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Talk given in Houston, Texas.
February 20, 1967.
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PLASTIC DESIGN OF MEMBERS IN BRACED MULTI-STORY FRAMES

by

V. Levi
G. C. Driscoll

INTRODUCTION

Research is being conducted at Lehigh University on the ultimate strength of braced multi-story frames.

We will describe some concepts of the behavior of parts of multi-story frames. From this behavior, realistic boundary conditions may be assumed to aid in applying recently developed theories of structural behavior to the analysis and design of multi-story frames. Some of the resulting steps in design will be discussed in order to show how we visualize possible applications to design.

We will show that in designing for ultimate strength some economy of weight can be achieved.

SLIDE 1

Here we have one of the frames of a tier building; in this case, the frame has 6 stories and 4 bays.

This bracing, however, could be staggered in any convenient manner. (We need not brace the same bay at each story level; or even a bay in the same frame.)
The frame can be loaded by vertical loading, which can vary from bay to bay and story to story. If the structure can reach an ultimate load of 1.85 times the working load its performance will be considered satisfactory.

The frame can also be loaded by a combination of vertical loading (w) and horizontal loading (H). An ultimate set of loads 1.40 times the combined vertical and horizontal loads will be considered adequate.

The frame is to be proportioned so that the girders have the smallest possible cross sections.

This is achieved by designing each beam so that it will form a mechanism upon application of dead load plus live load times a factor of 1.85.

This occurs when hinges form at both ends of the beam and near its center.

It is assumed that the effect of compressive loading and the reduction in yield stress due to shear are negligible.

Mechanisms will form in the girders, if the columns can be expected to behave themselves under all likely combinations of loading.
The formation of plastic hinges tends to isolate the columns. Any pair of unbalanced plastic moments at a joint tends to act on the continuous column as an external moment.

**SLIDE 4**

When all girders adjacent to a particular continuous column form mechanisms, there will therefore result external moments at the various joints.

These moments will be equal to the difference between the plastic moments of the adjacent beams.

These plastic moments can be different due to unequal spans and/or unequal loadings.

**SLIDE 5**

Such a continuous column is shown in this slide. It is subjected to axial loads and external moments at each support. The vertical loads in this column will be calculated from the reactions of the girders.

This column must be analyzed for stability.

**SLIDE 6**

If the magnitude of any one of the external moments $M$ is plotted against any one of the joint rotations and all other external moments are kept constant, a curve such as this one results from a theory derived on the basis of a recently published paper on the strength of restrained columns.

(When signs of moments are such that single curvature is predominant) If not
From this curve one can determine the maximum unbalanced moment at the one joint which can be resisted in conjunction with the other external joint moments.

The corresponding joint rotation is one which will occur if there has been no inelastic strain reversal in the column during the loading process.

If this maximum moment is equal to or greater than the one which acts on the structure, the column is stable.

**SLIDE 7**

In many instances partial loading in which a portion of the possible live load is absent tends to produce the most critical bending condition for a column segment. For interior columns the critical partial loading is brought about by "checker board" loading in the vicinity of each column. Two of the girders framing into the column will form mechanisms and the other two will remain elastic under only dead load times a factor of 1.85. "Checker board" loading brings about single curvature bending which is the most critical for stability.

Although "checker board" loading results in external joint moments larger than those resulting from full loading, it also provides more restraints in the form of the beams which remain elastic. The stability of a column segment subjected to bending about one of its principal axes _**CANNOT BE INVESTIGATED**_ unless the full measure of restraint of the rest of the structure is taken into consideration.
In order to simplify the analysis, the column segment is assumed to be restrained only by members immediately adjacent to it.

The effect of the rest of the structure is accounted for as an assumption on the end conditions of the restraining members.

These assumptions can usually be made conservatively.

The analysis for excessive bending is now the analysis of a sub-assemblage consisting of the members framing into two joints.

The assumptions of end conditions for the restraining members are made by specifying a ratio of end moments for each member.

If a plot of one of the external moments versus one of the end rotations is made with the other external joint moment remaining constant, the maximum joint external moment for which stable equilibrium is possible can be determined.

It is FUNDAMENTAL TO KNOW that this maximum joint moment is usually different from the one that would correspond to the maximum end moment of the column segment. This indicates the inadequacy of basing design on the maximum carrying capacity of a pin ended column.

The problems discussed up to this time are realistic ONLY IF the structure as a whole is loaded so that it does not sway.

THIS IS USUALLY NOT THE CASE.
SLIDE 10
If the "checker-board" loading in a frame is accompanied by a preponderance of loading on one side of the frame there will be sway.

SLIDE 11
The sub-assemblage will be similar to the no sway one.

The top of the column segment will move a distance $\Delta$ (laterally) with respect to the bottom.

Two additional assumptions are:
1. The sub-assemblage is not helped in its resistance to sway by the rest of the story.
2. The story below the one in question does not sway.

These two assumptions tend to counteract each other, as the one is conservative while the second is unconservatieve.

In such a case the column deflects ($\Delta$) until shear equilibrium has been achieved.

Since there are no external horizontal forces on the sub-assemblage the moments at the ends of the column segment will differ by $P\Delta$. The deflection $\Delta$ is of interest only insofar as it is used as part of the process for determining ultimate strength.
The column segment's freedom to sway forces into a more critical bending configuration.

**SLIDE 12**

A plot of \( \frac{\Delta}{L} \) versus \( M \) yields a curve from which the maximum external moment at one joint consistent with stable equilibrium can be determined.

The point at which the curve crosses the ordinate is the one corresponding to the external moments which will result in no sway. (i.e. the column end moments will be equal.)

**SLIDE 13**

If there is to be bracing at a given story level, it is possible to design it such that for any given deflection \( \Delta \) the bracing will react with a shear equal to the column compressive force \( P \) times \( \Delta \).

\( P \) is the compressive force which will act on the column segment at the end of the loading process.

**SLIDE 14**

Since there are no external horizontal forces acting on the system, \( M_A = M_B \).

Sway will cause one restraint to rotate to a greater angle and the other restraint to a smaller angle. (One works harder and one works less). The result of this is the achievement of swayed position of equilibrium where the column is of symmetric single curvature bending.
The outer columns are necessarily bent in double curvature.

Therefore, partial loading tends to force them into less of an antisymmetric bending configuration.

A conservative approximation would be to assume that one end of each interior column is pinned.

For the slenderness ratios which occur in most multi-story frames, plastic hinges will tend to form at the column ends, at failure.

When a frame is loaded symmetrically until its members (columns & girders) are bent into the inelastic range, there is the possibility that one or more of the bays will trigger buckling of the entire frame.

Through plastification, the stiffnesses of the columns and beams in the frame have been reduced.

At a certain point in the loading one or more stories can find a position of equilibrium in a swayed position.

The loading which brings this about is referred to as the buckling (bifurcation) load.

This is the problem which has just been described in more detail by Prof. Lu, for small frames.
It is possible to brace the frame against this occurrence.

The only way in which an adjacent swayed equilibrium position is possible is if the change in column shears due to sway divided by the story height is equal to the summation of \( \frac{P \Delta}{L} \) of all the column segments in the story.

If bracing is inserted at each story level which for any \( \Delta \) will react with a shear force equal to the summation of \( \frac{P \Delta}{L} \), then it will be impossible for the frame to buckle.

Additional bracing may be necessary to prevent failure due to combined vertical load and wind.

The analysis of such a problem for a multi-story frame is unsolved but is under study.

As before, it is possible to find a meaningful portion of the structure, which with conservative end conditions, can be analyzed by presently available theory.

This will give an upper bound on the bracing area needed.

A conservative assumption would be that each bay, treated as a pin-ended multi-bay frame, must resist a lateral force equal to the resultant story shear.
No restraint is afforded by the columns above or below.

Axial loads are applied at the column tops.

Since ours is a weak beam design, hinges can be expected to occur at the leeward ends of each girder.

**SLIDE 20**

A limiting case of this problem will occur when the combination of loading is such that hinges form near the center of each girder prior to instability.

This will then be equivalent to a simple plastic theory mechanism.

If the resulting shear resisted, either by the mechanism or by the frame at its limit of stability, is less than the applied shear; bracing which can resist the difference in shears must be added.

The analyses of the various problems described are essential to the understanding and rational design of braced multi-story frames.

However, an adequate design procedure can be obtained by the solution of simplifications of the problems described.

Cases where these simplified problems digress too far from reality can then be analyzed more precisely by posing as realistic a problem as possible.

We continue with a proposed design procedure.
SLIDE 21

All beams are designed as fixed ended for their ultimate load

\[ M_p = \frac{W_{D+L}^2}{16} \]

SLIDE 22

Bracing is designed so that

\[ A_{br} \geq \frac{1}{E} \frac{l^3}{hL^2} \sum P \]

is the minimum bracing area in tension.

If the bracing can resist compression

\[ \frac{L}{R} \leq \frac{L_c}{R_c} \text{crit.} \]

This amount of bracing only insures that the secondary moments \( P\Delta \) will not be a factor in the ultimate strength of members.

SLIDE 23

This slide represents the subassemblage for which interior columns will be designed. This ensures the overall stability of the column as part of a subassemblage of a braced frame.

SLIDE 24

Exterior columns will be designed to form a hinge at one end such that

\[ M_{pb} \leq M_{pc} \]
SLIDE 25

The end rotations of all columns (restraints too, must be limited so that

\[ \theta \leq \theta_{cr} \]

SLIDE 26

The slenderness ratio of a column must never be greater than

\[ \frac{h}{r} \leq \left( \frac{h}{r} \right)_{cr} \]

This insures a lowerbound on buckling about the weak axis.

SLIDE 27

Bracing may be checked for adequacy against wind by the analysis of each story as a multi-bay pin ended frame.

Due to the lack of an adequate lateral torsional buckling theory for restrained columns, it is assumed that this type of instability is either non-occurant or that adequate lateral bracing is present.

SLIDE 28

A comparative design of a ten-story five-bay braced frame shows that a design in which:

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In SUMMARY

1. We have given descriptions of physical (Structural mechanics) problems involved in the analysis and design of braced multi-story frames.

2. We have shown the results of some ultimate strength analyses of particular examples.

3. We have proposed in sketch a design procedure for the design of members in braced multi-story frames.

4. We have shown that in designing members for their ultimate strength as part of subassemblies we were able to achieve some economy in weight, for a particular frame.
**Interior Columns**

\[ \frac{1}{r} \leq \left( \frac{1}{r} \right)_{\text{crit}} \]

**Check Weak Axis Buckling**

**Exterior Columns**

\[ M_p \leq M_{pc} \]

**Check Wind**

**Check Local Buckling**

\[ \theta_e \leq \theta_{pc} \]
\[ \theta_{GA} = \theta_{BC} \]

\[ M_{BA} < M_{BC} \quad \text{STABLE} \]

\[ M_{BA} > M_{BC} \quad \text{FAILURE} \]

\[ M_{BA} = M_{BC} \quad \text{BUCKLING} \]
Frame Subjected to Wind

\((V + \frac{\delta}{L} P)\) \((P - \frac{\delta}{L} V) = P(1 - \frac{\delta}{L} V)\)

\[ M_B = M_A + P_S + VL \]

Column Subjected to Wind
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