR GIRDERS

The discussion of girder moments in Art. 1.5 (Fig. 1.4) and the statics equations in Art. 5.1 may be blended to give a method for balancing the moments in a girder, while conserving the girder sway moment. We will assume that the girder has a constant plastic moment capacity $M_p$ along its length. Then the moment factors $A$, $B$, and $C$ in Eq. (5.8) are the ratios of the girder moment to the plastic moment. These moment factors are bounded by $-1 \leq (A, B, C) \leq +1$, if we neglect potential strain hardening effects.

In the definition of the dimensionless load parameter $R$, Eq. (5.9), we will take the load factor $F$ to be either, the required load factor $F_{1R}$ for gravity loading, or $F_{2R}$ for combined loading. If, under factored gravity loading with $F = F_{1R}$, the girder forms a three-hinged mechanism, then $-A = B = C = +1$ and $R = +1$ from Eq. (5.9). This is a lower bound for $R$ under gravity loading ($R \geq 1$), because the girder must be at least strong enough to carry the factored gravity load $F_{1R} W_L g$. 
Conditions Applied in Girder Plastic Moment Balance

We are primarily interested in the girder moments for the combined loading condition. The moment balancing problem for combined loading may be described in graphical form with reference to the moment diagram in Fig. 5.1(d). The statically determinate part of this diagram is fixed by the known factored gravity load $F_{2R} w L_g$. Our problem is to locate the redundant fixing line (Art. 1.5) with three conditions in mind:

Condition (1) - The end-moment sum $M_A + M_B$ must conserve the girder sway moment $M_G$ assigned in the sway moment distribution (Art. 4.4).

Condition (2) - The plastic moment must not be exceeded at any girder cross-section.

Condition (3) - The plastic moment should be the minimum required to support the factored gravity load plus the assigned sway moments.

Suppose we start with an arbitrary horizontal fixing line, shown dashed, in Fig. 5.1(d). If $M_G$ is positive (wind from left to right), we must rotate the fixing line clockwise until the end-moment sum $M_A + M_B = M_G (1 - d_c/L)$. In effect, this determines the slope of the fixing line. Any fixing line with the required slope satisfies condition (1). Condition (2) is also satisfied if we take $M_p$ as the maximum vertical distance between the statically determinate moment diagram and any fixing line with the required slope. If we consider a family of parallel fixing lines with the required slope, one and only one of these lines will give equal maximum moments at two girder
cross-sections. This uniquely defined fixing line satisfies all three of the above conditions and serves to balance the maximum girder moments. Hence the description, plastic moment balance.

We are justified in locating the redundant fixing to satisfy conditions (1 to 3) if the girder provides sufficient rotation capacity to permit redistribution of the girder moments. Secondary design checks which guarantee adequate rotation capacity must be applied in this as in any plastic design method (Refs. 3 and 5).

Briefly stated, the plastic moment balance for a girder is the statically derived process of locating the redundant fixing line to satisfy an assigned girder sway moment and the minimum plastic moment condition.

The plastic moment balance may be stated in algebraic terms by assigning values for two of the three moment factors A, B, C in Eq. (5.8). If the girder has a constant plastic moment capacity along its length the plastic moment condition is satisfied for any values of A, B, C within the bounds

\[-1 \leq A \leq 1, \quad -1 \leq B \leq 1, \quad -1 \leq C \leq 1 \] (5.19)

These bounds may be combined with one of the conditions:

\[ B = C = +1 \quad \text{and} \quad -1 \leq A \leq 1 \] (5.20a)

or

\[ -A = C = +1 \quad \text{and} \quad -1 \leq B \leq 1 \] (5.20b)

or

\[ A = B = +1 \quad \text{and} \quad C = A \] (5.20c)

or

\[ A = B = -1 \quad \text{and} \quad C = -B \] (5.20d)
to give equal plastic moments at two girder cross-sections. Moment diagrams for each of Eqs. (5.20) are shown in Fig. 5.2.

Cases (a) and (b) in Fig. 5.2 may be applied to girders in the middle level zone, several floors below the roof of an unbraced frame. In this height zone, the sagging plastic hinge at C normally occurs between midspan and the leeward end of the girder, causing the girder sway mechanism in Fig. 1.5(a). Cases (c) and (d) in Fig. 5.2 may occur on the bottom levels of a tall unbraced frame. Here the sagging plastic hinge moves to the windward end of the girder. Cases (a) and (c) correspond to wind from left to right.

Plastic Moment Envelopes

It is useful to describe the girder plastic moment balance in graphical form, using plastic moment envelopes. The dashed curves in Fig. 5.2 are termed positive plastic moment envelopes. The positive plastic moment envelope for a girder with uniform plastic moment capacity $M_p$ is obtained by translating the statically determinate, simple span, gravity load moment diagram upward through a distance $M_p$. If the plastic moment capacity varies with distance along the girder, this variation determines the local distance between the statically determinate moment diagram and the positive plastic moment envelope.

The significance of the positive plastic moment envelope is evident from the following facts: (1) Any redundant fixing line (Art. 1.5) which is everywhere below the positive plastic moment envelope yields maximum positive girder moments which are less than $M_p$. (2) If the fixing line is tangent to the positive plastic moment envelope, the
maximum positive girder moment $M_C = M_p$ at the point of tangency.

(3) The fixing line cannot cross the positive plastic moment envelope because this would violate the yield condition of simple plastic theory. These facts follow from the idea that the moment at any girder cross-section is the vertical distance from the statically determinate moment diagram to the redundant fixing line.

To complete the envelope concept, we can plot a second negative plastic moment envelope by shifting the statically determinate moment diagram downward through a distance $M_p$. (Note that the different positive and negative plastic moment capacities for a composite girder should be considered in establishing the positive and negative plastic moment envelopes.) Any straight line which does not cross the positive or negative plastic moment envelopes is an admissible redundant fixing line, satisfying the equilibrium and yield conditions of simple plastic theory. Plastic hinges may form at points where the fixing line touches a plastic moment envelope. This is illustrated in Fig. 5.2. This figure is concerned with girders of uniform section so negative plastic moment envelopes are not needed. Note that as the plastic moment capacity of the girder increases, the positive plastic moment envelopes shift upward and permit a larger inclination of the redundant fixing line, thus giving a larger girder sway moment capacity.

If graphical methods are preferred, the plastic moment envelope approach provides a generally valid method for performing the plastic moment balance. Any distribution of gravity loads and variation of non-uniform plastic moment capacity may be effectively considered in a graphical plastic moment balance.
Mechanism Factor

If the sagging hinge occurs within the span as in Fig. 5.2 (a and b), the limits on Eq. (5.14) indicate that the sway moment coefficient $G$ is bounded by

$$-8 \leq G \leq 8$$  \hspace{1cm} (5.21)

The upper limit corresponds to the bound between case (a) and case (c) in Fig. 5.2. If $G$ exceeds these bounds then the girder forms a sway mechanism with plastic hinges at each end under combined loading. Thus, it is a simple matter to distinguish between the hinge patterns and mechanisms in the upper and lower parts of Fig. 5.2.

We may now find the plastic moment capacity required for each of the combined load conditions in Fig. 5.2 by solving certain of the statics relations in Art. 5.1 for $R$ and using $M_p = R M_{pm}$ from Eq. (5.9). For example, consider Fig. 5.2(a) and the corresponding plastic moment factors in Eq. (5.20a). Substituting these factors in Eq. (5.14a) and solving for $R$ gives

$$R = \left(1 + \frac{G}{8}\right)^2 \quad \text{for} \quad 0 \leq G \leq 8$$ \hspace{1cm} (5.22a)

The upper bound on $R$ is then $R \leq 4$. A lower bound on $R$ will be obtained later. If we repeat this process using Eqs. (5.14b) and (5.20b) we obtain

$$R = \left(1 - \frac{G}{8}\right)^2 \quad \text{for} \quad -8 \leq G \leq 0$$ \hspace{1cm} (5.22b)

for the moment diagram in Fig. 5.2(b). Note that Eq. (5.14) is used here because the maximum positive moment occurs between the ends of the girder.
Equation (5.12) may be applied when the maximum girder moments occur at the ends. Then the plastic moment factors in Eqs. (5.20c, d) yield

\[ R = \frac{G}{2} \quad \text{for } G \geq 8 \]  
(5.22c)

\[ R = \frac{-G}{2} \quad \text{for } G \leq -8 \]  
(5.22d)

which correspond to cases (c) and (d) in Fig. 5.2. It is obvious that Eqs. (5.22b and d) are redundant if we replace \( G \) by its absolute value in Eq. (5.22a and c).

Now consider the lower bound on \( R \) for combined loading. On the levels near the roof of an unbraced frame the plastic moment capacity of the girders may be controlled by the factored gravity load \( F_{1R} w_L g \). Then \( M_p = F_{1R} w_L g \frac{2}{16} \) is the required plastic moment capacity. If we use this value of \( M_p \) in Eq. (5.9) with \( F = F_{2R} \) for the combined loading condition we obtain

\[ R_{LB} = \frac{F_{1R}}{F_{2R}} \]  
(5.23)

where the subscript \( LB \) denotes lower bound. The value of \( R \) for the combined loading condition cannot be less than \( R_{LB} \) because the girder must have at least the moment capacity required to carry the factored gravity load. For the required load factors \( F_{1R} = 1.70 \) and \( F_{2R} = F_{1R}/1.33 \), the lower bound on \( R \) for combined loading is \( R_{LB} = 1.33 \).

Thus to determine the required plastic moment capacity for a girder of uniform \( M_p \), consistent with an assigned sway moment coefficient \( G \), we may use Eq. (5.22) to find \( R \) for the combined load.
condition. If \( R > R_{LB} \), then \( M_p = R M_{pm} \) is the required plastic moment as controlled by the combined load. But if \( R \leq R_{LB} \), then \( M_p = R_{LB} M_{pm} \) is the required plastic moment as controlled by gravity loading.

In the context of this plastic moment balancing discussion, the value of \( R \) from Eq. (5.22) indicates both the controlling loading condition and the girder mechanism for the controlling loading condition. For this reason, \( R \) as determined from Eq. (5.22) is aptly termed the mechanism factor for the plastic moment balance. The mechanism factors are associated with the corresponding moment diagrams in Fig. 5.2.

It may be noted that if \( F_2 > F_1 R \) (as suggested in Ref. 8), Eq. (5.23) gives \( R_{LB} < 1 \). Then any nonzero value of \( G \) in Eq. (5.22) gives \( R > 1 > R_{LB} \) and combined loading controls the design of all girders. This idea serves to simplify the preliminary design but it will not be pursued further in this study because it appears to be an uneconomical and unnecessary procedure.

Normally, the load factor ratio in Eq. (5.23) gives a value of \( R_{LB} < 4 \). Then we may distinguish three cases which govern the value of \( M_p \) in terms of \( R \) from Eq. (5.22):

\[
\begin{align*}
\text{Case I:} & \quad R_{LB} < R < 4 \\
\text{Case II:} & \quad R \geq 4 \\
\text{Case III:} & \quad R \leq R_{LB}
\end{align*}
\] (5.24)

The moment diagrams in Fig. (5.2a and b) belong to case I in which the maximum positive moment at the sagging hinge occurs between the ends.
of the girder. Case II is associated with Fig. (5.2c and d) where the sagging hinge occurs at the windward end of the girder. Case III is obtained when gravity loading controls \( M_p \).

**Design Chart for Girder Mechanisms**

A graph of the mechanism factor \( R = \frac{M_p}{M_{pm}} \) versus the sway moment coefficient \( G = (M_G/M_{pm})(1 - d_c/L) \) is given in Fig. 5.3 which applies to girders with uniform plastic moment capacity. The value \( R_{LB} = 1.33 \) for the required load factors \( F_{1R} = 1.70 \) and \( F_{2R} = F_{1R}/1.33 \) is indicated on the vertical axis. The corresponding value of

\[
G = 8\left(\sqrt{R} - 1\right) \quad \text{for} \quad R_{LB} \leq R \leq 4
\]

from Eq. (5.22a), is \( G = 1.24 \). The domains for the mechanism factor cases which control \( M_p \), as defined in Eq. (5.24), are indicated in the insets together with the corresponding girder mechanisms. Equations (5.22a and c) give the curve in Fig. 5.3 which consists of a parabola, with vertex at \( (G = -8, R = 0) \), in the domain \( 0 \geq G \geq 8 \); and a linear tangent with slope \( dR/dG = 1/2 \) in the domain \( G \geq 8 \).

If \( G \) and \( R \) fall within the dashed rectangle defined by \( R_{LB} \) in Fig. 5.3, then the required plastic moment capacity \( M_p = R_{LB} M_{pm} \) is controlled by gravity loading and the maximum sway moment capacity of the girder can be found from Eq. (5.25) with \( R = R_{LB} \). If \( G \) or \( R \) fall outside of the dashed rectangle, then the required plastic moment capacity \( M_p = R M_{pm} \), is controlled by combined loading.
Figure 5.3 considers positive values of the sway moment coefficient $G$ for wind from left to right. The graph of $R$ versus $G$ for negative values of $G$ is the mirror image of Fig. 5.3, reflected about the vertical $R$ axis. The same mirror image idea also applies to the moment diagrams in Fig. 5.2. In mathematical terms, the mirror image corresponds to using the absolute value of $G$ in Eq. (5.22).

We may use Fig. 5.3 in two different ways. First we can find the plastic moment required to resist known uniformly distributed gravity loads plus sway moments by entering Fig. 5.3 with the sway moment coefficient $G = \left(\frac{M_g}{M_{pm}}\right)(1 - \frac{d}{L})$. The curve gives the mechanism factor $R$ from which we find $M_p = R M_{pm}$. Second, we may find the sway moment capacity $M_g$ for a girder with known $M_p$ and gravity loads, by entering Fig. 5.3 with $R$ and finding $G$. In either case, the governing loading condition, and mechanism are immediately evident.

When combined loading controls $M_p$, we may define an effective load coefficient $F_{1E} > F_{1R}$ such that

$$M_p = F_{1E} \frac{1}{16} w L_g^2$$

Thus $F_{1E}$ plays the role of a load factor which however has only a superficial relation to the required gravity load factor $F_{1R}$. We may relate $F_{1E}$ to the mechanism factor $R$ using Eq. (5.9) with $F = F_{2R}$ for combined loading, in the form

$$F_{1E} = R F_{2R}$$

The effective load coefficient idea is included here to show how the description of plastic moment balancing in this article is related to
the work in Ref. (70) and Chapter 14 of Ref. (6). The parameter \( F_{1E} \) seemed appropriate in the initial work on plastic moment balancing (Ref. 20) but it can lead to some confusion between required gravity load factors and effective load coefficients. The mechanism factor \( R \) from Eq. (5.22) has a more basic meaning in plastic moment balancing, and is used in this study in place of the effective load coefficient \( F_{1E} \).

**Steps in Plastic Moment Balance**

The steps in the plastic moment balance for a girder with uniform \( M_p \) are summarized in Fig. 5.3. We begin with the uniformly distributed (dead plus reduced live) working load \( w \), the clear span \( L \), the required load factors \( F_{1R} \) for gravity loading and \( F_{2R} \) for combined loading, the assigned sway moment \( M_G = M_{JA} + M_{JB} \), and the column depth to span ratio \( d_c/L \). The plastic moment balance proceeds in four steps.

**Step 1** - Find the load parameter \( M_{pm} \) and the sway moment coefficient \( G \) using the equations recorded in Fig. 5.3. The ordinate of the simple beam moment diagram is fixed by \( M_{pm} \) and the slope of the redundant fixing line is set by \( G \).

**Step 2** - Find the mechanism factor \( R \) from Eq. (5.22) or the curve in Fig. 5.3. The plastic moment balance is performed in this step.

**Step 3** - Find the lower bound \( R_{LB} = F_{1R}/F_{2R} \) for the mechanism factor. At this stage the controlling load, and mechanism are obtained from Fig. 5.3.
Step 4 - Find the required $M_p$ from the simple formulas at the bottom of Fig. 5.3.

It is evident that the plastic moment balancing process is appropriate for manual or for computer application.

**Design Refinements**

Girder sizes may be selected to satisfy the required plastic moment. Several girders on a level will frequently provide some excess plastic moment capacity because of the discrete distribution of $M_p$ for $W$ shapes. This excess moment capacity can be utilized by selecting $W$ shapes for all but one bay on a level. The sway moment capacity of these shapes can be determined by using Fig. 5.3 to find $G$. Then the sway moment in the remaining bay, which is required to conserve the sway moment sum on the level, can be used to select the remaining girder size. This amounts to redistributing the sway moments on the level to suit a particular set of girder sizes. Example 16.2 in Ref. 6 illustrates this optional design refinement. Justification for this refined design procedure depends in part on the degree of refinement included in the sway moment sum for the level.

**5.3 Restricted Hinge Patterns for Girders**

The plastic moment balance in Art. 5.2 gives the requires $M_p$ for each girder. If the selected $W$ shape provides no excess plastic moment capacity, the girder forms a mechanism under the controlling factored load. When the mechanism forms, the girder contribution to sway stiffness in the adjacent stories is lost. Article 1.9 suggests
that sway stiffness may be conserved by using a restricted hinge pattern for one or more girders on a level. A girder with a restricted hinge pattern is intended to approach but not to reach a mechanism condition at ultimate load. This is a means for controlling the \( P_{ex} \) effect under combined loading and sidesway frame buckling under gravity loading in the preliminary design phase, before members are selected. This article demonstrates how restricted girder hinge patterns can be specified in the plastic moment balance for girders with uniform \( M_p \).

One approach to restricting a girder hinge pattern is to find the \( M_p \) required for a mechanism and then select a \( W \) shape with larger plastic moment capacity. It is then possible to find a unique redundant fixing line which satisfies conditions (1) and (2) in Art. 5.2, but which results in only one negative moment hinge for combined loading or two such hinges for gravity loading. The maximum positive girder moment \( M_C \) corresponding to this unique fixing line is less than \( M_p \) but the moment factor \( C = M_C/M_p \) is not immediately known for combined loading.

If \( C \) is close to unity, a small redistribution of sway moments on the level can result in girder mechanisms in the bay or bays where girder mechanisms are not intended. Thus, it is desirable to control the value of the positive moment factor \( C \). In other words, instead of finding \( M_p \) from the mechanism condition which we want to avoid, we can find \( M_p \) from the restricted hinge pattern which we want to occur, by specifying the value of \( C < 1 \).
Attempts to restrict negative hinges at the ends of a girder are analytically possible. Unless the girder is proportioned to remain elastic at ultimate load, it is quite likely that a hinge will form first at the leeward end of the girder, regardless of analytical assumptions to the contrary. Since the girder end-moments also influence joint equilibrium and column moments, attempts to restrict negative end hinges can give misleading results.

**Conditions Applied in Girder Plastic Moment Balance**

Article 4.2 defines the three conditions which are satisfied in the plastic moment balance. Here, we add a fourth condition which may be applied to achieve a restricted hinge pattern.

**Condition (4)** - The maximum positive girder moment must not exceed \( C M_p \) where \( C \leq 1 \) is the moment factor for positive moment.

Figure 5.4 graphically indicates the results of the restricted hinge condition for combined loading. Hinges with plastic moment capacity \( M_p \) are assumed to occur at the leeward end of the girders. The sway moment resisted by a girder depends on how far the fixing line rotates about the leeward hinge. The maximum possible rotation of the fixing line is limited by the positive plastic moment envelopes. These envelopes are shown as dashed curves at a distance \( M_p \) above the simple beam moment diagrams in Fig. 5.4. The limiting fixing lines are also shown dashed.

A second positive moment envelope is shown as a dash-dot curve at a distance \( C M_p \) above the simple beam moment diagram or at a distance...
(1 - C) \( M_p \) below the dashed positive plastic moment envelope in Fig. 5.4. The maximum positive moment is limited to \( C M_p \) for the solid fixing lines which are tangent to the dash-dot positive moment envelopes. This enforces condition (4) in addition to conditions (1) to (3) in Art. 5.2.

Restricted girder hinge patterns tend to reduce the sway moment, Eq. (5.6), resulting from the end-moment sum, Eq. (5.7). The end-moment sum is reduced by an amount larger than \( (1 - C) M_p \) in Fig. 5.4 (a and b) where the maximum positive moment occurs between midspan and the windward end of the girder. In Fig. 5.4 (c and d) the maximum positive moment occurs at the windward end and the end-moment sum is reduced by \( (1 - C) M_p \). A larger plastic moment capacity is required in each case to conserve an assigned sway moment. The increase in \( M_p \) is a function of the positive moment factor \( C \).

When the restricted hinge condition is applied, the moment factors in Eq. (5.20) of Art. 5.2 must be replaced by

\[
B = +1, \quad 0 < C \leq +1, \quad \text{and} \quad -1 \leq A \leq C \quad (5.28a)
\]

or

\[
A = -1, \quad 0 < C \leq +1, \quad \text{and} \quad -C \leq B \leq +1 \quad (5.28b)
\]

or

\[
B = +1, \quad 0 < C \leq +1, \quad \text{and} \quad A = +C \quad (5.28c)
\]

or

\[
A = -1, \quad 0 < C \leq +1, \quad \text{and} \quad B = -C \quad (5.28d)
\]

as indicated in Fig. 5.4. These equations express the restricted hinge pattern concept in algebraic form.
Restricted Mechanism Factor

We may combine Eq. (5.28) with appropriate statics equations in Art. 5.1 to obtain the mechanism factor $R$ in $M_p = R M_{pm}$. Following the procedure in Art. 5.2 we obtain the results

\[
R = \frac{2}{C + 1} \left( 1 + \frac{G}{\theta} \right)^2 \quad \text{for} \quad 0 \leq G \leq \theta \quad (5.29a)
\]

\[
R = \frac{2}{C + 1} \left( 1 - \frac{G}{\theta} \right)^2 \quad \text{for} \quad -\theta \leq G \leq 0 \quad (5.29b)
\]

\[
R = \frac{G}{C + 1} \quad \text{for} \quad G > \theta \quad (5.29c)
\]

\[
R = \frac{-G}{C + 1} \quad \text{for} \quad G < -\theta \quad (5.29d)
\]

which apply for combined loading. These equations express the mechanism factor $R$ in terms of the sway moment coefficient $G$ from Eq. (5.7) and the positive moment factor $C$. If we set $C = 1$, then Eqs. (5.29a and 5.20) are identical.

The mechanism factor equations (5.29) are related to the girder moment diagrams in Fig. 5.4. If $C < 1$ these moment diagrams do not correspond to girder mechanisms but rather to restricted girder hinge patterns. For this reason, values of $R$ from Eq. (5.29) for $C < 1$ are termed restricted mechanism factors.

A lower bound for $R$ under combined loading may be established by first considering minimum plastic moment requirements for the gravity load condition as in Art. 5.2. It may be advisable to specify
restricted hinge patterns for gravity loading in an effort to control sidesway frame buckling (Art. 1.9) in the preliminary design stage. Again, attempts to restrict negative end hinges can give misleading results. Hence we are led to application of condition (4) for restricted hinge patterns under gravity loading. However, the positive moment factors for gravity load and combined load need not be identical. The subscript 1 will be used to identify the positive moment factor

\[ C_1 = \frac{M_c}{M_p} \]

for gravity loading.

Figure 5.5 graphically illustrates the results of the restricted hinge condition for gravity loading. Hinges with plastic moment capacity \( M_p \) are assumed at each end of the girder but the maximum positive moment \( M_c = C_1 M_p \) produces no positive hinge at midspan under the factored gravity load \( F_{LR} w L_g \) if \( C_1 < 1 \). The dashed redundant fixing line in Fig. 5.5 corresponds to a three-hinged gravity load mechanism and the standard value \( C_1 = 1.0 \). The solid fixing line for a restricted hinge pattern is below that for the gravity load mechanism. If we take \( C_1 = 0.5 \), the solid fixing line corresponds to the elastic fixed-end moment condition.

For any positive moment factor in the range \( 0 \leq C_1 \leq 1 \), the plastic moment required for factored gravity loading with a restricted hinge pattern is

\[ M_p = \frac{2}{1 + C_1} \frac{1}{16} F_{LR} w L_g^2 \]

which indicates the inverse relation between \( M_p \) and \( C_1 \). The lower bound for \( R \) under combined loading is obtained from Eq. (5.9) with \( F = F_{2R} \),
and Eq. (5.30), with the result

\[
R_{LB} = \frac{2}{1 + C_1} \left( \frac{F_1R}{F_2R} \right)
\]  

(5.31)

This result reduces to Eq. (5.23) for \( C_1 = 1 \) and gives values between 1.0 and 1.33 times Eq. (5.23) for \( 1 > C_1 > 0.5 \).

**Design Chart for Restricted Hinge Patterns in Girders**

A graph relating the restricted mechanism factor \( R \) to the sway moment coefficient \( G \) is shown in Fig. 5.6 which applies to girders with uniform plastic moment capacity. The vertical axis in this figure uses the product \( R (C + 1) \). Otherwise, there is little difference between Fig. 5.6 for restricted girder hinge patterns and Fig. 5.3 for girder mechanisms. If \( C = 1 \), it is easier to use Fig. 5.3 but identical results are obtained from Fig. 5.6.

All of the remarks describing Fig. 5.3 in Art. 5.2 may be extended to Fig. 5.6 if we replace Eq. (5.25) by

\[
G = 8 \left( \sqrt{\frac{C + 1}{2} R - 1} \right) \quad \text{for} \quad R_{LB} \leq \frac{C + 1}{2} R \leq 4 \quad (5.32)
\]

and take \( R_{LB} \) from Eq. (5.31). Equation (5.32) gives the sway moment coefficient \( G \) in terms of the positive moment factor \( C \) for combined load and the restricted mechanism factor \( R = M_p/M_{pm} \). Thus the sway moment capacity for a girder with known \( M_p \) is found by entering Fig. 5.6 with the product \( R (C + 1) \) on the vertical axis and finding \( G \) on the horizontal axis. In particular, the sway moment capacity for a girder
with \( M_p \) controlled by a restricted gravity load hinge pattern \((R = R_{LB})\) can be obtained from

\[
G = 8 \left( \sqrt{\frac{C + 1}{C_1 + 1} \cdot \frac{F_{1R}}{F_{2R}}} - 1 \right)
\]  

(5.33)

if \( R_{LB} \leq 4 \) in Eq. (5.31). The required load factors \( F_{1R} = 1.70 \), \( F_{2R} = F_{1R}/1.33 \), and the positive moment factors \( C = C_1 = 1.0 \) give \( R_{LB} (C_1 + 1) = 2.67 \) from Eq. (5.31) and \( G = 1.24 \) from Eq. (5.33). These values are represented by the dashed rectangle on the graph in Fig. 5.6.

The restricted mechanisms in the insets in Fig. 5.6 include solid circles to indicate plastic hinge locations and open circles to indicate where maximum positive moments occur. Hinges are restricted (avoided) at these open circles if the positive moment factors for the controlling load are less than unity. These moment factors are indicated above the mechanisms in Fig. 5.6. The three cases which govern the value of \( M_p \) in terms of \( R \) from Eq. (5.29) are

| Case I; \( R_{LB} (C_1 + 1) < R (C + 1) < 8 \) |
| Case II; \( R (C + 1) \geq 8 \) |
| Case III; \( R (C + 1) \leq R_{LB} (C_1 + 1) \) |

(5.34)

The domains for each case are recorded in the insets in Fig. 5.6.

To establish the gravity load domain, Case III, in Fig. 5.6 we take \( R = R_{LB} \) from Eq. (5.31) and enter on the vertical axis with the product.
and read the corresponding value of \( G \) from the graph. Alternatively, we may find the limiting value of \( G \) from Eq. (5.33) if \( R_{LB} \leq 4 \). If \( R \) or \( G \) fall within the gravity load domain, Case III, the required plastic moment is \( M_p = R_{LB} M_{pm} \). When \( R \) or \( G \) fall within the combined load domain, Cases I or II, we find the required plastic moment from

\[
M_p = R_{pm}
\]

The restricted hinge condition extends the scope and validity of the plastic moment balancing process with a trivial increase in design effort. The four step plastic moment balance for a girder with uniform plastic moment and a restricted hinge pattern, defined by the positive moment factors \( C \) and \( C_1 \), is summarized in Fig. 5.6. Restricted girder hinge patterns and moment factors are further considered in Chapter 8.

### 5.4 GIRDER MOMENTS

In addition to the required plastic moment capacity we also need values for the girder end-moments and the girder joint moments; both for the controlling and the non-controlling load conditions. This article considers the girder moments in the following sequence:

1. Gravity load moments
   a. Gravity load controls \( M_p \)
   b. Combined load controls \( M_p \)
2. Combined load moments
   a. Combined load controls $M_p$
   b. Gravity load controls $M_p$

3. Joint moments for all load conditions.

It will be assumed that wind acts from left to right for combined load. Moment diagrams for wind acting from right to left are the mirror image of those considered in this article. This is valid if the same sway moment distribution is assumed for each wind direction, and if the girder is of uniform cross-section. It is further assumed that restricted girder hinge patterns (Art. 5.3) are used to find $M_p$. The moment equations may be specialized for girder mechanisms (Art. 5.2) by taking $C = C_1 = 1$ in appropriate terms.

Gravity Load Moments

The moment diagram for the factored gravity load $F_{1R}wL_g$ is shown in the inset on Fig. 5.7. For purpose of preliminary design we shall assume that the gravity load moments are symmetrical about midspan so that the left end-moment $M_A = -M_B$. Since the midspan ordinate of the simple beam moment diagram is $2M_{pm} \left( \frac{F_{1R}}{F_{2R}} \right)$ the sum of the left end moment $M_B$ and the maximum positive moment $M_C$ must be

$$\frac{(M_B + M_C)/M_{pm}}{F_{1R}/F_{2R}} = 2$$

(5.36)

in dimensionless form. It is helpful to note that $M_{pm} = F_{2R}wL_g^2/16$ and that the nondimensionalizing factor $M_{pm} \left( \frac{F_{1R}}{F_{2R}} \right)$ is simply the plastic moment $F_{1R}wL_g^2/16$ required for a factored gravity load.
mechanism. Equation (5.36) applies regardless of which loading condition controls $M_p$. We now proceed to find nondimensional expressions for $M_B$ and $M_C$.

If gravity load controls $M_p$, with a restricted girder mechanism governed by the positive moment factor $C_1$, then $M_B = M_p$ and $M_C = C_1 M_p$. The dimensionless equations

$$
\frac{M_B}{M_{pm}} = \frac{2}{C_1 + 1} \quad (5.37a)
$$

$$
\frac{M_C}{M_{pm}} = \frac{2 C_1}{C_1 + 1} \quad (5.37b)
$$

sum to 2 as required by Eq. (5.36). The equation for $M_B$ derives from $M_B = M_p = R_{LB} M_{pm}$ with $R_{LB}$ from Eq. (5.31). We also find the limit

$$
\frac{R}{F_{1R}/F_{2R}} \leq \frac{2}{C_1 + 1} \quad (5.37c)
$$

from Eq. (5.31). When the restricted mechanism factor $R$ satisfies this limit, gravity load controls $M_p$, $M_B$, and $M_C$.

Now consider the case where combined loading controls $M_p$. As a preliminary design approximation we may use the elastic fixed-end moment diagram for the gravity load $F_{1R} w_L g$ if the plastic moment condition is not violated. Once member sizes have been selected, the fixed-end moments can be distributed, if necessary for a moment check in adjacent columns and girders. In dimensionless form the fixed-end state gives
and the plastic moment condition limits $M_B$ to

$$\frac{M_B}{M_{pm}} \leq \frac{R}{F_{1R}/F_{2R}}$$

The gravity load moment plot in Fig. 5.7 graphically illustrates Eqs. (5.37 to 5.39) together with the plastic moment equations

$$\frac{M_p}{M_{pm}} = \frac{2}{C_1 + 1} \quad \text{for } R \leq R_{LB}$$

$$\frac{M_p}{M_{pm}} = \frac{R}{F_{1R}/F_{2R}} \quad \text{for } R \geq R_{LB}$$

This plot uses the ratio $R \cdot (F_{1R}/F_{2R})$ as the abscissa and

$(M/M_{pm}) \cdot (F_{1R}/F_{2R})$ as the ordinate, with $M = M_B, M_C$, or $M_p$. The domains for the three restricted mechanism cases defined in Eq. (5.34) are recorded at the bottom of the graph. The boundary between Case I and Case II depends on the load factors and the positive moment coefficient $C$ for combined load. The values $F_{1R} = 1.70, F_{2R} = F_{1R}/1.33$ and $C = 0.8$ are used to plot this boundary in Fig. 5.7. Except for the Case (I, II) boundary, the data in Fig. 5.7 is independent of the load factors unless $F_{2R} > F_{1R}$. This is the reason for choosing $(F_{1R}/F_{2R})$ as a nondimensionalizing parameter.
Several features of Fig. 5.7 deserve comment. The sum of the ordinates for $M_B$ and $M_C$ is always 2 as required by Eq. (5.36). In the gravity load domain (Case III) the values of $M_B$ and $M_C$ depend upon the positive moment factor $C_1$ for gravity load. Lines for $C_1 = 0.8$ (restricted hinge pattern) and $C_1 = 1.0$ (mechanism) are labeled in Fig. 5.7. Note that $M_C \leq M_B$ for $C_1 \leq 1.0$. If $1.0 \leq C_1 \leq 0.5$, the lines for $M_B$ and $M_C$ lie between the ordinates $4/3$ and $2/3$. The heavy dashed portion of these lines in the gravity load domain corresponds to Eq. (5.37) but only the point at $R = R_{LB}$ on these lines is used in practice. Values of $C_1 < 0.5$ are unrealistic for a girder with uniform cross-section because the gravity load moments would then tend to be governed by the fixed-end condition rather than by the plastic moment balance. In the gravity load domain and in the initial stage of the combined load domain (Case I) $M_B$ and $M_p$ coincide. It would obviously be unrealistic to retain the assumption $M_B = M_p$ for gravity loading throughout the combined load domain. Thus, Eq. (5.39) ceases to govern $M_B$ when the end-moment reaches the clear span fixed-end moment condition. This occurs when

$$\frac{R}{F_{1R}/F_{2R}} = \frac{4}{3}$$

(5.38c)

from Eqs. (5.38a and 5.39). Figure 5.7 completely summarizes the preliminary design assumptions for factored gravity load moments caused by uniformly distributed loads.
Combined Load Moments

Two statics relations which are of value in determining girder moments due to combined load, with wind from left to right, are

\[
\frac{M_A}{M_{pm}} = G - \frac{M_B}{M_{pm}} \tag{5.41}
\]

and

\[
\begin{align*}
\frac{M_C}{M_{pm}} &= 2 \left(1 + \frac{G}{8}\right)^2 - \frac{M_B}{M_{pm}} \quad \text{for } 0 \leq G \leq 8 \tag{5.42a} \\
\frac{M_C}{M_{pm}} &= G - \frac{M_B}{M_{pm}} \quad \text{for } G > 8 \tag{5.42b}
\end{align*}
\]

These equations give the left (windward) end-moment \( M_A \) and the maximum positive moment \( M_C \) in terms of the right (leeward) end-moment \( M_B \).

Equation (5.41) follows directly from the definition of the sway moment coefficient \( G \) in Eq. (5.7). To obtain Eq. (5.42) we may use \( R = \frac{M_p}{M_{pm}} \) in Eq. (5.14a) for \( 0 \leq G \leq 8 \) and \( M_C = M_A \) for \( G > 8 \). The terms \( 2 \left(1 + \frac{G}{8}\right)^2 \) and \( G \) in Eq. (5.42) may be replaced by \( R \left(C + 1\right) \) from Eq. (5.29) to give

\[
\frac{M_C}{M_{pm}} = R \left(C + 1\right) - \frac{M_B}{M_{pm}} \quad \text{for } R \left(C + 1\right) \geq 2 \tag{5.42c}
\]

Note that \( R \) in Eq. (5.42c) is a sway moment parameter which may be less than \( R_{LB} \). That is, we may obtain the product \( R \left(C + 1\right) \) from the curve in Fig. 5.6 for any value of \( G \), regardless of the controlling loading case. Equations (5.41 and 5.42) are statics equations which are independent of \( M_p \) or the controlling mechanism or load.
Thus, the moments for any combined load state, defined by the gravity load parameter \( M_{\text{pm}} = \frac{F_{\text{2R}} w L g^2}{16} \) and the sway moment \( M_G = G M_{\text{pm}} / (1 - \frac{d_c}{L}) \), are completely determined once the right-end moment \( M_B \) is found. If the combined loading case governs the plastic moment, \( M_p = R M_{\text{pm}} \), then \( M_B = M_p \) when a complete or restricted girder mechanism forms with wind from left to right. However, if \( M_p = R_{LB} M_{\text{pm}} \) is controlled by the factored gravity load, then \( M_B \) may be less than \( M_p \) for sufficiently small values of the sway moment. We want to know: (1) is \( M_B \) plastic or elastic and (2) how do we estimate the elastic value of \( M_B \)?

The elastic response of a girder to combined load is considered in Fig. 5.8. The girder moments due to the gravity load \( F_{\text{2R}} w L g \) are approximated by the elastic fixed-end moment diagram. This gives the dot-dash fixing line in Fig. 5.8(b) with fixed-end moments of \( (4/3) M_{\text{pm}} \). When small sway moments are applied in the elastic range, the fixing line is assumed to rotate about the elastic pivot which is located at midspan on the fixing line for the fixed-end moments. This conforms to the assumptions of the cantilever and portal methods of wind analysis and gives equal sway moments of \( (G/2) M_{\text{pm}} \) at each end of the girder.

In the elastic range the right end-moment

\[
\frac{M_B}{M_{\text{pm}}} = \frac{4}{3} \left( 1 + \frac{G}{2} \right)
\]  

(5.43)
from Fig. 5.8(b). This includes the gravity load contribution \((4/3)\)
and the sway moment contribution \((G/2)\) and answers our second question.
As the sway moment increases, the fixing line continues to rotate until
\[ M_B \] reaches \( M_p \) as in Fig. 5.8(c). This limit of elastic sway response
occurs when the sway moment coefficient reaches

\[
G_{ES} = 2 \left( R - \frac{4}{3} \right), \quad R > \frac{4}{3}
\]  

(5.44)

for \( M_B = M_p = R M_{pm} \) in Eq. (5.43). The subscript \( ES \) on \( G_{ES} \) denotes
"elastic sway" moment coefficient. When \( G \) exceeds \( G_{ES} \), the fixing
line rotates about the leeward plastic hinge, because this hinge re-
distributes the sway moments.

The elastic and plastic leeward end-moment domains are bounded
by Eq. (5.44) in Fig. 5.9. The value of the mechanism factor \( R \) may be
obtained from \( M_p \) for a \( W \) shape or from the plastic moment balance in
Fig. 5.6. In the latter case, the \( R \) and \( G \) coordinates will normally
fall within the plastic domain in Fig. 5.9 if combined loading governs
\( M_p \). However, this is not a universal rule, as discussed in Chapter 8.
Here it suffices to comment that the positive moment factor \( C \) may be
chosen to obtain entirely elastic sway response, for the purpose of
controlling frame stability.

If gravity loading controls \( M_p \) in the plastic moment balance
we may further refine the elastic-plastic leeward end-moment boundary
in Eq. (5.44). In this case, the required plastic moment \( M_p \) equals
\[ R_{LB} M_{pm} \] with \( R_{LB} \) from Eq. (5.31). Using this value of \( R_{LB} \) in Eq.
(5.44) gives
\[ G_{ES} = 2 \left( \frac{2 \frac{F_{1R}}{C_1 + 1}}{\frac{F_{2R}}{F_{2R}^2}} - \frac{4}{3} \right) \quad \text{for} \; R = R_{LB} \; \text{and} \; G \geq 0 \] (5.45)

This is the value of \( G \) at which a girder with \( M_p \) controlled by gravity load reaches the first leeward plastic hinge under increasing sway moments (Fig. 1.4e) according to our elastic sway response assumptions.

Graphs of the sway moment coefficient \( G \) versus the load factor ratio \( F_{1R}/F_{2R} \), for three values of the positive moment factor \( C_1 \) (for gravity load), are plotted in Fig. 5.10. The inset in this figure shows how each \( C_1 \) line defines the boundary between the elastic and plastic leeward end-moment domains.

The intercept for \( G = 0 \) in Eq. (5.45) is

\[ F_{1R}/F_{2R} = 2 \frac{(C_1 + 1)}{3} \] (5.46)

If the load factor ratio is less than this value, plastic hinges are produced at each end of the girder under the factored gravity load \( F_{2R} w_L g \) and zero sway moment. Then the fixed-end moment state is never reached except for \( C_1 = 0.5 \), and sway moments cause unloading of the windward plastic hinge. Numerical values for the \( G = 0 \) intercept are recorded below the graph in Fig. 5.10.

In this discussion of the girder moments for combined load, it has been assumed that plastic hinges form first at the leeward end of the girder. This assumption is valid for uniformly distributed loads and girders with uniform plastic moment capacity if the fixed-end moment state is accepted as a reasonable approximation for the gravity load moments. The moment diagrams in Fig. 5.8 indicate that, in the
elastic range, the inequalities $M_A < M_B$ and $M_C < M_B$ are always satisfied when the fixing line rotates clockwise about the elastic pivot for wind from left to right. Then $M_B$ must be the first moment to reach $M_p$.

The steps used to obtain the moment diagram for combined load, with wind from left to right, are summarized in the following outline:

**Step 1** - Check Fig. 5.9 or Eq. (5.44) to establish whether the leeward end-moment $M_B$ is plastic or elastic. (Normally this step can be deleted, but see the discussion of restricted hinge patterns in Chapter 8.) Then determine $M_B$.

**Step 2** - Use Eqs. (5.41 and 5.42) to obtain $M_A$ and $M_C$.

**Step 3** - Use Eqs. (5.13 and 5.18) to locate the sections of maximum positive moment and zero moment.

**Joint Moments**

The girder joint moments $M_{jA}$ and $M_{jB}$ for combined loading can be obtained from Eq. (5.4) once the end-moments $M_A$ and $M_B$ are known. This equation can be further simplified, using Eq. (5.7), to the form

$$
\frac{M_{jA}}{M_{pm}} = \frac{M_A}{M_{pm}} - (4 - \frac{1}{2} G) \frac{d_c/L}{1 - d_c/L}
$$

$$
\frac{M_{jB}}{M_{pm}} = \frac{M_B}{M_{pm}} + (4 + \frac{1}{2} G) \frac{d_c/L}{1 - d_c/L}
$$

(5.47a)

(5.47b)
which includes the sway moment coefficient \( G \). If we use \( M_{jA} = M_G - M_{jB} \) from Eq. (5.6) only the second of Eqs. (5.47) is needed. Thus the joint moments are a function of the sway moment \( M_G \) and only one end-moment \( M_B \).

The girder joint moments for the gravity loading condition \( (G = 0) \) are given by

\[
\frac{M_{jB}}{M_{pm}} = -\frac{M_{jA}}{M_{pm}} = \frac{M_B}{M_{pm}} + 4 \frac{F_{1R}}{F_{2R}} \frac{d_c/L}{1 - d_c/L} \tag{5.48}
\]

with \( M_{pm} = F_{2R} \frac{L}{g} \). The values of \( M_B/M_{pm} \) in Eq. (5.48) depend on the restricted mechanism factor \( R \) and can be obtained from Fig. 5.7 or the following conditions:

\[
\text{If } R \leq \frac{4}{3} \left( \frac{F_{1R}}{F_{2R}} \right), \quad \frac{M_B}{M_{pm}} = R = \frac{M_p}{M_{pm}} \geq R_{LB} \tag{5.49a}
\]

\[
\text{If } R \geq \frac{4}{3} \left( \frac{F_{1R}}{F_{2R}} \right), \quad \frac{M_B}{M_{pm}} = 4 \frac{F_{1R}}{F_{2R}} \tag{5.49b}
\]

These conditions limit the girder end-moment \( M_B \) to the required plastic moment in Eq. (5.49a), or the fixed-end moment in Eq. (5.49b). If the load factor ratio \( F_{1R}/F_{2R} \) equals 1.33, the mechanism factor boundary between Eqs. (5.49a and b) is \( R = 1.78 \). The joint moments due to the gravity load shear \( \frac{1}{2} F_{1R} \frac{L}{g} \) are included by the last term in Eq. (5.48).

5.5 SUMMARY

This chapter explains the plastic moment balance for uniformly loaded girders with uniform \( M_p \). The most important results are included
in Figs. 5.3 and 5.6 which summarize the four steps in the plastic moment balance and in Figs. 5.7 to 5.10 which help to define the moment diagrams for gravity and for combined load. The eight variables which must be specified to begin the plastic moment balance are the working load, clear span, load factors \( F_{1R} \) and \( F_{2R} \), and \( d_c/L \) ratio; the sway moment \( M_G \); and the positive moment factors \( C_1 \) for gravity load and \( C \) for combined load. The first five variables are specified by the frame geometry and loads. The sway moment distribution, considered in Chapter 4 gives \( M_G \). The last two variables must be assigned, with consideration given to the behavior desired, as explained in Chapter 8.

Operations tables for the girder plastic moment balance and the joint moments are provided in Tables A6 and A7 in Appendix 1. These operations tables explain the sequence of the plastic moment balancing examples in Tables A8 and A9.

It is evident that some of the results obtained here can be derived in a more direct manner by earlier application of plastic moment conditions. Although concise derivations have their place, a less direct but more comprehensive development is purposely used in this study by first considering girder statics and later stating the conditions which enforce the plastic moment balance.

The results in Fig. 5.3 and Eqs. (5.47 and 5.48) may be compared with Eqs. (14.12 to 25) and Fig. 14.8 in Ref. 6. That reference does not intend to consider the restricted hinge pattern concept described in Art. 5.3. Some of the data needed in plastic moment balancing can be extracted from the equations in Ref. 16 (including
discussion), from Figs. 8.51 and 8.52 in Ref. 26 and from Figs. 2 and 8 in Ref. 98. It is quite logical that other investigators have considered the same equilibrium and plastic moment conditions, which form the basis for plastic moment balancing, because these conditions are fundamental determinants of structural behavior.

After completing the plastic moment balance for each girder on a level, we may proceed to the next step which is the joint balancing operation.
6. JOINT EQUILIBRIUM

The joint balance is the third major part of the plastic moment balancing method, and provides the analytical link between the first two parts of the process which are considered in isolation from each other in Chapters 4 and 5. The analytical link of the joint balance is analogous to the physical link which the joints of a rigid frame provide in making the girders and columns work together to carry loads.

This chapter describes four joint balancing methods which are basically similar but which may be used to accomplish different objectives. One objective is the ability to duplicate the results of refined methods of inelastic analysis by properly choosing certain distribution parameters in plastic moment balancing. A second objective is to distribute the frame moments so as to conserve double curvature bending in the columns under combined loading. The means for accomplishing these objectives are described. The chapter concludes with a comparison of the four joint balancing methods.

6.1 CONTROLLED JOINT BALANCE

The joint balance may begin with any initial set of girder and column moments which satisfy certain girder and story equilibrium
requirements. These requirements are:

1. The initial column moments below level (I) must sum to the portion $D_v(I) \sum M_C(I)$ of the total sway moment in the story below level (I) which is distributed to the upper end of the columns in story (I).

2. The initial column moments above level (I) must sum to the portion $(1 - D_v(I-1)) \sum M_C(I-1)$ of the total sway moment in the story above level (I) which is distributed to the lower end of the columns in story (I-1).

3. The girder moments $M_{jA}(J,I)$ plus $M_{jB}(J,I)$ must sum to the sway moment $\sum M_G(I)$ for level (I).

These initial moment requirements are expressed mathematically by Eqs. (4.10 to 4.13). The two-stage process of distributing the total girder sway moment on a level to each girder, using Eq. (4.13 and 4.14), followed by the girder plastic moment balance, gives a set of girder moments which satisfy requirement (3). The girder moments obtained in the girder plastic moment balance need not be altered when the joints are balanced. Of course, the girder moments can be redistributed by revising the horizontal distribution factors $D_C$ and repeating the girder plastic moment balance, if desired. Once an acceptable girder moment distribution is obtained, the joint balancing operation holds the girder moments constant. The purpose of the joint balance is then to find column moments which satisfy both joint equilibrium and story equilibrium.
The initial column moments for requirements (1) and (2) are obtained from

\[
M_{iU}(J,I) = D_{U}(J,I) D_{V}(I) \sum M_{C}(I) \tag{6.1}
\]

\[
M_{iL}(J,I-1) = D_{L}(J,I-1) (1 - D_{V}(I-1)) \sum M_{C}(I-1) \tag{6.2}
\]

Note that the subscript \(i\) is used to distinguish an initial joint moment from the final joint moment (subscript \(j\)) which is obtained later from the joint balance. The terms \(D_{U}\) and \(D_{L}\) are initial column moment distribution factors which must satisfy

\[
\sum_{(J)} D_{U}(J,I) = \sum_{(J)} D_{L}(J,I-1) = 1 \tag{6.3}
\]

at each level \((I)\), according to requirements (1) and (2). In the portal method of wind analysis the product \(D_{U} D_{V}\) for a column is proportional to the sum of the adjacent girder spans. In plastic moment balancing the column moment distribution factors may be assigned any values consistent with Eq. (6.3). For instance, the joint balance for the two-bay frame in Example 14.1 of Ref. 6 uses \(D_{V} = 1/2\) and \(D_{U} = D_{L} = 1/3\) under the assumption that the initial column moments are equally distributed between the ends of 3 columns in each story.

The initial distribution of column moments is illustrated in Fig. 6.1 which shows the positive joint moments in two stories of a frame. All moments in this sketch carry numerical subscripts (omitted for clarity) and may have different values from member to member. The story sway moments \(\sum M_{C}\) are shown at the left center of each story.
These sway moments are distributed, using the vertical distribution factors in the triangles above and below, to the upper and lower ends of the columns. This vertical distribution gives the column moment sums, \( \sum M_{jL} \) and \( \sum M_{jU} \) on the left in Fig. 6.1. The initial column moments \( M_{iL} \) and \( M_{iU} \) are obtained from the column moments sums, using the column moment distribution factors \( D_L \) and \( D_U \) in the triangles adjacent to each column. The initial column moments satisfy the story equilibrium requirements (1) and (2) but do not necessarily conserve joint equilibrium.

The moments which are considered to act on a joint at the beginning of the joint balance are the column and girder moments from requirements (1) to (3) plus a hypothetical external moment \( M_{iE} \) which conserves joint equilibrium. The external moment is determined from the joint moment sum

\[
M_{jA(J,I)} + M_{jB(J-1,I)} + M_{iU(J,I)} + M_{iL(J,I-1)} + M_{iE(J,I)} = 0 \quad (6.4)
\]

at joint \((J,I)\). The joint balancing operation consists of distributing the portion \( D_j M_{iE} \) of the external moment to the column above and the portion \((1 - D_j) M_{iE}\) to the column below. The symbol \( D_j \) is termed the joint balancing ratio which must have the same value for each joint on one level of the frame. This requirement is demonstrated below.

The results of the joint balance are the final column moments

\[
\begin{align*}
M_{jU(J,I)} &= M_{iU(J,I)} + (1 - D_j(I)) M_{iE(J,I)} \\
M_{jL(J,I-1)} &= M_{iL(J,I-1)} + D_j(I) M_{iE(J,I)}
\end{align*}
\quad (6.5)
\quad (6.6)
\]
To demonstrate that these column moments satisfy joint equilibrium we can form the sum

\[ M_{jU}(J,I) + M_{jL}(J,I-1) = M_{iU}(J,I) + M_{iL}(J,I-1) + M_{iE}(J,I) \]

and substitute \( M_{iE} \) from Eq. (6.4) to obtain

\[ M_{jU}(J,I) + M_{jL}(J,I-1) = -M_{jA}(J,I) - M_{jB}(J-1,I) \]  \( (6.7) \)

This indicates that the final moments on the joint sum to zero for any value of the joint balancing ratio \( D_j \). Thus joint equilibrium is satisfied.

Now consider the story equilibrium conditions defined by Eqs. (4.10 and 4.11). The sum of the column moments \( M_{jU} \) below level \( I \), using Eq. (6.5) is

\[ \sum_{(J)} M_{jU}(J,I) = \sum_{(J)} M_{iU}(J,I) + \sum_{(J)} (1 - D_j(I)) M_{iE}(J,I) \]  \( (6.8) \)

where the first sum on the right side is \( D_v(I) \sum M_C(I) \) according to requirement (1). To evaluate the second term, we may sum the initial joint equilibrium condition, Eq. (6.4), for all joints on level \( I \). The girder moments contribute

\[ \sum_{(J)} (M_{jA}(J,I) + M_{jB}(J-1,I)) = \sum M_G(I) \]  \( (6.9) \)

from Eq. (4.13). The initial column moments from Eq. (6.1 and 6.2) give

\[ \sum_{(J)} (M_{iU}(J,I) + M_{iL}(J,I-1)) = D_v(I) \sum M_C(I) + (1 - D_v(I-1)) \sum M_C(I-1) \]  \( (6.10) \)
since the column moment distribution factors $D_U$ and $D_L$ must sum to unity as in Eq. (6.3). Then the sum of the external moments $M_{iE}$ in Eq. (6.4) is minus one times the sum of Eqs. (6.9 and 6.10). But if we use Eq. (4.12) for the girder moment sum $\sum M_G(I)$ on level (I), it follows that

$$\sum_{(J)} M_{iE}(J,I) = 0 \quad (6.11)$$

That is, the hypothetical external moments $M_{iE}$ have a vanishing resultant on each level. It is helpful to note that Eq. (6.11) is valid for any set of moments which satisfy requirements (1) to (3).

Now Eq. (6.8) can be written in the form

$$\sum_{(J)} M_{jU}(J,I) = D_V(I) \sum M_C(I) + \sum_{(J)} D_J(I) M_{iE}(J,I) \quad (6.12)$$

The first line in Eq. (6.12) is identical with Eq. (4.10). The second line vanishes in view of Eq. (6.11) if, and only if, the joint balancing ratios $D_J$ are identical for each joint on the level. Using this as the only necessary condition on $D_J$ we have shown that the column moments $M_{jU}$ below level (I), from Eq. (6.5), do in fact satisfy the story equilibrium condition of Eq. (4.10). The reader may demonstrate a similar conclusion for the column moments $M_{jL}$ above level (I) starting with Eq. (6.6).

The joint balancing operation is summarized in Fig. 6.2. Figures 6.2(a and b) indicate the initial and final moments on joint (J,I), with all moments shown in their positive sense. The girder
moments from the girder plastic moment balance are held constant. The column moments are obtained in three steps:

**Step 1** - Use the sway moments $\sum M_C$, the vertical distribution factors $D_V$, and the column moment distribution factors $D_U$ and $D_L$ to obtain the initial column moments $M_{iU}$ and $M_{iL}$.

**Step 2** - Find the external moment $M_{iE}$ required for joint equilibrium by multiplying the sum of the girder and initial column moments by minus one.

**Step 3** - Use the joint balancing ratio $D_j$ and the external moment $M_{iE}$ to obtain the balancing moments. The sum of the balancing moments and the initial column moments gives the final column moments $M_{jU}$ and $M_{jL}$.

The similarity of this plastic joint balancing process to elastic moment distribution is evident. This is quite logical and desirable because both plastic moment balancing and elastic moment distribution are based on the same foundation - equilibrium. Any method which does not conserve equilibrium is suspect, for there is not a more certain determinant of structural behavior.

In elastic moment distribution the initial moments are termed fixed-end moments and the external (clamping) moment is simply the fixed-end moment sum for a joint. This external moment is distributed to the members in proportion to their elastic flexural stiffness. In plastic
moment balancing we are free to assign distribution factors and balancing ratios without reference to flexural stiffness if the frame members are sufficiently ductile; that is, able to bend under nearly constant moment in the plastic range. Then plastic moment redistribution can be relied on, within reasonable limits, to alter the initially elastic distribution of moments.

We have accomplished two objectives by formulating the joint balancing operation of plastic moment balancing in a form which is analogous to elastic moment distribution. First, the joint balance uses familiar concepts. Second, some ideas gleaned from elastic moment distribution or similar methods of elastic analysis may be profitably carried over to plastic moment balancing. This will be demonstrated in Chapter 7.

The joint balancing method in this article introduces two additional groups of parameters into the preliminary design process. These are the column moment distribution factors $D_U$ and $D_L$ and the joint balancing ratios $D_j$. These parameters provide the numerical means for specifying how the columns are to share in resisting girder moments. Any information on elastic column moment distributions may be used as a guide in assigning values to these parameters subject to Eq. (6.3).

There is some logic to using distribution and balancing factors derived from elastic analyses because the more the plastic moment distribution departs from the original elastic moment distribution, the greater is the element of plastic redistribution of moment. Moment redistribution implies rotation capacity and plastic hinge induced
sway stiffness reductions. If rotation capacity is very limited or if sway stiffness reductions are excessive, the ultimate load capacity of the frame may be impaired. This points to the need for further inelastic sway studies to suggest limitations on the values assumed for the column moment distribution factors and the joint balancing ratios. Even after further study it appears likely that structural judgment will be needed to assign distribution factors. However, the need for structural judgment is a universal feature of any design method.

It is germane to comment that while elastic distribution factors may provide a guide in assigning $D_U$, $D_L$, and $D_J$, it is not necessary to perform an involved elastic analysis to find them. Nor is there a firm basis for placing strict elastic limitations on these parameters for a ductile material like steel. Inelastic column behavior can be profitably and safety utilized as indicated in Fig. 1.6 so long as inelastic assumptions are not carried to extremes.

It is also pertinent to note that less ductile materials or members may place more restrictive limitations on the departure from elastic behavior which can be safely utilized in structural design. Regardless of what restrictions on departures from elastic behavior are considered necessary, the plastic moment balancing approach may be tailored to satisfy these restrictions. This may be necessary for a reinforced or prestressed concrete frame.

The method of joint balancing described in Fig. 6.2 and using requirements (1) to (3) plus Eqs. (6.1 to 6.6) is designated as method I for reference. The method avoids any moment carry-over between joints.
but provides versatility and computational control in determining column moments. This control is obtained by selecting values for the column moment distribution factors \( D_U \) and \( D_L \). Hence the term \textit{controlled joint balance}.

Joint balancing method I is but one of several methods that may be used to find column moments which satisfy the joint and story equilibrium conditions once the girder moments are established from the girder plastic moment balance. An interesting alternative joint balancing method (designated as method II) is the following:

\textbf{Step 1} - Assign final column moments \( M_{jU} \) and \( M_{jL} \) at all but one joint on a level, subject to the joint equilibrium condition, Eq. (6.7). Note that \( M_{iE} = 0 \) at the balanced joints if we take \( M_{iU} = M_{jU} \) and \( M_{iL} = M_{jL} \) in Eq. (6.4).

\textbf{Step 2} - Use Eqs. (4.10 and 4.11) to determine final column moments at the last joint on the level. This enforces the story equilibrium conditions on the joint balance, and in view of Eq. (6.11), joint equilibrium is also satisfied at the last joint.

This alternative approach to joint balancing completely eliminates the column moment distribution factors and the joint balancing ratio. It is difficult to conceive of a more simple method of joint balancing, but method II appears to be rather arbitrary. The basis for selecting the final column moments needs further definition.
This alternative is included here to suggest an idea for further study and to indicate the versatility of plastic moment balancing.

6.2 AUTOMATIC JOINT BALANCE

The joint balancing operation can be formulated in a manner which is free of column moment distribution factors or assigned final column moments, as in methods I and II of Art. 6.1. Two such formulations are described in this article. We assume (as in Art. 6.1) that final girder moments $M_{jA}$ and $M_{jB}$ are obtained from the girder plastic moment balance. The purpose of the joint balance is then to find column moments which satisfy both joint equilibrium (Eq. (6.7)) and story equilibrium (Eqs. 4.10 and 4.11).

To begin the description of automatic joint balancing, it is convenient to define the symbol

$$M_{jG(J,I)} = M_{jB(J-1,I)} + M_{jA(J,I)} \quad (6.13)$$

for the sum of the girder moments on joint $(J,I)$. The sum of the girder moments on the joints at level $(I)$ must yield

$$\sum J M_{jG(J,I)} = \sum M_{G(I)} \quad (6.14)$$

where $\sum M_{G(I)}$ is the sway moment sum for the girders from Eq. (4.12). This result is valid for any set of girder distribution factors $D_G$ (used to obtain $M_G$ in the girder plastic moment balance) which sums to unity on level $(I)$, as per Eq. (4.14). In addition, the joint
equilibrium condition of Eq. (6.7) takes the form

$$M_{jU(J,I)} + M_{jL(J,I-1)} = -M_{jG(J,I)} \quad (6.15)$$

using Eq. (6.13). Note that the sum of the girder moments is known for each joint on a level once the girder plastic moment balance (Chapter 5) has been performed for each girder on the level.

The first method of automatic joint balancing (designated as method III) obtains the final column moments from

$$M_{jU(J,I)} = D_f(J,I) \left[ D_V(I) \sum M_C(I) \right] \quad (6.16)$$

$$M_{jL(J,I-1)} = D_f(J,I) \left[ (1 - D_V(I-1)) \sum M_C(I-1) \right] \quad (6.17)$$

The bracketed terms are the sway moment components distributed to level (I) from the stories below and above this level (Art. 4.3). The column moment distribution factors $D_f(J,I)$ at joint (J) on level (I) are computed from the girder moment sums $M_{jG}$ in the form

$$D_f(J,I) = \frac{M_{jG(J,I)}}{\sum M_G(I)} \quad (6.18)$$

These distribution factors serve to distribute the bracketed sway moment components in Eq. (6.16 and 6.17) to the columns. Note that the same factor $D_f$ is used above and below joint (J) but that $D_f$ varies from joint to joint on level (I) as implied by the double subscript (J,I) on $D_f$. The distribution factors $D_U$ and $D_L$ in joint balancing method I are replaced by $D_f$ in method III. Note also that $D_f$ from
Eq. (6.18) is uniquely and automatically determined from the girder plastic moment balance on level (I). Hence the description, automatic joint balance.

Now we demonstrate that joint balancing method III satisfies joint and story equilibrium requirements. First, we sum the distribution factors $D_f$ from Eq. (6.18) for the joints on level (I). Using Eq. (6.14) we find

$$\sum_{(J)} D_f(J, I) = 1 \quad (6.19)$$

Then the sum of the column moments from Eq. (6.16 and 6.17) is identical with the required column moment sums in Eq. (4.10 and 4.11). Any set of distribution factors $D_f$ which sums to unity on level (I), as in Eq. (6.19), thus conserves story equilibrium in the joint balance.

To investigate joint equilibrium conditions we use Eq. (6.16 to 6.18) to obtain the sum of the final column moments on joint (J, I):

$$M_{jU}(J, I) + M_{jL}(J, I-1) =$$

$$M_{jG}(J, I) \left[ \left\{ D_V(I) \sum M_C(I) + (1 - D_V(I-1)) \sum M_C(I-1) \right\} \right]$$

$$\sum M_G(I)$$

According to Eq. (4.12) for $\sum M_G(I)$ the bracketed term has the value minus one. Then the final column moments on joint (J, I) sum to $-M_{jG}(J, I)$ as per Eq. (6.15). Thus joint equilibrium is automatically satisfied if the distribution factors $D_f$ are obtained from Eq. (6.18).

Joint balancing method III is summarized in Fig. 6.3. Figure 6.3 (a) shows the girder moments obtained from the girder plastic moment
balance. These girder moments are held constant in the joint balancing operation. Figure 6.3(b) shows the final moments on the balanced joint. The column moments are obtained in three steps:

**Step 1** - Sum the girder moments $M_{jG}$ on the joint.

**Step 2** - Determine the column moment distribution factor $D_f$ as the ratio of $M_{jG}$ to the total sway moment sum $\sum M_G$ for the girders on the level. The sign on $D_f$ depends on the sign on $M_{jG}$.

**Step 3** - Obtain the final column moments as the product of $D_f$ and the sway moment components distributed to the level from the stories above and below that level.

The results of the joint balance may be checked by summing the final moments on the joint.

The result of joint balancing method III is to give final column moments at each joint on level (I) in the constant ratio

$$R_{LU(I)} = \frac{M_{jL(J,I-1)}}{M_{jU(J,I)}} = \frac{1 - D_v(I-1)}{D_v(I)} \frac{\sum M_{C(I-1)}}{\sum M_{C(I)}}$$

(6.20)

from Eqs. (6.16 and 6.17). We are free to assign the vertical distribution factors $D_v$ in this ratio but the story sway moment ratio is governed by frame geometry and load parameters plus the sway deflection index in the stories above and below level (I). The sway moment ratio varies from zero at the roof ($I = 1$) and 0.3 to 0.5 at level 2, to nearly unity at the lower levels of a tall unbraced frame. Note that
at the roof \((I = 1)\), \(R_{LU(1)} = 0\). The term joint moment ratio aptly describes \(R_{LU}\). The joint moment ratio is frequently close to unity.

We can provide a relative computational control on the column moments above and below level \((I)\) by assigning the value of \(R_{LU(I)}\) in Eq. (6.20). This equation then provides a relation between \(D_V\) above and below level \((I)\) in the form

\[
D_V(I) = R_{LU(I)} \left(1 - D_V(I-1)\right) \frac{\sum M_C(I-1)}{\sum M_C(I)}
\]  

Equation (6.21) may be used as a recurrence formula for the vertical distribution factors. For example, we can assign \(D_V(1) = 0.5\) in the story below the roof and \(R_{LU(2)} = 1.0\) at level 2. Then the value of \(D_V(2)\) below level 2, which is consistent with these assigned values, can be determined from Eq. (6.21). With \(D_V(2)\) known and an assigned value of \(R_{LU(3)}\) we can again apply Eq. (6.21) to find the vertical distribution factor \(D_V(3)\) below level 3. This process may be continued to the bottom level of the frame or may be terminated at any level, and reestablished at a lower level, if desired.

Joint balancing method III automatically distributes the sway moments above and below a level to the columns using Eqs. (6.16 and 6.17). The same results can be obtained by distributing the girder moment \(M_{jG}\), Eq. (6.13), to the columns above and below a joint. This introduces joint balancing method IV.
In method IV the final column moments are given by

\[ M_{jU}(J,I) = D_{j(I)} M_{jG}(J,I) \quad (6.22) \]

\[ M_{jL}(J,I-1) = -(1 + D_{j(I)}) M_{jG}(J,I) \quad (6.23) \]

where the joint moment sum \( M_{jG} \) is obtained from the girder plastic moment balance. The joint balancing ratio

\[ D_{j(I)} = D_{v(I)} \frac{\sum M_{C(I)}}{\sum M_{G(I)}} \quad (6.24) \]

has the same (negative) value for each joint on level \((I)\). Note that Eqs. (6.22 to 6.24) are simply a rearrangement of Eqs. (6.16 to 6.19) so the joint and story equilibrium requirements are satisfied in method IV. Again the column moments are uniquely and automatically determined from the results of the girder plastic moment balance on level \((I)\).

Joint balancing method IV is summarized in Fig. 6.4. Figure 6.4(a) shows the girder moments obtained from the girder plastic moment balance. These girder moments are held constant in the joint balancing operation. Figure 6.4(b) shows the final moments on the balanced joint. The column moments are obtained in three steps:

**Step 1** - Sum the girder moments on the joint to obtain \( M_{jG} \).

**Step 2** - Determine the joint balancing ratio \( D_{j(I)} \) for level \((I)\) as the ratio of the sway moment distributed to level \((I)\) from the story below, to the total sway moment sum for the girders on
level (I). The alternative expression for $D_j$ in Fig. 6.4 is Eq. (6.26).

**Step 3** - The column moment below each joint is $M_{jU} = D_j M_{jG}$ and above each joint $M_{jL} = -M_{jG} - M_{jU}$, using Eq. (6.15).

The joint balance for level (I) may be checked by summing the column moments $M_{jL}$ and $M_{jU}$ for all joints. These column moment sums must be equal to the sway moments distributed to level (I) from the stories above and below this level.

Joint balancing method IV gives final column moments at each joint on level (I) in the constant ratio

$$ R_{LU(I)} = \frac{M_{jL(J,I-1)}}{M_{jU(J,I)}} = \frac{- (1 + D_j(I))}{D_j(I)} \tag{6.25} $$

from Eqs. (6.22 and 6.23). We are free to assign $R_{LU}$ at each level to maintain a relative computational control over the column moments. Then the joint balancing ratio for level (I) may be obtained from

$$ D_j(I) = -1 / (1 + R_{LU(I)}) \tag{6.26} $$

if the same value of $R_{LU}$ is used in the recurrence relation, Eq. (6.21) for the vertical distribution factors.

Regardless of how it is determined, the value of $D_j(I)$ must be consistent with the sway moments and the vertical distribution factors in order to enforce story equilibrium in the joint balance for method IV.
From Eqs. (6.24 and 4.12)

\[ D_j(I) = \frac{-D_v(I) \sum M_c(I)}{D_v(I) \sum M_c(I) + (1 - D_v(I-1)) \sum M_c(I-1)} \]  (6.27)

Note that at the roof \( I = 1 \), \( D_j(I) = -1 \). Equation (6.27) is simply an expanded form of Eqs. (6.24 or 6.26).

### 6.3 DOUBLE CURVATURE COLUMN BENDING

It is of some interest to consider the column moment ratio

\[ q_j = M_{jl} / M_{jU} \] when joint balancing method III is used together with the vertical distribution factor recurrence relation of Eq. (6.21). From Eqs. (6.16 and 6.17)

\[ q_j(J,I) \frac{M_{jL(J,I)}}{M_{jU(J,I)}} = \frac{D_f(J,I + 1)}{D_f(J,I)} \left( 1 - \frac{D_v(I)}{D_v(I)} \right) \]  (6.28)

If both terms on the right side of this equation are positive, then \( q_j > 0 \) and the columns are bent in double curvature. Note that the sway moment \( \sum M_c(I) \) below level \( I \) cancels in Eq. (6.28).

The column moment distribution factors \( D_f \) are defined by Eq. (6.18). It is always possible and usually desirable to choose girder distribution factors \( D_g \) in Eq. (4.13) which make \( D_f \) positive at each joint on a level except at the windward column joint. The sign on \( D_f \) at the windward column joint changes from negative to positive at the level \( I_w \) where the windward moment on the girder in the windward bay changes sign as in Fig. 1.4(j). If we exclude the windward column
between levels \((I_w - 1)\) and \((I_w)\) and all of the columns in the bottom story, it is then possible to distribute the moments under combined load so as to make the \(D_f\) ratio in Eq. (6.28) positive for all remaining columns. In mathematical terms

\[
\frac{D_f(J,I + 1)}{D_f(J,I)} > 0 \quad (6.29)
\]

with the exclusions mentioned above.

Now consider the vertical distribution factor ratio in Eq. (6.28) and assume that the recurrence relation of Eq. (6.21) is used to obtain \(D_V\) in each story except the bottom story. It is logical to expect the sway moment ratio in Eq. (6.21) to be bounded by

\[
0 < \frac{\sum M_c(I-1)}{\sum M_c(I)} \leq 1, \quad \text{for } I \geq 2 \quad (6.30)
\]

if all of the horizontal loads act in the same direction. (This may exclude dynamic or seismic loading.) We are also free to choose the joint moment ratio \(R_{LU(I)}\) in the range

\[
0 < R_{LU(I)} \leq 1, \quad \text{for } I \geq 2 \quad (6.31)
\]

in this recurrence relation.

If the vertical distribution factor in the story above level \((I)\) has the bounds

\[
0 < D_V(I-1) < 1 \quad (6.32a)
\]
we can rearrange this inequality to the form

$$0 < 1 - D_V(I-1) < 1$$

(6.32b)

The bounds in Eqs. (6.30 to 6.32) establish that each term on the right side of Eq. (6.21) may be considered as a positive number which does not exceed unity. Then $D_V(I)$ must also be bounded by

$$0 < D_V(I) < 1, \text{ for } I \geq 1$$

(6.32c)

We are free to assign the vertical distribution factor $D_V(I)$ in the top story to satisfy the bounds in Eq. (6.32a) so Eq. (6.32c) is valid for $I \geq 1$.

We have demonstrated, by mathematical induction, that the vertical distribution factors $D_V(I)$ in Eq. (6.28) may always be assigned as positive numbers less than unity. Then the vertical distribution ratio

$$\left(1 - D_V(I)\right)/D_V(I) > 0$$

(6.33)

is positive in Eq. (6.28).

From Eqs. (6.28, 6.29, and 6.33) we have the following double curvature column proposition:

It is possible to distribute the frame moments due to combined load so as to obtain double curvature bending in every column of an unbraced frame, except in the bottom story and in one windward column.
Note carefully that this proposition does not imply that the columns will or must be bent in double curvature. Rather the proposition maintains that it is possible to distribute the combined load frame moments to achieve the double curvature column objective with equilibrium considered in the deflected state. It is advantageous to distribute the combined load moments in this manner so as to minimize the possibility of lateral-torsional column instability.

We assume that rotation capacity requirements at plastic hinges in girders do not become excessive for this double-curvature-column distribution of moments. Girder rotation capacity requirements deserve attention for the case of a long and a short girder span in adjacent bays. Vertical deflection limits for the long span girder may eliminate any problems with rotation capacity but also may make it more difficult to distribute the moments so as to conserve double curvature bending in the columns. This points to the need for considering a wide range of frame geometry and load parameters in studies which intend to investigate the scope and limitations of a proposed structural design method.

Consider now the exceptions to the double curvature column proposition. The windward column above level (I_w) violates inequality (6.29) and may exhibit single curvature bending. This column must function in both a windward and a leeward column role. Frequently the leeward column role controls the size of this column and the windward condition results in single curvature bending under reduced axial load and moments. If this is the case, the tendency for lateral-torsional buckling in single curvature bending is not demanding unless this column has a large slenderness ratio about either axis.
Hence we conclude that lateral-torisonal buckling considerations under combined loading may focus on the bottom story columns if the frame moments are distributed according to the double curvature column proposition and if large column slenderness ratios are avoided. The distribution of moments for the bottom story columns is considered in Chapter 7. These columns are excluded in Eq. (6.29) because \( D_{f(I+1)} \) in the bottom story depends on the column rotational restraint at the foundation, rather than on Eq. (6.18).

Note that the double curvature column conclusion does not extend to gravity loading because the sway moments in Eq. (6.30) vanish under symmetrical gravity loads.

There is a strong economic motivation for distributing the frame moments according to the double curvature column proposition when combined loading controls the column sizes. Double curvature bending reduces the controlling column moments and automatically gives a column moment distribution reasonably similar to a theoretical distribution for minimum column weight. In addition, double curvature bending reduces the need for increasing column sizes, or for providing lateral bracing, to control lateral-torsional instability. Finally, the rotation capacity of columns in double curvature bending is superior, relative to other states of bending. Since the columns may account for three-quarters of the material cost of a tall unbraced frame, it is economically advantageous to control column material requirements in the preliminary design stage, even at the expense of increasing some girder moments on lower levels.
To obtain these advantages of double curvature column bending we may proceed as follows:

**Step 1** - Find the sway moment \( \sum M_C(I) \) below each level (I), using Eq. (4.9).

**Step 2** - Assign the vertical distribution factor \( D_V(I) \) in the range \( 0 < D_V(I) < 1 \) for the top story and the joint moment ratio \( R_{LU}(I) \) for each level. The value \( R_{LU} = 1.0 \) is a reasonable assumption particularly when columns are designed in two-story tiers with splices above finished floor level. However we are not confined to unit joint moment ratios.

**Step 3** - Use the recurrence relation, Eq. (6.21), to obtain vertical distribution factors \( D_V(I) \) below each level, excluding the bottom story.

**Step 4** - Find the girder sway moment sum \( \sum M_G(I) \) at level (I) as described in Art. 4.4 and distribute this sum to each girder using assigned girder distribution factors \( D_G \). The \( D_G \) factors may be estimated, subject to Eq. (4.14).

**Step 5** - Perform the girder plastic moment balance as indicated in Chapter 5.

**Step 6** - Balance the joints using method III described in Fig. 6.3. Determine the column moment distribution factors \( D_f \) from Eq. (6.18) using the sum of the girder moments on each joint. If negative values of \( D_f \) are found for any
joint other than upper level windward column joints, attempt to redistribute the girder moments on the level by revising the values of \( D_G \) in step 4.

If difficulty is encountered in distributing the girder moments in step 4 so as to obtain positive column moment distribution factors in step 6, we can seek a girder moment distribution on successive levels which conserves the sign on \( D_f \) for most columns. If the ratio

\[
\frac{D_f(J,I+1)}{D_f(J,I)}
\]

is positive, column (J) below level (I) is bent in double curvature. If this \( D_f \) ratio is negative, then both \( D_f(J,I) \) and \( D_f(J,I+1) \) should be close to zero and relatively small single curvature column moments are obtained from the joint balance.

When joint balancing method III was first explored, it was considered to be rather inefficient in controlling column moments. This inefficiency is remedied by step 3. However, it should be noted that method III as described in Fig. 6.3 stands by itself and may be applied with any set of vertical distribution factors.

If we wish to use joint balancing method IV together with the double-curvature-column distribution of frame moments we must make the column moment ratio \( q_j = \frac{M_{jL}}{M_{jU}} \) positive. From Eqs. (6.22, 6.23, and 6.26)

\[
q_j(J,I) = \frac{M_{jL(J,I)}}{M_{jU(J,I)}} = \frac{1 + R_{LU(I)}}{1 + (1/R_{LU(I+1)})} \left( \frac{M_{jG(J,I+1)}}{M_{jG(J,I)}} \right)
\]  

(6.34)
The first term in this equation is positive if positive values of $R_{LU}$ are assigned. Then double curvature bending is obtained for column (J) below level (I) if the girder moments on levels (I) and (I+1) are distributed so as to make

$$\frac{M_{JG(J,I+1)}}{M_{JG(J,I)}} > 0$$  \hspace{1cm} (6.35)

This is a slightly different formulation of Eq. (6.29).

Thus joint balancing method IV may be used together with the procedure previously described for establishing frame moments which conserve double curvature column bending. It is only necessary to revise step 6 to the following:

**Step 6** - Balance the joints using method IV described in Fig. 6.4. If the girder moments $M_{JG}$ on the joints above and below a column have different signs, attempt to redistribute the girder moments by revising the values of $D_{G}$ in step 4. If this proves difficult, then try to make the ratio in Eq. (6.35) a small negative value.

This discussion of frame moment distributions which conserve double curvature column bending illustrates the versatility of the automatic joint balancing procedures. The simple device of assigning positive joint moment ratios $R_{LU}$ at each level is all that is needed to generate vertical distribution factors $D_{V}$ from Eq. (6.21), column moment distribution factors $D_{f}$ from Eq. (6.18) for method III, and
joint balancing ratios $D_j$ from Eq. (6.26) for method IV. In addition the joint moment ratios $R_{LU}$ provide a direct computational control over the relative column moments above and below each level right from the beginning of the preliminary design process.

6.4 COMPARISON OF JOINT BALANCING METHODS

Four joint balancing methods are described in Arts. 6.1 and 6.2. These methods have three features in common.

(1) The girder moments are obtained from the plastic moment balance for the girders on one level and are held constant in the joint balance.

(2) No carry-over of moments between joints or from joints to midspan of girders is needed.

(3) The column moments satisfy all necessary joint and story equilibrium conditions.

Method I introduces two column moment distribution factors $D_U$ and $D_L$ and a joint balancing ratio $D_j$ to determine the column moments. This method affords a maximum of computational control over the column moment distribution. The results of any elastic-plastic or inelastic frame analysis may be duplicated in plastic moment balancing by proper choice of the distribution factors and other initial data. Then if the frame geometry or load parameters are varied within small limits, the previously determined distribution factors may be used to obtain a reasonably accurate and rapid preliminary moment distribution which approximately corresponds to the compatibility condition.
It appears possible in future studies to correlate the distribution factors with frame geometry and load parameters, using on the one hand refined methods of inelastic sway analysis, and on the other hand simple structural models like that described in Art. 14.6 of Ref. 6. If it is considered appropriate to state limitations in a design specification on the extent to which plastic behavior may be utilized in the design of unbraced frames, the numerical parameters in method I may provide a reasonable framework for formulating these limitations. In other words, joint balancing method I provides the pattern for translating research information on the structural behavior of unbraced frames into a form suitable for manual or computer design.

The remaining three joint balancing methods may be considered as special cases of method I. Method II indicates how the joint and story equilibrium conditions may be satisfied without direct reference to distribution or balancing factors. In an analysis problem involving known member sizes, column plastic moment capacities $M_{pc}$ can be estimated. Method II provides the means for fitting an equilibrium distribution of column moments into available moment capacity envelopes.

Joint balancing methods III and IV are described as automatic methods because the column moment distribution follows uniquely from the results of the girder plastic moment balance and the vertical distribution of sway moments. These two methods provide identical results and are simply different formulations of the same idea; which is that the sway moment components above and below a level may be distributed to the columns at each joint in proportion to the sum of the girder moments on the joint. Further study is needed to substantiate this.
idea and to suggest its limitations and relative economy. In the meantime, numerical examples suggest that the automatic joint balancing methods yield column moments which are a reasonable basis for preliminary design.

The reason for presenting two joint balancing methods which give the same results is that the different formulations point to different features of the plastic moment balancing method. For example, method III leads directly to the recurrence relation for the vertical distribution factors $D_v$ in Eq. (6.21). Method IV most clearly indicates the relation between the joint balancing ratio $D_j$ and the joint moment ratio $R_{LU}$ in Eq. (6.26). Incidentally, method IV is more efficient than method III in a computational sense. Note that a single joint balancing ratio $D_j$ applies to all joints on one level in method IV while different distribution factors $D_f$ are needed for each joint in method III. The choice between these joint balancing methods is one of personal preference in manual calculations or convenience in computer programming.

6.5 SUMMARY

This chapter describes and compares four joint balancing methods. Each method begins with the joint moments from the girder plastic moment balance. The purpose of the joint balance is to determine column moments which satisfy joint and story equilibrium. No moment carry-over is needed in this operation.
The most direct joint balancing method is method IV in which the sum of the girder moments on a joint is distributed to the columns above and below the joint. This method uses one joint balancing ratio for the joints on each level which depends on the sway moment sums for the level. A method is described for arranging the plastic moment balance so as to achieve double curvature bending in most of the columns of a multi-story frame.

Numerical examples which illustrate joint balancing methods I and IV are given in Tables A10 to A12 in Appendix 1. After completing the joint balance, column end-moments are determined using Eq. (4.5).

Chapters 4, 5, and 6 complete the description of frame statics, the girder plastic moment balance, and the joint balance. These are the three major operations in plastic moment balancing. After completing these operations, tentative member sizes can be selected.

Refinements in the choice of preliminary design parameters for bottom story columns and for girders are considered in Chapters 7 and 8.
7. BOTTOM STORY COLUMNS

The plastic moment balance for the bottom story of an unbraced frame may follow the same procedure as that used in the stories above with one modification which concerns the column base detail. If this detail provides no resistance to moment (pinned base), we must use $D_v = 1.0$ in the bottom story because all of the story sway moment must then be resisted at the column tops. On the other hand, if the columns are rigidly connected to a rigid foundation (fixed base) we should assign $D_v \leq 0.5$ in the bottom story. In the limiting case of infinitely rigid girders on the floor above and rigid column bases, exactly half of the bottom story sway moment is distributed to the column tops. Less than half of the sway moment is attracted to the column tops if the girders are less than perfectly rigid while the column bases remain nearly fixed against rotation.

The actual condition of rotational restraint at the column bases may be expected to lie between the pinned and fixed conditions. This is particularly true if the column base detail is designed to transmit moment from the column to the foundation and if the foundation consists of spread footings or friction piles. Under these conditions the column bases will be more nearly fixed than pinned but will rotate under column moments.
This chapter discusses the behavior which results from rotation at the ends of bottom story columns. An approximate method is presented for modifying the bottom story moment balance to consider rotation effects.

7.1 COLUMN BASE ROTATION

In the following discussion, the behavior of bottom story columns with rotation-fixed bases and tops is compared with the more realistic assumption of rotationally restrained column bases and tops. It is assumed that the column base detail and foundation provide a larger ratio of column moment to column rotation than the girders on the floor above. In other words the foundation is considered to be stiffer than the girders in resisting rotation of the bottom story columns. Note that the girders on the bottom level of a multi-story frame must resist rotation of two column segments, one above and one below this level.

If the bottom story columns are fixed against rotation at both ends, sway in the bottom story causes anti-symmetrical double curvature bending with a column moment ratio \( q_j = \frac{M_jL}{M_jU} = +1.0 \). One effect of rotation at either end of these columns is to change the column moment ratio. This indicates the possibility of single curvature bending \((q_j < 0)\) in windward bottom story columns, or in columns on the windward side of a long-span interior bay, under combined load. Lateral-torsional stability may be reviewed for these single curvature columns with consideration given to the nearly complete torsional fixity provided by many column base details. It is helpful to have some
indication of single curvature bending in the bottom story during the preliminary design so that column sizes can be selected to conserve lateral-torsional stability.

If the foundation is rotationally stiffer than the girders on the floor above, the assumption of rotation-fixed column ends (top and bottom) may tend to underestimate the elastic moments at the base of some bottom story columns. If this effect is not considered in the preliminary design, the bottom story leeward columns may form plastic hinges at a load somewhat less than the intended ultimate combined load capacity. Then we must rely on rotation capacity at these column hinges to redistribute the bottom story sway moments caused by increasing loads.

Several factors may influence the ability of bottom story columns to redistribute moment.

(1) Torsional fixity at the column base plus encasement in basement walls (for exterior columns) or strong but ductile fireproofing (for interior columns) help to preserve lateral-torsional stability and rotation capacity.

(2) Most of the capacity of bottom story columns is used to carry axial load. That is, these columns usually have large $P/P_y$ ratios (0.8 or more) at ultimate combined load, particularly on the leeward side of a frame. This tends to reduce rotation capacity.
(3) If large $P/P_y$ ratios and intermediate slender-ness ratios (say $h/r_C > 30$ in the plane of the frame) are both present, the maximum column moment capacity decreases as the column moment ratio $q$ approaches 0.6 or smaller values. (See Fig. 11.10 in Ref. 6 in which $q$ is the ratio of the smaller to the larger column moment.) In addition, the column rotation capacity may be relatively limited.

(4) Plastic hinge induced sway stiffness reductions in the bottom story detract from the stability of the frame and place increasing demands on moment redistribution under increasing loads, particularly in view of the maximum gravity loads in this story relative to those above.

(5) The width-to-thickness ratio of the plate elements in bottom story columns normally is the range where local buckling is of little concern. Thus, local buckling is not likely to control the available rotation capacity in stocky bottom story column shapes.

Items (1) and (5) tend to help and items (2) to (4) to hinder moment redistribution in bottom story columns. Other factors, such as differential dead load settlement plus sweep and erection tolerances, may play a uncertain role which contributes to the need for load factor or safety margin requirements in any design method.
The discussion in this article suggests that somewhat more refined ideas are appropriate in the preliminary design of bottom story columns because of the behavior which may accompany large column loads. In summary this behavior may include:

(1) Reduced plastic moment and rotation capacity under unfavorable column moment gradients.

(2) Lateral-torsional instability in single curvature bending.

(3) Reduced sway stiffness due to plastic hinges at the base of leeward columns.

Regardless of the design method (or even the material) used for the bottom story columns, these columns must carry large axial loads. Since we cannot completely avoid the behavior which accompanies large column loads, it is appropriate to consider ways to predict and control this behavior in the preliminary design by using refinements in the bottom story moment balance. Refinements in the bottom story moment balance are not absolutely essential in the statical sense. However, if refined ideas lead to a closer estimate of required column sizes in the bottom story without a significant increase in the preliminary design effort, these refinements are worth considering. The remaining articles in this chapter outline a refined approach for the bottom story columns which is compatible with the plastic moment balancing method.

The basis for the refined approach is the assumption that the bottom story columns may be treated as elastic for purposes of preliminary design. This does not mean that inelastic behavior cannot be tolerated in these columns. Instead, the elastic assumption is
considered as a guide in distributing moments in the bottom story columns. These columns may then be proportioned, using plastic design criteria, for the controlling combination of axial load and moment obtained from the preliminary design moment balance. It is understood that subsequent inelastic sway checks are a part of the final structural design process. These sway checks are considered necessary to demonstrate either that the elastic assumptions are valid, or that the preliminary design needs revision.

7.2 ELASTIC COLUMN RELATIONS

The elastic distribution of moments in a deflected column will be studied with the aid of Fig. 7.1. All forces, moments, and rotations are shown in the positive direction in this figure. The direction of the column moments is usually determined by the angle from the chord to the end-tangent. Thus, if a column deflects as depicted in Fig. 7.1 the column moments are negative. The deflected state is defined by the joint rotations $\theta_{jU}$ and $\theta_{jL}$ at the upper and lower joints and the chord rotation $\Delta/h$ between the joints. If the competing effects of axial load stiffness reduction and the stiffness contribution of the connections are neglected, the column moments at the upper and lower joints are

$$M_{jU} = \frac{EI_c}{h} \left( 4\theta_{jU} + 2\theta_{jL} - \frac{\Delta}{h} \right)$$

$$M_{jL} = \frac{EI_c}{h} \left( 2\theta_{jU} + 4\theta_{jL} - \frac{\Delta}{h} \right)$$

(7.1)

where $I_c$ is the moment of inertia of the column in the plane of the frame. These expressions are first order elastic slope deflection equations.
The column base introduces an additional relation between the moment $M_{jL}$ and rotation $\theta_{jL}$ at the base plate of a bottom story column. This relation is conveniently expressed in the form

$$M_{jL} = -4 R_f \frac{E I_c}{h} \theta_{jL} \quad (7.2)$$

where $R_f$ is a dimensionless rotational restraint coefficient for the foundation. The value of $R_f$ may range from zero for an idealized pinned-base detail to infinity for an absolutely rigid base. Intermediate values of $R_f$ may be estimated from soil properties and column base details. An elastic method for estimating rotational restraint coefficients is presented in Ref. 99. (The coefficient $R_f$ in Eq. (7.2) is related to the parameters $\tau$ and $\lambda$ in Ref. 99 by $R_f = 1.5 \tau \lambda$.)

Tentative elastic calculations indicate that the bounds $1.0 < R_f < \infty$ are likely to include the practical range of rotational restraint. If $R_f$ is within these bounds, the parameters used in the bottom story moment balance are relatively insensitive to $R_f$. The influence of varying rotational restraint is considered later in this chapter. The negative sign in Eq. (7.2) is explained by the fact that the column moment and rotation at a restrained column base must be opposite in sense, while both the moment on the column and the rotation are taken as positive when clockwise.

The base restraint relation may be combined with the slope-deflection equations to eliminate the base rotation. Using Eqs. (7.1 and 7.2) we have
for the bottom story columns where the symbol

\[ R_b = \frac{1}{1 + R_f} \quad 0 \leq R_b \leq 1 \]  

(7.4)

includes the effect of base restraint. For convenient reference, \( R_b \) is termed the base factor. Values of the base factor are bounded by zero for a fixed column base (\( \Theta_{jL} = 0, R_f = \infty \)) and unity for a pinned base (\( M_{jL} = R_f = 0 \)). For \( R_f = 1.0 \), Eq. (7.4) gives \( R_b = 0.5 \).

Equations (7.3) are useful in studying two features of the moment distribution in bottom story columns. These features are

(1) the column moment ratio \( q_j = \frac{M_{jL}}{M_{jU}} \) and (2) the change in moment \( \delta M_{jL} \) at the column base which occurs when the moment and rotation at the top of the column are varied by \( \delta M_{jU} \) and \( \delta \Theta_{jU} \) with the chord rotation held constant. These features are related to the moment balancing process in Art. 7.3.

The column moment ratio, from Eq. (7.3) is

\[ q_j = \frac{M_{jL}}{M_{jU}} = \frac{2(1 - R_b) (D_r - 3)}{(4 - R_b) D_r - 3 (2 - R_b)} \]  

(7.5)

The two parameters which contribute to the column moment ratio are the base factor \( R_b \) and the deflection ratio

\[ D_r = \frac{\Theta_{jU}}{\Delta/h} \]  

(7.6)
We may interpret the deflection ratio as a relative dimensionless measure of the resistance to joint rotation $\theta_{jU}$ provided by the girders on the floor above the bottom story. If these girders are taken to be infinitely stiff, as in Fig. 1.3(b), then $D_r = \theta_{jU} = 0$. If these girders are considered to provide no rotational restraint to the columns, then $M_{jU} = 0$ in Eq. (7.3) and $D_r = (6 - 3R_b)/(4 - R_b)$. The deflected column in Fig. 7.1 corresponds to the typical conditions $\theta_{jU} < \Delta/h$ and $D_r < 1$.

To indicate how the deflected shape of a column is related to the deflection ratio $D_r$, consider Fig. 7.2. This figure shows the graph of Eq. (7.5) for columns with a rotation fixed base ($R_b = 0$), together with sketches of the deflected columns and their moment diagrams. The graph has a vertical asymptote at $q_j = 0.5$ which corresponds to columns with no sway ($\Delta/h = 0$, $D_r = \pm \infty$). The no sway condition may occur under symmetrical gravity loading. The graph in Fig. 7.2 also has a horizontal asymptote at $D_r = 1.5$ which corresponds to columns with zero moment at the upper end ($M_{jU} = 0$, $q_j = \infty$).

Consider the bottom story fixed base columns of a symmetrical frame under combined load with wind acting from left to right. Before wind loads are applied, the moments due to gravity load in the windward and leeward columns are indicated in sketches (1) and (7) of Fig. 7.2. The interior columns have similar moment diagrams with generally smaller moments due to gravity load.

As wind loads are applied (with gravity loads held constant), the windward column moments change as indicated by the diagrams below sketch (1), and the leeward column moments vary as shown in the diagrams.
above sketch (7). The interior column moments start with sketch (1) or (7) and proceed toward the center in Fig. 7.2 under increasing wind loads.

The moment diagrams below sketch (3) in Fig. 7.2 give column shears which act in the direction of the story shear for wind from left to right. Most of the bottom story columns must have combined load moment diagrams below sketch (3) in order to satisfy shear equilibrium in the story. Shear equilibrium could be satisfied if the column moment diagrams are assumed between sketches (3) and (4) but this places the columns in single curvature bending. The combined load moments in many interior and leeward columns may be expected to lie between sketches (5) and (6). Then the deflection ratio is within the range $0 \leq D_r \leq 1$. The windward column moments due to combined load may be expected to lie somewhere between sketches (1) and (6) which indicates that $D_r \geq 0$ for these columns. Note that the range of moments between sketches (2) and (4) is the range of single curvature bending.

From this discussion we tentatively conclude that the deflection ratio range

$$0 \leq D_r \leq 1$$  \hspace{1cm} (7.7)

may be anticipated for most bottom story columns with rotation fixed bases. The deflection ratio may tend to exceed this range for those exterior columns with gravity load moments which are large relative to the wind moments. Further study is needed to suggest the limitations on frame geometry and load parameters which correspond to the deflection ratio range in Eq. (7.7). Note that the conventional assumption of
rotation fixed ends for bottom story columns is considered in Fig. 7.2, sketch (6) with \( D_r = 0 \). This assumption does not give a very good estimate of fixed base column moments under combined load in the elastic range, if the tops of the columns can rotate.

Graphs of \( D_r \) versus \( q_j \) for columns with rotationally restrained bases \((0 < R_b < 1)\) are similar to Fig. 7.2 except that the asymptotes are closer to the coordinate axes. For example, if we take \( R_f = 1.0 \) for the rotational restraint in Eq. (7.2) then the base factor \( R_b = 0.5 \). These values give the asymptotes \( D_r = 9/7 \) and \( q_j = 2/7 \) which intersect at point B in Fig. 7.2. The dashed curve in this figure is for \( R_f = 1.0 \). Note that the values \( D_r = 3.0 \) and \( 1.0 \) give \( q_j = 0 \) and \( 2.0 \) from Eq. (7.5) for any base factor. This indicates that points (2) and (5) in Fig. 7.2 remain fixed as the base factor varies. If the column base is pinned \((R_f = 0 \text{ and } R_b = 1)\) the graph of \( D_r \) versus \( q_j \) degenerates to the vertical line \( q_j = 0 \).

If we consider practical values of the base restrain coefficient \( R_f \) to be larger than unity, then the column moment ratio \( q_j \) is bounded by the solid and dashed curves in Fig. 7.2. From this we conclude that the column moment ratio is relatively insensitive to the rotational restraint at the base of a column if the deflection ratio \( D_r \) satisfies Eq. (7.7). Crude estimates of \( R_f \) and \( D_r \) are apparently sufficient to establish values of the column moment ratio \( q_j \). The value of \( q_j \) can be used as a guide in the bottom story moment balance as indicated in Art. 7.3.
Now we turn to the second feature of the moments in bottom story columns. The carry-over moment concept is well established in elastic moment distribution. This same concept may be used to advantage in plastic moment balancing for bottom story columns. When the joints at the top of these columns are balanced, using method I in Chapter 6, the column moments change from their initial value $M_{IU}$ to the final value $M_{jU}$. The resulting change in moment $\delta M_{jU}$ produces a change $\delta M_{jL}$ at the base of the columns. Since the sway deflections are considered to be fixed in moment balancing, this moment carry-over from column top to column base may be estimated by holding the chord rotation constant in the bottom story. Then the carry-over moment, from Eq. (7.3) is

$$\delta M_{jL} = \left[ \frac{2(1 - R_b)}{4 - R_b} \right] \delta M_{jU}$$

where the bracketed term is the carry-over factor. This carry-over factor ranges from zero for pinned base columns ($R_b = 1$) to 0.5 for columns with fixed bases ($R_b = 0$). For a column with rotational restraint coefficient $R_f = 1.0$ ($R_b = 0.5$) the carry-over factor is 0.286.

### 7.3 BOTTOM STORY MOMENT BALANCE

The ideas used in the bottom story moment balance are described in six steps.

**Step 1** - Assume an initial deflected state for the bottom story defined by the chord rotation $\Delta/h$ and the deflection ratio $D_r$. Also assign the base factor
The chord rotation is assigned for all stories in plastic moment balancing to include estimated \( \Phi \) effects in the moment balance. In the bottom story we refine the deflection assumption by estimating the influence of the girders and the foundation on the vertical distribution of moments. This is simply a refinement of the conventional fixed-end assumption for the columns shown in sketch (6) of Fig. 7.2.

**Step 2** - Estimate the vertical distribution factor \( D_v \) for the bottom story column moments. The vertical distribution factor, defined in Eq. (4.10), can be expressed as

\[
D_v(I) = \frac{\sum_{J} M_{ju}(J,I)}{\sum_{J} M_{ju}(J,I) + \sum_{J} M_{jL}(J,I)} = \frac{1}{1 + \overline{q}_j}
\]

where \( \overline{q}_j \) is a weighted column moment ratio for the bottom story. We can find \( \overline{q}_j \) from

\[
\overline{q}_j = \frac{\sum_{J} M_{jL}(J,I)}{\sum_{J} M_{ju}(J,I)} = \sum_{J} \left( \frac{M_{ju}(J,I)}{\sum_{J} M_{ju}(J,I)} \right) \]

using the moment ratio \( q_j(J) \) from Eq. (7.5) for column (J). The value of \( q_j(J) \) is weighted by the fraction \( \frac{M_{ju}(J,I)}{\sum_{J} M_{ju}(J,I)} \) for column (J).
The discussion in Art. 7.2 suggests that most of the bottom story columns will have moment diagrams between sketches (5) and (6) in Fig. 7.2 for the fixed base condition. It is further suggested that the column moment ratio \( q_j = \frac{M_jL}{M_jU} \) is relatively insensitive to the rotational restraint at the base of a column, at least within the practical range of base restraint. If these premises are acceptable as a first approximation, the values of \( q_j(J) \) in Eq. (7.10) are limited to a small range for most of the bottom story columns. We may assign a common value of \( q_j(J) \) to all of the bottom story columns for the purpose of estimating \( D_V \). Then Eq. (7.10) reduces to \( \tilde{q}_j = q_j \).

To find the moment ratio \( \tilde{q}_j \) for the bottom story we can use Eq. (7.5). This amounts to redefining the parameters \( D_r \) and \( R_b \) in Art. 7.2. Instead of considering these parameters for an individual column, we regard them as parameters which determine the vertical distribution of moments in the bottom story.

For example, we can associate the joint rotation \( \Theta_{jU} \) in the definition for \( D_r \), Eq. (7.6), with the joint rotation \( \Delta_g / h \) in Fig. 1.3(c). Then \( D_r \) represents the influence of the girders on the vertical distribution of moments. Similarly the joint rotation \( \Theta_{jL} \) in Eq. (7.2) can be associated with the joint rotation \( \Delta_g / h \) at the base of the columns in Fig. 1.3(c). Then \( R_b \) reflects the influence of rotational restraint at the base of the columns on the vertical distribution of moments. Although the same joint rotation appears at the upper and lower ends of the columns in Fig. 1.3(c) there is no reason why the rotational restraint at the top and bottom of the bottom story columns must be the same.
The deflection ratio range in Eq. (7.7) is an appropriate guide in estimating $D_r$. Within this range, $D_r$ is inversely proportional to the rotational stiffness of the girders on the level above the bottom story. Although little is known about these girders at the beginning of the preliminary design, their influence on the bottom story columns can be tentatively anticipated in assigning the value of $D_r$.

Individual girders may differ in their contribution to resisting rotation of the columns, but this is not the effect considered in assigning $D_r$. Instead, we are interested in the combined restraining effect of all of the girders on the level above the bottom story. The aim in assigning $D_r$ is to estimate the combined effect of the girders on the vertical distribution of column moments.

Note that if the column moment sums $\sum M_{jU}$ and $\sum M_{jL}$ are determined from a refined inelastic sway analysis of the bottom story, $D_V$ and $\bar{q}_j$ may be calculated from Eqs. (7.9 and 7.10). This same value of $D_V$ can then be used in the bottom story moment balance for frames with similar frame geometry, load, and restraint parameters. This indicates how future studies may be used to improve on the ideas presented here.

Vertical distribution factors are considered in Art. 7.4. The remaining steps in the bottom story moment balance are as follows.

Step 3 - Find the sway moment $\sum M_{c(I)}$ in the bottom story and distribute the story sway moment using Eqs. (4.9 and 4.10).

Step 4 - Perform the plastic moment balance for the girders on the floor above the bottom story as described in Chapter 5.
Step 5 - Balance the joints on this floor using method I in Chapter 6. In this step the initial moments at the top of the bottom story columns are estimated using

\[ M_{jU(I,J)} = D_{U(I,J)} D_{V(I)} \sum M_C(I) \] (7.11)

and the balancing moments in these columns take the form

\[ \delta M_{jU(I,J)} = (1 - D_{j(I)}) M_{iE(J,I)} \] (7.12)

The column moment distribution factors \( D_{U(I,J)} \) and the joint balancing ratio \( D_j \) must be assigned. These factors control the horizontal distribution of moments between the bottom story columns. If it is desired to limit rotation capacity requirements for these columns then approximate elastic moment distribution methods may be used as a guide in assigning the values of \( D_U \) and \( D_j \). Alternatively, the results of previous inelastic sway analyses, which do not involve large rotation capacity requirements, may be used as the guide. Regardless of how the column moment distribution factors are obtained, they must sum to unity as per Eq. (6.3).

Note that the joint balance changes the moments at the top of the bottom story columns. During this process the joints rotate. We are not concerned with the amount of this joint balancing rotation except that it should be sufficient to establish joint equilibrium.
Step 6 - The moments at the base of the bottom story columns are

\[ M_{jL}(J,I) = q_j M_{iU}(J,I) + \left[ \frac{2}{4 - \frac{R_b}{R_b}} \right] \delta M_{jU}(J,I) \] (7.13)

using the initial and balancing moments at the top of the columns from Eqs. (7.11 and 7.12). The first term of Eq. (7.13) is the initial moment and the second term the carry-over moment at the column base. The initial moments are consistent with the vertical distribution factor \( D_V \), Eq. (7.9), and the distribution of initial moments between the columns in Eq. (7.11). The carry-over moment in Eq. (7.13) accounts for the rotation at top and base of the columns which occur when the joints are balanced.

These six steps represent an approximate elastic modification of the bottom story moment balance. The modification is formulated so as to satisfy joint and story equilibrium requirements and to preserve the basic operations of plastic moment balancing (steps 3 to 5). The aim of this elastic modification is to delay (but not prevent) the formation of plastic hinges in bottom story columns and to indicate the possibility of single curvature bending. Further inelastic sway studies are needed to indicate how well these aims are satisfied by the elastic modification and to suggest improvements or simplifications. For example, the bottom story moment balance could be simplified by using automatic joint balancing methods III or IV of Chapter 6 in
step (5). This requires modifications in step (6) which remain to be formulated.

It is important to note that the same carry-over factor must be used for each bottom story column in step (6). Otherwise, the carry-over operation changes the total sway moment \( \sum M_c(I) \) in the bottom story. From Eqs. (7.12 and 7.13) the moment carried over to the base of column \( (J) \) is

\[
\delta M_{jL(J,I)} = \left[ \frac{2 (1 - R_b)}{4 - R_b} \right] (1 - D_j(I)) M_{iE(J,I)} \quad (7.14)
\]

The total change in the column base moments is the sum of \( \delta M_{jL} \). If the bracketed carry-over factor is the same for each column, the sum of \( \delta M_{jL} \) is zero because the sum of the external moments \( M_{iE(J,I)} \) on the level above the base must vanish as in Eq. (6.11). Thus the total bottom story sway moment \( \sum M_c(I) \) and the vertical distribution of moment in this story are conserved in the column moment carry-over operation. The effect of the carry-over operation is to redistribute the column base moments in a manner which depends on the joint balance for the level above the bottom story and on the base factor.

The bottom story moment balance is graphically described in Fig. 7.3. Sketches (a) and (b) are moment diagrams for windward and leeward columns with wind from left to right. The dashed lines represent initial moments prior to the joint balance. The initial moments from sketch (5) of Fig. 7.2 are reproduced in Fig. 7.3. These initial moments correspond to \( R_b = 0 \) (fixed base) and \( D_r = 1.0 \) (flexible girders). From Eq. (7.5) the column moment ratio \( q_j = 2.0 \) for the
bottom story and Eq. (7.9) gives the vertical distribution factor

\[ D_V = \frac{1}{3}. \]

The initial moments need not be identical for each column but they do give the same column moment ratio. The point of inflection \((M = 0)\) for the initial moment diagrams is at a distance \((1 - D_V) h\) from the base. Any set of column moments with the same moment ratio

\[ q_j = \frac{M_{jL}}{M_{jU}} \]

which balances the total sway moment \(\sum M_C(I)\) in the bottom story is an admissible set of initial moments.

The joint balance and moment carry-over operations may change the column moment diagrams from the dashed to the solid lines in Fig. 7.3. These solid lines pivot about a point on the initial moment diagram at a distance \(p h\) above the base. The pivot factor \(p\) follows from Eq. (7.8) in the form

\[ p = \frac{\delta M_{jL}}{\delta M_{jL} + \delta M_{jU}} = \frac{2 (1 - R_b)}{3 (2 - R_b)} \quad 0 \leq p \leq \frac{1}{3} \quad (7.15) \]

and is bounded by zero for \(R_b = 1.0\) (pinned base) and \(1/3\) for \(R_b = 0\) (fixed base). The latter value of \(p\) is used in Fig. 7.3.

It is of interest to note that the same initial moments are obtained in Fig. 7.3 if we change the base factor from zero to \(R_b = 0.5\) \((R_f = 1.0)\) with the deflection ratio held constant. However, the final base moments vary with \(R_b\) because the pivot factor changes to \(p = 2/9\), thus reducing the moment carried over to the more flexible base.

The joint balancing moments \(\delta M_{jU}\) for the two columns in Fig. 7.3 are equal and opposite. The same is true of the carry-over moments \(\delta M_{jL}\). The sum of the joint balancing moments must always vanish as the result of Eq. (7.11). The carry-over moment sum is also zero if the
same carry-over factor or pivot factor $p$ is used for all bottom story columns.

The result of the carry-over operation is to change the moment at the base of a bottom story column by an amount which is less than the change in moment during the joint balance at the top of the column. The carry-over operation may also cause

(1) A decrease in the maximum moment and single curvature bending for windward columns, as in Fig. 7.3(a) and

(2) An increase in the base moment in leeward columns as in Fig. 7.3(b).

If the initial leeward column moments are between sketches (5) and (6) in Fig. 7.2 the increased base moment may be larger than the moment at the top of the leeward column. The carry-over operation then serves to delay the formation of plastic hinges at the base of leeward columns by requiring a larger column moment capacity in the preliminary design. Of course, if the balancing moments $6M_{ju}$ in Fig. 7.3 are in the opposite direction, the results of the carry-over operation are reversed.

The elastic column ideas presented in this article are intended for use as a guide in plastic moment balancing. However, these ideas may also have application in a conventional allowable stress design method.
7.4 VERTICAL DISTRIBUTION FACTOR

The vertical distribution factor $D_v$ in the modified bottom story moment balance depends upon two stiffness quantities which are (1) the girder stiffness, as measured by the deflection ratio $D_r$, and (2) the rotational restraint at the base of the bottom story columns, as measured by the base factor $R_b$. These stiffness parameters are defined, relative to the columns, in Art. 7.2. Using Eqs. (7.5 and 7.9), we can obtain $D_v$ as a function of the stiffness parameters in the form

$$D_v = \frac{(4 - R_b) D_r - 3 (2 - R_b)}{3 (2 - R_b) D_r - 3 (4 - 3R_b)}$$

(7.16)

The column moment ratio for the initial moment state is then

$$\bar{q}_j = \frac{1}{D_v} - 1$$

(7.17)

This indicates that the coefficients on the moment terms in Eq. (7.13) for the moment at the base of the bottom story columns are functions of $D_r$ and $R_b$. The moment terms $M_{iU}$ and $\delta M_{jU}$ in this equation depend on the horizontal distribution of the story sway moment between the bottom story columns.

**Design Chart for Vertical Distribution Factor**

Plots of $D_v$ versus $R_b$ for selected values of the deflection ratio $D_r$ are shown in Fig. 7.4, using Eq. (7.16). The values of $D_r$ range from minus infinity for the top dashed curve to +1.25 for the bottom dashed curve. Values of $D_v$ above the top dashed curve are impossible unless the bottom story sways into the wind. The dashed curve for $D_r = 1.25$ indicates the unusually low values of $D_v$.
corresponding to very flexible girders. The curves labeled \( D_r = 0 \) and \( D_r = 1.0 \) include the deflection ratio range in Eq. (7.7). These curves are shown as heavy solid lines to indicate that they represent the tentative practical range for the deflection ratio.

The left and right sides of the graph in Fig. 7.4 correspond to fixed and pinned base conditions. In Art. 7.2 it is suggested that the foundation rotational restraint coefficient is usually in the range \( 1.0 \leq R_f \leq \infty \). Based on this premise, the base factor is limited to the range \( 0 \leq R_b \leq 0.5 \) as indicated above the curves in Fig. 7.4. The points (1-3-4-2) define the region of interest in Fig. 7.4. This region maps into the area between the solid and dashed curves in Fig. 7.2. From Fig. 7.4 the vertical distribution factor may be expected to lie between \( D_v = 0.6 \) at point (3) and \( D_v = 0.33 \) at point (4) for many frames.

An increase in girder stiffness reduces the rotation of the joints on the level above the bottom story and decreases the deflection ratio \( D_r \). The result is to increase the vertical distribution factor \( D_v \). On the other hand, an increase in base restraint tends to decrease \( D_v \). The influence of increasing girder and base stiffness is indicated by the arrows above the graph in Fig. 7.4. The five sketches at the top of this figure show how the column moment and column deflection diagrams vary with the base factor and deflection ratio. Sketches (1) to (4) bound the diagrams of practical interest. The numbers above and below the moment diagrams are relative column moments with unit sums. Notice the 67 percent increase in the relative base moment.
for the transition from sketch (3), representing rigid girders and a flexible base, to sketch (2) for flexible girders and a rigid base, in Fig. (7.4).

The curves in Fig. 7.4 show that $D_v$ is not sensitive to the base factor in the range $0 \leq R_b \leq 0.5$. This suggests that we can approximate the vertical distribution factors by using $R_b = 0.3$ in Eq. (7.16), with the result

$$D_v \approx \frac{D_r - 1.4}{1.4 D_r - 2.5}$$

This result gives values of $D_v$ which differ by a maximum of 12 percent relative to Eq. (7.16). Since both $R_b$ and $D_r$ must be estimated, differences of this magnitude are not excessive for purposes of preliminary design. The bracketed carry-over factor in Eqs. (7.8 and 7.13) is approximately 0.4 for $R_b = 0.3$.

7.5 EVALUATION

The modified bottom story moment balance introduces two additional parameters into the plastic moment balancing method. These are the base factor $R_b$ and the deflection ratio $D_r$. The discussion of Fig. 7.2 in Art. 7.2 suggests that the structural behavior of the bottom story is not sensitive to these parameters if they are assigned with reasonable judgment in the preliminary design. However, this tentative conclusion is based on elastic approximations of structural behavior.
A more convincing evaluation of the influence of base restraint and joint rotation could be obtained from an (inelastic) analytical and experimental investigation of bottom story column-and-girder subassemblages with non-linear rotational base restraint. Such an investigation would also provide a more rational basis for assigning values of \( R_b \) and \( D_r \). The minimal amount of information on the interaction between a frame and its foundation is a fundamental gap in the basis for any method of designing bottom story columns.

Reference 99 gives an indication of the results which may be expected from further study of base restraint. This paper uses elastic second order slope deflection equations with a tangent modulus stiffness modification to study the in-plane sidesway buckling of one- and two-story, single bay unbraced frames with elastically restrained column bases. The paper concludes that a small amount of base restraint can significantly increase the elastic sidesway frame buckling load. The practical result of this study is to encourage the use of simple and economical column base details and to reduce the frame stiffness requirements needed to control sidesway buckling of one and two story frames. If similar conclusions can be demonstrated for bottom story columns in multi-story unbraced frames under gravity and combined loads, the result may be to reduce the material, fabrication, and erection costs of the heaviest frame members.

In the meantime, the modified bottom story moment balance provides a convenient and adaptable method for the preliminary design of bottom story columns. The conventional assumption of fixed ends for
bottom story columns is included in this method by taking $D_r = R_b = 0$. If judgment suggests that something less than complete rotational restraint can be anticipated, the parameters $D_r$ and $R_b$ provide a simple means for applying this judgment in the preliminary design stage. Otherwise the modified bottom story moment balance introduces no additional design effort into the plastic moment balancing method.
8. PRELIMINARY GIRDER DESIGN

PARAMETERS

The plastic moment balancing parameters which influence the design of girders are the positive moment coefficients $C$ and $C_1$ (Art. 5.3) and the girder sway moment distribution factors $D_G$ (Art. 4.4). This chapter explores the objectives which can be accomplished in assigning these parameters.

8.1 POSITIVE MOMENT FACTORS

The statical relations for restricted girder hinge patterns are described in Art. 5.3 and summarized in Fig. 5.6. The purpose of restricting the hinge pattern in a girder is to conserve sway stiffness under ultimate gravity or combined loading. This is accomplished by specifying the value of the positive moment factors $C < 1$ for combined load and $C_1 < 1$ for gravity load. This requires a plastic moment capacity larger than the minimum $M_p$ needed for a girder mechanism under gravity or combined loading. Then the preliminary design girder moment diagram for the controlling load is determined by a negative girder end-moment equal to the required plastic moment $M_p$ and a maximum positive girder moment $M = C M_p$ or $C_1 M_p$. No girder mechanism develops under these conditions if the distribution of moments in the frame is reasonably similar to that assigned in the preliminary design. This
means that the girder can still contribute to the sway stiffness of the frame at ultimate load.

In Chapter 5, the only limitations given for the moment factors $C$ and $C_1$ are $0 < C \leq 1$ and $0.5 \leq C_1 \leq 1$. These limits apply for girders with uniform plastic moment capacity. The lower limit on $C_1$ corresponds to the clear span fixed-end moment diagram under gravity loading. It is logical to expect a more restrictive lower limit on the positive moment factor $C$ for combined loading.

It is helpful to study what happens when small values of $C$ are used in the plastic moment balance; that is when $M_p$ is noticeable larger than the minimum plastic moment for a girder mechanism under combined loading. Assume that combined loading controls $M_p$ and in addition that the restricted mechanism factor $R$ and the sway moment coefficient $G$ satisfy the elastic-plastic boundary condition of Eq. (5.44) for the leeward end-moment $M_B$. The combined load girder moment diagram in Fig. 5.8(c) illustrates the elastic-plastic boundary condition and gives the restricted mechanism factor

$$ R = \frac{M_p}{M_{pm}} = \frac{4}{3} + \frac{G}{2} $$

(8.1)

where $M_{pm} = F_{2R} w L_g^2 / 16$. If combined load controls $M_p$, then Eqs. (5.29(a) and (c)) also relate $R$, $G$, and the positive moment factor $C$. From Eqs. (5.29) and (8.1) the values of $C$ for the elastic-plastic boundary are
\[
C = \frac{2 \left(1 + \frac{G}{8}\right)^2}{4/3 + G/2} - 1 \quad \text{for } 0 \leq G \leq 8 \tag{8.2a}
\]

\[
C = \frac{G}{4/3 + G/2} - 1 \quad \text{for } G > 8 \tag{8.2b}
\]

If the value of \(C\) from Eq. (8.2) is used, the assumed elastic sway response in Fig. 5.8 indicates that the girder moments remain elastic until the controlling combined load condition is reached. No plastic hinges are formed until the sway moment increases to \(M_G = G M_p/(1 - d_c/L)\), at which time the leeward end-moment \(M_B\) first reaches \(M_p\) and the maximum positive moment \(M_C = C M_p\). Since the positive plastic moment capacity is not fully utilized, the girder can resist a larger sway moment if necessary.

Figure 8.1 is an example of the results obtained by using the positive moment factor from Eq. (8.2) in the plastic moment balance. The data used to begin this example is

- Sway moment coefficient \(G = 8.0\)
- Positive moment factor \(C = 0.5\)

The box at the bottom of Fig. 8.1 includes the sequence of the plastic moment balance and the results

- Restricted mechanism factor \(R = 16/3\)
- Required plastic moment \(M_p = (16/3) M_{pm}\)
- Maximum positive moment \(C M_p = (8/3) M_{pm}\)

Since \(G = 8\) the maximum positive moment section is at the windward end \(A\) and \(M_C = M_A\). The \(M/M_{pm}\) axis at the right side of Fig. 8.1 provides a
nondimensional scale for the girder moment diagram. The steps in drawing the moment diagram are explained in Art. 1.5. Note that the solid sloping fixing line in Fig. 8.1 is tangent to the dashed positive moment envelope (Art. 5.3) labelled $C = 0.5$. This envelope is drawn at a distance $0.5 M_p$ above the simple span gravity load moment diagram for the girder.

Consider now the sway behavior of the girder in Fig. 8.1 with the gravity load $F_{2R} w_L$ held constant. As sway moments are applied and $G$ increases from 0 to 8, the fixing line rotates about the elastic pivot from the fixed-end position (shown as a horizontal dash-dot line) to the controlling combined load position with $M_B = M_p$. The entire sway response is in the elastic range! No plastic moment redistribution occurs and no rotation capacity is needed. Furthermore, the girder contribution to sway stiffness is retained at its elastic value up to the controlling combined load condition. Note that the $R, G$ coordinates for this example lie on the elastic-plastic boundary at point B in Fig. 5.9.

It is emphasized that the elastic pivot idea, described in Art. 5.4, is a convenient preliminary design approximation to describe the elastic sway response of a girder. The portal and cantilever methods of preliminary elastic wind analysis provide the precedent for the elastic pivot assumption. Some departure from the assumed elastic sway response can be expected for girders in irregular frames but the elastic pivot idea is a reasonable first approximation which can be refined in later stages of the design process if this is considered necessary. Some perspective on the accuracy of the basic wind load and gravity load is helpful in deciding whether such design refinements are warranted or meaningful. Design practice definitely is not an exact science. This
is particularly true of the preliminary wind load analysis for an un-braced multi-story frame.

If the girder in Fig. 8.1 is designed to form a sway mechanism under combined load \((C = 1)\), the mechanism factor changes to \(R = 4\).
The girder end-moments are then \(M_B = -M_A = M_p = 4 \cdot m\) and the sloping fixing line shifts upward to the position shown dotted in Fig. 8.1.
The results of increasing \(C\) from 0.5 to 1.0 are to decrease \(M_p\) from the plastic moment balance by \(\frac{16}{3} - 4) \div \frac{16}{3}\) or 25 percent and to decrease the sway stiffness of the frame in the stories above and below the girder.

In effect, the example in Fig. 8.1 represents a conventional elastic design situation and appears to defeat the plastic design concept. However, this is not the case. We may interpret this example as a limiting condition which may be used together with more economically designed plastic girders in adjacent bays on one level. The elastic girder then serves to conserve sway stiffness and limit the \(P_A\) moments in the stories above and below the level.

If the positive moment factor \(C\) is assigned in the range \((1.0 \leq C \leq 0.5)\) the fixing lines from the girder plastic moment balance are bounded by the dotted and solid sloping lines in Fig. 8.1.
According to condition (1) in Art. 5.2, all fixing lines for a constant sway moment coefficient \(G\) are parallel. As \(C\) is decreased from the mechanism value \(C = 1.0\) to the value from Eq. 8.2, the fixing line shifts parallel to itself from the dotted to the solid sloping lines in Fig. 8.1. The value of \(M_p\) from the girder plastic moment balance increases with decreasing values of \(C\). The increased plastic moment
requirement means that more of the total sway moment is resisted elastically and that deterioration of sway stiffness due to plastic hinges is less pronounced. This of course assumes that the distribution of sway moments in the frame is reasonably similar to that assigned in the preliminary design.

The example in Fig. 8.1 is considered a limiting condition in the plastic moment balance for the following reason. For the value $G = 8$ used in this example, the positive moment limit imposed by $C = 0.5$ is reached at the same time that the controlling combined load condition is attained. If a value of $C < 0.5$ is assigned in the girder plastic moment balance for $G = 8$, the apparent fixing line is parallel to and below the solid sloping line in Fig. 8.1. This implies that the point of inflection in the girder sway moment diagram is to the left of the elastic pivot. (The sway moment at any girder cross-section is the vertical distance from the horizontal dot-dashed fixing line for the fixed-end moment state, to the sloping fixing line determined from the plastic moment balance.) In fact if we assign $C = 0$ in the girder plastic moment balance, the entire girder is apparently subjected to negative moment except at the windward end where $M_A = 0$. This is obviously an extreme and unlikely situation.

The smallest value of the positive moment factor $C$ which makes the fixing line pass through the elastic pivot is considered to be a reasonable (but not an absolute) lower bound on $C$. This lower bound on $C$ is given by Eq. 8.2. In essence, it is assumed that the girder wind moments obtained from the portal method are a limiting case. The girder plastic moment capacity for the portal wind moment distribution
is obtained from Eq. 8.1. If values of \( \text{C} \) larger than Eq. 8.2 are assigned, the required \( M_p \) from the plastic moment balance is less than the corresponding plastic moment requirement for the portal wind moment distribution.

A graph of the positive moment factor \( \text{C} \) versus the sway moment coefficient \( \text{G} \) from Eq. (8.2) is shown in Fig. 8.2 to define the conditions separating completely elastic girder sway response from elastic-plastic behavior. This graph is based on the assumption that the elastic sway response of the girder follows the elastic pivot assumption described in Fig. 5.8 and Art. 5.4. The notation "Elastic-Plastic Domain" in Fig. 8.2 indicates that the leeward girder end-moment \( M_B \) reaches \( M_p \) before the maximum positive girder moment \( M_C \) reaches \( C M_p \). In this "Elastic-Plastic Domain," rotation capacity at the leeward hinge is required to redistribute the girder moments under increasing sway effects until the sway moment reaches \( M_G = G M_{pm} / (1 - d / L) \). The notation "Elastic Domain" defines the range of \( \text{C} \) and \( \text{G} \) values in the girder moment balance which result in entirely elastic sway response if the required plastic moment is computed from \( M_p = R M_{pm} \) using Fig. 5.6. In the elastic domain there is little to be gained by using a girder with \( M_p \) larger than that obtained from Eq. (8.1) for combined load. If a girder with larger \( M_p \) is used, the only advantage gained is a marginal elastic increase in sway stiffness.

The values \( \text{G} = 8 \) and \( \text{C} = 0.5 \) used in Fig. 8.1 fall on the border between the plastic and elastic domains in Fig. 8.2. Equation (8.2b) defines the elastic-plastic boundary in Fig. 8.2 for \( \text{G} \geq 8 \) and asymptotically approaches the vertical line \( \text{C} = 1 \) as \( \text{G} \) goes to infinity.
For a large sway moment, say $G = 40$ (five times the sway moment in Fig. 8.1), Eq. (8.2b) gives $C = 7/8$. These values are recorded in the top right corner of Fig. 8.2. Values of $G$ near the top of Fig. 8.2 are obtained for a girder with a large sway moment relative to the gravity load moments. The value $G = \infty$ corresponds to zero gravity load. It is evident that a relatively small increase in $M_p$ above that for a girder panel mechanism (Fig. 1.5b) is required to ensure that all moments can be resisted in the elastic range, without sway stiffness deterioration.

The elastic-plastic boundary is defined by Eq. (8.2a) for $G \leq 8$. In this region of smaller sway moments, the limiting values of $C$ range from $1/2$ to $1/3$ as indicated in Fig. 8.2. The combined load girder moment diagram corresponding to the minimum coordinates $G = 8/3$ and $C = 1/3$ is shown in Fig. 8.3 together with the calculations for the plastic moment balance. The windward end-moment $M_A$ equals zero from Eq. (5.41) and the maximum positive moment $M_C = (1/3)M_p$ occurs at a distance $X = (2/3)L$ from the leeward end of the girder, using Eq. (5.13). Values of $G$ and $C$ on the elastic-plastic boundary in Fig. 8.2 above the coordinates $G = 8/3, C = 1/3$ give positive windward end moments $M_A$ and below these coordinates, $M_A$ is negative.

For sufficiently small values of the sway moment coefficient $G$, the required plastic moment capacity $M_p = R_{LB} M_{pm}$ is controlled by the factored gravity load $F_{1R} w L g$ rather than by combined loading. If gravity load controls $M_p$, the maximum sway moment coefficient can be determined from Eq. (5.33). This equation is used to plot the dashed curves at the bottom of Fig. 8.2. These curves define the gravity load
domain, which depends upon the required load factors for gravity and combined load and upon the positive moment factors $C$ and $C_1$. Curves for three values of $C_1$ (for gravity load) are shown in Fig. 8.2 using the load factors $F_{1R} = 1.70$ and $F_{2R} = F_{1R}/1.33$ in Eq. (5.33). Other load factors will shift the $C_1$ curves. The required plastic moment is controlled by gravity loading for $(C, G)$ coordinates below the $C_1$ curves.

An example will help to explain the significance of the $C_1$ curves in Fig. 8.2. Suppose we decide to use a restricted girder hinge pattern for gravity load with $C_1 = 0.50$ and $F_{1R}/F_{2R} = 1.33$. Also assume that the distribution of sway moments under combined loading gives $G = 0.9$ for the girder. Entering Fig. 8.2 with these values we find $C = 0.4$. If this value of $C$ is used in the plastic moment balance for combined load, the same required plastic moment is adequate for gravity and combined loading. If a smaller value of $C$ is assigned in the plastic moment balance, combined loading controls $M_p$. Values of $C$ and $G$ on the line $C_1 = 0.5$ in Fig. 8.2 result in a girder which is just elastic under factored gravity loading. This is based on an assumed fixed-end moment diagram due to gravity loads.

The intersection of the solid elastic-plastic boundary curve with the dashed $C_1$ curves in Fig. 8.2 correlates with the boundary lines in Fig. 5.10. For example, the dashed curve labelled $C_1 = 0.5$ intersects the solid elastic-plastic boundary curve at $G = 0.9$ and $C = 0.4$ for $F_{1R}/F_{2R} = 1.33$ in Fig. 8.2. The line labelled $C_1 = 0.5$ in Fig. 5.10 gives the same value of $G$ for this load factor ratio. If we use the positive moment factors $C_1 = 0.5$ for gravity load and $C = 0.4$ for combined load in the preliminary design of a girder with sway moment
coefficient $G = 0.9$ and load factor ratio $F_{1R}/F_{2R} = 1.33$ (point A in Figs. 8.2 and 5.10) we may expect the girder to remain elastic under both factored gravity and factored combined loading. The full elastic sway stiffness contribution of this girder is then available at ultimate load if the distribution of moments in the frame follows that used in the preliminary design.

8.2 ELASTIC SWAY RATIO

If the positive moment factor $C$ and sway moment coefficient $G$ are in the plastic domain in Fig. 8.2, part of the girder sway response is elastic (Fig. 5.8) and the remainder is in the elastic-plastic range with one plastic hinge at the leeward end of the girder. It is helpful to know what portion of the sway response is elastic because this gives a rough idea of the deterioration of sway stiffness due to the formation of leeward plastic hinges in girders. If we define

$$G_{ES} = \text{sway moment coefficient at formation of leeward plastic hinge under combined load (subscript ES for elastic sway), and}$$

$$E_{SR} = G_{ES}/G = \text{elastic sway ratio}$$

then $E_{SR}$ indicates what portion of the girder sway moment $M_G = G_{pm}^{\text{le}}/(1 - d_c/L)$ is resisted in the elastic range.

Using Fig. 5.8(c) or Eq. (5.44) we have

$$G_{ES} = 2 (R - 4/3) \quad (8.3)$$
The restricted mechanism factor $R$ is given by Eq. (5.29) if a restricted girder hinge pattern controls the plastic moment under combined load. The elastic sway ratio equations are obtained from Eqs. (8.3 and 5.29) with the results

$$E_{SR} = \frac{2}{G} \left( \frac{2}{C + 1} \left( \frac{1 + G/8}{2} - \frac{4}{3} \right) \right)$$ \text{ for } G \leq 8 \quad (8.4a)$$

$$E_{SR} = \frac{2}{G} \left( \frac{G}{C + 1} - \frac{4}{3} \right)$$ \text{ for } G > 8 \quad (8.4b)$$

Figure 8.4 gives plots of $G$ versus $E_{SR}$ for six values of the positive moment factor $C$. The region between the curve labeled $C = 1.0$ and the vertical line $E_{SR} = 1.0$ in Fig. 8.4 is the Elastic-Plastic Domain in Fig. 8.2. The elastic-plastic boundary in Fig. 8.2 maps into the vertical line $E_{SR} = 1.0$ at the right side of Fig. 8.4.

Positive moment coefficients in the range $1 \leq C < 0.5$ may give negative values of $E_{SR}$ for small values of $G < 1.24$ in Eq. (8.4a). Negative values of $E_{SR}$ indicate that the factored gravity load $F_{2R} w_L g$ causes negative plastic hinges at both ends of the girder before any sway moments are applied. Values of $G$ below the dashed $C_1$ curves in Fig. 8.4 indicate the range of sway moments for which a restricted hinge pattern under gravity load controls the required plastic moment. These dashed curves are obtained by cross-plotting the $C_1$ curves from Fig. 8.2.

The curve labeled $C = 1.0$ in Fig. 8.4 corresponds to the combined load girder sway or panel mechanisms shown in the insets. This
curve illustrates an interesting feature of girder mechanisms for combined load. As the total sway moment resisted by a girder mechanism increases, relative to the gravity load end-moment, an increasing portion of the sway response is in the elastic range. For example, a three-fold increase in the sway moment coefficient from $G = 8/3$ to $G = 8$ results in a two-fold increase in the portion of the total sway moment which is carried elastically. This implies that the rotation capacity required at the leeward plastic hinge, to develop a girder sway mechanism, decreases with increasing values of $G$. As rotation capacity requirements decrease, it may be possible to relax the plastic design limitations on width-to-thickness ratios for the flange and web elements of $W$ shapes. Furthermore, note that the moment gradient in the vicinity of the leeward hinge increases with increasing values of $G$. This indicates that the yield zone adjacent to the leeward hinge is more confined, thus decreasing the tendency for local and lateral-torsional buckling of the bottom (compression) flange.

Note that girder rotation capacity requirements are related to the loading sequence and the order of hinge formation. Hinges which form late in the loading sequence require less rotation capacity than those which form first. The elastic sway ratio for a girder sway mechanism ($C = 1.0$) gives a relative idea of the plastic hinge rotation at a leeward (first formed) girder hinge which is required to just develop the second positive moment hinge in that girder. Once the girder mechanism had formed, the girder may "ride" with the frame under constant gravity load and increasing sway. This requires additional
rotation at the leeward girder hinge but does not involve uncontrolled vertical deflection of the girder.

The center girder in the frame described in Fig. 1.6 illustrates this behavior. Two different rotation capacity requirements are of interest at the leeward hinge numbered (1). At ultimate load this hinge need have only the rotation capacity required to form a mechanism in the center girder. (This hinge rotation requirement is relatively small compared with the rotation which causes strain hardening.) A larger hinge rotation is required to reach the frame mechanism condition.

Current ideas on rotation capacity requirements are based on hinge rotation studies for continuous beams and single story frames. (18) The ultimate combined load capacity of a single story frame is frequently controlled by a complete frame mechanism. But this is not true of many multi-story frames. At ultimate load the multi-story frame does not form a complete frame mechanism. In fact, the shear versus sway curve in Fig. 1.6 suggests that the deformed shape at ultimate load may not be too far removed from the elastic state. This means that relatively small hinge rotations are in evidence at ultimate load. Much larger hinge rotations are needed to reach the frame mechanism condition.

The point of this discussion is that rotation capacity requirements for girders in multi-story frames may vary by a considerable margin between ultimate load and the frame mechanism load. Furthermore, the rotation capacity needed in single story frame girders may
not give a realistic indication of the plastic hinge rotation at ultimate combined load in multi-story frame girders. This is a direct result of two features of the multi-story frame. In addition to the incomplete mechanism feature, it is also true that the girder sway moments are large, relative to the gravity load moments, for many of those girders which are controlled by combined loading. (Otherwise the combined load girders would have to be only slightly stronger than those controlled by gravity loading.)

As the relative moment contribution of the gravity load decreases, so also does the rotation capacity needed to form a girder sway mechanism. In the limiting case of large sway moments and no gravity load, plastic hinges form simultaneously at each end of the girder. No rotation capacity is needed to form the girder sway mechanism. Note that in this limiting case, the elastic sway ratio $E_{SR}$ equals 1, which indicates zero required rotation capacity for the sway mechanism.

Further study should be given to plastic design rotation capacity requirements for girder sway mechanisms and restricted girder hinge patterns. It may be possible to justify using at least some of the non-compact W economy shapes in Ref. 5 for plastically designed girders with large sway moment coefficients using ASTM A441 or similar steels. This tentative suggestion is based on the idea that rotation capacity requirements can be controlled, at the beginning of the preliminary design, by assigning the elastic sway ratio $E_{SR}$. The positive moment factor $C$ then depends upon the sway moment coefficient $G$ as indicated in Fig. 8.4.
It is germane to comment that rotation capacity requirements at a leeward girder hinge depend in part on the rotation of the leeward girder-to-column joint. If this joint rotation is conservatively taken equal to the chord rotation $\Delta/h$ in the story below, then rotation capacity requirements at a leeward girder hinge also are related to the sway stiffness of the frame. Design provisions which are intended to control sway stiffness also provide some control over rotation capacity requirements. It seems reasonable to expect that if the sway deflection at ultimate load is limited to some (as yet undetermined) fraction of the story height, we need have little concern for girder rotation capacity requirements. Further study is needed to substantiate this tentative idea.

The primary purpose of Fig. 8.4 is to indicate how we can preset the extent to which plastic behavior is utilized in the design of girders using the plastic moment balancing method. To do this we enter Fig. 8.4 with the sway moment coefficient $G$ and the desired elastic sway ratio $E_{SR}'$, and find the positive moment factor $C$. This value of $C$ is used as initial data in the girder plastic moment balance.

It is evident that the required plastic moment increases with the elastic sway ratio for a constant value of $G$. The following girder designs give an idea of the increase in material weight which is required by increasing values of the elastic sway ratio.
The gravity load and sway moment are the same for each girder and are similar to the load and moment in bay CD at level 20 of Frame C (Ref. 6). Girder Gl is designed to form a mechanism under combined loading but nearly half of the sway moment is carried in the elastic range. Girders G2 and G3 are designed using a restricted hinge pattern as indicated by the smaller values of C. This increases the material weight by 10 percent and the elastic range of sway response by 26 percent for G2 and 61 percent for G3, relative to girder Gl. Only a modest increase in material weight is needed to considerably extend the elastic range of sway response.

The elastic sway ratio $E_{SR}$ is approximately related to the intermediate elastic limit load factor $\alpha$ in paragraphs 19 and 45 of Ref. 61. The curves in Figs. 7 and 8 of this reference indicate the substantial decrease in the PA amplification of moment which is obtained by increasing the elastic range of sway response for girders.
This suggests that the elastic sway ratio is an effective preliminary design parameter for controlling $\Delta$ effects.

It appears to be an accepted fact that all members of an unbraced multi-story frame cannot be expected to work simultaneously at the limit of their plastic bending capacity under ultimate combined loading. This is simply a restatement of the idea that the ultimate combined load capacity is reached before a frame mechanism is completely developed. The elastic sway ratio is a convenient preliminary design parameter which serves to limit the amount of plastic bending in girders to something less than the plastic mechanism capacity. It remains to demonstrate, in future inelastic sway studies, how the elastic sway ratio should be chosen in preliminary design so as to control frame sway behavior at ultimate load.

One of the problems involved in applying limit design concepts to a concrete frame is the control of rotation capacity requirements. It is tentatively suggested that the elastic sway ratio may be used to provide this control for girders in unbraced multi-story concrete frames.

8.3 GIRDER DISTRIBUTION FACTORS

The distribution of sway moments to the girders on one level is discussed in Art. 4.4. The only statical limitation on the girder distribution factors $D_G$ is that they must sum to unity on each level as per Eq. (4.14). In the present article we indicate some of the ideas which can be used in assigning the values of $D_G$. 
Consider the top levels of an unbraced frame. The plastic moment capacity of the girders on these levels is normally controlled by gravity loading. We can avoid having to increase the size of these girders for combined loading by assigning

\[ D_G(J,I) = \frac{M_{pm}(J,I)}{\sum_{(J)} M_{pm}(J,I)} \]  

(8.5)

on several levels below the roof. That is the girder sway moments may be distributed in proportion to the product of total gravity load \( w L_g \) and clear span \( L \) in each bay. This sway moment distribution differs from the conventional portal distribution which depends only on the bay spans. If the plastic moment \( M_p = R_{LB} M_{pm} \) is controlled by gravity loading the maximum sway moment capacity of the upper level girders can be determined using \( G \) from Eq. (5.33) in the equation

\[ M_G(J,I) = G \frac{M_{pm}(J,I)}{(1 - d_c/L)(J)} \]  

(8.6)

Note that the limiting sway moment coefficient \( G \) in Eq. (5.33) depends upon the positive moment factors \( C \) and \( C_1 \) and upon the load factor ratio \( F_{1R}/F_{2R} \). This limiting value of \( G \) for gravity load girders is independent of girder span or gravity load so \( M_G \) in Eq. (8.6) is proportional to \( M_{pm} \). The same is true of \( D_G \) in Eq. (8.5) which indicates the logic of this expression. We could modify Eq. (8.5) by including the \( d_c/L \) ratio but this is an insignificant refinement unless \( d_c/L \) exceeds 0.1.

The factored gravity load and span parameter \( M_{pm} = F_{2R} w L_g^2/16 \) in all equations of Chapter 8.
The positive moment factors \( C = C_1 = 1.0 \) for a girder mechanism and the load factors \( F_{1R} = 1.70 \) and \( F_{2R} = F_{1R}/1.33 \) give \( G = 1.24 \) from Eq. (5.33). If we change \( C \) to \( C = 0.50 \) the result is \( G = 0 \) and the girders are controlled by combined loading. This indicates the typical range for \( G \) in Eq. (8.6).

The girder distribution factors from Eq. (8.5) are appropriate from level (I) at the roof to the lowest level on which the gravity load girders are adequate for combined load. Call this level (II). On any level between (I) = 1 and (II) the maximum sway moment capacity of the gravity load girders is

\[
\text{Max. } \sum_{G(I)}^{} = G \sum_{pm(J,I)}^{M}(1 - d_c/L)(J) \quad (8.7)
\]

from Eq. (8.6). It is assumed that no excess plastic moment capacity is provided above that required by gravity loading. The gravity load girders are adequate for combined load from the roof to the lowest level (II) for which \( \text{Max. } \sum_{G} \) is less than the required girder sway moment sum \( \sum_{G(I)}^{M} \) from Eq. (4.12). Note that the story sway moments \( \sum_{C(I)}^{M} \) in Eq. (4.12) include the \( P\Delta \) contribution. Note also that \( M_{pm} \) in Eq. (8.7) includes the effect of live load reduction. Thus, both \( P\Delta \) effects and live load reductions are accounted for in determining the level (II).

This method of determining the height zone for which the girders are governed by gravity loading uses data which is needed in other portions of the preliminary design. In other words there are few additional calculations required which makes the process efficient.
Furthermore, the method involves no special conditions of equal spans, story heights, or loads and is not based on a particular mechanism or equal story sway assumptions. Both gravity load girder mechanisms and restricted girder hinge patterns may be considered in determining G from Eq. (5.33) for use in Eq. (8.7). It is assumed that the same values of C and C_1 are assigned to each girder on one level, so that the gravity load limit on G is a constant for that level. Otherwise, variations of G should be considered in the summation.

Below level (I_1) at least some of the girders are controlled by combined loading. There are several ideas which might be considered in assigning the girder distribution factors D_G in this height zone. These ideas are summarized in List 8.1.

**LIST 8.1**

1. Continue to use D_G from Eq. (8.5).
2. Make the increase in girder and column material, above that needed for gravity load, a minimum.
3. Use the same girder depth in each bay.
4. Use a shallow girder in one bay.
5. Use a sway moment distribution which results in desirable sway behavior, based on inelastic or elastic-plastic sway studies.
6. Vary the girder sway moments to conserve double-curvature bending in the columns.
7. Distribute most of the sway moment to one bay.
8. Use girders of variable section.
   a. Negative moment cover plates for small wind moments.
   b. Wind bracing stubs or tees for larger wind moments.
   c. Composite girders.
Any of these ideas may be selected to suit the needs of a particular design problem using the plastic moment balancing method.

For example, we can select item (3) to reduce fabrication cost and minimize construction depth. Item (4) may be used to fit a large suspended duct into the minimum construction space or to suit other architectural depth limits in one bay. Items (5) and (6) concentrate on controlling structural behavior and safety. We can develop a vertical vierendeel truss in one bay to carry wind loads using item (7). This truss arrangement may be used in every second or third frame by transferring wind shear to the trussed frames through the floor system. Item (8) provides a considerable advantage in material weight but some of this advantage may be offset in higher fabrication costs for wind bracing at the ends of the girders. The substantial increase in stiffness provided by composite girders may be used to control sway effects in a tall unbraced frame.

List 8.1 is included to suggest some of the varying considerations which are met in designing multi-story frames and to indicate the versatility of plastic moment balancing as a method for considering these requirements in preliminary design. Space limitations prevent a complete treatment of the ideas in List 8.1. However, the first three items in this list are of particular interest and will be pursued further in Arts. 8.4 to 8.6. Item (6) is considered in Art. 6.3.

Variations from Assigned Sway Moment Distribution

When each girder on a level is designed to reach a mechanism under combined load, we can be reasonably assured that the horizontal
distribution of sway moments assigned in the preliminary design will be closely realized in the frame. This assumes that undesired behavior does not interfere with plastic redistribution of moments. The frame then has little freedom to seek a girder moment distribution substantially different from that assumed in the preliminary design unless the PΔ effect is overestimated. The situation is somewhat different when a restricted hinge pattern is applied in the preliminary design for one or more girders. Then the distribution of girder moments in the frame depends in part on elastic compatibility in addition to plastic behavior.

If the level includes a short, stiff girder span, this girder will attract sway moment in the elastic range. The first leeward girder hinge and the first complete girder mechanism is likely to form in the short span. This behavior is illustrated in Fig. 1.6. It is possible to assign a restricted hinge pattern for the short girder in the plastic moment balance but the frame is likely to redistribute the sway moments assigned in the moment balance. Nevertheless, the girders on the level will have sufficient capacity to resist the total sway moment \( \sum M_G \) used in the preliminary design. Furthermore, if the total sway moment is not exceeded, at least one of the girders (probably the girder with the smallest I/L ratio) will not form a mechanism. In short, if sufficient girder moment capacity is provided in the preliminary design, the girders will find a way to carry the loads. This is the basis for the Kist dictum quoted in Art. 2.1.
8.4 PROPORTIONAL GIRDER DESIGN METHOD

Consider item (1) in List 8.1. If Eq. (8.5) is used to assign the girder distribution factors $D_G$ on every level of the frame, the result is to make the sway moment coefficient $G$ nearly constant for each girder on one level. To find the value of $G$ for level (I) we substitute $D_G$ from Eq. (8.5) into $M_{G(J,I)} = D_G(J,I) \sum M_G(I)$. Then Eq. (8.6) gives

$$G(J,I) = \frac{\sum M_G(I)}{\sum M_{pm}(J,I)} \left(1 - \frac{d}{L}ight)_{(J)}$$

(8.8)

On one level $G(J,I)$ varies with $(1 - d_c/L)(J)$, which is nearly constant. As we proceed down the frame, $G$ increases with the sway moment sum $\sum M_G(I)$.

The value of $G$ from Eq. (8.8) can be used in Eqs. (5.29) or Fig. 5.6 to find the restricted mechanism product $R (C + 1)$ for level (I). If the same positive moment factor $C$ is used in the plastit moment balance for each girder on level (I), the required plastic moment $M_p$ for combined load is directly proportional to $M_{pm}$ for each girder.

The preliminary girder design, using item (1) in List 8.1 involves the following steps.
LIST 8.2

1. Compute \( M_{pm} = \frac{F_2RwL^2}{g} \) for each girder and assign the positive moment factors \( C \) for combined load and \( C_1 \) for gravity load at each level. Refer to Art. 8.2 in assigning these factors.

2. Determine the girder distribution factors \( D_G \) from Eq. (8.5). The girder sway moments \( M_G = D_G \sum M_G(I) \).

3. Find the limiting values of \( R_{LB} \) and \( G \) for gravity load girders from Eqs. (5.31 and 5.33).

4. Determine the sway moment coefficient \( G \) for each level \( (I) \) from Eq. (8.8). This step is simplified by using a constant value of \( (1 - d_c/L) \) at each level.

5. Starting from the roof, the lowest level (largest \( I \)) which gives \( G \) from step (4) less than the gravity load limit on \( G \) from step (3) is the level \( (I_1) \). Below this level the girders are controlled by combined load.

6. For \( (I) > (I_1) \) use the sway moment coefficients from step (4) and \( C \) from step (1) to find the restricted mechanism factor \( R \) for each level. This step performs the plastic moment balance using Eqs. (5.29) or Fig. 5.6.

7. The required plastic moments for all girders are obtained from

\[
M_p = R_{LB} M_{pm} \quad \text{for} \quad 1 \leq (I) \leq (I_1)
\]

\[
M_p = R M_{pm} \quad \text{for} \quad (I) > (I_1).
\]
Notice that $M_p$ in step (7) is proportional to $M_{pm}$ for each girder on one level. The term proportional girder design aptly describes the results of the steps in List 8.2. Note also that the same values of $M_p$ are adequate for wind from the left and from the right. The column moments and axial loads obtained from the joint balance will change with the wind direction.

Further study is needed to evaluate the inelastic sway behavior and economy which results from a proportional girder design for various frame geometry and load parameters. In the case of a tall, slender, unbraced frame, it is suggested that the positive moment factor $C$ may be assigned to limit working load sway deflection in the preliminary design stage. This function of $C$ is an addition to the ideas in Art. 8.2.

It is helpful to comment that a proportional girder design can be performed for isolated levels. In this case, the girder distribution factors from Eq. (8.5) are used in Eq. (4.16) to estimate the column loads due to sway. The values of $G$ from step (4) in List 8.2 may be used to guide in the selection of the isolated levels. According to Eq. (5.13), $G$ determines the distance from the leeward end of a girder to the section of maximum positive moment under combined load. The level $(I_2')$ at which the section of maximum positive girder moments first reaches the windward end of the girders can be determined from the values of $G$ in step (4) of List 8.2. The first level which gives $G \geq 8$ is the level $(I_2')$. Below level $(I_2')$ the mechanism factor $R$ in step (7) of List 8.2 varies linearly with $G$ from step (4).
The level \( (I_w) \) defined in Art. 6.3 is the first level for which
\[
\frac{M_A}{M_{pm}} = G - R. \text{(Eq. 5.41)}
\]
is positive. The condition \( G = R \) may be combined with Eq. (5.29a) to give
\[
G = 16 \left( C + \frac{1}{2} - \sqrt{C(C + 1)} \right) \quad \frac{1}{3} \leq C \leq 1 \quad (8.9)
\]
This is the value of \( G \) which makes the windward girder end-moment \( M_A = 0 \).
Equation (8.9) gives the following values for \( G \).

<table>
<thead>
<tr>
<th>( C )</th>
<th>1.0</th>
<th>0.75</th>
<th>0.50</th>
<th>1/3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( G )</td>
<td>1.38</td>
<td>1.67</td>
<td>2.14</td>
<td>8/3</td>
</tr>
</tbody>
</table>

Figure 8.3 indicates the girder moment diagram for \( G = 8/3 \) and \( C = 1/3 \).
The level \( (I_w) \) is the first level at which \( G \) from step (4) of List 8.2 exceeds \( G \) from Eq. (8.9). The windward column above level \( (I_w) \) is bent in single curvature under combined load. A proportional girder design may result in single curvature bending for other columns if the frame has two adjacent bay spans which are quite different.

### 8.5 SHORT GIRDER DISTRIBUTION

One result of the proportional girder design method in List 8.2 is to require that the size of every girder on and below level \( (I_1 + 1) \) be increased for combined load. This is not necessarily the most economical way to provide the girder capacity required for combined loading; which leads us to item (2) in List 8.1.

A method for optimizing the design of 2 and 3 bay multi-story frames is proposed in Ref. 70. This reference uses plastic moment
balancing to minimize a plastic moment weight function \( \sum M_p L \) for girders plus \( \sum M_{pc} h \) for columns) by iteration on a digital computer. The method can be extended to frames with more than 3 bays using a linear programming or Lagrange multiplier approach. A hypothetical increase in the story shear is used to account for \( P/A \) effects in Ref. 70 which also indicates provisions for enforcing girder depth limitations in the plastic moment balance. The use of \( M_{pc} \) in the column weight function would appear to suppress the column contribution to material weight for large \( P/P_y \) ratios. Since the lower story columns are the major contribution to material weight, the column plastic moment weight function deserves careful review.

Other minimum weight studies suggest that the weight function \( \sum M_p \times \text{length} \) is not sensitive to variations in plastic moment ratios if the moment distribution is not too far removed from the theoretical minimum weight solution. Thus calculations which seek the theoretical minimum weight solution are of value in research studies but lose some of their appeal in practice in view of the marginal material economy which results. Quoting from Ref. 60 (closure):

"The writers suggest that the true value of minimum weight design procedures is to lead the designer into the vicinity of the lightest solution from which point he can make selections of realistic members without concern for being wasteful. In making such selections, conflicting practical restraints will cause the designer to deviate from the theoretical design."

The results of the study reported in Ref. 70 suggest simple guides for assigning the girder sway moment distribution below level \( I_L \). These guides give a preliminary design moment distribution close to the
theoretical minimum plastic moment weight function solution. Similar
guides are included in Ref. 60. The basic idea in distributing the girder
sway moments is to strengthen short interior girder spans first. Gravity
load girders are used in the remaining spans. The total girder sway
moment capacity of the gravity load girders on one level can be deter­
mined from Eq. (8.7) by omitting the term for the short girder span.
The short girder must provide the remaining girder sway moment capacity
on the level. After repeating this process on several levels, the short
girder may reach a practical depth limit. The sway moment distribution
is then shifted to the next short girder span.

Note that item (7) in List 8.1 is a special case of this short
girder distribution scheme in which a single bay of the frame is used
to provide the girder sway moment capacity not supplied by the gravity
load girders in the remaining bays. The resulting vertical vierendeel
truss may be an economical wind bracing system for some frames, depending
on frame geometry and architectural requirements. The influence of
chord shortening on upper story girders and columns and uplift at the
foundation deserve attention for a tall slender vierendeel wind bracing
system.

8.6 CONSTANT DEPTH GIRDERS

The short girder distribution scheme may result in girders of
different depth in adjacent bays. It is sometimes desirable to distrib­
ute the girder sway moments below level \( I_1 \) so as to give each girder
the same nominal depth on one level. This approach serves to simplify
connections and to reduce fabrication costs. Girders of constant
depth also minimize the construction space between finished floor and ceiling and reduce the cost of all vertical mechanical services and architectural materials. This is the motivation for item (3) in List 8.1.

Rather than seek the girder distribution factors $D_G$ for a constant depth sway moment distribution, it is more convenient and direct to ask: What plastic moment capacity is required for constant depth girders below level $(I_1)$? The answer to this question depends upon the sway moment coefficient $G$ in Eq. (8.6) and upon the level $(I)$. The first several levels below level $(I_1)$ are a transition zone between the gravity load girders on level $(I_1)$ and constant depth girders on lower levels. Below this transition zone are two zones (an upper and a lower zone) of constant depth girders which depend on the smallest sway moment coefficient $G$ on a typical level $(I)$. These upper and lower zones are defined later in this article.

To simplify this discussion we shall assume that the same plastic moment capacity $M_p$ is used for each girder on a level. This obviously results in constant depth girders on the level. However, with $M_p$ known it is frequently possible to use the plastic modulus table in Ref. 5 to select a set of girders with the same nominal depth but with different plastic moment capacities close to $M_p$. If the economy sections in the plastic modulus table are selected, we may be able to reduce the total material weight for the girders on one level without sacrificing the constant depth feature. Thus $M_p$ in the equations which follow is to be interpreted as a guide in selecting constant depth girders rather than a
minimum plastic moment requirement. For this reason we may tolerate some simplifying approximations in estimating \( M_p \).

Consider first the levels near the bottom of a tall unbraced frame. It is logical to expect large girder sway moments sums \( \sum M_g(I) \) (Eq. 4.12) on these levels. We may define the lower zone of constant depth girders by requiring that the sway moment coefficient

\[
G(j, i) = \frac{M_{G(j, i)} (1 - d_c/L)(j)}{M_{pm(j, i)}} \geq 8 \tag{8.10}
\]

for every girder on level \((I)\). This assumes that the maximum positive moment under combined load occurs at the windward end of each constant depth girder on level \((I)\).

The restricted mechanism factor \( R = \frac{M_p}{M_{pm}} \) is determined by Eq. (5.29c) or the linear part of the graph in Fig. 5.6 if \( G \geq 8 \). Using Eqs. (8.6 and 5.29c) we obtain the girder sway moment sum

\[
\sum M_g(I) = \left( \sum \left( M_p(j, i) \frac{C(j, i) + 1}{(1 - d_c/L)(j, i)} \right) \right) \geq 8 \tag{8.11}
\]

where \( C(j, i) \) is the positive moment factor in span \((J)\) on level \((I)\). We want to hold \( M_p \) constant on level \((I)\) so we can factor \( M_p(I) = M_p(j, i) \) from this sum. Since the \( d_c/L \) ratio is small \((d_c/L < 0.1)\) we can also consider \((1 - d_c/L)(J, I)\) to be a constant on level \((I)\) with little error.

Now Eq. (8.11) gives the plastic moment

\[
M_p(I) = \frac{(1 - d_c/L)(I) \sum M_g(I)}{\left( \sum (C(j, i) + 1) \right)} , \quad G(j, i) \geq 8 \tag{8.12}
\]
for constant depth girders on level (I). This plastic moment varies linearly with the sway moment sum $\sum M_g(I)$. We can use $M_p$ from Eq. (8.12) to select a tentative set of constant depth girders with varying plastic moment capacity $M_p(J,I)$ and check the sway moment capacity of this tentative set using Eq. (8.11).

Equation (8.10) was used to define the lower constant depth girder zone. A more useful definition is obtained by computing the sway moment coefficient $G(J,I)$ from Eq. (8.12). Using $R = M_p/M_{pm}$ and Eq. (5.29c) we have the result

$$G(J, I) = \left( \frac{C(J, I) + 1}{\sum_{(J)} (C(J, I) + 1)} \right) \left( \frac{(1 - d_c/L)(I)}{M_{pm}(J, I)} \right) G(J, I) \geq 8$$

(8.13)

where all quantities are available at the beginning of the preliminary design. Equation (8.13) gives the sway moment coefficient in each span for a constant $M_p(I)$ from Eq. (8.12). To determine whether level (I) is in the lower zone of constant depth girders we select the girder with the largest value of $M_p$ on the level. If this value of $M_{pm}$ yields $G \geq 8$ in Eq. (8.13), level (I) is in the lower zone and $M_p(I)$ for constant depth girders is determined by Eq. (8.12). An alternative procedure is to set $G = 8$ and solve Eq. (8.13) for $\sum M_g(I)$ using the largest value of $M_{pm}$ in the several bays of the frame. This gives

$$\text{Lower Zone} \sum M_g(I) > \left( \frac{8}{\sum_{(J)} (C(J, I) + 1)} \right) \left( \frac{M_{pm}(J, I)}{(1 - d_c/L)(I)} \right)$$

(8.14a)
The smallest value of $\sum M_G(I)$ which satisfies this inequality determines the top level of the lower zone of constant girder depth. For convenient reference, call this level $I_{LCD}$ with subscript LCD for "lower zone, constant depth". Note that if the frame has $N_B$ bays and if the same positive moment coefficient $C_{(J,I)}$ is assigned in each bay, Eq. (8.14a) reduces to

$$\sum_{\text{Lower Zone}} M_G(I) \geq 8 N_B \frac{M_{pm(J,I)}}{(1 - d_c/L)(I)} \quad (8.14b)$$

The levels above $I_{LCD}$ are in the upper zone of constant girder depth. Repeating the lower zone analysis for $G < 8$ gives

$$M_{p(I)} = \left( \frac{(1 - d_c/L)(I) \sum M_G(I) + 8 \sum M_{pm(J,I)}}{8 \sum (C_{(J,I)} + 1) M_{pm(J,I)}} \right)^2$$

(8.15)

for constant depth girders at level (I) in the upper zone. This expression is more involved than Eq. (8.12) because the maximum positive girder moments do not occur at the windward end of the girders. Nevertheless, all of the girders on one level can be designed from one application of Eq. (8.15). If the gravity loads on successive levels do not vary, the only term in Eq. (8.15) which changes with (I) is $\sum M_G(I)$. The involved nature of this equation is a small price to pay for designing all of the constant depth girders on several levels of the upper zone.
If we select girders with the same nominal depth but with varying plastic moment capacity $M_p(J,I)$ on level (I) in the upper zone, the sway moment capacity of these girders can be checked using

$$
\sum M_{G(I)} = 8 \sum (J) \sqrt{\frac{1}{2} \left(C(J,I) + 1\right) \frac{M_{p(J,I)}}{M_{pm(J,I)}} \frac{M_p(J,I)}{M_{pm(J,I)}} \left(1 - \frac{d}{L}(J,I)\right)}
$$

(8.16)

for $G < 8$. An alternative to this expression is the following checking sequence which starts with the known plastic moments $M_p(J,I)$ on level (I).

**LIST 8.3**

1. Compute the mechanism factor

$$R(J,I) = \frac{M_p(J,I)}{M_{pm(J,I)}}$$

for each girder on level (I).

2. Enter Fig. 5.6 with the product $R(J,I)(C(J,I) + 1)$ on the vertical axis and find the sway moment coefficient $G(J,I)$ on the horizontal axis.

3. Determine the sway moment capacity for the girders on level (I) from

$$\sum M_{G(I)} = \sum G(J,I) \frac{M_{pm(J,I)}}{(1 - \frac{d}{L})(J,I)}$$

(8.17)

This sway moment sum must be larger than $\sum M_{G(I)}$ from Eq. (4.12).
If this sway moment checking sequence is used there is no need to differentiate between the upper and lower zones of constant girder depth because Fig. 5.6 is valid for all positive values of G. Nevertheless, Eqs. (8.12 and 8.15) are useful in estimating plastic moment capacities for constant depth girders in the lower and upper zones.

No attempt will be made here to rigorously define the top and bottom levels of the upper zone because the defining equations are involved and not very useful. Instead, the following ideas are suggested to indicate appropriate limits of the upper zone of constant girder depth.

It is evident that the largest gravity load girder on level \( I_1 \) indicates the smallest practical depth for girders of constant depth in the upper zone. We can enter the plastic modulus table in Ref. 5 and determine the plastic moment for the lightest \( W \) economy shape with this nominal depth. Call this plastic moment \( M_{pES} \) (subscript ES for economy shape). It is assumed that the \( W \) economy shape satisfies all width-to-thickness requirements necessary to conserve rotation capacity. Note that \( M_{pES} \) may be 40 percent less than the plastic moment required for the largest gravity load girder on level \( I_1 \). Next we find \( M_{p(I)} \) from Eq. (8.15) for several levels below \( I_1 \). The first level which gives \( M_{p(I)} \geq M_{pES} \) is the top level \( I_{UCD} \) of the upper zone. The subscript UCD stands for "upper zone, constant depth". On level \( I_{UCD} \) we can use the largest gravity load girder from level \( I_1 \) together with constant depth \( W \) economy shape girders in the remaining bays. The economy shape girders should have plastic moments approximately equal to \( M_{p(I)} \).
The levels between \( I_1 \) and \( I_{UCD} \) are the transition zone mentioned previously. We can use the largest gravity load girder from level \( I_1 \) in this zone. The girders in the remaining bays may be selected from the \( W \) economy shapes and checked for sway moment capacity using the procedure in List 8.3.

Several levels below \( I_{UCD} \) the plastic moments from Eqs. (8.12 and 8.15) are nearly equal. This results from the nearly linear nature of the graph in Fig. 5.6 for \( G \) in the range \( 4 < G < 8 \). For most practical purposes we can relax the limit on \( G \) in Eqs. (8.12 and 8.13) to \( G > 4 \) with the understanding that the constant depth girders should be checked for sway moment capacity using List 8.3. The upper and lower zones of constant girder depth gradually merge when Eq. (8.13) gives the smallest value of \( G(J, I) \) in the range \( 4 < G < 8 \).

**Analysis Versus Design**

It is significant to note that List 8.3 constitutes a procedure for the combined load analysis (rather than design) of the girders on one level of an unbraced frame with known member sizes. This girder analysis may be followed by a joint balance using method II in Chapter 6. With some trial and error it is possible to construct moment capacity envelopes for the columns to guide the method II joint balance. Thus the plastic moment balancing method may be used both for the design and for the ultimate strength analysis of unbraced multi-story frames. It is assumed that reasonable estimates of sway deflection and vertical distribution factors are included in the analysis problem. Once these estimates are available, plastic moment balancing provides a systematic method for fitting an equilibrium moment distribution (in the deflected
state) into available moment capacity envelopes. This analysis application deserves further investigation. The primary emphasis in this study is on the design problem.

8.7 CONSTANT DEPTH DISTRIBUTION FACTORS

It is of interest to determine the girder distribution factors $D_G$ in the lower zone of constant girder depth. By definition

$$D_G(J,I) = \frac{M_G(J,I)}{ \sum M_G(I) } = \frac{G(J,I) M_{pm}(J,I)}{(1 - d_{c/L})(J,I) \sum M_G(I)}$$

In the lower zone, the sway moment coefficients from Eq. (8.13) give the simple result

$$D_G(J,I) = \frac{C(J,I) + 1}{\sum_{(J)} (C(J,I) + 1)}$$ (8.18a)

If the frame has $N_B$ bays and if a single positive moment factor $C(I) = C(J,I)$ is assigned in each bay at level (I), we have

$$D_G(J,I) = \frac{1}{N_B}$$ (8.18b)

in the lower zone of constant girder depth.

In other words, constant depth girders result from the simple device of distributing the girder sway moment sum equally between the $N_B$ bays, regardless of how the bay spans may vary. The required plastic
moment capacity of the constant depth girders should be in the vicinity of

\[ M_p(I) = \frac{(1 - \frac{d}{L})(I) \sum M_G(I)}{(C(I) + 1) N_B} \]  

(8.19)

from Eq. (8.12) with \( C(I) = C(J,I) \). Equation (8.19) is definitely valid when \( \sum M_G(I) \) satisfies inequality (8.14b) and approximately valid when

\[ \sum M_G(I) \geq 4 N_B \frac{M_{pm}(J,I)}{(1 - \frac{d}{L})(I)} \]  

(8.20)

It is understood that \( M_{pm}(J,I) \) is the largest value of \( F_{2R_w L_g^2/16} \) on level (I) in Eq. (8.20).

The girder distribution factors may thus be established as follows:

**LIST 8.4**

1. Use the first 5 steps in List 8.2 to determine the lowest level \((I_1)\) on which the girders are controlled by gravity loading.
2. Find the highest level \((I_{GD})\) which satisfies Eq. (8.20).
3. From the roof to level \((I_1)\) use

\[ D_{G}(J,I) = \frac{M_{pm}(J,I)}{\sum_{(J)} M_{pm}(J,I)} \]
4. From level \( I_{CD} \) to the bottom level use

\[
D_G(J,I) = \frac{1}{N_B}
\]

5. Between levels \( (I_1) \) and \( (I_{CD}) \) use a transition in the girder distribution factors from step (3) to step (4).

The result of the procedure in List 8.4 is a constant-girder-depth distribution of girder sway moments below level \( I_{CD} \). Note that no girders need be sized in this procedure. We can further simplify the process by taking \( I_{CD} = (I_1 + 1) \) and discarding steps (2) and (5) if the difference between \( D_G \) in steps (3) and (4) is not drastic. However we should not expect to obtain constant depth girders on level \( I_1 + 1 \) unless the frame geometry and loads are regular.

The steps in List 8.4 may be applied to any unbraced frame regardless of varying bay spans, story heights, or wind load distribution. Any reasonable assumptions for positive moment factors may be used so long as the same factors are assigned to each bay on one level. The values of \( C(I) \) may vary from level to level, but should logically remain constant or increase with increasing distance from the roof.

**8.8 SUMMARY**

This chapter indicates how we can control the design of girders to accomplish several objectives. The elastic sway ratio in Art. 8.2 provides a numerical measure of the extent of plastic behavior which is utilized in the girder design. This parameter may be assigned at the beginning of the preliminary design to limit sway stiffness and
rotation capacity requirements for girders. The extent to which sway stiffness and rotation capacity requirements are limited by the elastic sway ratio remains to be demonstrated in future investigations.

List 8.1 suggests several ideas which may be used in distributing sway moments to the girders. The first three ideas in this list lead to relatively simple rules or formulas for assigning girder distribution factors. Items (5) and (6) in List 8.1 suggest how the results of future multi-story frame research can be used to improve the rationality and dependability of plastic moment balancing.
9. EVALUATION AND FUTURE WORK

9.1 EVALUATION

Plastic moment balancing may be described as a refined formulation of equilibrium for unbraced multi-story frames. The refinements are formulated to consider the influence on frame statics of sway deflection, finite joint size, and plastic girder mechanisms or restricted girder hinge patterns. The basic statics conditions for a rigid frame include; (1) girder equilibrium, (2) joint equilibrium, and (3) story equilibrium. It is useful to contrast the plastic moment balancing method, and its predecessor plastic moment distribution (40) with respect to the manner in which these statics conditions are enforced.

In plastic moment distribution all three statics conditions are considered together in one operation. The purpose of the moment distribution is to satisfy joint equilibrium by redistributing initial unbalanced girder and column moments on the joints. Girder equilibrium is conserved by using plastic moment carry-over factors between both ends or between midspan and one end of the girder. This assumes that maximum positive girder moments occur close to midspan. Story equilibrium is conserved by using carry-over moments between the ends of columns or between stories. With some trial it is possible to arrive at any reasonable equilibrium distribution of moments in a multi-story
frame using plastic moment distribution. The sequence of the statics calculations is arbitrary.

In plastic moment balancing, the three statics conditions are separately considered in the following sequence:

**Step 1** - Distribute the total sway moment $\sum M_G(I)$ in each story to the upper and lower levels. This gives the column moment sums $\sum M_{jU}(I)$ and $\sum M_{jL}(I)$ in each story and the total girder sway moment $\sum M_G(I)$ on each level. These moment sums are conserved in the plastic moment balance.

**Step 2** - Distribute the total girder sway moment $\sum M_G(I)$ on each level to the girders. The result is the sway moment $M_G = M_{jA} + M_{jB}$ for each girder.

**Step 3** - Perform the plastic moment balance for each girder.
This gives the maximum required plastic moment capacity as controlled by factored gravity or combined load, using girder mechanisms or restricted girder hinge patterns. The girder moments $M_{jA}$ and $M_{jB}$, on the left and right joints, conserve the sway moment $M_G$ from step 2.

**Step 4** - Balance each joint, holding the girder moments $M_{jA}$ and $M_{jB}$ constant. The results are the column moments $M_{jU}$ and $M_{jL}$ below and above each joint.
Girder equilibrium is satisfied in steps 2 and 3. Step 4 determines column moments which satisfy joint equilibrium and which conserve the column moment sums in step 1. Then story equilibrium is enforced by steps 1 and 4.

The interesting feature of this statics sequence is that it isolates the girder equilibrium condition. After the gravity load and sway moment for a girder are established, the girder moment diagram for each loading condition can be determined without reference to the rest of the frame as described in Chapter 5. Once determined, the girder moments need no further modification. This eliminates any moment carry-over between the ends of a girder and simplifies the joint balance operation. Furthermore, a single plastic moment balance for each girder automatically determines the controlling loading condition and plastic moment capacity. Girder plastic moments are based on the clear span between column flanges.

Four joint balancing methods are considered in Chapter 5 and other methods are possible. The main idea in the joint balance is to satisfy both story and joint equilibrium without changing the girder moments. There is no need for column moment carry-over or for transfer of moments between stories in plastic moment balancing. In effect, each joint balance is an isolated operation.

The purpose of considering the three statics conditions separately in plastic moment balancing is to organize the equilibrium calculations in a meaningful and simplifying sequence. The definite pattern of enforcing the statics conditions in plastic moment balancing
contrasts with the arbitrary statics sequence in plastic moment distribution. However, there is no essential limitation on the results of the plastic moment balance. Any reasonable distribution of moments can be produced. In fact, the relative moment distribution is specified in a meaningful parametric form at the beginning of the plastic moment balance. If the plastic moment balancing parameters are assigned values derived from refined inelastic sway investigations, the results of the plastic moment balance provide a reasonable and rapid prediction of inelastic statical behavior. From this viewpoint, plastic moment balancing is at least as rational a preliminary design approach as most available allowable stress or plastic design methods.

The basis for plastic moment balancing is a modified form of the lower bound theorem of simple plastic theory. The modifications account for the most significant in-plane deflection effects in unbraced multi-story frames. This represents an extension of plastic theory to include the element of frame stability in preliminary design. In addition, the means for controlling frame instability is built into plastic moment balancing through the judicious choice of positive moment factors for girders. Once the preliminary design is completed, the frame with tentative member sizes can be reviewed to demonstrate satisfactory stability behavior.

Plastic moment balancing is not a new or revolutionary concept. Instead it is a restatement of a tried and tested design philosophy which appears to have originated in elementary form over half a century ago. Nor does plastic moment balancing require advanced mathematical concepts. Probably the most unfamiliar element in the plastic moment
balancing formulation is the numerical double subscript notation \((J,I)\), defined in Art. 3.4. This is an essential bookkeeping device for identifying the many similar quantities which describe the statics of a multi-story frame.

The organized statics sequence and the bookkeeping system of plastic moment balancing have two results. First, the method can be mastered with little more than a firm statics foundation. This is certainly a minimal qualification for one who is to design multi-story frames. Second, the systematic nature of the method renders it feasible for manual or computer application. In fact, it was the effort to organize the plastic moment balancing method into a form suitable for the combination of a small computer (say 8000 memory locations) and a relatively large frame (say 40 stories and 8 bays) which led to the four step statics sequence. A two-stage preliminary design program is available and will be described in a future report. Here it suffices to comment that the plastic moment balancing statics sequence can be applied to one level of a frame at a time so that a minimum amount of data need be available in the computer core at any one stage of the calculations. It is interesting to note that the digital computer does not antiquate simple design methods but does considerably extend the scope of their application.

Several design objectives are considered in Arts. 6.3, 8.2, and 8.3. These design objectives include double curvature bending of columns, girders of constant depth on one level, and limitations on the extent to which plastic behavior is utilized in the preliminary design of girders. A valuable feature of plastic moment balancing is
the fact that some design objectives can be specified at the beginning of the preliminary design with the assurance that the results of the design will reasonably approximate these objectives. Another interesting feature of plastic moment balancing is that isolated floors and stories can be designed. The effects of parts of the frame above an isolated story are approximated in a simple manner as described in Art. 4.5. Chapter 7 suggests ideas for the design of bottom story columns including the effect of base restraint.

In a sense, the plastic moment balancing method represents a useful link between structural research which aims to improve the safety and economy of buildings, and the practicing designer who must execute designs with dispatch. By this we mean that research results may be stated in a form which is of use in design at the beginning of the task, that is, in the preliminary design stage. Plastic moment balancing serves to define certain parameters derivable from frame research which have direct application in preliminary frame design. These parameters are:

**LIST 9.1**

1. The sway deflection at ultimate load, and at the mechanism load.

2. The vertical distribution factors $D_V$ (Chapters 4, 6, and 7).

3. The girder (horizontal) distribution factors $D_G$ (Chapters 4, 6, and 8).
4. The positive moment factors $C$ and $C_1$ (Chapters 5 and 8) and the elastic sway ratio (Art. 8.2).

5. The joint balancing parameters (Chapter 6).

6. The bottom story column base restraint factor $R_b$ and the deflection ratio $D_r$ (Chapter 7).

Once these preliminary design parameters are assigned, the mechanics and results of the preliminary design are unique and void of trial and error or iteration. The preliminary design moments, shears, and axial forces may be used to select tentative member sizes. If these members provide no optional excess capacity, the behavior of the frame under a single application of factored design loads is a unique result of the preliminary design parameters, at least in theory.

9.2 FUTURE WORK

It is the task of future research to establish and correlate the relation between the preliminary design parameters and the working load and ultimate load behavior of unbraced frames which are proportioned using plastic moment balancing. A particularly vital need, from the viewpoint of plastic moment balancing, is data on sway deflection at ultimate load. Specifically, we need a method or model which estimates ultimate load sway deflection from frame geometry and load data. It is reasonable to expect that items (2) to (4) in List 9.1 may also enter into this initial sway deflection prediction model.
Other questions concerning plastic moment balancing include:

**LIST 9.2**

1. What are appropriate restricted hinge patterns in girders and how do these patterns depend on frame geometry and load parameters? How do restricted hinge patterns influence inelastic frame stability under combined loading and frame buckling under gravity loads?

2. What limits on the preliminary design parameters in List 9.1 are appropriate for inclusion in a building code or design specification?

3. What conditions of frame geometry and load combine to produce significant column shortening effects in top stories of an unbraced frame and how can these effects be estimated in the plastic moment balance?

4. Is it possible to modify the plastic moment balancing statics relations to include the horizontal shear and vertical reactions developed by shear walls or bracing in the plane of the frame? Can the horizontal shear be predicted on the basis of sway deflection estimates?

5. What values of the positive moment factor $C$ are required at ultimate combined load to limit the working load sway deflection to an assigned value? How are these values of $C$ related to frame geometry and load parameters?
6. If the girder distribution factors $D_G$ are assigned so as to conserve double curvature bending in the columns (Art. 6.3) is it possible to simplify lateral-torsional buckling checks for the columns?

Some of the items in List 9.2 can be studied using inelastic or elastic-plastic sway analysis methods which are recently available.\(^{(10,71,72)}\) The effect of finite joint size should be included in the sway analysis (Art. 4.1) since this effect controls the location of plastic hinges and increases the stiffness and load capacity of girders and columns.

Reference is made in this study to the application of plastic moment balancing in the design of composite girders and girders with variable section (Arts. 5.2 and 8.3). No details are given to describe this application. It is suggested that the positive moment factors $C$ and $C_1$ can be redefined to extend the scope of the girder plastic moment balance to include composite or variable section girders. This appears to be an attractive topic for future study from the viewpoint of sway stiffness and economy.

9.3 SUMMARY

The central idea in this dissertation is the definition of the factors which should be considered in the preliminary design of unbraced multi-story frames. Chapter 1 shows how $\Delta P$ effects are intensified by plastic-hinge-induced sway stiffness reduction and suggests restricted hinge patterns in girders as the means for limiting $\Delta P$ effects. This
chapter also indicates that either the mechanism or the ultimate load conditions may be selected as the preliminary design criterion in the plastic moment balancing method. If the mechanism condition is used one should consider the shear reduction between ultimate and mechanism loads and should use larger sway assumptions to account for PA effects together with girder mechanisms, in the preliminary design. If the ultimate load criterion is selected, smaller sway assumptions should be combined with restricted girder hinge patterns in the preliminary design (Fig. 1.6).

Available methods of plastic design and analysis are compared on the basis of consideration given to PA effects, restricted hinge patterns, and inelastic columns in Chapter 2. This chapter includes an evaluation of current elastic-plastic frame stability concepts. It is indicated that the mechanism condition is a valid preliminary design criterion for combined load if adequate provision is made for PA effects. This is true in spite of the fact that incremental sway stiffness is negative when a frame sway mechanism forms.

The ultimate load condition with restricted girder hinge patterns is the preferred preliminary design criterion for three reasons:

(1) The story shear capacity when a frame sway mechanism forms may not give an accurate indication of the ultimate story shear capacity because of the variable shear reduction between the ultimate and the mechanism conditions.
The story shear versus sway behavior is influenced by secondary factors (Art. 1.6). These secondary factors are more active in the vicinity of the mechanism condition than at ultimate load because of the larger deformations, forces, and moments which are required to reach the mechanism condition. In addition, it seems unnecessary to require that members provide the larger deformation capacities needed to reach the mechanism state when the smaller deformations which occur at ultimate load are adequate for the load carrying function.

The sway at ultimate combined load appears to be more consistent than the sway when a frame mechanism forms (see Fig. 1 of Ref. 100). Further sway subassemblage studies are needed to substantiate the tentative idea that the ultimate load sway is primarily a function of frame geometry and load parameters.

The plastic moment balancing formulation in Chapters 4 to 7 introduces a series of preliminary design parameters which determine how the frame members are to share in resisting the frame loads. List 9.1 summarizes these parameters, which provide the pattern for translating the results of frame research investigations into a form which is of use in preliminary design. This is a primary purpose of the equilibrium formulation in this study. The preliminary design
parameters provide the facility for specifying the relative distribution of moments in the frame at the beginning of the preliminary design.

Another objective of this preliminary design study is to organize the statics calculations in a definite sequence which isolates the story, girder, and joint equilibrium conditions. This objective is accomplished by the four step statics sequence outlined in Art. 9.1. The definite statics sequence in plastic moment balancing contrasts with the arbitrary sequence of joint balancing and moment carry-over in plastic moment distribution. No moment carry-over is needed in plastic moment balancing and no versatility is lost by deciding in advance how the frame moments should be distributed.

The key step in plastic moment balancing is the girder moment balance described in Chapter 5. This chapter states the conditions applied in the girder plastic moment balance and defines the girder sway moment coefficient and mechanism factor. Positive moment factors are defined and used to extend the girder plastic moment balance to the case of restricted hinge patterns. The plastic moment envelope concept is introduced to suggest how the moment balancing process can be used for composite girders and non-uniform load distributions.

Four joint balancing methods are described in Chapter 6, to achieve different objectives. One objective is to duplicate the relative moment distribution obtained from refined inelastic sway analyses. From this viewpoint, the equilibrium, yield, and compatibility conditions can be approximately satisfied in the preliminary design.
A second objective is to fit an equilibrium distribution of moments into available moment capacity envelopes. This is the plastic analysis problem. A third objective is to seek a moment distribution which conserves double curvature bending in most columns (Art. 6.3).

Chapter 7 uses elastic concepts to guide in the selection of preliminary design parameters for bottom story columns. Consideration is given to column base restraint and relative stiffness of bottom floor girders.

The elastic sway ratio, defined in Chapter 8, provides the facility for deciding in advance, the relative degree of plastic behavior which is utilized in the preliminary design of girders for combined loading. The girder moments can be selected anywhere between the portal wind moment distribution (point of inflection at midspan) and the girder mechanism distribution of moments. The purpose for limiting the relative degree of plastic behavior in the preliminary design is to conserve sway stiffness at ultimate combined load. Rotation capacity requirements are limited in the process. It is tentatively suggested that sway deflection limits at working load may also be considered in assigning the elastic sway ratio.

Chapter 8 indicates some of the design objectives which can be considered in distributing sway moments to the girders. This chapter describes a method for determining the lowest level on which girder sizes are governed by gravity loading, with consideration given to variable bay dimensions, live load reduction, and \( P\Delta \) effects. The largest girder on this level indicates the smallest practical depth of
constant depth girders on lower levels. Sway moment distribution factors for constant depth girders are given in Chapter 8, and discussed in Appendix 1.

Clear spans and heights are used in the design of all members in this study in recognition of the fact that plastic hinges form outside of adequately designed rigid girder-to-column-flange joints. The effect of finite joint size on joint equilibrium and column end-moments is considered in Art. 4.1 and Appendix 2.

The method used to approximate in-plane frame stability effects in the preliminary design is outlined in Art. 1.3. The preliminary design begins with an assumed sway deflection in each story at ultimate load or at the mechanism load. This investigation does not intend to develop an initial sway prediction model but this is felt to be an important topic for further study. The design example in Appendix 1 tentatively suggests that preliminary member sizes are not especially sensitive to the initial sway assumption if reasonable provisions are made for restricted hinge patterns in girders.

In the absence of a reliable initial sway prediction model, plastic moment balancing is regarded as a preliminary design method which must be followed by an inelastic (10) or elastic-plastic (71) sway analysis. The purpose of the sway analysis is to check the initial sway estimate used to begin the plastic moment balance. In effect, the sway analysis is an ultimate load frame stability check. If initial sway estimates are not reasonable verified in this sway check, it is necessary to either repeat the preliminary design or to estimate
revised member sizes. The second preliminary design is more likely to be successful in giving valid sway checks for two reasons. First, the results of the previous sway check give an indication of appropriate initial sway data. Second, inadequate members may be evident from the sway check. An improved moment distribution can then be assigned in the second preliminary design. In this sequence, plastic moment balancing is an iterative design process. It is emphasized that iteration is required only to correct inappropriate initial sway estimates.

If future research can develop a reliable initial sway prediction model, the sway check may be considered as redundant. The product of this initial sway prediction model and plastic moment balancing may then closely approximate a direct design method for unbraced multi-story frames.

In summary, there are several questions which remain to be explored concerning the design of unbraced multi-story frames. Rather than providing answers to these questions, it is suggested that plastic moment balancing is a reasonable framework for defining the problems and guiding in their solution.
**SYMBOLS**

A  Moment factor \( \frac{M_A}{M_p} \) at left end of girder.

B  Moment factor \( \frac{M_B}{M_p} \) at right end of girder.

C  Moment factor \( \frac{M_C}{M_p} \) for maximum positive girder moment due to combined load.

C_l  Moment factor \( \frac{M_C}{M_p} \) for maximum positive girder moment due to gravity load.

D_G(J,I)  Factor defining the portion of the girder sway moment sum on level (I) which is assigned to girder (J). (Girder distribution factor, horizontal distribution factor)

D_f  Factor defining the portion of the column moment sum above and below one level which is distributed to the columns above and below one joint. (Column moment distribution factor, joint balancing method III)

D_j  Factor defining the portion of the initial external moment \( M_{IE} \) which is distributed to the column above a joint. (Joint balancing ratio, method I)

Factor defining the portion of the girder moment \( M_{IG} \) which is distributed to the column below the joint. (Joint balancing ratio, method IV)

D_L(J,I)  Factor defining the portion of the column moment sum at the lower end of the columns below level (I) which is distributed to column (J) to give the initial column moment \( M_{IL} \) at the beginning of the joint balance, method I. (Initial column moment distribution factor)

D_r  Deflection ratio \( = \frac{\theta_{ju}}{\Delta/h} \) for bottom story columns.
SYMBOLS (continued)

\( D_U(J,I) \)  Factor defining the portion of the column moment sum at the upper end of the columns below level (I) which is distributed to column (J) to give the initial column moment \( M_{iU} \) at the beginning of the joint balance, method I. (Initial column moment distribution factor)

\( D_V(I) \)  Factor defining the portion of the story sway moment assigned to the upper end of the columns in the story below level (I). (Vertical distribution factor)

d  Depth of section. Subscripts b and c denote girder and column depths, respectively. Subscripts \( b_U \) and \( b_L \) denote depth of girders at upper and lower end of column, respectively.

\( E_{SR} \)  Elastic sway ratio. (Ratio of elastic sway moment to total sway moment for girder, \( = \frac{G_{ES}}{G} \))

F  Load factor. Subscripts 1 and 2 denote gravity load and combined load factor, respectively. Subscript R indicates required load factor established by building code.

\( F_{1R}/F_{2R} \)  Load factor ratio.

f  Shape factor (the ratio of plastic moment to yield moment)

G  Girder sway moment coefficient \( (M_G/M_{pm}) \times (1 - d_c/L) \).

\( G_{ES} \)  Sway moment coefficient at formation of leeward plastic hinge in girder under combined load.

\( H(I) \)  Factored horizontal (wind) load at level (I).

\( H_{T(I)} \)  Factored wind shear in story below level (I).

\( \delta H_{T} \)  Change in story shear force.

\( h_c \)  Clear column height between girder flanges.

\( h(I) \)  Story height between center of joints below level (I).

\( (I) \)  Integer subscript denoting level or floor number starting from roof.
SYMBOLS (continued)

(1) Lowest level on which gravity load girders are adequate for combined load.

(2) Level at which maximum positive girder moment reaches windward end of girder.

(LCD) Top level of lower zone of constant girder depth.

(UCD) Top level of upper zone of constant girder depth.

(W) Integer level at which girder moment $M_A$ on windward column joint first becomes positive.

(I) Moment of inertia of column in the plane of the frame.

(J) Integer subscript denoting column, joint, or span number starting from left side of frame.

(J,1) Subscript added to any symbol to denote column or joint or span (J), (numbered from left to right), at level (I) (numbered from roof to foundation). Joint or span number always given first in double subscript.

(j) Prefix subscript denoting symbol which applies to a joint.

(K) Incremental sway stiffness of story (rate of change in story shear with story sway).

(L) Span length between column centers.

(L) Clear span of girder.

(M) Bending moment (sign convention defined in Art. 3.4).

($\delta M$) Change in moment.

($M_A$) Moment at left end of girder (face of column).

($M_B$) Moment at right end of girder (face of column).

($M_C$) Maximum positive moment along girder.

($M_{CC(I)}$) Resultant moment couple due to sway loads $P_s$ in columns below level (I).
### SYMBOLS (continued)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_G$</td>
<td>Sum of the girder moments $M_{jA} + M_{jB}$ (girder sway moment).</td>
</tr>
<tr>
<td>$M_I$</td>
<td>Initial moment about center of joint. Subscript $iL$ and $iU$ denote initial moments from columns above and below joint, respectively. (Joint balancing method I)</td>
</tr>
<tr>
<td>$M_{iE}$</td>
<td>Initial external moment on joint which conserves joint equilibrium with girder moments and initial column moments. (Joint balancing method I)</td>
</tr>
<tr>
<td>$M_j$</td>
<td>Moment about center of joint. Subscripts $jA$ and $jB$ denote moments from girders on right and left sides of joint, respectively. Subscript $jL$ and $jU$ denote moments from columns above and below joint, respectively.</td>
</tr>
<tr>
<td>$\delta M_j$</td>
<td>Change in moment on joint.</td>
</tr>
<tr>
<td>$M_{jG}$</td>
<td>Sum of the girder moments $M_{jA} + M_{jB}$ on a joint. (Girder joint moment, methods III and IV).</td>
</tr>
<tr>
<td>$M_L$</td>
<td>Moment at lower end of column (above girder flange).</td>
</tr>
<tr>
<td>$M_{OT(I)}$</td>
<td>Overturning moment at level (I) due to factored wind loads and $PA$ effect.</td>
</tr>
<tr>
<td>$M_P$</td>
<td>Plastic moment.</td>
</tr>
<tr>
<td>$M_{pc}$</td>
<td>Plastic moment modified to include the effect of axial compression.</td>
</tr>
<tr>
<td>$M_{pES}$</td>
<td>Plastic moment for lightest $W$ economy shape in a given depth series.</td>
</tr>
<tr>
<td>$M_{pm}$</td>
<td>$F_{2R} w L^2/16$ (2 $M_{pm}$ is the maximum moment due to factored gravity load at midspan of a girder with simple supports).</td>
</tr>
<tr>
<td>$M_S$</td>
<td>Moment at midspan of simply supported girder.</td>
</tr>
<tr>
<td>$M_U$</td>
<td>Moment at upper end of column (below girder flange).</td>
</tr>
</tbody>
</table>
SYMBOLS (continued)

\[ \sum M_C(I) \] Sum of the column moments in the story below level (I). (Story sway moment) The sum includes all columns in the story and the column moments are referred to the center of the joints.

\[ \sum M_G(I) \] Sum of the girder moments on level (I). (Girder sway moment sum.) The sum includes all girders on level (I) and the girder moments are referred to the center of the joints.

\[ \sum M_{jU} \] Sum of the column moments (referred to joint centers) at the upper end of the columns in one story.

\[ \sum M_{jL} \] Sum of the column moments (referred to joint centers) of the lower end of the columns in one story.

\[ N_B \] Number of bays.

\[ P \] Axial load.

Subscripts L and U denote axial load from column above and below joint, respectively.

Subscript S denotes axial load due to wind and sway. (Sway load)

\[ P_T(I) \] Total factored gravity load in the columns below level (I).

\[ P_Y \] Axial force corresponding to yield stress level.

\[ P_A \] Total factored gravity load times story sway.

\[ p \] Pivot factor for carry-over moments in bottom story.

\[ Q \] Horizontal column shear force.

Subscript o and \( \Delta \) denote shear in vertical and swayed column, respectively.

Subscripts L and U denote shear from column above and below joint, respectively.
SYMBOLS (continued)

q   Column moment ratio, positive when column is bent in double curvature.

qj  Column moment ratio $M_{jL}/M_{jU}$.

$q_j$  Weighted column moment ratio for bottom story.

R   Mechanism factor $M_p/M_{pm}$.

$R_b$  Base factor $= 1/(1 + R_f)$.

$R_f$  Dimensionless rotational restraint coefficient for the foundation.

$R_{LB}$  Lower bound for mechanism factor.

$R_{LU}$  Ratio $M_{jL}/M_{jU}$ of the column moments at a joint. (Joint moment ratio)

$rc$  Radius of gyration for column in the plane of bending.

V   Vertical shear force in girder.

Subscripts A and B denote shear reaction at left and right ends of girder, respectively.

$W(I)$  Total factored gravity load on level (I).

$W_{cr}(n)$  Deteriorated critical load parameter
(Hypothetical load parameter corresponding to elastic sidesway buckling of frame with n true hinges).

w  Uniformly distributed gravity load (including live load reduction) on a girder under working load conditions.

X  Distance from leeward end of girder to section of zero shear. (If $X \leq L_g$, maximum positive moment occurs at distance X from leeward end of girder.)

$X_{pi}$  Distance from leeward end of girder to point of inflection ($M = 0$).
SYMBOLS (continued)

Δ Horizontal deflection between floors. (Story sway, wind drift)

Δc Story sway due to column bending.

Δg Story sway due to girder bending.

δΔ Change in story sway.

Δ/h Column chord rotation (Sway deflection index)

θ End slope.

θj Joint rotation.

Subscripts A and B denote joint rotation at left and right end of girder, respectively.

Subscripts U and L denote joint rotation at upper and lower end of column, respectively.

δθ Change in rotation.
Fig. 1.1 Column forces and moments

Fig. 1.2 Relative reduction in column shear capacity due to sway
Fig. 1.3 Sway deflection approximations for an unbraced rigid frame
Fig. 1.4 Stages in the formation of plastic hinges in girders under combined load.
Fig. 1.5 Girder mechanisms under combined load.
**Elevation at Level 20 below roof**

Hinges at level 20 for wind from left shown thus:
- Hinge forms before ultimate wind shear
- Hinge forms after ultimate wind shear

---

**Fig. 1.6 Story shear versus sway behavior (from Ref. 10)**
Fig. 2.1 Second order elastic-plastic load versus sway curve for an unbraced frame under proportional combined loads.
Fig. 3.1 Frame geometry and notation.
Fig. 3.2 Notation and sign convention for moments.
Fig. 4.1 Forces acting on an interior girder-to-column-flange joint.
Fig. 4.2 Forces and moments on a rigidly framed column.
Fig. 4.3 Vertical and horizontal distribution of sway moments.
Fig. 4.4 Story moment equilibrium condition.

\[
M_G(j-1, I) = D_G(j-1, I) \sum M_G(z) \\
= M_G(j-I, I)/L(j-I) \\
= M_G(j, I)/L(j)
\]

Fig. 4.5 Equilibrium between column sway loads and girder shears due to sway.
Fig. 5.1 Forces and moments on a rigidly framed girder.
Plastic Moment Factors

<table>
<thead>
<tr>
<th>Condition</th>
<th>Factor</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B = C = +1 )</td>
<td>( R = (1 + \frac{G}{8})^2 )</td>
<td>(-1 \leq A \leq +1)</td>
</tr>
<tr>
<td>(-A = C = +1)</td>
<td>( R = (1 - \frac{G}{8})^2 )</td>
<td>(-1 \leq B \leq +1)</td>
</tr>
<tr>
<td>( A = B = +1 )</td>
<td>( R = \frac{G}{2} )</td>
<td>( C = A )</td>
</tr>
<tr>
<td>( A = B = -1 )</td>
<td>( R = -\frac{G}{2} )</td>
<td>( C = -B )</td>
</tr>
</tbody>
</table>

Diagram:

- Diagram A: Plastic moment envelope
- Diagram B: Moment diagram with forces
- Diagram C: Moment diagram with moment capacities

Diagrams illustrate the plastic moment envelope and moment factors for different conditions.
For $G \geq 8$
\[ R = G/2 \]

For $G \leq 8$
\[ R = (1 + G/8)^2 \]

\[ R_{LB} = F_{1R}/F_{2R} \]

\[ M_p = R M_{pm} \]

\[ M_p = R_{LB} M_{pm} \]

Fig. 5.3 Design chart for girder mechanisms.
Fig. 5.4 Plastic moment balance for restricted girder hinge patterns.
Fig. 5.5 Restricted hinge condition for gravity loading.
Fig. 5.6 Design chart for restricted girder hinge patterns.
Fig. 5.7 Girder moments under gravity loading.
Fig. 5.8 Assumptions for elastic sway response of a girder.
Plastic

\[ M_B = M_p = R M_{pm} \]

Elastic

\[ M_B = \left( \frac{4}{3} + \frac{G}{2} \right) M_{pm} \]

Fig. 5.9 Elastic and plastic leeward end-moment domains for a girder as a function of the mechanism factor \( R = \frac{M_p}{M_{pm}} \).
Fig. 5.10 Elastic and plastic leeward end-moment domains for a girder as a function of the load factor ratio $\frac{F_{1R}}{F_{2R}}$. 

$$G = \frac{1}{\frac{4}{(C_1 + 1)}}$$

$$M_B = M_P = R_{LB} M_{pm}$$

$$Elastic$$

$$M_B = \left(\frac{4}{3} + \frac{G}{2}\right) M_{pm}$$

$$C_1 = 0.5, 0.75, 1.0$$

Note: This graph applies when $M_P = R_{LB} M_{pm}$.
Fig. 6.1 Distribution of initial column moments.
**Initial column moments**

\[
M_{jL}(J,I-1) = \frac{D_L(J,I-1)}{D_L(J,I-1)} (1 - D_V(I-1)) \Sigma M_C(I-1)
\]

**Column moment distribution factors**

\[
M_{jB}(J,I) \quad M_{jA}(J,I) \quad M_{jU}(J,I)
\]

**External joint moment**

**Final column moments**

\[
M_{jL}(J,I-1) = M_{jL}(J,I-1) + D_j(I) M_{jE}(J,I)
\]

**Joint balancing ratio**

\[
M_{jB}(J,I) \quad M_{jA}(J,I) \quad M_{jU}(J,I)
\]

**Balancing moments**

**Fig. 6.2 Joint balance, Method I**
1. Sum girder moments

\[ M_{jG}(J,I) = M_{jB}(J-1,I) + M_{jA}(J,I) \]

\[ M_{jB}(J-1,I) \quad M_{jA}(J,I) \]

2. Column moment distribution factor

\[ D_f(J,I) = \frac{M_{jG}(J,I)}{\sum M_{jG}(I)} \]

3. Final column moments

\[ M_{jL}(J,I-1) = D_f(J,I) \left( 1 - D_V(I-1) \right) \sum M_{jC}(I-1) \]

\[ M_{jB}(J-1,I) \quad M_{jA}(J,I) \]

\[ M_{jU}(J,I) = D_f(J,I) D_V(I) \sum M_{jC}(I) \]

Fig. 6.3 Joint balance, Method III
1. Sum girder moments
\[ M_{jG}(J, I) = M_{jB}(J-1, I) + M_{jA}(J, I) \]

2. Joint balancing factor for level
\[ D_j(I) = \frac{D_V(I) \sum M_C(I)}{\sum M_G(I)} \]
\[ = \frac{-1}{1 + R_{LU}(I)} \]

3. Final column moments
\[ M_{jG}(J, I) \rightarrow = -(1 + D_j(I)) M_{jG}(J, I) \]
\[ = M_{jG}(J, I) - M_{jU}(J, I) \]
\[ M_{jU}(J, I) \rightarrow = D_j(I) M_{jG}(J, I) \]

Fig. 6.4 Joint balance, Method IV
Fig. 7.1 Forces and moments on bottom story column.
Fig. 7.2 Deflection ratio versus moment ratio for fixed base columns.
Fig. 7.3 Bottom story moment balance.
Fig. 7.4 Design chart for vertical distribution factor.
Given: \( G = 8, C = 0.5 \)

Plastic moment balance

Chart: \( R (C+I) = 8 \)

\[ R = \frac{16}{3} \]

\[ M_p = RM_{pm} = \left(\frac{16}{3}\right) M_{pm} \]

\[ CM_p = \left(\frac{8}{3}\right) M_{pm} \]

\[ M_B = M_p = \frac{16}{3} M_{pm} \]

Fig. 8.1 Plastic moment balance for \( G = 8, C = 0.5 \)
Fig. 8.2 Elastic and elastic-plastic domains for girder sway response
Given: \( G = \frac{8}{3}, \ C = \frac{1}{3} \)

Plastic moment balance

Chart: \( R(C+1) = \frac{32}{9} \)

\[
R = \frac{8}{3} \\
M_p = RM_{pm} = \frac{8}{3} M_{pm} \\
CM_p = \frac{8}{9} M_{pm}
\]

Fig. 8.3 Plastic moment balance for \( G = 8/3, \ C = 1/3 \)
Fig. 8.4 Elastic sway ratio for girders
APPENDIX I

The purpose of this appendix is to indicate the nature of preliminary design calculations for an unbraced multi-story frame using the plastic moment balancing method. The 24-story 3-bay unbraced frame considered in the design example is designated as Frame C in Chapters 6, 16, and 19 of Ref. 6. A sway subassemblage analysis for level 20 of this frame is performed in Ref. 10. This reference is the source of the member sizes and story shear versus sway curve in Fig. 1.6. Chapter 21 of Ref. 2 includes portions of an allowable stress design for Frame C.

This appendix illustrates portions of the preliminary design calculations for Frame C in tabular form as follows:

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>Preliminary design data: Source; Ref. 6, Example 6.3</td>
</tr>
<tr>
<td></td>
<td>Defines frame geometry and working loads.</td>
</tr>
<tr>
<td>A2</td>
<td>Preliminary design parameters</td>
</tr>
<tr>
<td></td>
<td>Summarizes the parameters assigned in the plastic moment balance.</td>
</tr>
<tr>
<td>A3</td>
<td>Vertical distribution of sway moments</td>
</tr>
<tr>
<td></td>
<td>Calculations for story sway moments ( \Sigma M_C ) and girder sway moment sums ( \Sigma M_G ).</td>
</tr>
<tr>
<td>A4</td>
<td>Horizontal distribution of sway moments</td>
</tr>
<tr>
<td></td>
<td>Calculations for girder sway moments ( M_G ).</td>
</tr>
<tr>
<td>A5</td>
<td>Floor girder data</td>
</tr>
<tr>
<td></td>
<td>Calculations for ( M_{pm} = \frac{1}{16} F_{2R} w L^2 ).</td>
</tr>
</tbody>
</table>
Table  Description
A6 Girder plastic moment balance operations table
Outlines the tabular sequence and equations in the girder plastic moment balance.
A7 Girder joint moments operations table
Outlines the tabular sequence and equations used to find the joint moments applied by the girders.
A8 Girder plastic moment balance and joint moments
Illustrates the girder plastic moment balance for several cases:
<table>
<thead>
<tr>
<th>Level</th>
<th>Girder controlled by</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Gravity load mechanism</td>
</tr>
<tr>
<td>12</td>
<td>Combined load mechanism</td>
</tr>
<tr>
<td>16,20</td>
<td>Combined load, restricted hinge pattern</td>
</tr>
</tbody>
</table>
A9 Steps in joint balance - Method I
Indicates step-by-step calculations.
A10 Joint balance - Method I
Illustrates the joint balance at level 20 using Method I, for three loading conditions.
A11 Joint balance - Method IV
Illustrates the joint balance at level 20 using automatic Method IV, for three loading conditions.
A12 Constant depth girders

The moment diagrams at level 20 in Figs. A1.1 and A1.2 are a graphical summary of the results obtained in the preliminary design. These results provide the basis for selecting tentative member sizes. The design checks which follow the preliminary design phase are considered in Refs. 6 and 10 and are not included in this investigation.
Initial Data

The first step in the preliminary design is to define the frame geometry and working loads. Table A1(6) indicates the frame elevation and bent spacing, the unit working loads, and the girder, joint, and wind loads. The gravity loads in the columns and the wind shear in each story can be determined from the data in Table A1. The dead loads from floors, columns, and exterior walls are included in determining maximum allowable live load reduction factors in this example. These routine calculations are omitted for brevity.

Preliminary Design Parameters

Table A2 lists the preliminary design parameters assigned by the designer in the plastic moment balance. The load factors in item 1 are taken from Ref. 6. The ultimate load is selected as the preliminary design criterion. Item 2 in Table A2 lists the sway deflection assumptions used to account for PA effects at ultimate load. The ultimate load criterion also includes assumptions for restricted hinge patterns in girders. Restricted hinge patterns are considered in assigning positive moment factors as indicated by item 5. The positive moment factors C < 1 for combined load are assumed to apply to each girder on levels 16 to 24. The corresponding restricted hinge pattern includes three plastic hinges at the leeward ends of the three girders on each level.

The assumptions for the sway and hinge pattern at ultimate load in Table A2 are conservative in comparison with the ultimate load behavior described in Fig. 1.6 at level 20 of Frame C. This indicates how the results of refined inelastic sway investigations can be used
to guide the selection of preliminary design parameters in the plastic moment balancing method. The positive moment factors in item 5 of Table A2 are used to illustrate ideas and are not regarded as firm design recommendations.

Item 3 in Table A2 lists the vertical distribution factors assigned in the plastic moment balance. The value $D_V = 0.5$ assigns half of the story sway moment to the upper and lower end of the columns. The vertical distribution factor in the bottom story depends on the assumptions for column base restraint and relative girder stiffness on level 24. If the column bases are considered to be fixed, the base factor $R_b$ is zero. Figure 7.4 gives vertical distribution factors in the range $0.33 \leq D_V \leq 0.50$ for girders that vary from flexible to infinitely stiff. The value $D_V = 0.33$ is assigned in the bottom story moment balance. This produces larger preliminary design moments at the base than at the top of the bottom story columns. Columns selected to resist the larger base moments will tend to delay the formation of plastic hinges and to reduce PA$\Delta$ effects in the bottom story.

Frame Statics

Tables A3 and A4 illustrate the frame statics calculations which are the first part of the plastic moment balancing process. The story sway moments for wind from left to right are computed and distributed in Table A3. This table begins with the wind shear $H_T$ and total gravity load $P_T$ (including column live load reduction) below each level, in columns 2 and 3. These columns are obtained in a routine manner from the working load data in Table A1. The values of the sway
Deflection index in column 4 are used to include estimated $P \Delta$ effects in the plastic moment balance. The terms in Eq. (4.9) for the story sway moment are calculated in columns 5 and 6. The $P_T \Delta$ moments in each story below level 4 contribute about 35 percent of the total story sway moment $\Sigma M_c$ in column 7.

The vertical distribution factors in column 8 of Table A3 are used to determine the portion of the story sway moment assigned to the upper end of the columns in each story. The product $D_V \Sigma M_c$ is entered in column 9 of Table A3. The data in columns 7 and 9 is combined to give the girder sway moment sum $\Sigma M_G$ for each level in column 10, using Eq. (4.12). This completes the vertical sway moment distribution.

The horizontal distribution of sway moments to individual girders is performed in Table A4, which is based on Eq. (4.13). The girder sway moment distribution factors $D_G$ assigned in this table vary from level to level and are selected with two ideas in mind. First, the minimum girder sizes needed to carry factored gravity loading should be used on as many levels as possible. Second, it is desired to use girders with the same nominal depth on lower levels where girder sizes are controlled by combined loading. Later work will indicate how these ideas are used to obtain the values of $D_G$ in Table A4. Regardless of how they are assigned the girder distribution factors are statically admissible if they sum to unity on each level. Table A4 concludes the frame statics part of the plastic moment balancing process.
Girder Plastic Moment Balance

The data needed to begin the girder plastic moment balance are the girder sway moments $M_G$ from Table A4 and the gravity load parameter $M_{pm} = \frac{1}{16} F_{2R} t w L^2 g$. Table A5 includes tabular calculations which give $M_{pm}$ in row 6. The remaining rows in this table are considered later.

An operations table and all equations for the girder plastic moment balance are outlined in Table A6. The first five rows are used to collect data from previous tables and to record the positive moment factors assigned for restricted hinge patterns. The sway moment coefficient $G$ is computed in row 6. Equations for the mechanism factor $R$ and the lower bound $R_{LB}$ are given in rows 7 and 8. Figure 5.6 may be used to find $R$ if desired. This is the key step in the girder plastic moment balance.

If the girder size is controlled by combined loading, the first equation in row 9 gives the required plastic moment $M_p$. The second equation in this row determines $M_p$ for gravity loading.

The entries in rows 10 and 11 of Table A6 can be used to select the girder sections for a specified yield strength $F_y$. This is an optional step which can be deferred until the end of the preliminary design.

An operations table and all equations needed for determining the joint moments applied by the girders are summarized in Table A7. This table considers combined loading in rows 1C to 6C and gravity loading in rows 1G to 5G. The equations in Table A7 are discussed in
Art. 5.4. The values of the mechanism factor R in rows 1C, 2C, and 2G in the joint moment operations table are taken from row 7 of Table A6.

To find the joint moments for combined load it is necessary to know whether the leeward end-moment \( M_B \) is plastic or elastic. This is determined by finding the elastic sway moment coefficient \( G_{ES} \) in row 1C of Table A7. The values of \( G_{ES} \) is compared with \( G \) in row 6 of Table A6. If \( G > G_{ES} \), the leeward end-moment is the plastic moment and Eq. (1) in row 2C of Table A7 applies. In the infrequent case that \( G < G_{ES} \), Eq. (2) in row 2C is used to estimate the elastic leeward end-moment. Row 3C gives the moment about the center of the leeward joint caused by the girder end-shear. The moments in rows 2C and 3C are in nondimensional form.

The leeward and windward joint moments \( M_{jB} \) and \( M_{jA} \) for combined loading with wind from left to right are obtained as indicated in rows 5C and 6C of Table A7. When wind is applied from right to left, the joint moments are reversed and multiplied by -1.

The calculations for the joint moments applied by the girders under gravity loading follow a similar pattern in the second part of Table A7. Row 1G is the elastic fixed-end moment in nondimensional form. The fixed-end moment is compared with \( R \) to establish whether the right end-moment is plastic or elastic, as indicated in row 2G. The joint moment caused by the gravity load end-shear is obtained in row 3G. The last row in Table A7 is the joint moment caused by gravity loading.
Table A8 illustrates the calculations for the girder plastic moment balance and joint moments at levels 4 and 12 of Frame C. The rows in this table use the same numbering system as the operations tables A6 and A7. The "item" column in Table A8 further identifies the nature of each row. The values $C = C_1 = 1.0$ in rows 4 and 5 indicate that girder mechanisms are assumed in the plastic moment balance at levels 4 and 12.

The calculations in Table A8 for level 4 indicate the nature of the plastic moment balance when gravity loading controls the girder sizes. Note that $R_{LB}$ in row 8 exceeds the mechanism factor $R$ in row 7. This is the clue that gravity load controls. The negative values of $C_{ES}$ in row 1C indicate that the factored load $F_2R_wL$ cause hinges at the ends of the girder under zero wind loads. The larger sway moments at level 12 reverse the relative values of $R$ and $R_{LB}$. The girders on this level are controlled by combined loading.

To indicate how restricted hinge patterns are recognized in the girder plastic moment balance, consider Table A9. This table applies to levels 16 and 20 of Frame C. The values of $C < 1$ in row 5 indicate that positive plastic hinges are avoided in the moment balance. The practical result is to increase the required girder sizes. For example, the sway moment $M_G = 937$ kip-ft. assigned to bay 2 at level 20 could be resisted by a 24B61 or a 21W62 girder if plastic hinges

*If $F_{1R} = 1.70$ is assigned for gravity loading, the ratio $F_{1R}/F_2R = 1.33$ in Art. 2.6 gives $F_2R = 1.28$. The load factor for combined loading is rounded off to $F_2R = 1.30$ in Ref. 6 and this Appendix. The value 1.28 implies a misleading element of accuracy.*
are assumed at each end. When the windward plastic hinge is restricted by using $C = 0.8$ in the plastic moment balance, the required girder size increases to a $24W68$.

Note that restricted hinge patterns involve no essential change in the moment balancing operations. The operations follow the same sequence as Tables A6 and A7. The girder plastic moment balance and joint moments complete the second part of the plastic moment balancing process.

**Joint Balance**

Joint balancing method I and IV from Chapter 6 are considered in this appendix. The joint balance begins with the moments from the girder plastic moment balance and holds these girder moments constant. The purpose of the joint balance is to find column moments which satisfy joint equilibrium and the story equilibrium conditions established during the frame statics phase.

The boxes at the top of Table A10 indicate the pattern used to record the results of the joint balance. The upper left and lower right quadrants at each joint are used for the girder moments $M_{jB}$ and $M_{jA}$. The lower left and upper right quadrants give the final column moments $M_{jU}$ and $M_{jL}$. One vertical column at the right side of the joint diagram is used to record the sway moment sums $\Sigma M_{jL}$, $\Sigma M_{G}$, and $\Sigma M_{jU}$ for the columns and girders.

Table A10 is used to explain the numerical sequence in joint balancing method I (Art. 6.1) at level 20 of Frame C. The row numbers at the left side of the diagrams will assist in the explanation. Rows
2 and 3 are reserved for the final results of the joint balance. The remaining rows are used to record intermediate data as follows:

- **Row 1**, Initial column moment distribution factors $D_U$ and $D_L$
- **Row 4**, Initial column moments $M_{iU}$ and $M_{iL}$
- **Row 5**, Balancing moments
- **Row 6**, External joint moments $M_{iE}$ and the joint balancing ratio $D_j$

The left and right side of each joint is used to record data for the column below and above, respectively.

The initial data needed to begin the joint balance includes the joint moments determined in the girder plastic moment balance and the moment sums for the columns and girders from the vertical distribution of sway moments. This initial data is recorded at the top of Table A10.

The center and bottom diagrams in Table A10 illustrate the method of joint balance in 6 steps.

**Step 1** - Assign the initial column moment distribution factors $D_U$ and $D_L$. These factors are recorded in row 1 above each joint. The conditions $\sum D_U = \sum D_L = 1$ must be satisfied in assigning the distribution factors.

**Step 2** - The product of the distribution factors and the column moment sums at the right side of the diagram gives the initial column moments $M_{iU}$ and $M_{iL}$, which are entered in row 4 below each joint.
Step 3 - Find the hypothetical external moment $M_{iE}$ at each joint which conserves joint equilibrium. This external moment is simply -1 times the algebraic sum of the girder and initial column moments in rows 2, 3, and 4, and is recorded in row 6 at each joint.

An intermediate check may be applied at this stage by noting that the sum of the external moments $M_{iE}$ in row 6 should be zero. In Table A10, $\Sigma M_{iE}$ is -1 due to a roundoff error.

Step 4 - Assign the joint balancing ratio $D_j$ and record this ratio in row 6. The same value of $D_j$ must be used for each joint on one level.

Step 5 - Distribute the portion $D_j M_{iE}$ of the external moment to the column above and the remainder of $M_{iE}$ to the column below. The results of this step are termed balancing moments and are entered in row 5 at each joint.

Step 6 - The final column moments are the algebraic sum of the initial moments in row 4 and the balancing moments in row 5. The final column moments are recorded in rows 2 and 3 at each joint.

Table A11 shows the joint balance at level 20 for three loading conditions. The joint balance for combined loading with wind from left to right is repeated from Table A10 for comparison with the other loading conditions. When wind acts from right to left, the girder
moments in each bay are reversed and multiplied by -1. The moment sums at the right center of Table All are also multiplied by -1. Under gravity loading, the column and girder moment sums are zero, so the joint balance reduces to steps 3 to 6.

The distribution factors in joint balancing method I may be varied to achieve different equilibrium distributions of column moments. To obtain this flexibility, the method involves recording 30 numbers at each interior joint for the three loading conditions. This can be reduced to 15 numbers using joint balancing method IV which is more direct but less flexible.

Table A12 illustrates the method IV joint balance at level 20 for three loading conditions. The same initial data (Table A10) is used to begin the joint balance for methods I and IV. The first step in method IV is to sum the girder moments \( M_{jG} = M_{jA} + M_{jB} \) on each joint. These are recorded in row 6 of Table A12. In the second step, the joint balancing ratio \( D_j = \frac{\Sigma M_{jU}}{\Sigma M_G} \) is computed and recorded at the right side of the diagram. Note that \( D_j \) is negative. The column moments are obtained from \( M_{jU} = D_j M_{jG} \) in row 3 and \( M_{jL} = -M_{jU} - M_{jG} \) in row 2. This completes the joint balance using method IV.

It is of interest to note that if the joint balancing ratio \( D_j \) in method I is assigned the value

\[
D_j = 1 + \frac{\Sigma M_{jU}}{\Sigma M_G}
\]  \hspace{1cm} (A1)

and if the initial column moment distribution factors \( D_L \) and \( D_U \) are equal above and below a joint, methods I and IV give identical results.
which are independent of $D_L$ and $D_U$. At level 20, Eq. (A1) gives

$$D_j = 1 - 0.513 = 0.487.$$ This is close to the value $D_j = 0.5$ in Table 11 which also uses $D_L = D_U$. This explains why Tables 11 and 12 give nearly the same column moments in this example.

Regardless of what method is used in the joint balance, the results may be checked against the column moment sums $\sum M_{jL}$ and $\sum M_{jU}$ in the right column and the condition $\sum M_j = 0$ at each joint. Following the joint balance, the column end-moments are obtained from Eq. (4.5).

**Girder Distribution Factors**

One refinement of the basic plastic moment balancing method used in this example is the choice of girder distribution factors in Table A4. The first idea considered in distributing the girder sway moments is to use the minimum girder sizes needed for gravity loading on as many levels as possible.

On the top levels of the frame, factored gravity loads control the required plastic moment capacity of the girders. The girders on these levels are adequate for combined loading if the girder sway moments are distributed using Eq. (8.5). Row 7 in Table A5 applies this equation to find girder distribution factors $D_G$ which are proportional to $M_{pm}$.

The lowest level on which the girder sizes are controlled by gravity loading is established next. The sway moment capacity of the gravity load girders can be estimated by taking $R = R_{LB} = 1.31$ in the girder plastic moment balance and finding the corresponding sway moment
coefficient $G = 1.15$ from Eq. (5.33) or Fig. 5.6. The positive moment coefficients $C = C_1 = 1.0$ are used in this calculation. It is assumed that the girders provide no excess plastic moment capacity above the value $RLB M_{pm}$ which is just adequate for factored gravity loading. The maximum sway moment sum for the gravity load girders on one level is evaluated in Table A5, row 9, using $G = 1.15$ in Eq. (8.7). The result is $\text{Max. } \Sigma M_G = 499$ kip-ft. This value is compared with the girder sway moment sums in column 10 of Table A3. The largest value of $\Sigma M_G(I)$ in column 10 which is less than $\text{Max. } \Sigma M_G$ indicates the lowest level ($I_1 = 4$) on which the gravity load girders are adequate for combined loading. The girder distribution factors in row 7 of Table A5 are appropriate on levels 1 to 4. This is the basis for the values of $D_G$ in Table A4 for level 4.

The second idea considered in distributing the girder sway moments is to use girders with the same nominal depth on lower levels where girder sizes are controlled by combined loading. The minimum depth for girders of constant depth is determined by the gravity load in bay 3. The girder in this bay must provide a minimum plastic moment capacity of $RLB M_{pm} = 315$ kip-ft. for factored gravity loading where $M_{pm}$ is obtained from Table A5, row 6. A $16\#58$ or an $18\#55$ or a $21B49$ can be used in bay 3. The $18\#55$ with $M_{pm} = 335$ kip-ft. is selected in this example. Therefore 18 in. is the minimum depth for constant depth girders below level 4.

Article 8.7 describes a procedure for selecting constant depth girder distribution factors. It is of interest to apply this procedure to Frame C which has varying bay sizes. Constant depth girders are
obtained by distributing the girder sway moments equally between the bays at and below the level \((I_{CD})\) determined by Eq. (8.20). Using the largest value of \(M_{pm}\) (for bay 3) in this equation gives \(\Sigma M_G = 3000\) kip-ft., which is approximately equal to the girder sway moment sum at level 24 in Table A3. Thus \((I_{CD}) = 24\) for Frame C. A similar frame with three 20-ft. bays would give \((I_{CD}) = 13\).

The irregular bay sizes for Frame C considerably extend the transition zone between \((I_1) = 4\) and \((I_{CD}) = 24\). Constant depth girders can be used in this zone but some initial calculations are needed to suggest distribution factors for constant depth girders. These calculations are considered next.

**Initial Estimate for Constant Depth Girders**

An estimate of the plastic moment capacity for constant depth girders is obtained in Table A13. This table is based on Eq. (8.15) which reduces to the expression in the box at the top of Table A13. The values of \(\Sigma M_{pm}\) and \(\Sigma \sqrt{M_{pm}}\) for Frame C are obtained from rows 6 and 8 in Table A5. The girder sway moment sums from Table A3 are the initial data in column 1 of Table A13. Column 4 gives an estimate of \(M_p\) for girders with identical plastic moment capacity in the three bays of Frame C. The sections in column 5 are tentative ASTM A36 steel girder sizes for combined loading. These girders are adequate for gravity loading in bays 1 and 2 but not in bay 3 between levels 5 and 10.

Two approximations used in Table A13 should be mentioned. First, the sway moment coefficients for bay 2 exceed the limit \(G < 8\)
which applies to Eq. (8.15). This makes the estimates of $M_p$ in Table A13 conservative. Second, no provisions for restricted hinge patterns are included in Table A13 which is based on a positive moment factor $C = 1.0$. If the factor $C = 0.8$ from Table A2 is used in Eq. (8.15) at level 20, $M_p^{(20)}$ increases by a factor of $2/(1 + 0.8) = 1.11$. These two approximations tend to cancel each other. The net result is that the tentative constant depth girder sections in Table A13 are a good approximation of the girder sizes obtained from the plastic moment balance at levels 12, 16, and 20 in Tables A8 and A9.

**Distribution Factors for Constant Depth Girders**

Gravity loading requires a girder with $M_p = R_{LB} M_{pm} = 315$ kip-ft. in bay 3. Level 12 is the first level in Table A13 which gives $M_p$ larger than 315 kip-ft. for constant depth girders. This suggests that the same girder size (an $18\times55$, $M_p = 335$ kip-ft.) is appropriate in bay 3 on levels 4 to 12. The girder sway moment distribution in Table A4 is arranged to utilize the sway moment capacity of an $18\times55$ in bay 3 and an $18\times50$ in bays 1 and 2 at level 12. A gradual transition in the girder distribution factors is assumed between levels 4 and 12. A series of shapes between an $18\times35$ and an $18\times50$ can be used in bays 1 and 2 together with an $18\times55$ in bay 3 to resist the girder sway moments on floors above level 12.

Below level 12 the girder distribution factors $D_G$ in Table A4 are varied gradually to approach the value $D_G = 1/3$ at level $(I_{CD}) = 24$. This sway moment distribution results in girders with constant nominal depths of 21 or 24 in. on each level.
The key step in a constant depth girder design is the selection of girder distribution factors $D_G$. Equation (8.15) is a valuable aid in this step for frames with irregular bay dimensions.

**Results of Preliminary Design**

The moment diagrams in Figs. AI.1 and AI.2 indicate the information obtained from the preliminary design at level 20. These diagrams represent a possible distribution of moments which conserves equilibrium in the assumed deflected state. Plastically designed girders and columns proportioned to resist these moments are a good first approximation of an adequate frame design, if the preliminary design parameters in Table A2 are assigned with reasonable judgment. The following is a summary of the preliminary member sizes at level 20 using ASTM A36 steel.

<table>
<thead>
<tr>
<th>Member</th>
<th>Section</th>
<th>Controlling Loading Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Girders 1 to 3</td>
<td>24W68</td>
<td>Combined load</td>
</tr>
<tr>
<td>Column 1</td>
<td>14W264</td>
<td>Combined load - wind right</td>
</tr>
<tr>
<td>Column 2</td>
<td>14W264</td>
<td>Combined load - wind right</td>
</tr>
<tr>
<td>Column 3</td>
<td>14W314</td>
<td>Combined load - wind left</td>
</tr>
<tr>
<td>Column 4</td>
<td>14W314</td>
<td>Gravity load</td>
</tr>
</tbody>
</table>

These members are similar to the design in Fig. 1.6 from Ref. 10. Individual member sizes differ by one W shape between the two designs.

The 24W76 girders in Fig. 1.6 were selected on the basis of girder mechanisms and an assumed sway index of $\Delta/h = 0.020$ (Chapter 16, Ref. 6). This set of preliminary design assumptions results in a frame with an ultimate story shear capacity of 162.4 kips below level
273.38

20 according to Ref. 10. The 24W68 girders obtained in this appendix are adequate for the factored story shear of 114.2 x 1.3 = 148.4 kips and an assumed sway index of $\Delta/h = 0.010$ together with a restricted hinge pattern. The relative insensitivity of girder sizes to the assumed sway is evident from this comparison.

Design checks at ultimate load and at working load are needed to verify that the preliminary member sizes are satisfactory. Any method of inelastic or elastic-plastic analysis may be used for the sway check at ultimate load. To be consistent with the plastic moment balancing calculations, the ultimate load analysis should consider plastic hinges at the ends of members (face of joint), PΔ effects, and axial load reduction of the plastic moment capacity of columns. Reference 10 suggests one method for investigating the story shear versus sway behavior of an isolated story in the inelastic range.

The sway check at working load is conveniently performed using the virtual work method described in Art. 21.9 of Ref. 2. The preliminary member sizes at level 20 result in a working load sway index of $\Delta/h = 0.0032$ below this level, based on centerline dimensions and members of zero depth. If the virtual work sway calculation is modified to consider members of finite depth, the working load sway index reduces to $\Delta/h = 0.0027$. These values indicate that the preliminary members provide adequate sway stiffness under working load conditions.
**TABLE A1**

**PRELIMINARY DESIGN DATA - FRAME C (Ref. 6)**

- **WORKING LOADS**
- **DEAD** (psf)
- **LIVE** (psf)

<table>
<thead>
<tr>
<th></th>
<th>Roof</th>
<th>Floor</th>
<th>Exter. wall</th>
<th>Wind</th>
<th>Columns</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(psf)</strong></td>
<td>95</td>
<td>120*</td>
<td>85</td>
<td>--</td>
<td>625 plf incl. fireproofing</td>
</tr>
</tbody>
</table>

*Includes partitions @ 20 psf

**GIRDER LOADS**

<table>
<thead>
<tr>
<th>Bay</th>
<th>Level</th>
<th>LLR (pct)</th>
<th>Working Load (kips/ft.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>3.00</td>
</tr>
<tr>
<td></td>
<td>2 to 24</td>
<td>38.4</td>
<td>4.36</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0</td>
<td>3.00</td>
</tr>
<tr>
<td></td>
<td>2 to 24</td>
<td>23.0</td>
<td>4.73</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>0</td>
<td>3.00</td>
</tr>
<tr>
<td></td>
<td>2 to 24</td>
<td>50.9</td>
<td>4.06</td>
</tr>
</tbody>
</table>

**Frame Elevation**

Bents @ 24' cc

**Joint Loads**

<table>
<thead>
<tr>
<th>Item</th>
<th>Col. 1</th>
<th>Col. 2</th>
<th>Col. 3</th>
<th>Col. 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>4' Parapet</td>
<td>8.2</td>
<td>---</td>
<td>---</td>
<td>8.2</td>
</tr>
<tr>
<td>Exter. wall</td>
<td>24.5</td>
<td>---</td>
<td>---</td>
<td>24.5</td>
</tr>
<tr>
<td>Column (12')</td>
<td>7.5</td>
<td>7.5</td>
<td>7.5</td>
<td>7.5</td>
</tr>
<tr>
<td>(15')</td>
<td>9.4</td>
<td>9.4</td>
<td>9.4</td>
<td>9.4</td>
</tr>
</tbody>
</table>

**Wind Loads**

<table>
<thead>
<tr>
<th>Level</th>
<th>H (kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.8</td>
</tr>
<tr>
<td>2 to 24</td>
<td>5.8</td>
</tr>
<tr>
<td>(1) Load Factors</td>
<td>Gravity Load</td>
</tr>
<tr>
<td>Combined Load</td>
<td>$F_{2R} = 1.30$</td>
</tr>
</tbody>
</table>

(2) Sway Deflection Index

| Below Level 1 | $\Delta/h = 0.0033$ |
| Below Level 2 | $\Delta/h = 0.0067$ |
| Below Level 3 to 24 | $\Delta/h = 0.0100$ |

(3) Vertical Distribution Factors

| Below Levels 1 to 23 | $D_v = 0.5$ |
| Below Levels 24 (bottom story) | $D_v = 0.33$ |

Assume Fixed Base

| $R_b = 0$ |

Assume Flexible Girders

| $D_r = 1.0$ |

(4) Girder Distribution Factors

| $D_G$ Table A4 |

(5) Positive Moment Factors

| Levels 1 to 15 Gravity Load | $C_l = 1.0$ |
| Combined Load | $C = 1.0$ |
| Levels 16 to 19 Gravity Load | $C_l = 1.0$ |
| Combined Load | $C = 0.9$ |
| Levels 20 to 24 Gravity Load | $C_l = 1.0$ |
| Combined Load | $C = 0.8$ |

(6) Joint Balancing Parameters

| Method I |
| Initial Column Moment Distribution factor | $D_L = 0.25$ |
| $D_U = 0.25$ |
| Joint Balancing Ratio | $D_J = 0.50$ |
# TABLE A3

## VERTICAL DISTRIBUTION OF SWAY MOMENTS - FRAME C

<table>
<thead>
<tr>
<th>Working Loads</th>
<th>Assign</th>
<th>(Sway Moments) x $P_{2R}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Story</td>
</tr>
<tr>
<td>Level 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Level 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Level 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Level 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Level 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Level 8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Level 12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Level 16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Level 20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Level 24</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Level 1 | 4.8 | 226 | 0.0033 | 75 | 12 | -87 | 0.5 | -44 | 44 |
| Level 2 | 10.6 | 578 | 0.0067 | 165 | 60 | -225 | 0.5 | -112 | 156 |
| Level 3 | 16.3 | 865 | 0.0100 | 225 | 135 | -390 | 0.5 | -195 | 307 |
| Level 4 | 22.1 | 1166 | 0.0100 | 344 | 182 | -526 | 0.5 | -263 | 458 |
| Level 5 | 27.8 | 1478 | 0.0100 | 434 | 231 | -665 | 0.5 | -332 | 595 |
| Level 8 | 39.4 | 2106 | 0.0100 | 614 | 329 | -943 | 0.5 | -472 | 1012 |
| Level 12 | 62.4 | 3362 | 0.0100 | 973 | 524 | -1498 | 0.5 | -749 | 1567 |
| Level 16 | 85.4 | 4617 | 0.0100 | 1333 | 720 | -2053 | 0.5 | -1026 | 2122 |
| Level 20 | 108.5 | 5873 | 0.0100 | 1692 | 916 | -2608 | 0.5 | -1304 | 2677 |
| Level 24 | 131.5 | 7128 | 0.0100 | 2052 | 1112 | -3164 | 0.5 | -1582 | 2958 |
### TABLE A4

**HORIZONTAL DISTRIBUTION OF SWAY MOMENTS TO GIRDERS - FRAME C**

<table>
<thead>
<tr>
<th>Level</th>
<th>$\Sigma M_G$ Tab. A3 (k-ft)</th>
<th>Bay 1</th>
<th>Bay 2</th>
<th>Bay 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$D_G$</td>
<td>$M_G$ (k-ft)</td>
<td>$D_G$</td>
<td>$M_G$ (k-ft)</td>
</tr>
<tr>
<td>4</td>
<td>458</td>
<td>0.31</td>
<td>142</td>
<td>0.11</td>
</tr>
<tr>
<td>8</td>
<td>1012</td>
<td>0.35</td>
<td>354</td>
<td>0.30</td>
</tr>
<tr>
<td>12</td>
<td>1567</td>
<td>0.37</td>
<td>580</td>
<td>0.40</td>
</tr>
<tr>
<td>16</td>
<td>2122</td>
<td>0.37</td>
<td>785</td>
<td>0.37</td>
</tr>
<tr>
<td>20</td>
<td>2677</td>
<td>0.35</td>
<td>937</td>
<td>0.35</td>
</tr>
<tr>
<td>24</td>
<td>2958</td>
<td>0.33</td>
<td>986</td>
<td>0.33</td>
</tr>
</tbody>
</table>

### TABLE A5

**FLOOR GIRDER DATA - FRAME C**

<table>
<thead>
<tr>
<th>Row</th>
<th>Bay</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>L</td>
<td>20.0</td>
<td>12.0</td>
<td>28.0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$d_c$ (assign)</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
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<tr>
<td>3</td>
<td>$d_c/L$</td>
<td>0.050</td>
<td>0.083</td>
<td>0.036</td>
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<tr>
<td>4</td>
<td>$L_g$</td>
<td>19.0</td>
<td>11.0</td>
<td>27.0</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>$F_{2R}$</td>
<td>5.66</td>
<td>6.15</td>
<td>5.27</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>$M_{pm}$ (k-ft)</td>
<td>127.9</td>
<td>46.5</td>
<td>240.5</td>
<td>414.9</td>
</tr>
<tr>
<td>7</td>
<td>$M_{pm}/\Sigma M_{pm}$</td>
<td>0.31</td>
<td>0.11</td>
<td>0.58</td>
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<td>8</td>
<td>$\sqrt{M_{pm}}$</td>
<td>11.3</td>
<td>6.8</td>
<td>15.5</td>
<td>33.6</td>
</tr>
<tr>
<td>9</td>
<td>$1.15 M_{pm} \frac{1}{l-d_c/L}$ (k-ft)</td>
<td>155</td>
<td>58</td>
<td>286</td>
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</table>
### TABLE A6

**GIRDER PLASTIC MOMENT BALANCE OPERATIONS TABLE**

<table>
<thead>
<tr>
<th>Row</th>
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<th>Source or Operation</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>$M_{pm}$</td>
<td>Table A5 $M_{pm} = \frac{1}{16} F_{2R} w L^2$</td>
</tr>
<tr>
<td>2</td>
<td>$d_c/L$</td>
<td>Table A5</td>
</tr>
<tr>
<td>3</td>
<td>$M_G$</td>
<td>Table A4 $M_G = D_G \Sigma M_G$</td>
</tr>
<tr>
<td>4</td>
<td>$C_1$</td>
<td>Assign positive moment factors for gravity and combined load</td>
</tr>
<tr>
<td>5</td>
<td>$C$</td>
<td>$G = \frac{M_G (1 - d_c/L)}{M_{pm}}$</td>
</tr>
<tr>
<td>6</td>
<td>$G$</td>
<td>$R = \frac{G}{C + 1}$ for $G \leq 8$</td>
</tr>
<tr>
<td>7</td>
<td>$R$</td>
<td>$R = \frac{2}{C + 1} \left(1 + \frac{G}{8}\right)^2$ for $G \leq 8$</td>
</tr>
<tr>
<td>8</td>
<td>$R_{LB}$</td>
<td>$R_{LB} = \frac{2}{C_1 + 1} \left(\frac{F_{1R}}{F_{2R}}\right)$</td>
</tr>
<tr>
<td>9</td>
<td>$M_p$</td>
<td>(1) $M_p = R M_{pm}$ for $R \geq R_{LB}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2) $M_p = R_{LB} M_{pm}$ for $R \leq R_{LB}$</td>
</tr>
</tbody>
</table>

Select girder sections

| 10  | $Z$ | $Z = 12 \frac{M_p}{F_y}$ (in.$^3$) |
| 11  | Section | Plastic modulus table |
## TABLE A7

**GIRDER JOINT MOMENTS OPERATIONS TABLE**

<table>
<thead>
<tr>
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<tr>
<td></td>
<td></td>
<td><strong>Combined Load - Wind Left</strong></td>
</tr>
<tr>
<td>1C</td>
<td>$G_{ES}$</td>
<td>Fig. 5.9 $G_{ES} = 2 (R - 1.33)$</td>
</tr>
<tr>
<td>2C</td>
<td>( \frac{M_B}{M_{pm}} )</td>
<td>(1) ( \frac{M_B}{M_{pm}} = R ) or ( \frac{R_{LB}}{M_{pm}} ) for $G &gt; G_{ES}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(Note 1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2) ( \frac{M_B}{M_{pm}} = \frac{4}{3} + \frac{G}{2} ) for $G &lt; G_{ES}$</td>
</tr>
<tr>
<td>3C</td>
<td>( \frac{V_B d_c}{2 M_{pm}} )</td>
<td>(4 + \frac{1}{2} G) ( \frac{d_c/L}{1 - d_c/L} )</td>
</tr>
<tr>
<td>4C</td>
<td>( \frac{M_jB}{M_{pm}} )</td>
<td>(Row 2C) + (Row 3C)</td>
</tr>
<tr>
<td>5C</td>
<td>( M_jB )</td>
<td>(Row 4C) ( M_{pm} )</td>
</tr>
<tr>
<td>6C</td>
<td>( M_jA )</td>
<td>( M_G - M_jB )</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Gravity Load</strong></td>
</tr>
<tr>
<td>1G</td>
<td>( \frac{FEM}{M_{pm}} )</td>
<td>( \frac{4}{3} ) ( \frac{F_{1R}}{F_{2R}} )</td>
</tr>
<tr>
<td>2G</td>
<td>( \frac{M_B}{M_{pm}} )</td>
<td>(1) ( \frac{M_B}{M_{pm}} = R ) or ( \frac{R_{LB}}{M_{pm}} ) for $R &lt; \frac{FEM}{M_{pm}}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(Note 1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2) ( \frac{M_B}{M_{pm}} = \frac{FEM}{M_{pm}} ) for $R &gt; \frac{FEM}{M_{pm}}$</td>
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<tr>
<td>3G</td>
<td>( \frac{V_B d_c}{2 M_{pm}} )</td>
<td>(4 ) ( \frac{F_{1R}}{F_{2R}} ) ( \frac{d_c/L}{1 - d_c/L} )</td>
</tr>
<tr>
<td>4G</td>
<td>( \frac{M_jB}{M_{pm}} )</td>
<td>(Row 2G) + (Row 3G)</td>
</tr>
<tr>
<td>5G</td>
<td>( M_jB = -M_jA )</td>
<td>(Row 4G) ( M_{pm} )</td>
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**Note 1:** Use the larger value
### TABLE A8

**GIRDER PLASTIC MOMENT BALANCE - JOINT MOMENTS**

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</tr>
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<td>1</td>
<td>$M^{\prime}_{pm}$</td>
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<td>0.083</td>
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<td>$M^{G}_{C}$</td>
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<td>142</td>
<td>50</td>
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<td>$C_{1}$</td>
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<td>1.0</td>
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<td>$C$</td>
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<td>1.0</td>
<td>1.0</td>
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<td>$G$</td>
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<td>1.26</td>
</tr>
<tr>
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<td>$R^{LB}$</td>
<td></td>
<td>1.31</td>
<td>1.31</td>
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<tr>
<td>9</td>
<td>$M^{p}_{P}$</td>
<td>k-ft</td>
<td>167</td>
<td>60.8</td>
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<td>in.$^{3}$</td>
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**Combined Load**

**Joint Moments - Wind Left**

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<td>$G^{ES}_{G}$</td>
<td>(-) (-) (-)</td>
</tr>
<tr>
<td>2C</td>
<td>$M^{B/M}_{pm}$</td>
<td>1.31 1.31 1.31</td>
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<tr>
<td>3C</td>
<td>$V^{B}<em>{B} d</em>{c/2} M^{pm}_{pm}$</td>
<td>0.24 0.41 0.17</td>
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<tr>
<td>4C</td>
<td>$M^{JB/M}_{pm}$</td>
<td>1.55 1.72 1.48</td>
</tr>
<tr>
<td>5C</td>
<td>$M^{J}_{J}$</td>
<td>198 80 355</td>
</tr>
<tr>
<td>6C</td>
<td>$M^{J}_{A}$</td>
<td>-56 -30 -89</td>
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</tbody>
</table>

**Gravity Load**

**Joint Moments**

<p>| | | |</p>
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</tr>
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<tbody>
<tr>
<td>1G</td>
<td>$F^{E/M}_{pm}$</td>
<td>1.75 1.75 1.75</td>
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<td>2G</td>
<td>$M^{B/M}_{pm}$</td>
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<tr>
<td>3G</td>
<td>$V^{B}<em>{B} d</em>{c/2} M^{pm}_{pm}$</td>
<td>0.28 0.47 0.20</td>
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<td>4G</td>
<td>$M^{JB/M}_{pm}$</td>
<td>1.59 1.78 1.51</td>
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<td>5G</td>
<td>$M^{J}<em>{J} = -M^{J}</em>{A}$</td>
<td>204 83 362</td>
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**TABLE A9**

**GIRDER PLASTIC MOMENT BALANCE - JOINT MOMENTS**

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<td>Unit</td>
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<tr>
<td>1</td>
<td>$M_{pm}$</td>
<td>k-ft</td>
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<tr>
<td>2</td>
<td>$d_c/L$</td>
<td></td>
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<tr>
<td>3</td>
<td>$M_G$</td>
<td>k-ft</td>
</tr>
<tr>
<td>4</td>
<td>$C_1$</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>$C$</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>$G$</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>$R$</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>$R_{LB}$</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>$M_p$</td>
<td>k-ft</td>
</tr>
<tr>
<td>10</td>
<td>$Z$</td>
<td>in.³</td>
</tr>
<tr>
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<td>Section</td>
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</table>

**Combined Load - Joint Moments - Wind Left**

<table>
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<tr>
<th>Row</th>
<th>Item</th>
<th>Unit</th>
<th>Bay 1</th>
<th>Bay 2</th>
<th>Bay 3</th>
<th>Bay 1</th>
<th>Bay 2</th>
<th>Bay 3</th>
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<tbody>
<tr>
<td>1C</td>
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<td>3.64</td>
<td>13.64</td>
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<td>8.15</td>
<td>1.72</td>
<td>3.89</td>
<td>10.26</td>
<td>2.19</td>
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<tr>
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<td>$V_B d_c/2 M_{pm}$</td>
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<td>0.36</td>
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<td>0.19</td>
<td>0.39</td>
<td>1.20</td>
<td>0.21</td>
</tr>
<tr>
<td>4C</td>
<td>$M_{JB'/pm}$</td>
<td></td>
<td>3.51</td>
<td>9.21</td>
<td>1.91</td>
<td>4.28</td>
<td>11.46</td>
<td>2.40</td>
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<td>5C</td>
<td>$M_{JB}$</td>
<td>k-ft</td>
<td>449</td>
<td>429</td>
<td>458</td>
<td>547</td>
<td>533</td>
<td>576</td>
</tr>
<tr>
<td>6C</td>
<td>$M_{JA}$</td>
<td>k-ft</td>
<td>336</td>
<td>356</td>
<td>94</td>
<td>390</td>
<td>404</td>
<td>227</td>
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</table>

**Gravity Load - Joint Moments**

<table>
<thead>
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<th>Row</th>
<th>Item</th>
<th>Unit</th>
<th>Bay 1</th>
<th>Bay 2</th>
<th>Bay 3</th>
<th>Bay 1</th>
<th>Bay 2</th>
<th>Bay 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1G</td>
<td>$FEM/M_{pm}$</td>
<td></td>
<td>1.75</td>
<td>1.75</td>
<td>1.75</td>
<td>1.75</td>
<td>1.75</td>
<td>1.75</td>
</tr>
<tr>
<td>2G</td>
<td>$M_B/M_{pm}$</td>
<td></td>
<td>1.75</td>
<td>1.75</td>
<td>1.72</td>
<td>1.75</td>
<td>1.75</td>
<td>1.75</td>
</tr>
<tr>
<td>3G</td>
<td>$V_B d_c/2 M_{pm}$</td>
<td>k-ft</td>
<td>0.28</td>
<td>0.47</td>
<td>0.20</td>
<td>0.28</td>
<td>0.47</td>
<td>0.20</td>
</tr>
<tr>
<td>4G</td>
<td>$M_{JB'/pm}$</td>
<td>k-ft</td>
<td>2.03</td>
<td>2.22</td>
<td>1.92</td>
<td>2.03</td>
<td>2.22</td>
<td>1.95</td>
</tr>
<tr>
<td>5G</td>
<td>$M_{JB} - M_{JA}$</td>
<td>k-ft</td>
<td>260</td>
<td>103</td>
<td>460</td>
<td>260</td>
<td>103</td>
<td>467</td>
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</table>
### TABLE A10

**STEPS IN JOINT BALANCE - METHOD I**

<table>
<thead>
<tr>
<th>Steps 1 to 3</th>
<th>Assign $D_u$, $D_l$</th>
</tr>
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<tbody>
<tr>
<td>(1) 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25</td>
<td>$-1304$</td>
</tr>
<tr>
<td>(2)</td>
<td>+547 +533 +576</td>
</tr>
<tr>
<td>(3)</td>
<td>+390 +404 +227</td>
</tr>
<tr>
<td>(4)</td>
<td>-343 -343 -343 -343 -343 -343</td>
</tr>
<tr>
<td>(5)</td>
<td>+279 -282 -91</td>
</tr>
<tr>
<td>(6)</td>
<td>+390-343-326</td>
</tr>
</tbody>
</table>

$M_{jE} = -(+390-343-326)$

$M_{jU} = 0.25(-1373)$

### Steps 4 to 6

| (1) 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 | $-1304$ |
| (2) | -186 +547 -467 +533 -371 +576 -280 |
| (3) | -204 +390 -484 +404 -389 +227 -296 |
| (4) | -343 -326 -343 -326 -343 -326 -343 -326 |
| (5) | +139 +140 -141 -141 -46 -45 +47 +46 |
| (6) | +279 0.5 -282 0.5 -91 0.5 +93 0.5 |

$M_{jL} = 0.5(+279)$

$M_{jU} = -343 +47$
**TABLE A11**

**JOINT BALANCE - METHOD I - LEVEL 20**

### Level 20 - Wind left

<table>
<thead>
<tr>
<th></th>
<th>M(j_B)</th>
<th>M(j_L)</th>
<th>M(j_B)</th>
<th>M(j_L)</th>
<th>M(j_B)</th>
<th>M(j_L)</th>
<th>M(j_B)</th>
<th>M(j_L)</th>
<th>M(j_B)</th>
<th>M(j_L)</th>
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<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>2</td>
<td>-186</td>
<td>+547</td>
<td>-467</td>
<td>+533</td>
<td>-371</td>
<td>+576</td>
<td>-280</td>
<td>+390</td>
<td>-484</td>
<td>+404</td>
<td>-389</td>
<td>+227</td>
</tr>
<tr>
<td>3</td>
<td>-204</td>
<td>+390</td>
<td>-484</td>
<td>+404</td>
<td>-389</td>
<td>+227</td>
<td>-296</td>
<td>+390</td>
<td>-484</td>
<td>+404</td>
<td>-389</td>
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<td>-141</td>
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<td>+46</td>
<td>+139</td>
<td>+140</td>
<td>-141</td>
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<tr>
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<td>-282</td>
<td>0.5</td>
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<td>+93</td>
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<td>+279</td>
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### Level 20 - Wind right

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<th>M(j_L)</th>
<th>M(j_B)</th>
<th>M(j_L)</th>
<th>M(j_B)</th>
<th>M(j_L)</th>
<th>M(j_B)</th>
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</tr>
<tr>
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<td>+453</td>
<td>-404</td>
<td>+481</td>
<td>-227</td>
<td>+105</td>
<td>-2677</td>
<td>+1304</td>
<td>+260</td>
<td>-78</td>
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### Level 20 - Gravity load

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<th>M(j_B)</th>
<th>M(j_L)</th>
<th>M(j_B)</th>
<th>M(j_L)</th>
<th>M(j_B)</th>
<th>M(j_L)</th>
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<td>2</td>
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## Table A12

**Joint Balance - Method IV - Level 20**

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<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>( \Sigma M_{jL} )</th>
<th>( \Sigma M_{jU} )</th>
<th>( \Sigma M_G )</th>
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<tr>
<td>(2)</td>
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<td>+547</td>
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<td>-200</td>
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<td>-488</td>
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<td>-389</td>
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<td>(6)</td>
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**Level 20 - Wind left**

\( D_j = -0.513 \)

**Level 20 - Wind right**

\( D_j = -0.513 \)

**Level 20 - Gravity load**

\( D_j = 0.5 \)
TABLE A13

CONSTANT DEPTH GIRDER - FRAME C

Eq. (8.15) Use $d_c/L = 0.05$  $C_{(J,I)} = 1.0$

$$
\begin{align*}
\left\{ \begin{array}{l}
\Sigma M_{pm} = 414.9 \text{ k-ft} \\
\Sigma \sqrt{M_{pm}} = 33.6
\end{array} \right.  \frac{\Sigma M_{pm}}{\Sigma \sqrt{M_{pm}}} = 12.3
\right\} \text{ Table A5}
\end{align*}
$$

$$
M_{p(I)} = (0.00353 \Sigma M_G(I) + 12.3)^2 
$$

for $G < 8$

<table>
<thead>
<tr>
<th>Level</th>
<th>$\Sigma M_G$ (k-ft)</th>
<th>$0.00353 \times (1)$</th>
<th>$(2) + 12.3$</th>
<th>$(3) \times (3)$</th>
<th>$M_{p(I)}$ (k-ft)</th>
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<td>4</td>
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<td>5</td>
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<td>14.9</td>
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<td>873</td>
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<td>15.4</td>
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<td>16.0</td>
<td>256</td>
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<td>1151</td>
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<td>16.5</td>
<td>272</td>
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<td>10</td>
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<tr>
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<td>1706</td>
<td>6.2</td>
<td>18.5</td>
<td>342</td>
<td>21W55</td>
<td></td>
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<tr>
<td>14</td>
<td>1845</td>
<td>6.7</td>
<td>19.0</td>
<td>361</td>
<td>21W55</td>
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<tr>
<td>15</td>
<td>1984</td>
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<td>19.5</td>
<td>380</td>
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<td>16</td>
<td>2122</td>
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Fig. A1.1 Girder moments for combined load
Preliminary design - Frame C at Level 20
Fig. A1.2 Column moments for combined load
Preliminary design - Frame C at Level 20
APPENDIX II

This appendix includes two examples which deal with the approximations concerning joint rotation in Chapter 4. The first example considers the influence of joint rotation on joint equilibrium in conjunction with Art. 4.1. The second example illustrates how the joint rotation assumption in Eq. (c) of Art. 4.2 affects column end-moments.

Example 1: Influence of Joint Rotation on Joint Equilibrium

Figure A2.1 gives data extracted from the example in Ref. 10. This example investigates the load versus sway-deflection behavior at the twentieth level below the roof of a 24 story, 3 bay unbraced frame, designed by the plastic method. Of interest here is the potential moment \( P \theta_j d_b / 2 \) about the center of the joint due to rotation of joint C at level 20. Data for column C below level 20 is summarized in Fig. A2.1. This column carries the largest axial load in the story below level 20 (for wind from left to right) and has a large \( P/y \) ratio (0.85). Two values of the sway deflection index in the story are also included in Fig. A2.1. It is not difficult to show that the rotation of joint C must be less than the sway-deflection index in the story below. The moment \( P \theta_j d_b / 2 \) on joint C due to rotation of this joint may be conservatively estimated by taking \( \theta_j \approx \Delta / h \). (At ultimate load \( \theta_j = 0.54 \ (\Delta / h) \))
The moment $PQ_j d_b/2$ due to rotation of joint C is less than 15.5 kip-ft. when the story develops its maximum resistance to sway at $\Delta/h = 0.005$. This moment represents 4 percent of the reduced plastic moment capacity of column C below level 20. A smaller moment $PQ_j d_b/2$ is applied to joint C from the column above level 20. If we consider the extreme situation which occurs when the story reaches the plastic mechanism condition at $\Delta/h = 0.009$, the moment contribution due to rotation of joint C does not exceed 28 kip-ft., or 7.3 percent of the reduced plastic moment capacity of column C below level 20.

Conclusion:

This example illustrates the fact that the moment $PQ_j d_b/2$ due to joint rotation in Eq. 4.1 is rarely significant when compared with the moment capacity of the columns. An example involving deep girders, relative to the story height ($d_b/h = 1/6$) and large axial load relative to the plastic load $P_y$, has purposely been selected to give a conservative estimate of the moment $PQ_j d_b/2$ due to joint rotation. However, it should be mentioned that if unusually large joint rotations are possible (say $\Theta_j = 0.03$), the moment due to joint rotation may reach values on the order of 10 percent or more of the column moment capacity. It is difficult to concoct examples for multi-story frames of practical proportions to illustrate this point because such large joint rotations simply do not occur until long after the ultimate load capacity of a frame is exhausted.
Example 2: Influence of Joint Rotation on Column End-Moments

Suppose we are given the data:

\[ M_{jU} = M_{jL} = P\Delta = M \]
\[ Q_{jU} = Q_{jL} = 1.5 \Delta/h \]
\[ d_{bU} = d_{bL} = h/8 \]

for a column bent in double curvature with large joint rotations relative to \( \Delta/h \). Note that this data violates the joint rotation assumption in Eq. (c) of Art. 4.2. We want to compare the end-moments obtained using Eqs. (b) and (4.5) in Art. 4.2. The shear force \( Q = -3M/h \) and the shear couple \( Q d_b/2 = -(3/16)M \) while the Py contribution is

\[ Py = P \left( 1.5\Delta/h \right) (d_b/2) = (3/32)M \]

Then Eq. (b) gives \( M_L = (29/32)M \) and Eq. (4.5) yields \( (28/32)M \) for \( M_L \). These answers differ by an insignificant 3.5 percent. If the joint rotations are decreased to \( \Theta_{jU} = \Theta_{jL} = 0.5 \Delta/h \) we get \( M_L = (27/32)M \) from Eq. (b) and no change in \( M_L \) from Eq. (4.5). The resulting conservative error in \( M_L \) from Eq. (4.5) is then a trivial 3.7 percent.

We conclude that Eq. (4.5) gives reasonable estimates of the column end-moments, except possibly for columns in single curvature bending with large joint rotations.
Loading condition: Combined Load - wind left

Data for Column C - below level 20

Axial load $P = 3096$ kips $P/Py = 0.85$ $h/r_x = 21$

Reduced plastic moment $M_{pc} = 385$ kip-ft.

Sway Deflection - below level 20

1. At ultimate shear in story, $\Delta/h = 0.005$
2. At plastic mechanism in story, $\Delta/h = 0.009$

Moment contribution - due to rotation of joint C at level 20

Assume joint rotation $Q_j = \Delta/h$

1. $\frac{1}{2} P \theta d_b = 3096 \times 0.005 \times 24/(2 \times 12) = 15.5$ kip-ft. $(0.040 M_{pc})$
2. $\frac{1}{2} P \theta d_b = 3096 \times 0.009 \times 24/(2 \times 12) = 28$ kip-ft. $(0.073 M_{pc})$

Fig. A2.1 Influence of joint rotation on joint equilibrium
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