Plastic Design of Multi-Story Frames

PLASTIC DESIGN OF MULTI-STORY FRAMES
--BRACED FRAMES

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ABSTRACT

This paper presents a procedure for the plastic design of braced multi-story frames. It is intended to be used as an office guide in design. For this reason, detailed explanations, derivations, and experimental verifications of design equations and concepts are not included. They can be found in the references listed at the end of the paper.

Only rigid frames, diagonally braced in the major plane are covered. Floor systems are assumed to be simply supported and perpendicular to the frame. Design procedures are included for high-strength steel members and composite steel and concrete beams. Design examples are given to aid the designer in understanding the procedure. Calculations are done in a tabular form to suggest a possible means of organization for manual computations.
1. INTRODUCTION

This report presents a method for the plastic design of braced, rigidly jointed, multi-story frames. The method is in a form suitable for design office application, but it is recommended that the reports "Plastic Design of Multi-Story Frames--Lecture Notes and Design Aids" be used along with this report when performing actual design. The frames considered are diagonally braced in one bay. Other types and arrangements of bracing are permissible but are not considered. Members in the plane of the frame are assumed to conform to AISC Type 1 construction. Out-of-plane members (floor and roof beams) are assumed to be simply supported and perpendicular to the plane of the frame.

Design procedures are presented for all beams, columns, and diagonal bracing in the plane of the frame. Requirements are given for the design of floor system members which must provide lateral bracing for the beams. Out-of-plane bracing for the columns is considered. The effect of cladding is not considered. Connections are discussed and the drift of the frame is checked. Two types of steel are considered in the design procedure as presented: ASTM A36 and ASTM A441. Other steels can be used when their adequacy in the plastic range is established. Examples at the end of each step of the procedure are given to aid in the understanding of the design method and to suggest a tabular form for organizing manual computations.

*Superscripts are used to denote reference numbers.*
Some of the recommendations presented in this report differ with the limitations of Part II of the 1963 AISC Specification. That specification properly limited the application of plastic design to one- and two-story buildings of ASTM A7 and A36 structural steels. A maximum limit was placed on column loads simply because information was lacking on the behavior of columns loaded above that limit. Similarly the use of plastic design in multi-story frames was limited to members in the floor systems of frames where sidesway is prevented by means of a bracing system other than the beams and columns.

The results of research have removed many of the uncertainties which necessitated the foregoing limitations. In preparing this report the authors have made recommendations which will lead to the design of multi-story building frames having safety factors consistent with established practice. The proper application of plastic design principles can enable the designer to effect savings through the elimination of material which contributes unnecessary strength in parts of the structure where this excess strength can be effectively used.

Where the recommendations of this paper exceed the limits stated in the AISC Specification, it will require a change in the Specification to make these recommendations fully acceptable in areas where building codes demand adherence to the Specification. Wherever possible, differences from the current AISC Specification will be noted in this paper.
The American Institute of Steel Construction is studying changes to its Specification based on the research cited. While some changes in numerical values can be anticipated, the final concepts of their revised specification should prove to be consistent with the concepts presented here. Meanwhile, the designer must remain constantly aware of the variances from the Specification and either conform with it or obtain permission from the proper regulatory body to use the recommendations as presented here.

The design of braced multi-story frames can be divided into a series of consecutive steps. In the order of discussion, these steps are: preparation of design data, preliminary design of members, final design of members, deflection and drift checks and connection design.
2. PREPARATION OF DESIGN DATA

The preparatory steps of multi-story frame design are:
dimensional layout, member identification, assignment and distribu­tion of the load systems, application of load factors, and the tabulation of preliminary design data.

Dimensional layout is based on the functional requirements of providing space and services for the occupants of the building. Architectural considerations and the economy of the total building installation will control.

For convenience in design, members should be identified by some systematic notation. The procedure used in this report is to assign numbers to the roof, floor levels and the ground level starting with the number "one" at the roof. Since the design method works from the top to the bottom of the frame, the numbering system starts from the top and proceeds downward. The word "level" is used to mean the floor. Thus level 5 is the fifth floor from the top counting the roof as level one. Columns are designated by letters, starting with A on the left. Thus the left column in the top story would be designated A1-A2.

The load systems can usually be assigned from a regional building code which will define the minimum loads. In the absence of a regional building code, one of the available recommended uniform building codes can be used. The assignment of loads is at all times at the discretion of the engineer--the codes only
suggest minimums. In this paper excerpts from the American Standard Building Code Requirements for Minimum Design Loads in Buildings and other structures (A58.1-1955) sponsored by the National Bureau of Standards are used. After the selection of load systems for the total structure, the designer must assign an appropriate portion of the load to each plane frame of the structure and to each member. In principle this should be done for each combination of loads which could be applied separately. For a preliminary design of an indeterminate frame, little or no information about member sizes is available immediately so an approximate distribution of the loads is most practical. A good rule of thumb to follow is that loads should be distributed so that a path is provided for all loads to be transmitted to the ground. A recommended manner of assuming the distribution of loads systems is as follows:

1. Floor dead and live loads are distributed through the floor system to each bent as uniformly distributed loads or as concentrated loads coming from simply supported floor members.

2. Exterior wall dead loads are distributed to each bent as concentrated loads on the bent at connection points of spandrel beams.

3. Lateral loads from wind and/or earthquake are distributed through each bent as concentrated loads at the level of the girders.
4. Lateral loads applied at certain unbraced bents may be distributed to sway-resisting bents through the diaphragm action of the floor system. The lateral loads assigned to the unbraced bents are added to the lateral loads on the sway-resisting bents at the same level as concentrated loads.

Some caution must be exercised in assuming loads to be distributed to the frame arbitrarily in cases where stiffnesses vary considerably between similar members. For instance, buildings with some unbraced and some sway-resisting bents might share the loads in a manner quite different from the uniform distribution assumed if the stiffnesses of the bent are arranged unsymmetrically.

Following the assignment and determination of the distribution of load systems, the first step which makes the design a plastic design is the multiplication of the working loads by the appropriate load factors. The factored loads represent a computed ultimate load for design purposes. The strength of the actual structure should be sufficient so that failure would not occur at a lower load if the actual working loads were increased proportionally to the ultimate level. Application of load factors is done after any adjustments are made in the working loads for live load reduction or partial loading. For gravity type loading, the factor used in this paper is 1.70 and for combination gravity and lateral loads the factor used is 1.30. These load factors are somewhat lower than those given in Part II of the AISC Specification, in recognition of improved knowledge of the strength of
beams and columns gained since the original issuance of that part of the Specification. The gravity load factor is essentially in agreement with the factor of safety included in Part I of the AISC Specification. The combined loading case load factor is three-fourths of the gravity loading case load factor rounded off to 1.30.

The preparation of design data so far discussed can be done in tabular form. This is shown in Example 1. This example consists of:

1) A presentation of the dimensions and working loads per unit area for the frame.
2) Conversion of the loads per unit area into loads per foot on girders considering a typical form of live load reduction.
3) Conversion of the loads per unit area into concentrated loads at each level of each column also considering a typical form of live load reduction.
4) Conversion of wind loads per unit area into concentrated loads at the joints.

The final loads are presented in three forms:

1) Working loads,
2) Factored load of 1.70 times working load for use in designing for gravity load alone,
3) Factored load of 1.30 times working load for use in designing for gravity plus wind load.
Example 1

Calculate and tabulate loads on girders and columns for design of frame

<table>
<thead>
<tr>
<th>Level 1</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>30' 24' 24'</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Bent Spacing = 24 ft

Working Loads:
- Roof: \( w_L = 30 \text{ psf} \)
- Floor: \( w_L = 80 \text{ psf} \)
- Exterior Wall: \( w_D = 45 \text{ psf} \)
- Wind: \( 20 \text{ psf} \)

1) Girder Loads

**Roof Girders (Level I)**

<table>
<thead>
<tr>
<th>L. L.</th>
<th>0.030 ksf ( \times 24 \text{ ft} )</th>
<th>= 0.72 k/ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>D. L.</td>
<td>0.060 ( \times 24 )</td>
<td>= 1.44</td>
</tr>
</tbody>
</table>

Total load (Working) = 2.16 k/ft
Floor Girders (Level 2 to 10)
(Percent Live Load Reduction by ASA A58.1)

<table>
<thead>
<tr>
<th>Formula</th>
<th>Bay</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AB</td>
</tr>
<tr>
<td>Floor Area Served</td>
<td>24 x Span</td>
</tr>
<tr>
<td>% L.L. Reduction</td>
<td>0.08 x Area</td>
</tr>
<tr>
<td>Max. L.L. Reduction</td>
<td>(\frac{100(D+L)}{4.33L})</td>
</tr>
</tbody>
</table>

L.L. \(0.539 \times 0.080 \text{ ksf} \times 24 \text{ ft} = 1.03 \text{ k/ft}\)

D.L. \(0.080 \text{ ksf} \times 24 \text{ ft} = 1.92 \text{ k/ft}\)

Total Load (Working) = 2.95 k/ft

Table 1
Uniformly Distributed Loads on Girders of Frame

<table>
<thead>
<tr>
<th>Working Load</th>
<th>Factored Load</th>
<th>Factored Load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.70 x Working</td>
<td>1.30 x Working</td>
</tr>
<tr>
<td></td>
<td>Gravity Load Case</td>
<td>Gravity + Wind Load Case</td>
</tr>
<tr>
<td>w (k/ft)</td>
<td>w (k/ft)</td>
<td>w (k/ft)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Roof Girders</th>
<th>Working Load</th>
<th>Factored Load</th>
<th>Factored Load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.44</td>
<td>2.45</td>
<td>1.87</td>
</tr>
<tr>
<td>Live</td>
<td>0.72</td>
<td>1.22</td>
<td>0.94</td>
</tr>
<tr>
<td>Dead</td>
<td>2.16</td>
<td>3.67</td>
<td>2.81</td>
</tr>
<tr>
<td>Total</td>
<td>2.95</td>
<td>5.02</td>
<td>3.85</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Floor Girders</th>
<th>Working Load</th>
<th>Factored Load</th>
<th>Factored Load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.92</td>
<td>3.26</td>
<td>2.50</td>
</tr>
<tr>
<td>Live</td>
<td>1.03</td>
<td>1.76</td>
<td>1.35</td>
</tr>
<tr>
<td>Dead</td>
<td>2.95</td>
<td>5.02</td>
<td>3.85</td>
</tr>
<tr>
<td>Total</td>
<td>2.95</td>
<td>5.02</td>
<td>3.85</td>
</tr>
</tbody>
</table>
2) **Column Loads**

(Percent Live Load Reduction by ASA A58.1)

**Max. L. L. Reduction Permitted:** \( \frac{100(D+L)}{4.33L} = 46.1\% \)

<table>
<thead>
<tr>
<th>Level 2</th>
<th>Col. A</th>
<th>Col. B</th>
<th>Col. C</th>
<th>Col. D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Floor Area Served</td>
<td>360</td>
<td>648</td>
<td>576</td>
<td>288</td>
</tr>
<tr>
<td>% L. L. Reduction (0.08 x Area)</td>
<td>28.8</td>
<td>46.1</td>
<td>46.1</td>
<td>23.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Levels 3-10</th>
<th>Floor Area Served</th>
<th>% L. L. Reduction (0.08 x Area)</th>
</tr>
</thead>
<tbody>
<tr>
<td>720</td>
<td>1296</td>
<td>1152</td>
</tr>
<tr>
<td>46.1</td>
<td>46.1</td>
<td>46.1</td>
</tr>
</tbody>
</table>

Floor Area Served > 576 sq. ft. Max. Reduction = 46.1%

Wall Load on Ea. Story at Exterior Col. = 45 psf x 12 x 24 ft = 13.0 k

Dead Weight of Column (average) = 200 lb/ft

Dead Weight of Fireproofing = 50 lb/ft

Calculation of column loads based on tributary area of floor served is given in Table 2. Increments of loads from girder to column are calculated assuming simple support conditions for the girders.

The intensity of floor load considered in calculating increment of load in the column is indicated following the description of the item calculated. In columns between Levels 2 and 3, each live load reduction decimal \( R \) is different. The reduced live load is calculated by multiplying the full live load value by \((1-R)\). The total loads for the top two stories are calculated in the table.

For stories below the top two, the total column loads are equal to the total load in the column of Levels 1 to 2 plus the indicated increment times the number of floors above the column.
## Table 2
Calculation of Column Loads for Frame Based on Tributary Area of Floor (Working Loads)

<table>
<thead>
<tr>
<th>Item Calculated</th>
<th>Load Intensity (k/ft)</th>
<th>Col. A</th>
<th>Col. B</th>
<th>Col. C</th>
<th>Col. D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>P</td>
<td>P</td>
<td>P</td>
<td>P</td>
</tr>
<tr>
<td>From Level 1 to 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Roof Total Load</td>
<td>2.16</td>
<td>32.4</td>
<td>58.3</td>
<td>51.9</td>
<td>25.9</td>
</tr>
<tr>
<td>Wt. of Col. + Fireproofing</td>
<td>0.25</td>
<td>3.0</td>
<td>3.0</td>
<td>3.0</td>
<td>3.0</td>
</tr>
<tr>
<td>Total Load (Level 1-2)</td>
<td></td>
<td>35.4</td>
<td>61.3</td>
<td>54.9</td>
<td>28.9</td>
</tr>
<tr>
<td>From Level 2 to 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D.L. from Floor Girder</td>
<td>1.92</td>
<td>28.8</td>
<td>51.9</td>
<td>46.1</td>
<td>23.1</td>
</tr>
<tr>
<td>Red. L.L. from Floor Gird.</td>
<td>(1-R)1.92</td>
<td>20.5</td>
<td>28.0</td>
<td>24.9</td>
<td>17.7</td>
</tr>
<tr>
<td>Wt. of Col. + Fireproofing</td>
<td>0.25</td>
<td>3.0</td>
<td>3.0</td>
<td>3.0</td>
<td>3.0</td>
</tr>
<tr>
<td>Wall Load on Exterior Col.</td>
<td></td>
<td>13.0</td>
<td>—</td>
<td>—</td>
<td>13.0</td>
</tr>
<tr>
<td>Total Increment (Level 2-3)</td>
<td></td>
<td>45.3</td>
<td>82.9</td>
<td>74.0</td>
<td>56.8</td>
</tr>
<tr>
<td>Total Load (Level 2-3)</td>
<td></td>
<td>100.7</td>
<td>144.2</td>
<td>128.9</td>
<td>85.7</td>
</tr>
</tbody>
</table>

### Increment per Story

<table>
<thead>
<tr>
<th>From Level 3 to 5</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>D.L. from Floor Girder</td>
<td>1.92</td>
<td>28.8</td>
<td>51.9</td>
<td>46.1</td>
</tr>
<tr>
<td>Red. L.L. from Floor Gird.</td>
<td>1.03</td>
<td>15.5</td>
<td>28.0</td>
<td>24.9</td>
</tr>
<tr>
<td>Wt. of Col. + Fireproofing</td>
<td>0.25</td>
<td>3.0</td>
<td>3.0</td>
<td>3.0</td>
</tr>
<tr>
<td>Wall Load on Exterior Col.</td>
<td></td>
<td>13.0</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Total Increment (Each Story)</td>
<td></td>
<td>60.3</td>
<td>82.9</td>
<td>74.0</td>
</tr>
<tr>
<td>Multiplier for Total</td>
<td>N-1</td>
<td>N-1</td>
<td>N-1</td>
<td>N-1</td>
</tr>
<tr>
<td>Limits of N*</td>
<td>3 5N/10</td>
<td>3 5N/10</td>
<td>3 5N/10</td>
<td>3 5N/10</td>
</tr>
<tr>
<td>Extra Col. Load (Level 10-11)</td>
<td>0.25</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
</tr>
</tbody>
</table>

* N = Number of floor level at top of column considered.

Load at Any Level = Total Load (Level 1-2) + (N-1) (Increment Each Story)
Table 3
Gravity Loads in Columns of Frame
Based on Tributary Area of Floors
(Working Loads)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A1-A2</td>
<td>35.4k</td>
<td>B1-B2</td>
<td>61.3k</td>
<td>C1-C2</td>
<td>54.9</td>
<td>D1-D2</td>
<td>28.9</td>
<td>180.5</td>
</tr>
<tr>
<td>A2-A3</td>
<td>100.7</td>
<td>B2-B3</td>
<td>144.2</td>
<td>C2-C3</td>
<td>128.9</td>
<td>D2-D3</td>
<td>85.7</td>
<td>459.5</td>
</tr>
<tr>
<td>A3-A4</td>
<td>156.0</td>
<td>B3-B4</td>
<td>227.1</td>
<td>C3-C4</td>
<td>202.9</td>
<td>D3-D4</td>
<td>131.8</td>
<td>717.8</td>
</tr>
<tr>
<td>A4-A5</td>
<td>216.3</td>
<td>B4-B5</td>
<td>310.0</td>
<td>C4-C5</td>
<td>276.9</td>
<td>D4-D5</td>
<td>183.3</td>
<td>986.5</td>
</tr>
<tr>
<td>A5-A6</td>
<td>276.6</td>
<td>B5-B6</td>
<td>392.9</td>
<td>C5-C6</td>
<td>350.9</td>
<td>D5-D6</td>
<td>234.8</td>
<td>1255.2</td>
</tr>
<tr>
<td>A6-A7</td>
<td>336.9</td>
<td>B6-B7</td>
<td>475.8</td>
<td>C6-C7</td>
<td>424.9</td>
<td>D6-D7</td>
<td>286.2</td>
<td>1523.8</td>
</tr>
<tr>
<td>A7-A8</td>
<td>397.2</td>
<td>B7-B8</td>
<td>558.7</td>
<td>C7-C8</td>
<td>498.9</td>
<td>D7-D8</td>
<td>337.7</td>
<td>1792.5</td>
</tr>
<tr>
<td>A8-A9</td>
<td>457.5</td>
<td>B8-B9</td>
<td>641.6</td>
<td>C8-C9</td>
<td>572.9</td>
<td>D8-D9</td>
<td>389.2</td>
<td>2061.2</td>
</tr>
<tr>
<td>A9-A10</td>
<td>517.8</td>
<td>B9-B10</td>
<td>724.5</td>
<td>C9-C10</td>
<td>646.9</td>
<td>D9-D10</td>
<td>440.6</td>
<td>2329.9</td>
</tr>
<tr>
<td>A10-A11</td>
<td>578.9</td>
<td>B10-B11</td>
<td>808.2</td>
<td>C10-C11</td>
<td>721.7</td>
<td>D10-D11</td>
<td>492.9</td>
<td>2601.7</td>
</tr>
</tbody>
</table>
### Table 4

**Factored Gravity Loads in Columns of Frame**

**Based on Tributary Area of Floors**

**Gravity Load Case (1.70 x Working Load)**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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### Table 5

**Factored Gravity Loads in Columns of Frame Based on Tributary Area of Floors**

Gravity Plus Wind Load Case (1.30 x Working Load)

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</table>
3) Wind Loads.

\[ H = 0.020 \text{ ksf} \times 12 \text{ ft} \times 24 \text{ ft} = 5.76 \text{ kips} \]

**Top Story (Level 1 to 2)**

\[ 0.5H = 2.88 \text{ kips} \]

**Bottom Story (Level 10 to 11)**

\[ 1.1H = 6.33 \text{ kips} \]

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<th>Cumulative Horizontal Shear ΣH</th>
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<td>48.96</td>
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<tr>
<td>10 - 11</td>
<td>55.29</td>
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3. SELECTION OF PRELIMINARY MEMBER SIZES

3.1 Girders

The preliminary design of girders can be based on the factored gravity load. Since a girder is able to form plastic hinges at the face of the columns, the effective length of the girders can be assumed to be the clear span length. For preliminary design, the depth of column can be assumed to equal twelve or fourteen inches. Lower stories generally require a fourteen inch column. A fairly uniform column size is assumed in order to simplify splicing. The required plastic moment of a girder, under uniformly distributed load and forming the beam mechanism with hinges at the column faces (Fig. 1), can then be found by:

\[ M_p = \frac{w(L-d_c)^2}{16} \]  

where

- \( M_p \) = plastic moment capacity required
- \( w \) = factored gravity load
- \( L \) = centerline span of girder
- \( d_c \) = depth of column
- \( (L-d_c) \) = clear span of girder

When choosing member sizes, the geometric proportions of the component plates of the members should be considered in order to avoid early local buckling. To ensure that local buckling does not occur until the plastic moment is reached and adequate additional hinge rotation has occurred, certain width-to-thickness
limitations are imposed on the flange. The depth-to-thickness ratio is also limited since the web also tends to buckle locally. It is recommended that the following ratios be used as maximums when checking local buckling tendencies in rolled wide-flange beams

\[ \sigma_y \quad b/t \quad d/w \]

\[
\begin{align*}
36 \text{ ksi} & \quad 17 & \quad 70 \\
50 \text{ ksi} & \quad 14 & \quad 60
\end{align*}
\]

The values given for steel with a yield point strength of 50 ksi are based on recent research. The 1963 AISC Specification does not permit the application of plastic design to such steels. General equations for these ratios can be found in Ref. 1.

3.2 **Columns**

The preliminary design of columns is based on axial load, moment introduced from the girders, and position in the frame. For most braced columns, the moments from the incoming girders are assumed to be distributed equally to the column above and the column below the girder. The moment coming from a girder is approximated by:

\[ M = \frac{w(L-d_c)(L+3d_c)}{16} \]  \hspace{1cm} (2)

This moment is assumed to act at the centerline of these columns (Fig. 1). It is based on the beam end moment and the moment of the shear force about the centerline of the column.
By using the maximum pair of preliminary moments and thrusts for the gravity or the combined loading case, trial column sections may be selected from reduced plastic moment tables (Part II - Design Aids Book). This can be most easily done by selecting a trial section with an axial yield load \( P_y \) greater than the factored axial load. The ratio of the factored axial load to the axial yield load \( P/P_y \) is then calculated. For this value of \( P/P_y \) the reduced plastic moment tables give a reduced plastic moment \( M_{pc} \) for the column section being considered. The moment-capacity of the section is reduced because of the axial force. For the given axial load, the column has a definite moment capacity which must be greater than the moments introduced from the girders in order to avoid failure. This design procedure is consistent with the current AISC Specification. Recent research has extended the procedure to new steels such as ASTM A441.

A special problem often arises at the corner joints between girders and columns at the roof level. Here the moment requirement based on a symmetrical beam mechanism (Fig. 1) in the girder might be somewhat greater than the moment capacity of a column which is suitable for all other probable loads. In this area the required revised girder moment is based on the mechanism shown in Fig. 2 and is given by:

\[
M_p = \frac{w(L-d_c)^2(2L+d_c)}{8(3L-d_c)} - \frac{kM_p(L-d_c)}{(3L-d_c)} \tag{3}
\]

*Since the Design Aids Book is referred to often in this paper, it will be understood that it is Reference 2 and no further mention of the reference number will be made.*
where \( kM_p \) = column plastic moment (determined for the corresponding value of \( P/P_y \)).

A new girder section is then chosen using the new \( M_p \) value as found above. The procedure for selecting preliminary members is illustrated in Example 2. Example 2 consists of three tables of tabulated calculations and trial designs. Using the factored gravity loads found in Example 1, trial girder sections are selected in Table 7 based on \( M_p \) values calculated by Eq. 1. Table 8 presents the tabular calculation of column end moments using Eq. 2. Trial column sections to resist the axial loads from Table 4 and the moments calculated in Table 8 are selected (Table 9) using Part II of the Design Aids. Preliminary design checks on the trial column sections are then made using Parts I and II of the Design Aids.
Example 2

Select girder and trial column sections for the frame in Example 1 using tabulated calculations.

For trial design, assume column depth is 12 in. Girders selected will be slightly larger than necessary if actual column is 14 in. Use A36 steel girders.

Table 7 - Trial Girders

<table>
<thead>
<tr>
<th>Girder No</th>
<th>Factored w</th>
<th>Lg</th>
<th>Required Mp</th>
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<td>Units</td>
<td>Lg</td>
<td>kip-ft</td>
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<td>Source</td>
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<td>121.3</td>
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<td>121.3</td>
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<tr>
<td>A2-B2</td>
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<td>29</td>
<td>264</td>
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<tr>
<td>B2-C2</td>
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<td>23</td>
<td>166</td>
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<tr>
<td>C2-D2</td>
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<td>23</td>
<td>166</td>
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Required Z | Section | Actual Z |
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<tr>
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<tr>
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<tr>
<td>A1-B1</td>
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<td>16 WF 36</td>
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<td>C2-D2</td>
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<td>16 WF 36</td>
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</table>
* May have to be made larger if exterior column is inadequate to resist roof girder end moment.

Assume $d_c = 12$ in. In step ④ $K = 1$ for level 1. $K = \frac{1}{2}$ for all other levels. (Constant Multiplier to use in Eq. 2).

Table 8 - Column Moments Due to Girder Loads

<table>
<thead>
<tr>
<th>Girder No.</th>
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<th>L - $d_c$</th>
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<td>feet</td>
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<td>23</td>
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<table>
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<td>—</td>
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<td>—</td>
<td>—</td>
<td>D2-D3</td>
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</table>
* May be reduced and girder size increased if column selected for story 2-3 is too small to accept this moment.

** Same column moment above and below girders for all lower stories having same girder moments.

Table 9 - Trial Columns

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4. DESIGN OF BRACING SYSTEM

4.1 Design Requirements

In the design of braced multi-story frames, it is assumed that the bracing will carry all the lateral forces (load factor = 1.30) plus the overturning moment. This is equivalent to assuming real hinges at the ends of all columns in each level and is usually conservative since the analysis does not consider the stiffness of the frame itself. This Chapter will consider only direct structural bracing with diagonal bracing used for illustration (Fig. 3). Bracing is assumed to be in one bay only although it may be distributed to more than one bay. The frame is assumed to be braced as long as each story is braced in at least one bay (not necessarily the same bay) in each story. The bracing is assumed to have no compressive strength because it will usually be quite slender and have an elastic buckling load much below its tensile strength.

Bracing is designed on the basis of strength to resist story shears, and of stiffness against the tendency of the frame to buckle sideways under gravity load alone. Design may also be controlled by maximum reasonable bracing slenderness. The design equations for these conditions are as follows:
1. **Strength Requirement**

The braces resist all the lateral forces (Fig. 4) by direct tension. The area required can be found by:

\[
A_{bil} = \frac{L_{bi}}{L} \sum_{i}^{1} H_i + \frac{L_{bi}}{E L h_i} \sum_{i}^{m+1} P_{ijl}
\]

(4)

where

- \(A_{bil}\) = required bracing area in i-th story for Condition (1)
- \(L\) = span of braced bay
- \(L_{bi}\) = length of brace in the i-th story
- \(\sum_{i}^{1} H_i\) = sum of the lateral forces above and including the i-th level (L.F. = 1.30), as found in Table 6, Example 1
- \(m\) = number of beams in a story
- \(\sum_{i}^{m+1} P_{ijl}\) = sum of the j column axial loads in the i-th story (L.F. = 1.30), as found in Table 5, Example 1

This brace requirement is derived from the assumption that the diagonal brace in a story assumed to be pin-jointed will just reach yield point stress when the frame carries its full factored lateral load at the same time that the factored gravity loads are in a sway displacement equal to that at which the diagonal brace will yield.
2. Frame Buckling Requirement

Under gravity load alone the frame may tend to buckle sideways. The bracing must provide sufficient stiffness to prevent this type of failure. The elastic force in the brace at a given sway counteracts the overturning effect of the gravity loads at the same sway. The required area is found by:

$$A_{bi2} = \frac{L_{bi}^3}{E} \frac{m+1}{L_i h_i} \sum_1^j P_{ij}$$  (5)

where

$$\sum_1^j P_{ij}$$ is the sum of the j column axial loads in the i-th story for Condition (2) (L.F. = 1.70).

3. Slenderness Requirement

It is usually required that the slenderness ratio of the brace be less than a certain maximum allowable value. The AISC Specification provide for a maximum slenderness ratio of 300 for bracing members in tension. This design condition can be expressed as

$$r_{bi} \geq \frac{L_{bi}}{300}$$  (6)

where $$r_{bi}$$ is the radius of gyration of the brace in the i-th story.

For adequate bracing, all three of the above conditions must be satisfied. Use of these equations is shown in Example 3 at the end of this chapter. Using Eqs. 4, 5, and 6, and the
tabulated horizontal forces (Table 6), the general criteria for diagonal bracing for the example frame are found. The final bracing requirements for each story are presented in tabular form and members are selected to satisfy these requirements.

Other types of bracing besides diagonal bracing can be used. The same basic requirements for the bracing member must be met. Reference 11 presents a detailed explanation of K bracing. Equations for other conditions and for other types of bracing are not presently available but can be derived by a procedure similar to the one in Refs. 1, 17.

4.2 Contribution of Frame Stiffness

The frame itself will contribute to the resistance to horizontal shear. This contribution has been neglected in designing the bracing members where it would not change the selected member sizes if it were included. However, it may be desirable to include the frame resistance to shear in computing the axial force in the beams in the braced bay when the final design of the girders is done. Using the equations for the moment at the left end of the beam and the moment at the end of the column, the shear resistance $H_{Fi}$ of the frame at the i-th level can be found by statics to be
This approximation is for a regular frame (Fig. 4) and uses the assumptions that the beam end moment is shared equally by the column above and below and that the moment at the bottom of the lower column is the same as at the top.

If the story deflection index under working loads is limited to 0.002 (including the contribution of the braces, the beams, and the columns, but exclusive of the P-Δ effect), the index under ultimate loading should be approximately \((1.3)(0.002) = 0.0026\). This index does not include the P-Δ effect. Conservatively the story rotation could be set equal to 0.004 at ultimate loads including the P-Δ effect for computational purposes.

When the columns have considerably greater stiffness than the beams, as would be the case in the lower stories of a tall frame, then the equation for shear resistance can be simplified to
The calculation of story resistance is illustrated in Example 4. This example uses the frame dimensions from Example 1 and the preliminary member sizes selected in Example 2 to calculate the horizontal frame resistance by Eq. 7. The frame's resistance is compared to the applied horizontal load and if the frame resistance is larger than the load no diagonal bracing is needed.

\[ H_{Fi} = \frac{3\pi R}{h_i} \sum_{l}^{m} k_{Bj} \]  

(8)
Example 3

Design bracing for the frame of Example 1.

Given: \( \sigma_y = 36 \text{ ksi} \); \( E = 29,000 \text{ ksi} \)

Use clear distances:
- \( h = 12 - 1 = 11 \text{ ft.} \)
- \( L = 24 - 1 = 23 \text{ ft} \)
- \( L_b = \sqrt{l^2 + 2.3^2} = 25.5 \text{ ft.} \)

\[
A_{b1} = \frac{L_{bi}}{\sigma_y L} \sum_i H_i + \frac{L_{bi}^3}{EL^2 h_i} \sum_i m_i \frac{P_{ij}}{L}
\]

\[
= \frac{25.5}{(36)(23)} \sum_i H_i + \frac{(25.5)^3}{29,000(23)^2(11)} \sum_i m_i \frac{P_{ij}}{L}
\]

\[
= 0.0308 \sum_i H_i + 0.0000984 \sum_i m_i \frac{P_{ij}}{L}
\]

Use load factor 1.30

\[
A_{b2} = \frac{L_{bi}^3}{EL^2 h_i} \sum_i P_{ij} = 0.0000984 \sum_i P_{ij}
\]

Use load factor 1.70
Slenderness Ratio Requirement

Use single angles

Unbraced length in plane of frame: \( \frac{L_b}{2} \)

\[ r_x = \frac{(25.5)(12)}{(300)} = 0.51 \text{ in.} \]

Unbraced length out-of-plane: \( L_b \)

\[ r_y = \frac{(25.5)(12)}{300} = 1.02 \text{ in.} \]

Bottom Story

\[ h = 15 - 1 = 14 \text{ ft.} \quad L = 23 \text{ ft.} \]

\[ L_b = \sqrt{14^2 + 23^2} = 26.95 \text{ ft.} \]

\[ A_{bl} = \frac{26.95}{(36)(23)} \sum H_i + \frac{(26.95)^3}{29,000(23)(23)(14)} \sum P_{ij} \]

\[ = 0.0326 \sum H_i + 0.0000917 \sum P_{ij} \]

\[ r_y = \frac{(12)(26.95)}{300} = 1.078 \text{ in.} \]

\[ r_x = 0.539 \text{ in.} \]

<table>
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<th>( n )</th>
<th>( \Sigma H_{1.3} )</th>
<th>( \Sigma P_{1.3} )</th>
<th>( 3.08 \times 10^2 \Sigma H )</th>
<th>( 9.84 \times 10^4 \Sigma P )</th>
<th>( A_{bl} )</th>
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<td>kip</td>
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<td>in(^2)</td>
<td>in(^2)</td>
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<td>$\Sigma P$</td>
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</table>

$A_{b2}$ will be less than $A_{b1}$ and therefore is not critical in this structure.

Using single angles (double angles can also be used) the area and the slenderness requirements on all levels are satisfied by

\[
\angle 6 \times 6 \times \frac{5}{16}
\]

\[
A = 3.65 \text{ in.}^2 \quad \text{r}_z = 1.20 \text{ in.}
\]

weight = 12.4 pounds per foot

**Total Weight of Bracing**

\[
= (9)(2)(2.55)(12.4) + (2)(2.95)(12.4)
\]

\[
= 3.18 \text{ ton}
\]
Example 4

Determine the horizontal frame resistance of the top story of the frame of Example 2.

Level 1

\[ k_{cj} = \frac{I_{ci}}{h_i} \quad \text{for column } j \]

\[ k_{BJ} = \frac{I_{Bi}}{L_i} \quad \text{for beam } j \]

\[ H_{F1} = \frac{12ER}{h_i} \sum_i \frac{k_{cj}}{1 + 4k_{cj} k_{BJ}} \]

where \( R = 0.004 \)

\[ H_{F1} = \frac{(12)(0.004)(30000)}{(12)(12)} \left[ \frac{2.96}{1 + 4(2.96)} + \frac{2.15}{1 + 4(2.15)} + \frac{0.573}{1 + 4(0.573)} \right] \]

\[ = 10 \left( 0.317 + 0.232 + 0.179 \right) \]

\[ = 7.28 \text{ kip} > 3.74 \text{ kip} \]

\[ \therefore \text{No bracing is needed for the top story.} \]
5. FINAL DESIGN OF GIRDERS

5.1 Dead and Live Load

The final design of girders for dead and live loads is exactly the same as used in preliminary design except that more accurate column depths can now be used. As in preliminary design

\[ Z = \frac{wl^2}{16\sigma_y} \]  \hspace{1cm} (9)

where \( L_g = L - d_c \)

\( d_c \) = average column depths for the adjoining columns as found in the preliminary design.

5.2 Dead, Live and Lateral Loads

Girders in the braced bay must be checked for axial load and if necessary redesigned as beam-columns. Assuming the frame is braced in one bay only, the axial force (Fig. 5) in a girder which is in equilibrium with the forces in the diagonal bracing proportioned according to Eq. 4 is:

\[ P_{gi} = \sum_{i=1}^{i} H_i + R_i \sum_{j=1}^{m+1} P_{ij} - H_{Fi} \]  \hspace{1cm} (10)

where

\( P_{gi} \) = axial girder force

\( R_i \) = the \( i \)-th story rotation \( (\Delta_i/h_i) \) which can be conservatively assumed to be equal to 0.004

\( H_i, H_{Fi}, P_{ij} \) = are as defined in Chapter 4
The load factor in this case is 1.30 because combined loading is being considered. When the girder in the braced bay is subjected to axial force, its ability to carry transverse load is reduced. The reduced transverse load capacity is given by:

\[ w = \frac{(P_e - P_{gi})(P_o - P_{gi})w_p}{P_o(P_e - 0.4 P_{gi})} \]  

(11)

where

- \( P_e \) = elastic buckling load = \( \frac{\pi^2 E r_x^2}{L_g^2} \)
- \( P_{gi} \) = thrust in the girder for the i-th level
- \( P_o \) = axial capacity of the beam section without bending moment
- \( w_p \) = \( \frac{16L\sigma_y}{L_g^2} \) = the transverse load capacity of the girder in the absence of axial load

This equation is based on the interaction equation for the strength of a fixed-end beam subjected to a uniformly distributed transverse load. The transverse load can also be found graphically from Fig. 6. Sample calculations of beam-column loads and the design of members are shown in Example 5 at the end of this chapter. The frame resistance \( H_{Fi} \) for sample levels is calculated as was done in Example 4 as well as by the simplification of Eq. 8. A second calculation using Eq. 10 and involving the applied load, the P-Δ effect, and the frame resistance is made to obtain the axial force in the girder of the braced bay.
Where the axial force in the girder exceeds 0.15 $P_y$, the girders selected in Example 2 are checked by Eq. 11 to see that the axial load and the transverse load can be carried simultaneously.

5.3 Shear

The influence of shear should also be considered. One limit usually proposed for the maximum attainable shear is that at which the whole web is fully yielded in shear. Then,

$$V_m = \tau_y A_w = \frac{\sigma_y w(d-2t)}{\sqrt{3}} \quad (12)$$

where

- $V_m =$ maximum shear
- $\tau_y =$ yield stress in shear
- $A_w =$ area of the web

If the approximation

$$1 - \frac{2t}{d} \approx 0.93$$

is used, the equation for maximum shear becomes

$$V_m = 0.54 \sigma_y wd \quad (13)$$

or for practical use

$$\begin{align*}
\sigma_y & \quad V_m \\
36 \text{ ksi} & \quad 19.5 \text{ wd} \\
50 \text{ ksi} & \quad 27 \text{ wd}
\end{align*}$$

No design adjustments are necessary if the applied shear force is less than the maximum attainable shear. If it is larger, either a deeper section or a section with a thicker web should be chosen. The web may also be stiffened by a doubler plate.
5.4 Lateral Bracing

Lateral bracing must be provided at and in the vicinity of plastic hinges so that the plastic moment can be maintained until straining has progressed to strain hardening and failure takes place by local buckling. Floor joists which frame into the beam to be braced can serve as bracing. The joists must provide axial strength and stiffness to resist the main member's tendency toward lateral deflection. The bracing members must also possess bending stiffness about their own major axis to resist the tendency of the main member to twist. Lateral bracing should be placed at each plastic hinge location and at distances $L_{cr}$ to either side of each hinge (Fig. 7) where $L_{cr}$ as found by analytical and experimental studies has been shown to be:

Uniform Moment ($\gamma > 0.7$)

$$L_{cr} = \frac{\pi r_y}{K \epsilon_y \sqrt{1 + \frac{0.56E}{E_{st}}}}$$

(14)

where

$\gamma = \text{ratio of end moments on the braced segments}$

(smaller end moment in braced span divided by the larger end moment)

$K = 0.54$ - if adjacent span is elastic

$K = 0.80$ - if adjacent span is yielded ($\gamma \geq 0.88$)

The critical length might be considered as the height of an imaginary column comprising the compression half of the beam. This column must reach the yield stress and permit additional
straining to nearly 0.8 of the strain-hardening strain. The critical length also depends somewhat on the restraining characteristics of the adjacent span of the member, which provides better restraint if it is largely elastic.

To formulate a practical design recommendation, an effective length factor \( K = 0.54 \) is assumed and Eq. 14 is evaluated for different yield stresses as:

\[
\begin{align*}
\sigma_y & \quad L_{cr} \\
36 \text{ ksi} & \quad 38 r_y \\
50 \text{ ksi} & \quad 28 r_y
\end{align*}
\]

Moment Gradient \((1.0 < \gamma < 0.7)\)

\[
L_{cr} = \frac{0.7\pi r_y}{\sqrt{\varepsilon_y}}
\]  \(15\)

This situation is more favorable than the uniform moment case because beams under moment gradient yield in a relatively non-critical location (at the end of the span) and thus tend to buckle locally before they buckle laterally. Based on the idea that local buckling will occur first, theory and experiments have shown that the plastic hinge angle which can be delivered would exceed the usual requirements.\(^{10,14}\)

For practical use the critical bracing spacing necessary to maintain the plastic moment until local buckling occurs can be expressed in terms of the radius of gyration for various steels as:
The values $L_{cr}$ given for the two types of moment distribution and steel are not in agreement with Part II of the AISC Specification. Recent research has shown that the values given provide adequate lateral bracing while using material more economically.

Bracing should also meet the following dimensional requirements:

1. Axial strength and stiffness to provide a tensile force to resist the lateral deflection tendency of the main member.

\[
\frac{L_b}{L_a} \geq \frac{0.19L_a}{b} \frac{A_b}{A_{br}}
\]

(16)

2. Flexural stiffness to resist the bending of the bracing member by the twisting of the main member.

\[
\frac{L_b}{d_b} \leq \frac{b^2t}{3\varepsilon_y A_d}
\]

(17)

where $A, b, t, d =$ dimensions of member to be braced

$L_b =$ length of bracing member

$d_b =$ depth of bracing member

$A_{br} = \frac{0.22b^2t}{L_L + L_R} =$ bracing area required to resist axial force caused by the main member trying to deflect in the out-of-plane direction
The bracing should be connected to the compression flange. In addition a vertical stiffener is needed. If the bracing can not be framed into the compression flange, then this flange should be held in place by braces, such as a pair of diagonal braces (Fig. 8). When bracing is possible on only one side of a girder it is still necessary to provide lateral bracing equivalent to bracing on two sides of the girder. This can be done by doubling the area and the moment of inertia of the bracing members. When the compression flange is completely embedded into a concrete slab, the member can be assumed to be fully restrained against lateral buckling. However, even in this case the local buckling requirements should be met and vertical stiffeners should be provided at hinge locations. Light sheeting or grating, intermittently tack welded to the top flange of the beam would in general not meet the flexural stiffness requirements and should thus not be counted on to provide adequate bracing. Bracing in the elastic portion of the beams can be spaced according to Part I of the AISC Specification. The design of lateral bracing is illustrated in Example 6 at the end of this chapter. In this example a fixed-end beam from a multi-story frame is designed in A36 steel to support a uniformly distributed load. Based on the solutions presented for Eqs. 14 and 15 the required maximum spacing of the lateral bracing is found. Minimum sizes of intersecting floor members which serve as bracing are determined by using Eqs. 16 and 17.
Example 5
With bracing in bay C-D, design Girders C2-D2 and C10-D10 as beam-columns.

**Level 2**

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<th>18WF45</th>
<th>16WF36</th>
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<td>8WF28</td>
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<td>8WF48</td>
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</table>

\[ h = 12 \text{ ft} \]

**Level 10**

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<td>14WF111</td>
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</tr>
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</table>

\[ h = 15 \text{ ft} \]

\[ 30' \quad 24' \quad 24' \]

\[ R = 0.004 \]

\[ k_{cj} = \frac{I_{ci}}{h_i} \quad \text{for column } j \]

\[ k_{Bj} = \frac{I_{Bi}}{L_i} \quad \text{for beam } j \]

\[ H_{Fi} = \frac{12ER}{h_i} \sum_{i}^{m} \frac{k_{cj}}{1 + 4k_{cj}k_{Bj}} \]
### Resistance of Frame to Horizontal Loads

<table>
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<tr>
<th>Operation</th>
<th>( k_{cj} )</th>
<th>( k_{bj} )</th>
<th>( 4k_{cj} )</th>
<th>( 4k_{cj}/k_{bj} )</th>
<th>( 1 + 5 )</th>
<th>( k_{cj}/6 )</th>
<th>Eq. 7</th>
<th>Eq. 8</th>
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<td>in(^3)</td>
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<tr>
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### Axial Force, \( P_g \)

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<th>( H_{F1} )</th>
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<th>( P_g/P_y )</th>
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For the tabular computation of the transverse load capacity of beam-columns, it is convenient to rearrange Eq. 11 to

\[
W = \frac{\sigma_e - \sigma}{\sigma_e - 0.4\sigma} \left(1 - \frac{\sigma}{\sigma_0}\right) W_p = \frac{N_1}{D} N_2 W_p
\]

where

\[
\sigma = \frac{P_g}{A}, \quad \sigma_e = \frac{\pi^2 E}{(L/r)^2},
\]

\[
\sigma_0 = \sigma_y \left[1 - \frac{\sigma_y}{4\pi^2 E} \left(\frac{L}{r}\right)^2\right]
\]

Design of Girder C10-D10 as a Beam-Column

<table>
<thead>
<tr>
<th>P_g</th>
<th>w</th>
<th>L_g</th>
<th>Section</th>
<th>A</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>kip</td>
<td>kip</td>
<td>in</td>
<td>16WF45</td>
<td>13.24</td>
<td>82.0</td>
</tr>
<tr>
<td>75.87</td>
<td>3.85</td>
<td>274.08</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>r_x</th>
<th>L/r</th>
<th>(L/r)^2</th>
<th>\sigma</th>
<th>\sigma_e</th>
<th>\sigma_0</th>
</tr>
</thead>
<tbody>
<tr>
<td>in</td>
<td>41.3</td>
<td>1706</td>
<td>5.73</td>
<td>167.8</td>
<td>34.03</td>
</tr>
<tr>
<td>6.64</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>\sigma/\sigma_0</th>
<th>N_1</th>
<th>N_2</th>
<th>D</th>
<th>W_p</th>
<th>w</th>
</tr>
</thead>
<tbody>
<tr>
<td>kip/in^2</td>
<td>163.77</td>
<td>0.832</td>
<td>165.51</td>
<td>7.54</td>
<td>6.2</td>
</tr>
</tbody>
</table>

Check for Local Buckling

\[
\frac{d}{w} \leq 70 - 100 \frac{P}{P_y} = 70 - 100 \frac{\sigma}{\sigma_y}
\]

46.6 < 70 - 100(5.73) = 54.1

\[\therefore \text{Use a 16WF45}\]
Example 6

A fixed-end beam of 30'-0" length is to support a uniformly distributed working load of 2 kip per ft. The distance between adjacent beams is 10'-0". Design the beam for A36 steel. Determine the location and the size of the floor joists or filler beams which serve as bracing members, using A36 steel. The load factor to be used is L.F. = 1.7.

\[ M_p \text{ required} = \frac{1}{2} \left( \frac{wL^2}{8} \right) \]

\[ w = (1.7)(2) = 3.4 \text{ kip/ft.} \]
\[ L = 30 \text{ ft.} \]

\[ M_p = \frac{1}{12} (3.4 \times 30)^2 = 191 \text{ kip-ft.} = 2290 \text{ kip-in.} \]

Required Plastic Modulus

\[ Z_r = \frac{M_p}{\sigma_y} = \frac{2290}{36} = 63.7 \text{ in.}^3 \]

Try 16WF36

\[ Z = 63.9 \text{ in.}^3 \]

\[ \frac{d}{W} = \frac{15.85}{0.299} = 53.0 < 70 \therefore \text{OK.} \]

\[ \frac{b}{t} = \frac{6.99}{0.428} = 16.3 < 17 \therefore \text{OK.} \]
Maximum Shear: \( \frac{wL}{2} = \frac{(3.4)(30)}{2} = 51.0 \text{ kip} \)

19.5 \( wd = 19.5(0.299)(15.85) = 92.5 > 51.0 \text{ kip} \) :: Shear OK

::: Use 16WF36 Beam

Spacing of lateral bracing:

In region of near uniform moment (at center):

\( L_{cr} = 38\text{ry} = 38(1.45) = 55\text{ in.} \)

In region of moment gradient (at ends):

\( L_{cr} = 65\text{ry} = 65(1.45) = 94\text{ in.} \)

\[ 15'-0'' = 180\text{ in.} \]

\[ 65\text{ in.} \quad 65\text{ in.} \quad 50\text{ in.} \]

\[ \text{Bracing Arrangement} \]

Design of Bracing

\( L_b = 10\text{ft.} = 120\text{ in.} \)

\[ d_b = \frac{3\varepsilon_y L_b A_d}{b^2} \]

\[ \varepsilon_y = \frac{36}{29000} = 0.00124 \]

\[ d_b = \frac{(3)(0.00124)(120)(10.59)(15.85)}{(6.99)(6.99)(0.428)} = 3.6\text{ in.} \]

Try 6B12 Bracing members
\[ A_{br} = \frac{0.22 b^2}{L_L + L_R} \]

\[ = \frac{(0.22)(6.99)^2(0.428)}{50 + 50} = 0.046 \text{ in}^2 < 3.53 \text{ in}^2 \]

\[ \frac{L_b}{L_a} = \frac{0.19 L_a}{b} \frac{A_b}{A_{br}} \]

\[ \frac{L_b}{L_a} = \frac{(0.19)(50)(3.53)}{(6.99)(0.046)} = 104 > \frac{120}{50} = 2.4 \]

\[ \therefore \text{Use GB12 Bracing members} \]

(minimum requirement)
6. FINAL DESIGN OF COLUMNS

In the preliminary design of columns, the member sizes were selected simply on the basis of their reduced plastic moment with no attention given to the various possibilities of buckling and instability failure associated with columns. In the final design process, member sizes will be checked or modified to take into account the possibilities of these types of failure. The following paragraph describes briefly the failure modes which occur in columns and should be considered in design.

If a column does not sway and has bracing between floors to prevent out-of-plane movement, it is called a "braced" column in this report. The possible failure modes for braced columns are:

1. Strong axis buckling, if only axial force is present (Fig. 9a).

2. Failure due to excessive bending in the plane of the frame (instability), if combined axial force and bending moment are present (Fig. 10).

If sway is prevented in a column but out-of-plane movement between floor levels is allowed, the column is called an "unbraced" column. The design conditions for an unbraced column are:

1. Weak axis buckling, if only axial force is present (Fig. 9b).

2. Lateral-torsional buckling, if combined axial force and bending moment are present (Fig. 11).
6.1 Columns Under Axial Load Only

The design of axially-loaded columns, both braced and unbraced, can be based on the column buckling formula given in the Column Research Council Guide. In terms of critical stress the column buckling formula can be written in the following form:

\[
\frac{\sigma_{cr}}{\sigma_y} = 1 - \frac{\sigma_y}{4\pi^2E} \left[ \frac{Kh}{r} \right]^2
\]

(18)

where

- \( \sigma_{cr} \) = critical stress
- \( \sigma_y \) = yield stress
- \( K \) = effective length factor
- \( h \) = height of the column
- \( r \) = radius of gyration

The buckling configuration of an individual column in a braced frame is similar to that of a pinned-end column. This is because the formation of plastic hinges at the ends of the beams where they are connected to the columns tends to reduce the end restraint on the column. Thus the effective length factor \( K \) can be taken equal to one, the value for simply-supported columns.

Part I of the Design Aids gives values of the critical stress computed from the column formula given above. For braced columns, the strong axis radius of gyration is used in computing the slenderness ratio. For unbraced columns the weak axis radius of gyration should be used. With the critical stress value from the table for the \( Kh/r \) value as calculated, the critical axial load \( P_{cr} \) can be found by:
The trial column section must have a critical load equal to or greater than the computed axial force corresponding to the design ultimate load.

Local buckling is an additional problem which can be avoided in axially loaded rolled wide-flange shapes if the b/t and d/w ratios do not exceed the following limits

<table>
<thead>
<tr>
<th>σ_y</th>
<th>b/t</th>
<th>d/w</th>
</tr>
</thead>
<tbody>
<tr>
<td>36 ksi</td>
<td>32</td>
<td>42</td>
</tr>
<tr>
<td>50 ksi</td>
<td>26</td>
<td>36</td>
</tr>
</tbody>
</table>

All shapes listed in Part II of the Design Aids meet these geometric restrictions for the designated steel.

6.2 Strength of Beam-Columns

A member loaded with both axial load and bending moment is called a beam-column. In multi-story frames most of the columns are really beam-columns. The axial load comes from the columns above the column in question and from the adjacent floor loads and the moment is transmitted from the connecting girders. A special case arises in interior columns with equal adjacent spans loaded equally on both sides. In this case the bending moments from the adjacent girders became equal and the columns are then loaded by axial loads only. These columns can be designed as axially loaded columns by the procedure described in Sect. 6.1.
For the purpose of design, beam-columns can be classified into two main categories--braced and unbraced. This section will describe the situation in which each category is found and the strength characteristics which may be expected from each. Design aids based on these strength characteristics will be described as well.

**Braced Beam-Columns**

Increasing loads and/or moments will cause a braced column to deflect in the plane of the frame. At some point in the loading process, excessive bending in the plane of the frame will cause failure to occur. The deflected shapes and the strength of the column are dependent on the type of loading and the position of the column in the frame. The important types of deflected shapes occur under the following conditions of column position in the frame and column loading:

1. Double curvature--exterior columns under full or partial (checkerboard) loading and interior columns with unequal adjacent spans under full or partial loading.

2. Single curvature--interior columns under partial loading.
The type of deflection configuration a column will take is consistent with the sign of the end moment ratio \( q \). For this report \( q \) will be defined as \( M_2/M_1 \) where \( M_1 \) is the larger (in absolute value) of the two moments and where clockwise applied moments are positive. \( q \) is positive when the column is bent in double curvature and negative when it is in single curvature. The deflection configuration is of interest because columns behave differently when they are bent in one configuration or the other. The strength of a single curvature column drops quickly as the rotation is increased and therefore the single curvature condition is a critical one for the design of beam-columns. Double curvature columns, however, can maintain their strength after the maximum value of moment is reached. This configuration constitutes a more favorable situation for the design of beam-columns.

**Unbraced Beam-Columns**

For columns bent about the strong axis but without bracing between floors in the weak direction, there is a tendency towards lateral-torsional buckling which may reduce the maximum moment capacity of a column and may impair its rotational capacity (Fig. 12). Lateral-torsional buckling strength depends largely on the slenderness ratio, end moment ratio, and sectional properties. For most practical problems the reduction in moment-carrying capacity has been found to be small.\(^{(30)}\)
Design Aids

To aid in the design of beam-columns, moment-rotation curves for various end moment and axial load to yield load ratios have been developed based on the Column Deflection Curve concept by using numerical integration. Part III of the Design Aids gives moment-rotation curves for various $P/P_y$ ratios and for end moment ratios $q$ of 0 and -1, and for columns with one end fixed. These end moment ratios were selected because the curves represent nearly correct results for most practical columns or else give conservative answers. The curves are for columns braced between floors in the weak direction and can be used to check whether the columns selected in the preliminary design have adequate capacity to resist the applied moment. The curves can also be used for unbraced columns if the additional problems of weak axis buckling and lateral-torsional buckling are considered.

In using the curves for double curvature columns, the maximum moment ordinate (in terms of $M/M_{pc}$, actual column moment to column plastic moment ratio) is given in the curve labeled $q = 0$ which has an $h/r$ value equal to one half the strong axis slenderness of the actual column. This curve provides the correct answer because a double curvature column (Fig. 13) behaves just like two shorter columns, each with one end pinned and having the sum of their lengths equal to the length of the double curvature column. For columns in single curvature, the effective $M/M_{pc}$ ratio is found from the chart with $q = -1$ and an $h/r$ ratio found by using the total height of the column in question.
Charts for the design of the bottom story columns are chosen on the basis of the individual end conditions of the columns, such as pinned, fixed, etc.

6.3 Design of Braced Columns

In this article, the design of braced columns is discussed for the case of full loading and for the case of partial or checkerboard loading. In addition the special case of columns in the bay which contains diagonal sway bracing is considered because of the extra forces accumulated there from the diagonal bracing.

The required maximum spacing of braces between floors to qualify a column as a braced column is as follows:

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_y$</th>
<th>$h/r_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Double Curvature</td>
<td>36 ksi</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td>50 ksi</td>
<td>55</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_y$</th>
<th>$h/r_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single Curvature</td>
<td>36 ksi</td>
<td>38</td>
</tr>
<tr>
<td></td>
<td>50 ksi</td>
<td>28</td>
</tr>
</tbody>
</table>

These columns should also be checked for weak axis buckling. The derivation of these values is given in Ref. 1. If the bracing spacing required exceeds the height of the column, the column acts as a braced column although no bracing is required. If
bracing is found to be required, it can be proportioned according to Eqs. 16 and 17 for the bracing of beams.

Local buckling must also be considered when designing beam-columns. The limiting b/t and d/w ratios for shapes to be used as beam columns are as follows:

<table>
<thead>
<tr>
<th>$\sigma_y$</th>
<th>b/t</th>
<th>d/w</th>
</tr>
</thead>
<tbody>
<tr>
<td>36 ksi</td>
<td>17</td>
<td>42</td>
</tr>
<tr>
<td>50 ksi</td>
<td>14</td>
<td>36</td>
</tr>
</tbody>
</table>

All shapes listed in Part II of the Design Aids meet these geometric restrictions for the designated steel.

**Full Loading**

One possible loading case for the design of beam-columns is the full loading case in which all possible loads are applied to the structure simultaneously.

The bending moment $M$ applied to an exterior column at each joint is computed for design purposes as $M_p + V d_c/2$ assuming each girder fails with plastic hinges at the two ends (column faces) and at the mid-span. This moment is to be resisted by the two adjacent column segments. It is appropriate in design calculations to assume the condition of symmetrical double curvature since the strength of a column with low slenderness ratios (say $h/r_x \leq 40$) is not significantly affected by a small variation in $q$. Since double curvature columns can maintain their maximum moment at the ends while the column rotates through a considerable angle, it is only necessary to satisfy a statical condition for
each joint (Fig. 14):

\[ M_{ma} + M_{mb} \geq M \]  

(20)

where \( M_{ma} \) and \( M_{mb} \) are the maximum moments that can be resisted by the columns above and below the joint as found in Part III of the Design Aids.

If the slenderness ratio is limited to about 40, the maximum moment may be taken as \( M_{pc} \) for \( P/P_y \) equal to or less than 0.7. For higher \( P/P_y \) ratios, the maximum moment will be less than \( M_{pc} \). The reduced \( M_{pc} \) value can be obtained from the moment-rotation curves of Part III of the Design Aids.

Interior columns with unequal adjacent spans or with adjacent girders designed for unequal loads must be designed to resist the net moment transmitted by the girders. The net moment at each joint can be expressed as \( M_{pl} - M_{pr} + (V_t - V_r) \cdot d_c/2 \). These moments cause double curvature bending in all the column segments. The situation is essentially the same as that for an exterior column, and the same design procedure may be followed. The design of a typical column for full loading is shown in Example 7 at the end of this chapter. Since the bays adjacent to Column C in the example frame are equal, the design of the interior column is governed by the axial load. Using Part I of the Design Aids and Eq. 19, the trial column section from Example 2 can be checked for adequacy.
Checkerboard Loading

The case of live loads in alternate bays and stories (checkerboard loading) is often considered in design since it produces more severe bending moment in the columns. The following discussion of column design for checkerboard loading will cover several cases of columns: exterior columns, interior columns with equal adjacent bays, and interior columns with unequal adjacent bays.

(a) Exterior Columns

A study of the moment-rotation curves given in Part III of the Design Aids shows that the strength of beam-columns with slenderness ratios smaller than 40 and subjected to axial forces less than \(0.6 P_y\) is not significantly affected by a change in the end moment ratio. The maximum resisting moment that can be developed at the ends is usually equal to \(M_{pc}\). Therefore, checkerboard loading does not require any increase in the column size.

For beam-columns subjected to axial forces higher than \(0.6P_y\), a change in the end moment ratio \(q\) usually results in a small change of the moment-carrying capacity. An estimate of \(q\) is therefore needed in the design. To estimate the \(q\) value of a column, it is first necessary to compute the moments applied to the two end joints by the girders. The moments including shear effects transmitted by the girders to the joints can be approximated by:
Full Loading

where

\[ M_F = M_p + \frac{w_F L g}{2} \cdot \frac{d_c}{2} \] (21)

Dead Loading

\[ M_D = \frac{w_D L g^2}{12} + \frac{w_D L g}{2} \cdot \frac{d_c}{2} \] (22)

where

- \( M_F \) = joint moment transmitted by the girder with full loading
- \( w_F \) = full factored load as found in Chapter 2
- \( M_D \) = joint moment transferred by the girder with dead load
- \( w_D \) = factored dead load
- \( M_p \) = plastic moment capacity of the girder with full loading

The moment transmitted by the girder with dead load only was found by considering the girder to be fixed at both ends and assuming elastic behavior. Using the moments as found above an estimate for the column end moment ratio \( q \) can be found by:

\[ q = \frac{M_D}{M_F} \] (23)

The maximum moment which can be applied to the column can then be found by using Fig. 15 and the end moment ratio from Eq. 23. The condition to be satisfied in the design is that the sum of the two resisting column moments should be equal to or greater than the moment applied by the girder with full loading (see Fig. 20). The axial force in the column may be adjusted for the removal of the live load from the adjacent girders. The design of a typical exterior column segment is shown in Example 8 at the end of this chapter. The joint moments corresponding to the given dead
and live loads are computed using Eq. 21 and 22. The end moment ratio is then estimated by Eq. 23. The resisting moments that the column section selected in Example 2 can develop are found using Part III of the Design Aids and added. The sum is compared to the maximum joint moment. If the resisting moment sum is equal to or greater than the maximum joint moment (Eq. 20), the trial column section is satisfactory.

(b) Interior Columns With Equal Adjacent Bays

Under dead load alone the interior column between equal bays remains essentially straight and the slope of the deflected girders near the interior column is approximately zero. Upon application of live load in a checkerboard pattern, unbalanced moments are immediately introduced at the interior joints. At each joint the unbalanced moment is resisted jointly by the columns above and below and by the adjacent girder carrying only dead load. The interior column segment may be bent in single curvature and the column end moment ratio $q$ can be conservatively assumed to equal -1.0.

To determine the resisting moment offered by the girder, it is necessary to consider the moment-rotation behavior. When the girder is loaded with dead load only, the rotation adjacent to the interior column is essentially zero and the moment introduced to the column is given by Eq. 22. The application of live load to alternate girders causes interior joints to rotate. This in turn causes the interior end of the girders with dead load only (restraining girders) to rotate until the girder forms a plastic hinge at
the face of the interior column. The maximum resisting moment is:

$$M_T = M_p + \frac{w_D L_g}{2} \cdot \frac{d_c}{2}$$  \hspace{1cm} (24)$$

where:

- $M_T =$ maximum resisting moment provided by the restraining girder
- $M_p =$ plastic moment capacity of the restraining girder

It should be noted that $M_p$ (Eq. 21) and $M_T$ (Eq. 24) are not the same. There are different loading conditions which cause the shear contribution to the moment to be different. In a two bay equal-span frame, the rotation of the girder at the formation of the plastic hinge can be estimated by assuming that the far end is pinned (Fig. 16). The rotation is given by:

$$\theta_{ph1} = \frac{2}{3} f \frac{\sigma_y}{E} \left[ 1 - \frac{4}{3} \cdot \frac{w_D}{w_T} \right] \frac{L_g}{d_g}$$  \hspace{1cm} (25)$$

where:

- $\theta_{ph1} =$ the rotation at which a plastic hinge forms in the restraining girder at the face of the interior column.
- $f =$ shape factor of the restraining girder.

In a frame of three or more bays; the end rotation of an interior girder at the formation of the plastic hinge is given by:

$$\theta_{ph2} = f \frac{\sigma_y}{E} \left[ 1 - \frac{4}{3} \cdot \frac{w_D}{w_T} \right] \frac{L_g}{d_g}$$  \hspace{1cm} (26)$$

$\theta_{ph1}$ and $\theta_{ph2}$ differ because of the restraining effect of an additional bay (Fig. 17) which is present in frames with more than two bays. The additional bay causes the far end of the
restraining girder to react differently.

The moment-resisting capacity of the restraining girder and the columns above and below the joint in question can be found by:

1. Calculate the plastic hinge rotation \( \theta_{ph} \) and the girder moments \( M_T \) and \( M_D \) for the girder carrying dead load only.

2. Draw the moment-rotation curve for the girder using the information from step 1 (see Fig. 18).

3. Determine the moment-rotation curves for the columns from Part III of the Design Aids assuming \( q = 1.0 \).

4. Sum the curves (Fig. 18) to obtain the joint moment-rotation curve and determine the maximum resisting moment of the joint when \( \theta = \theta_{ph} \) or when \( \theta \) equals one of the two peak rotations of the moment-rotation curves for the column segments above and below the joint.

The maximum applied joint moment at the centerline of the column can be found by:

\[
M_M = 1.15 M_p + \frac{w L q}{2} \cdot \frac{d_c}{2}
\]  

(27)

where \( M_M \) is the maximum girder moment including strain hardening at the centerline of the column. The applied joint moment includes the effect of strain hardening in the girder under both dead and live load. It has been found that strain hardening tends to increase the maximum plastic moment by approximately 10 to 15\%.(9)
(c) **Interior Columns With Unequal Adjacent Bays**

The procedure described above may also be used in the design of interior columns with unequal adjacent bays. But, in this case, the columns are not always bent in single curvature, and the end moment ratio \( q \) may not be close to -1.0. The variation of \( q \) with the ratios of adjacent span lengths and loads on the adjacent girders can be expressed graphically. The resulting curve (Fig. 19) can be used in design calculations to determine the end moment ratio for a particular column. A more detailed discussion on the design procedure can be found in Ref. 1. The design of a typical interior column is illustrated in Example 9 at the end of this chapter. Using dimensions, loads, and trial members from Examples 1 and 2, the maximum applied joint moments are found by Eq. 27. The moment-rotation curves are found for the column segments using Part III of the Design Aids. The dead load moment, the maximum moment, and the rotation at the maximum moment for the restraining girder are found using Eqs. 22, 24, and 25. The total resisting moment is then found by the procedure given in Art. 6.3(b) and is compared to the maximum applied joint moment. If the applied joint moment is larger, a new column section must be tried.

(d) **Columns in the Braced Bay**

The application of lateral loads causes tension forces to develop in the braces. In each story the vertical component of the bracing force introduces an additional compressive force to the leeward column of the braced bay. These forces accumulate from the top story down in the manner shown in Fig. 20. The columns
designed previously for gravity loading must be checked for their capacities to resist the axial forces and bending moments under combined gravity and lateral loading.

The cumulative compressive force in the leeward column of the i-th story can be computed from the following formula

$$H_{BF_i} = \frac{1}{L} \left[ h_1 (H_1 + \frac{\Delta_1}{h_1} w_1) + h_2 (H_1 + H_2 + \frac{\Delta_2}{h_1} w_1 + \frac{\Delta_2}{h_2} w_2) + \ldots + h_i (\sum H_1 + \frac{\Delta_i}{h_1} \sum w_1) \right]$$

(28)

where $F_{BF_i}$ = column force at the i-th level due to lateral load on the bracing

$\Delta_i$ = chord rotation of the i-th level (can be assumed equal to 0.004)

$w_i$ = the total gravity load acting on the i-level (L.F. = 1.3)

The force $F_{BF_i}$ must be added to the force due to gravity load for the combined load case. The total axial force may exceed the axial force present in the column when gravity load alone is applied to the frame. If this occurs, the column size selected previously for the case of gravity loading is no longer satisfactory, and a larger column must be used. 12

6.4 Design of Columns Unbraced Between Floors

The design procedure for unbraced columns is the same as that for braced columns except for the problem of lateral-torsional buckling which limits the maximum moment that can be applied to
The maximum moment which a column can support for a given axial load can be found from:

\[
\frac{P}{P_0} + \frac{C_m M_1}{M_o \left[1 + \frac{P}{P_e}\right]} = 1.0
\]  

(29)

where

- \( P \) = the applied axial load
- \( P_0 \) = critical axial load which can be supported by the column in the absence of moment (computed about the weak axis)
- \( C_m \) = equivalent moment coefficient = \( 0.6 - 0.4q \geq 0.4 \)
- \( M_1 \) = maximum moment which the column can support
- \( M_o \) = lateral buckling moment in the absence of axial force (can be estimated from Fig. 21)
- \( P_e \) = elastic buckling load in the plane of bending = \( \pi^2 EI/L^2 \)

The rotation \( \theta_{LT} \) at \( M_1 \) must be found from the moment-rotation curve for each column. The rotation \( \theta_{LT} \) is compared to the rotation value \( \theta \) corresponding to the maximum moment on the joint moment-rotation curve obtained by designing the column as if it were braced between floors. If \( \theta_{LT} \geq \theta \) the column size chosen will resist the applied loads. If \( \theta_{LT} < \theta \) the column size chosen can resist the moment corresponding to \( \theta_{LT} \) on the joint moment-rotation curve. Reference 11 has a more detailed discussion on the design of unbraced columns. Sample calculations are shown in Example 10. This example is similar to Example 9 except that there is no out-of-plane bracing between floors. The determination of the maximum moment applied to the joints and of the construction
of the moment-rotation curve for the restraining girder is the same. Using Eq. 29 and the moment-rotation curves, the maximum moments which the column segments can support and the rotations at these moments are found. The total resisting moment is then found by the procedure given in Art. 6.4 and is compared to the maximum applied joint moment. If the applied joint moment is larger, a new column section must be tried.
Example 7

Design Column Segments C5-C6 and C6-C7 for the case of full dead and live loading. The two column segments will be made from the same shape and will be braced in the perpendicular direction against out-of-plane deformation. Use A36 steel.

Lower column controls
From Tables 2 & 4 Example 1
\[ P_{C6-C7} = 597 + 1.7(74.0) = 723 \text{ k} \]

Try 14 WF 74
\[ A = 21.76 \text{ in}^2 \quad r_x = 6.05 \text{ in} \]
\[ h/r_x = 23.8 \]
From Part 1 - Design Aids
\[ \sigma_{cr} = 35.35 \text{ ksi} \]
Required Area = \[ 723/35.35 = 20.5 \text{ in}^2 \]
\[ 20.5 \text{ in}^2 < 21.76 \text{ in}^2 \]
\[ \therefore \text{Trial section satisfactory} \]
Example 8

Design Column Segments D7-D8 and D8-D9 for the checkerboard loading pattern shown. Lateral bracing is provided to prevent out-of-plane deformation. A single section of A36 steel will be used for both segments. (Note: The axial forces given have been adjusted for the removal of live load on Girders C7-D7 and C9-D9).

The joint moments M_D7, M_D8 and M_D9 corresponding to the given dead and live loads on the girders are first computed.
The end moment ratio of the two segments is estimated to be $\frac{M_{D7}}{M_{D8}} = 0.757$

Try 14 WF 74

$h = 144$ in  \quad $r_x = 6.05$ in  \quad $h/r_x = 23.8$

<table>
<thead>
<tr>
<th>Member</th>
<th>$P_{kip}$</th>
<th>$P_y_{kip}$</th>
<th>$P_{kip}/P_y$</th>
<th>$M_{m_{kip-ft}}$</th>
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<tr>
<td>D7-D8</td>
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<td>783</td>
<td>0.706</td>
<td>133</td>
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<tr>
<td>D8-D9</td>
<td>641</td>
<td>783</td>
<td>0.818</td>
<td>82</td>
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$\Sigma M_{m} = 133 + 82 = 215$ kip-ft $\approx 215.4$ kip-ft

\[ M_{D7} = M_{D8} = \frac{w_D L_D^2}{12} + \frac{w_D L_g d_c}{2} \]

\[ = \left(\frac{3.26}{12}\right)(22.8)^2 + \left(\frac{3.26}{2}\right)(22.8) \frac{7}{12} = 163 \text{ kip-ft} \]

\[ M_{D8} = M_p + \frac{w_T L_g d_c}{2} \]

\[ = 182 + \left(\frac{5.02}{2}\right)(22.8) \frac{7}{12} = 215.4 \text{ kip-ft} \]

*This includes a reduction due to a small axial force present in Girder C8-D8.*
Example 9

Column Segments C5-C6 and C6-C7 have been designed for the case of full loading and the 14WF 74 section (A36 steel) has been chosen for both columns. Redesign these columns for the checkerboard loading condition shown below. (Note: The axial forces given have been adjusted for the removal of the live load on Girders C5-D5, B6-C6 and C7-D7).

![Diagram of column segments]

Maximum Moment Applied at Joints

\[
M_{C5} = M_{C6} = M_{C7} = 1.15M_p + \frac{w_T L g}{2} \frac{d_c}{2} \\
= (1.15)(163) + \frac{(5.02)(22.8)}{2} \frac{7}{12} = 221 \text{k-ft}
\]
Moment-Rotation Curves for Columns

Try 14 WF 74

\( P_y = 783 \text{ kip} \)
\( r_x = 6.05 \text{ in.} \)

Column C5-C6
\[ \frac{P}{P_y} = \frac{576}{783} = 0.74 \]
\[ \frac{h}{r_x} \frac{144}{6.05} = 23.8 \]

\[ M_{C5} (\text{k-ft}) \]
\[ \Theta_{C6} (\text{Radian}) \]

Column C6-C7
\[ \frac{P}{P_y} = \frac{680}{783} = 0.87 \]

\[ M_{C7} (\text{k-ft}) \]
\[ \Theta (\text{Radian}) \]

Moment-Rotation Curve for Girder B6-C6

\[ M_D = \frac{w_D L_k^2}{12} + \frac{w_D L_k}{2} \frac{d_c}{2} \]
\[ = \frac{(3.26)(22.8)(22.8)}{12} + \frac{(3.26)(22.8)}{2} \frac{7}{12} \]
\[ = 163 \text{ kip-ft} \]

\[ M_T = M_p + \frac{w_D L_k}{2} \frac{d_c}{2} \]
\[ = 163 + \frac{(3.26)(22.8)}{2} \frac{7}{12} \]
\[ = 185 \text{ kip-ft} \]
The rotation at which $M_T$ is reached is $\Theta_{ph}$

$$\Theta_{ph} = 1.33 \times 10^{-3} \left(1 - \frac{4 \, \text{wD}}{3 \, \text{wT}}\right) \frac{L_g}{d_g}$$

$$= 1.33 \times 10^{-3} \left(1 - \frac{4}{3} \times \frac{3.26}{5.02}\right) \frac{22.8}{1.17}$$

$$= 0.00339 \text{ Radian}$$

![Graph of $M_{CB}$ vs $\Theta_{CG}$](image)

The Total Resisting Moment

![Graph of $M_{Joint}$ vs $\Theta_{CG}$](image)

$$M_R = 307 \text{ k-ft}$$

$307 > 22.1$ \:: \text{trial section satisfactory}

Satisfactory estimates corresponding to the peak rotation of the columns can also be obtained from the joint moment-rotation curve.
Example 10

Column Segments C5-C6 and C6-C7 have been designed for checkerboard loading and the 14WF74 section (A36 steel) was found to be satisfactory. Check these columns for checkerboard loading assuming there is no out-of-plane bracing between floors.

Maximum Moment Applied at Joints

\[ M_{C5} = M_{C6} = M_{C7} = 221\, \text{k-ft} \]

Moment-Rotation Curve for Girder B6-C6

\[ M_D = 163\, \text{k-ft} \quad M_T = 185\, \text{k-ft} \]

\[ \Theta_{ph1} = 0.0039 \, \text{Radian} \]

Moment-Rotation Curves for Columns

Try 14WF74

- Column C5-C6: \( \frac{P}{P_y} = 0.74 \)
- Column C6-C7: \( \frac{P}{P_y} = 0.87 \)

\[
\frac{P}{P_0} + \frac{C_M M_t}{M_0} \left[ \frac{1}{1 - \frac{P}{P_e}} \right] = 1.0
\]
For column C5-C6

\[ P_e = \frac{\pi^2EI}{L^2} = \frac{(3.14)^2(29000)(796.8)}{(12)^4} = 10990 \text{ k} \]

\[ P_0 = A \sigma_{cr} \]

\[ r_y = 2.48 \text{ in.} \quad \frac{h}{r_y} = 144 \quad = 58.1 \]

\[ P_0 = (32.18)(32.18) = 700 \text{ k} \]

\[ \frac{P}{P_e} = \frac{576}{10990} = 0.0524 \]

\[ \frac{P}{P_0} = \frac{576}{700} = 0.823 \]

\[ M_0 = xM_p = 0.95(376.8) = 358 \text{ k-ft} \]

\[ x \text{ is from Fig. 21} \]

\[ 0.823 + \frac{M_1}{358} \left[ \frac{1}{0.9476} \right] = 1.0 \]

\[ M_1 = 60.4 \text{ k-ft} \]

\[ M_{65} \quad 100 \]

\[ (\text{k-ft}) \quad 50 \]

\[ 0 \quad 0.01 \]

\[ \Theta_{c6} \quad \text{(Radian)} \]

For column C6-C7

\[ \frac{P}{P_e} = \frac{680}{10990} = 0.063 \]

\[ \frac{P}{P_0} = \frac{680}{700} = 0.97 \]

\[ 0.97 + \frac{M_1}{358} \left[ \frac{1}{0.937} \right] = 1.0 \]

\[ M_1 = 10.06 \text{ k-ft} \]
The Total Resisting Moment

\[ M_{\theta} (k\text{-ft}) \]

\[ \theta_{c6} (\text{Radian}) \]

\[ M_{\text{joint}} (k\text{-ft}) = 179 \text{ k-ft} \]

\[ \theta_{c6} (\text{Radian}) \]

179 \times 2.21

\[ M_R = 179 \text{ k-ft} \]

\[ \theta_{c6} (\text{Radian}) \]

\[ M_{\text{joint}} (k\text{-ft}) = 179 \text{ k-ft} \]

\[ \theta_{c6} (\text{Radian}) \]

.: The trial section is not satisfactory. The columns should have out-of-plane bracing between floors or a larger section should be used.
7. DEFLECTION AND DRIFT CHECKS

7.1 Beam Deflections

If beam deflections under working loads are excessive, they lead to cracking of plaster or to an objectionable appearance. For these reasons, beam deflections are often checked. It has been found that when the deflection due to live load is approximately less than 1/360 of the span, it leads to no undesirable effects except for possible vibrations. This limitation is, however, a matter of judgement, and the final decision depends on the engineer or the architect. Assuming that the moments at the ends of the beams are equal to the moment at mid-span for a uniformly distributed load, the deflection at working load can be found by:

\[
\delta = \frac{w_w L^4}{192 EI}
\]  

(30)

where \( \delta \) = deflection of the beam at mid-span in inches

\( w_w \) = working live load in kip/ft

In many cases it is sufficient to check the depth-to-span ratio of a girder instead of calculating an approximate deflection. For a given uniformly distributed live load \( w_L \), the deflection of a beam is

\[
\delta = \frac{k_1 w_L L^4}{EI}
\]  

(31)
where \( k_1 \) is a constant incorporating the end restraint. The total load, including the effect of live load, creates a bending stress in doubly symmetrical beams of

\[
\sigma_y = \frac{M_p}{z} = \frac{k_2 w_T L^2 d}{2fI}
\]

(32)

where

- \( k_2 \) = a constant incorporating the end restraint
- \( w_T \) = factored total load
- \( f \) = shape factor

Combining Eqs. 31 and 32 to eliminate \( I \) and assuming conservative values \( 7 k_1, k_2, w_L/w_T, E, f, \) and \( \delta = L/360 \) gives several solutions for \( d/L \) depending on the end conditions used in assuming \( k_1 \) and \( k_2 \). A reasonable value for floor beams in braced multi-story frames is the rounded off \( d/L \) ratio for continuous beams.

\[
\frac{d}{L} = \frac{\sigma_y}{1,000,000}
\]

(33)

The AISC Specification should be followed for beams subject to shock or vibration. For flat roofs there is not yet a completely satisfactory \( d/L \) ratio.

7.2 Frame Drift

The frame drift (sway caused by lateral load) at working loads can also be checked and compared to the amount of drift acceptable for the given type of frame. Drift at working loads can be calculated by considering the following effects:
(1) brace elongation
(2) beam shortening
(3) column elongation and shortening

In considering these effects, it is assumed that only horizontal forces cause deflections (P-Δ effect is neglected) and that only the braced bay contributes to the resistance to the horizontal forces by acting as a pin-jointed truss. Although, the effects causing frame drift act simultaneously, they may be considered separately for ease of derivation and computation with the final answer being the sum of the separate effects.

The drift component calculated from brace elongation is equal to the sway of a rigid pin-connected rectangle (Fig. 22) caused by the horizontal component of the force on the diagonal brace which is equal to the horizontal story shear. In terms of the chord rotation \( R (=\Delta/h) \) of the \( i \)-th story, the drift due to brace elongation is:

\[
R_{bi} = \frac{L_i^2}{A_{bi}Eh_iL_i^2} \sum_{1}^{i} \frac{H_i}{1.30}
\]

where \( H_i \) is the ultimate horizontal shear.

The drift of a pin-jointed rectangle caused by beam shortening due to the horizontal shear force (Fig. 23) can be found in terms of the chord rotation by:

\[
R_{Bi} = \frac{L}{A_{bi}Eh_i} \sum_{1}^{i} \frac{H_i}{1.30}
\]
It is assumed that the whole overturning moment due to the horizontal forces is resisted by the columns of the braced bay. Then the column force is equal to:

\[ F_{ci} = \frac{1}{L} \left[ h_1 H_1 + h_2 (H_1 + H_2) + \ldots + h_i \frac{i}{L} H_i \right] \]  (36)

This force will be compressive in the leeward column and tensile in the windward column. The sway of a pin-connected rectangle, with two vertical members shortening and elongating by amounts consistent with the cumulative couple from all story shears \( F_{ci} \), in terms of the column chord rotation in the \( i \)-th story is equal to:

\[ R_{ci} = \frac{2F_{ci} h_i}{A_{ci} E L} \]  (37)

The lowest story will have no rotation (Fig. 24), and the rotation of the higher stories will consist of the sum of the rotations in the stories below. The areas \( A_{bi}, A_{Bi}, A_{ci} \) used in the above equations should be the actual areas of the members selected. The sum of the horizontal forces \( \Sigma H_i \) was found previously in Chapter 2. The relative rotation at any floor is simply the sum of the contributions due to the three causes. The rotation of any floor is the sum of the relative rotations starting at the bottom:

\[ R_i = R_{bi} + R_{Bi} + R_{ci} \]  (38)

where \( R_i \) is the chord rotation of the \( i \)-th floor.
To satisfy the assumption of Chapter 4, the total rotation at ultimate load must be less than or equal to 0.004h. Other limitations might place more severe restrictions on the total rotation. A frequently used drift index is $\Delta_w = 0.002h$. Based on the initial drift check, the member sizes can be revised to provide appropriate frame stiffness as deemed necessary by the engineer. Since brace elongation usually causes the largest component of drift, bracing members might be the first ones to revise. A drift check for the frame used in the examples of this report is shown in Example 11. The drift due to brace elongation is found using the loads from Table 6 and Eq. 34. Equation 35 is used to find the drift due to beam shortening. The relative effect of column elongation and shortening at each level is found by Eq. 37. The actual effect of changes in column length is found by adding the relative effect at each level starting at the bottom with zero. The drift per level is found by adding the contributions due to the three causes (Eq. 38).
Example II

Determine the story deflection index \( R = \Delta / h \) for the frame used in previous example problems.

**Brace Elongation**

\[
R_{bi} = \frac{L_{bi}^3}{A_{bi} E L^2 h_i} \sum H_i
\]

\[
= \frac{(12^2 + 24^2)^{3/2}}{(3.65)(129 \times 10^3)(24)^2(12)} \sum H_i
\]

\[
= 2.62 \times 10^{-5} \sum H_i
\]

\[
= \frac{(15^2 + 24^2)^{3/2}}{(3.65)(129 \times 10^3)(24)^2(15)} \sum H_i
\]

\[
= 2.48 \times 10^{-5} \sum H_i
\]

<table>
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<tr>
<th>( n )</th>
<th>( \sum H_i )</th>
<th>( R_{bi} \times 10^{-5} )</th>
</tr>
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<td>137.0</td>
</tr>
</tbody>
</table>

**Beam Shortening**

\[
R_{Bi} = \frac{L}{A_{Bi} E h_i} \sum H_i
\]
\[ R_{Bi} = \frac{2.4}{(12 \times 29 \times 10^3)} \frac{\sum H_i}{A_{Bi}} \]

\[ = 6.9 \times 10^{-5} \frac{\sum H_i}{A_{Bi}} \]

\[ = \frac{2.4}{(15 \times 29 \times 10^3)} \frac{\sum H_i}{A_{Bi}} = 5.52 \times 10^{-5} \frac{\sum H_i}{A_{Bi}} \]

<table>
<thead>
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</table>

**Column Elongation and Shortening**

\[ F_{ci} = \frac{h \cdot \sum H_i}{L} = \frac{1}{2} \frac{\sum H_i}{A_{ci}} \]

\[ = \frac{5}{8} \sum \sum H_i \]

\[ R_{ci} = \frac{2 \cdot F_{ci} h_i}{A_{ci} \cdot EL} = \frac{2(12)}{2(24)(29 \times 10^3)} \frac{\sum H_i}{A_{ci}} \]

\[ = 1.723 \times 10^{-5} \frac{\Sigma H_i}{A_{ci}} \]

\[ = \frac{5(15)(2)}{8(24)(29 \times 10^3)} \frac{\sum \sum H_i}{A_{ci}} = 2.155 \frac{\sum \sum H_i}{A_{ci}} \]
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8. DESIGN OF CONNECTIONS

The principal requirements for connections are sufficient strength, adequate rotation capacity, over-all stiffness for maintaining the location of all structural units relative to each other, and economy of fabrication. The principles of design of welded rigid connections for plastically designed structures were well-established with the introduction of plastic design for single story frames.\textsuperscript{1,5,6,18,19,20,21} Relatively few new developments have been introduced in the intervening years. Therefore, work on connections for multi-story frames has consisted of simply identifying the applicable solutions from previous work and making an organized presentation of the necessary facts. The design equations and diagrams for various types of welded connections common to multi-story frames are shown in Table A. Sample design calculations are shown in Example 12. Using the equations given in Table A, three types of welded connections are designed. Part (a) is the design of a straight corner connection at the roof level. Part (b) is the design of an exterior beam-to-column connection and Part (c) is the design of an interior beam-to-column connection.

While all recommendations for the design of welded connections are expected to be amply conservative, current research may lead to further information on connections with conditions peculiar to multi-story frames such as very high column loads.
The use of bolts for all connections in a plastically designed multi-story frame is presently limited because of inadequate knowledge on the behavior of some of the connections. Information on the design of some types of moment resisting joints and splices is available in Refs. 1, 22, 23, 24, and 25.
Example 12

A. Design of a Corner Connection

\[ \theta = \tan^{-1} \frac{16}{13.68} = \tan^{-1} 1.169 \]
\[ \theta = 49.45^\circ \]

Required Web Thickness For Shear

\[ W = \frac{\sqrt{3} M_p}{\sigma_y d_b d_c} = \frac{\sqrt{3} (209.1)(12)}{736(16)(13.68)} = 0.551 \text{ in.} \]

\[ W_{furn} = 0.308 \text{ in.} < 0.551 \text{ in.} \]

\[ \therefore \text{Diagonal Stiffener Required} \]

\[ A_s = \frac{1}{\cos \theta} \left[ \frac{M_p}{\sigma_y d_b} - \frac{Wd_c}{\sqrt{3}} \right] \]

\[ = \frac{1}{0.65} \left[ \frac{(209.1)(12)}{(36)(16)} - \frac{(0.308)(13.68)}{\sqrt{3}} \right] \]

\[ = \frac{1}{0.65} (4.36 - 2.44) = 2.96 \text{ in}^2 \]

\[ \therefore \text{Use } 3 \frac{1}{2} \text{ in. } \times \frac{7}{16} \text{ in. Plates Each Side} \]
B. Design An Exterior Beam-to-Column Connection

Web Thickness to Prevent Web Crippling

\[
W = \frac{A_{bf}}{t_b + 5k_c} = \frac{(7.477)(0.499)}{0.499 + 5(1.125)} = 0.609\text{ in.}
\]

\[
W_{furn} = 0.308\text{ in.} < 0.609\text{ in.}
\]

\[
\therefore \text{Stiffener Required}
\]

\[
A_{st} = A_{bf} - W_c(t_b + 5k_c)
\]

\[
= 3.73 - 0.308(6.124) = 1.86\text{ in.}^2
\]

\[
\frac{b}{t} = 8.5 \quad t_{min} = \frac{b}{8.5} = \frac{3.5}{8.5} = 0.41\text{ in.}
\]

\[
\therefore \text{Use } \frac{7}{16}\text{ in.} \times 3\frac{1}{2}\text{ in. Stiffeners}
\]

\[
A_{st} = 3.06\text{ in.}^2 > 1.86\text{ in.}^2 \quad \therefore \text{OK}
\]
Check Tension Distortion
\[ t_c \geq 0.4 \sqrt{A_{bf}} = 0.4 \sqrt{3.73} = 0.77 \text{in.} \]
\[ t_{c_{furn}} = 0.528 \text{in.} < 0.77 \text{in.} \]
\[ \therefore \text{Stiffener Required} \]
Use Same Size Used for Compression

Required Web Thickness for Shear
\[
W_{\text{req}} = \frac{\sqrt{3} M_3}{\sigma_y d_b d_c} = \frac{\sqrt{3} (268.8 \times 12)}{(36 \times 13.68 \times 17.86)}
\]
\[= 0.635 \text{in.} \]
\[ W_{\text{furn}} = 0.308 \text{in.} < 0.635 \text{in.} \]
\[ \therefore \text{Diagonal Stiffener Required} \]

\[
A_s = \frac{1}{\cos \theta} \left[ \frac{(268.8 \times 12)}{(36 \times 17.86)} - \frac{0.308(13.68)}{\sqrt{3}} \right]
\]
\[= 4.25 \text{in}^2 \]
Use \( \frac{5}{8} \text{in.} \times 3 \frac{1}{2} \text{in.} \) Stiffeners Each Side of Web

C. Design An Interior Beam-to-Column Connection

\[
M_p = 268.8 \text{kip-ft} \quad d = 17.86 \text{in.}
\]
\[
18 \text{WF}45 \quad 14 \text{WF}34 \quad 12 \text{WF}40
\]
\[
d = 14.0 \text{in.} \quad M_p = 163.5 \text{kip-ft}
\]
\[
d = 11.94 \text{in.}
\]
Thickness to Prevent Web Crippling

\[ W = \frac{A_{bf}}{t_b + 5k_c} \]

\[ W_L = \frac{(7.477)(0.499)}{0.499 + 5(1.125)} = 0.61 \text{ in.} > 0.294 \text{ in.} \]

\[ W_R = \frac{(6.75)(0.453)}{0.453 + 5(0.9375)} = 0.596 \text{ in.} > 0.294 \text{ in.} \]

\[ \therefore \text{Stiffeners Required} \]

\[ A_{st_1} = A_{bf} - W_c(t_b - 5k_c) \]

\[ = 3.74 - 0.294(5.625) = 2.09 \text{ in.}^2 \]

\[ A_{st_2} = 3.06 - 0.294(5.141) = 1.55 \text{ in.}^2 \]

\[ \therefore \text{Use two stiffeners} \quad \frac{7}{16} \text{ in.} \times 3\frac{1}{2} \text{ in.} \text{ on each side of the column} \]

Check Tension Distortion

\[ t_c > 0.4 \sqrt{A_{bf}} \]

\[ t_{c_1} = 0.4 \sqrt{3.74} = 0.775 \text{ in.} > 0.516 \text{ in.} \]

\[ t_{c_2} = 0.4 \sqrt{3.058} = 0.701 \text{ in.} > 0.516 \text{ in.} \]

\[ \therefore \text{Stiffener Required} \]

Use Same Size Used for Compression

Check Web Thickness for Buckling

\[ W > \frac{d_c}{30} = \frac{11.94}{30} = 0.398 \text{ in.} > 0.294 \]

\[ \therefore \text{Stiffener Required} \]

Check Web Thickness for Shear

\[ W = \sqrt{\frac{3 \Delta M}{\sigma_y d_b d_c}} = \sqrt{\frac{3 \times (105.3)(12)}{(36)(11.94)(14.0)}} \]

\[ = 0.364 \text{ in.} > 0.294 \text{ in.} \]
Stiffeners Required

0.398 in. > 0.364 in.

Web Buckling Thickness Controls

Add \( \frac{1}{8} \) in. Doubler Plate to Column Web
Adequate braced multi-story frames can be designed in carbon and high strength steels using the procedure presented in this report. The plastic method gives the engineer a more rational design procedure than the current allowable-stress method. The procedure consists of: preparation of design data, preliminary design of members, final design of members, deflection and drift checks, and connection design. This procedure enables the designer to proportion a braced multi-story frame which can resist systems of factored loads of a certain magnitude greater than the working loads. The design is based on an equilibrium solution of the structure in which the plastic strength of the members is not exceeded. Special attention is given to means for assuring the stability of members and of the complete structure.

A final design of the ten-story frame that has been used for the example problems was made using both the allowable-stress method and the plastic method. The results of these designs are shown in Fig. 25 and a weight comparison is shown in Fig. 26. A total material savings of 8%, mainly in the girders, was obtained by the plastic method.

The authors recommend that changes be made in structural steel building design specifications and in building codes to permit application of this method in the design of multi-story buildings. The experimental evidence generated during the preparation of this
method assures that the use of this method is feasible without reducing safety standards below those acceptable in parts of present and past typical construction. It has been revealed that the American Institute of Steel Construction is studying changes to its Specification based on the research cited. Already several buildings are being designed and built under authorized building code variances.

A future paper will present a similar condensed report on the plastic design of unbraced multi-story frames.
NOMENCLATURE

\[ A \] = Area--subscripts \( c \) and \( s \) denote concrete and steel areas, respectively

\[ A_{Bi} \] = Area of beam in the \( i \)-th level

\[ A_{bi} \] = Area of bracing member in the \( i \)-th story

\[ A_{br} \] = Required area of bracing member

\[ A_{sr} \] = Area of reinforcing steel in negative moment region

\[ A_w \] = Area of the web

\[ a \] = Distance from neutral axis to top concrete fiber in composite member design

\[ b \] = Width of flange

\[ = \text{Effective width of concrete} \]

\[ C \] = Total compressive force on the concrete

\[ C' \] = Compressive force on the steel

\[ C_{1} \] = Compressive force to be resisted by studs in the negative moment region

\[ C_m \] = Equivalent moment coefficient (see Eq. 28)

\[ d \] = Depth of section--subscripts \( c \) and \( g \) denote column and girder depths, respectively

\[ E \] = Modulus of elasticity--subscripts \( c \) and \( s \) refer to concrete and steel moduli, respectively

\[ E_{st} \] = Strain-hardening modulus

\[ e \] = Moment arm in composite design

\[ F_{BF} \] = Force in column which equals vertical component of the bracing force
\( F_c \) = Force in a column
\( f \) = Shape factor
\( f'c \) = Compressive strength of concrete at 28 days
\( G \) = Modulus of elasticity in shear
\( H_i \) = Horizontal load at the i-th level
\( H_{Fi} \) = Horizontal shear resistance of a frame at the i-th level
\( h \) = Story height
\( I \) = Moment of inertia—subscript \( s \) refers to steel
\( I_{tr} \) = Transformed moment of inertia of composite section
\( k \) = Effective length factor (see Eq. 7)—subscripts \( cj \) and \( Bj \) refer to columns and beam in the i-th story and level, respectively
\( kM_p \) = Column plastic moment (see Fig. 2)
\( L \) = Span length from centerline of column to centerline of column
\( L_L \) = Distance left from bracing member in question to next bracing member
\( L_R \) = Distance right from bracing member in question to next bracing member
\( L_a \) = The larger of \( L_L \) and \( L_R \)
\( L_b \) = Length of bracing member
\( L_{cr} \) = Critical length between bracing
\( L_g \) = Clear span length of a girder
\( l \) = Length of the positive moment region
M = Bending moment--subscripts 1 and 2 refer to column end moments where M₁ is the larger in absolute value

Mₘ = Maximum column end moments at a joint--subscripts a and b denote moment from the column above and the column below, respectively

M₃ = Girder moment at centerline of column due to factored dead load only

M₆ = Girder moment at the centerline of column due to factored full load

M₇ = Maximum girder moment including strain hardening at the centerline of column

Mₐ = Lateral buckling moment

Mₚ = Plastic bending moment

Mₚₑ = Plastic column moment--reduced for axial load

Mₜ = Maximum resisting moment provided by the restraining girder

Mₜᵤ = Ultimate moment

N = Number of shear connectors

n = Modular ratio (Eₛ/Eₜ)

P = Axial load

Pᵥₑ = Critical axial load

Pᵥₑ = Elastic buckling load

Pᵥₙ = Axial force in girder

Pᵥₒ = Maximum axial load when no moment is present

Pᵥₚ = Axial yield force
q = Column end moment ratio
q_u = Ultimate load per shear connector
R = Chord rotation = Δ/h--subscripts b, B, and c refer to the chord rotation due to brace elongation, beam shortening, and column elongation and shortening, respectively
r = Radius of gyration--subscript b denotes a bracing member
T = Total tensile force on rolled steel member
t = Flange thickness
V = Shear force--subscripts l and r denote shear forces to the left and right of a joint, respectively
V_m = Maximum shear force
w = Uniformly distributed load
= Web thickness
w_D = Uniformly distributed factored dead load
w_F = Full factored load
w_P = Transverse load capacity in the absence of axial load
w_w = Working live load
Z = Plastic modulus
γ = Ratio of end moments on the braced segment
δ = Deflection of beam
Δ = Lateral deflection of a story
ε_y = Strain at yield point
θ_p_hl = Rotation of the windward end of a girder at which a plastic hinge forms at leeward end of the girder in a two bay frame
\( \theta_{ph2} \) = Rotation of the windward end of a girder at which a plastic hinge forms at leeward end of the girder in a frame of three or more bays

\( \tau_y \) = Yield stress in shear

\( \nu \) = Poisson's ratio

\( \sigma_{cr} \) = Critical stress

\( \sigma_y \) = Yield point stress
Composite construction can be used to advantage in multi-story frame design where savings can be obtained in the weight of steel and in construction depth. The stiffness of the floor system is also increased. The ultimate capacity of a composite beam is reached when full plastification of the cross section occurs. Full plastification is possible only if the shear connectors between the slab and the steel beam are sufficiently strong. Stud connectors will be considered here since they are most commonly used. The effective width to be used in the plastic design of composite sections is the approximation recommended by AISC Specification 1.11.1.

**Moment Capacity**

Full plastification of the composite section can occur in two possible modes: with the neutral axis in or with the neutral axis below the concrete slab. The location of the neutral axis (Fig. 27) must be known in order to determine which mode of plastification is present.

The neutral axis is located by the following equation derived from equilibrium between the maximum possible tensile force in the steel and a compressive force in the concrete slab.
\[ a = \frac{A_s \sigma_y}{0.85 f'_c b} \]  \hspace{1cm} (A1)

where

- \( a \) = distance of the neutral axis from the top surface of the concrete slab
- \( A_s \) = area of steel beam
- \( b \) = effective width of concrete slab
- \( f'_c \) = compressive strength of concrete at 28 days

If "\( a \)" is equal to or less than the slab thickness, it gives the correct location of the neutral axis and all quantities are calculated according to Model 1 below. If "\( a \)" is greater than the slab thickness, the neutral axis falls within the steel beam and all quantities are calculated according to Mode 2 below.

**Mode 1 - neutral axis in concrete slab**

\[ C = 0.85 f'_c b a \]  \hspace{1cm} (A2)

\[ T = A_s \sigma_y \]

\[ C = T \]

\[ M_u = T e = T \left[ \frac{d-a}{2} + t \right] \]  \hspace{1cm} (A3)

**Mode 2 - neutral axis below concrete slab**

The neutral axis lies at the bottom edge of the steel area in compression. The areas in tension and compression can be determined from the fact that the tension force must be half the combined capacity for tension and compression of the system.
\[ C = 0.85 f'_c bt \]  
\[ T = \frac{1}{2} (0.85 f'_c bt + A_s \sigma_y) \]  
\[ C' = T - C = \frac{1}{2} (A_s \sigma_y - 0.85 f'_c bt) \]  
\[ M_u = C' + C'e'' \]  
(See Fig. 27)  

Values of e' and e'' depend on the shape of the steel section and must be determined by considering the properties of the particular composite beam. Fortunately, the neutral axis falls in the slab most of the time and the more complex calculations of Mode 2 are unnecessary.

Terms used in the equations above are defined as follows:

- \( C \): compressive force in concrete slab
- \( C' \): compressive force in steel beam
- \( d \): depth of symmetrical steel section
- \( e \): moment arm between centroid of steel area in tension and centroid of concrete area in compression
- \( e' \): moment arm between centroid of concrete slab and centroid of steel area in tension
- \( e'' \): moment arm between centroids of steel areas in tension and compression
- \( t \): concrete slab thickness
- \( \sigma_y \): yield point stress of the steel beam
- \( M_u \): ultimate moment
Shear Connectors

The shear connectors act over the full length of the beam. In regions of positive moment, the force on the connectors is the compressive force $C$ on the concrete as found in Eq. A2 or A5. The horizontal shear force in the negative moment region which must be resisted by the studs is equal to the yield strength of the longitudinal reinforcement of the slab. This force can be found by:

$$C_1 = A_{sr} \sigma_{sr}$$  \hspace{1cm} (A6)

where

$C_1$ = compressive force to be resisted by studs in the negative moment region

$A_{sr}$ = area of reinforcing steel in negative moment region

$\sigma_{sr}$ = minimum yield point of the reinforcing steel

The length of the positive moment region can be found by:

$$l = L_g \sqrt{\frac{M_u}{M_u + M_p}}$$  \hspace{1cm} (A7)

where

$l$ = length of the positive moment region

$L_g$ = clear span length of the girder

$M_p$ = plastic moment of the steel member

The number of studs required is given by:

$$N = \frac{C}{q_u}$$  \hspace{1cm} (A8)
where $q_u$ is the ultimate capacity of a single connector in pounds. For design purposes the ultimate moment capacity of a stud connector can be found by:

$$930 \frac{d_s^2}{f_c^t}$$  \hspace{1cm} (A9)$$

where

- $d_s =$ diameter of stud in inches
- $f_c^t =$ compressive strength of concrete in psi

If longitudinal reinforcing steel is placed in the slab to help resist negative moments and the force in the reinforcing steel is small, then stud connectors should be placed in the negative moment region at approximately the same spacing as was used in the positive moment region. The use of the above equations is illustrated in Example A1 which is the design of girders on the second level of a frame as composite beams. The mode of plastification with the neutral axis in the concrete (Mode 1) is selected as the condition for design. Using Eq. A3 the ultimate moment is determined. The shear connectors are then designed using Eqs. A7, A8, and A9.

**Deflection of Composite Beams**

Deflection can be checked in composite beams to see that practical and aesthetic considerations are not exceeded. It has been found that when deflection due to live load is approximately less than $1/360$ of the span, it leads to no undesirable effects except for possible vibration. What constitutes excessive deflection though is up to the judgement of the engineer.
When checking the deflection of composite beams, deflection due to the following causes should be considered. Permanent deflection or creep will be caused by dead load plus any part of the live load which will remain on the structure most of the time. Shrinkage of the concrete will also cause permanent deflections in most beams. Short term deflection is caused by the remaining live load, the load which goes on and off the structure frequently. Shrinkage and creep deflection can be calculated by the simple procedures given in References 13 and 28. Composite beams with their preponderence of flange and relatively small web may exhibit considerable shear deflection due to loads and shear deflection should therefore be considered.

To calculate these deflections for composite beams, the first essential is to obtain a transformed moment of inertia in terms of one of the materials, in this report, steel. Two values of the transformed moment of inertia are generally determined. One is based on a modular ratio \( n \) of the steel-to-concrete for use in short term live load deflection calculations. The second uses a modular ratio with three times \( n \) replacing \( n \) to give a smaller value of \( I_{tr} \) for calculating the more severe deflection resulting from creep under long term loading. The transformed moment of inertia can be found by:

\[
I_{tr} = I_s + \frac{ba^3}{3n} + A_s \left( \frac{d}{a} + t - a \right)^2
\]  \hspace{1cm} (A10)
where 
\[ I_{tr} = \text{transformed moment of inertia of the composite section} \]
\[ n = \text{modulus of elasticity ratio} = \frac{E_s}{E_c} \]
\[ I_s = \text{moment of inertia of the steel beam} \]
\[ a = \sqrt{\frac{n A_s}{b} \left[ \frac{n A_s}{b} + d + 2t \right]} - \frac{n A_s}{b} \quad (A11) \]
\[ E_s = \text{modulus of elasticity of steel} \]
\[ E_c = \text{modulus of elasticity of concrete} \]

The second essential is to consider the appropriate boundary conditions and to use the transformed moment of inertia to obtain formulas for the bending deflection due to each cause. The following deflection equation assumes that the moments at the beam ends are equal to the moment at midspan.

\[ \delta = \frac{w_w L^4}{192 E I_{tr}} \quad (A12) \]

where 
\[ \delta = \text{deflection} \]
\[ w_w = \text{factored working load} \]

When this equation is used with the proper values of \( w_w \) and \( I_{tr} \), it can give good approximate values for short term live load deflection \( \delta_{st} \), total live load deflection \( \delta_L \), and long term or creep deflection \( \delta_{LT} \).

In the calculation of shear deflection, the shear may be assumed to be fully carried by the web of the steel section and uniformly distributed across the depth. For uniformly distributed
loading on beams with moment diagrams symmetrical about the beam centerline, the following equation for shear deflection $\delta_s$ is applicable:

$$\delta_s = \frac{w_w L^2}{8GA_w}$$  \hspace{1cm} (A13)

where
- $G$ = shear modulus of steel = $E/(1+v)$
- $A_w$ = area of the web of the steel beam

Shrinkage of concrete which is restrained by the connectors and the steel beam sets up a force in the concrete which is eccentric to the neutral axis of the combined section. This eccentric force causes a bending moment and curvature in the beam which results in a deflection. The expression for the centerline deflection of the composite beam for a constant shrinkage moment throughout its length is given by:

$$\delta_{sh} = \frac{ML^2}{8Es_{tr}}$$  \hspace{1cm} (A14)

where
- $\delta_{sh}$ = deflection due to shrinkage of concrete
- $M = P \cdot e$
- $e = a - t/2$
- $P = bt \cdot \varepsilon_{sh} E_c$

Deflection calculations for composite beams are illustrated in Example A2 at the end of this Appendix. Deflections are computed for the composite girders designed in Example A1.
As a special case in Example A2, half of the live load is assumed to remain on the structure long enough to be considered in the calculation of long term deflection or creep. The remaining half of the live load is assumed to be transient and cause the typical short term deflection.

In the example the section properties for short term deflection are calculated using Eqs. A10 and A11. The short term deflection caused by one half the live load is then calculated using Eq. A12. For comparison with the live load deflection limits, the deflection for full live load is calculated using the same properties and equation. In this case it equals twice the short term deflection.

The second stage of Example A2 includes the calculation of the long term deflection properties by Eqs. A10 and A11 plus the deflection by Eq. A12. Deflection due to shear is calculated for the total load using Eq. A13, and deflection due to an assumed shrinkage strain is calculated using Eq. A14.

The total deflection is the sum of the short term and the long term bending deflections due to dead and live load plus the deflections due to shear and shrinkage. It is shown that the deflection due to shear is 19 percent of the total indicating that shear deflection rates consideration in composite beams of these proportions.
Example A1

Design girders as composite beams \((L.F. = 1.7)\) for level shown. Use \(\sigma_y = 36 \text{ ksi}\), slab thickness \(t = 5\text{ in.}\), \(f_c' = 3 \text{ ksi}\).

Effective Width \(b = L_g / 4\) or \(b = (2 \times 8) t\)

Required \((M_p + M_u) = \frac{wL_q^2}{8}\)

The tabulation below is set up for Mode I (Eq. A3) that is, \(a < t\). If \(a > t\), use Eq. A5.

### Design of Composite Beams

<table>
<thead>
<tr>
<th></th>
<th>(L_g)</th>
<th>(w)</th>
<th>(b)</th>
<th>(M_u + M_p)</th>
<th>Section</th>
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<td>in</td>
<td>kip-in.</td>
<td></td>
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<tr>
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<td></td>
<td></td>
<td></td>
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</tr>
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<td>56.6</td>
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</tr>
<tr>
<td>B - C</td>
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<td>8.04</td>
<td>32.5</td>
<td>1785</td>
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<td>Units kip-in.</td>
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<td>2300</td>
<td>6870</td>
</tr>
</tbody>
</table>

Design connectors for Girder A-B

Distance between inflection points (length of positive moment diagram):

\[
\lambda = L \sqrt{\frac{M_u}{M_u+M_p}} = 18.86 \sqrt{\frac{3025}{4461}}
\]

\[
= 15.52 \text{ ft.}
\]

Number of studs for half \( \lambda \)

Use \( \frac{5}{8} \) in. studs \( q_u = 29.4(0.625)^2 \sqrt{3} = 20 \) kip

\[
N = \frac{1}{q_u} = \frac{275}{20} = 13.5 \text{ use 14}
\]

Spacing of studs = \( \frac{1}{N} \left[ \frac{\lambda}{2} \right] = \frac{(7.76 \times 12)}{14} = 6.65 \text{ in.} \)
Negative Moment Region

if longitudinal reinforcing used

\[ N = \frac{(9.43 - 7.76)(12)}{6.65} \approx 3 \text{ studs} \]
**EXAMPLE A.2**

Compute the total working load deflection for Girder A2-B2 designed as a composite beam (shored construction).

Data: \( L_g = 18.80 \text{ ft}, \ b = 50.0 \text{ in}, \ t = 5 \text{ in} \).

\( f_c = 3 \text{ ksi} \)

Short term loading, \( E_c = 32000 \text{ ksi}, \ n = 9 \)

Long term loading (creep) \( n = 3n = 27 \)

Shrinkage strain \( \varepsilon_{sh} = 0.00016 \)

\[
\begin{align*}
\text{Working loads} & \quad \text{Dead} \ W_d = 2.88 \text{ k/ft} \\
& \quad \text{Live} \ W_L = 1.48 \text{ k/ft} \\
& \quad \text{Total} \ W = 4.36 \text{ k/ft}
\end{align*}
\]

Steel section: \( 14 B 20 \) (\( d_s = 7.05 \text{ in}, \ I_s = 242.8 \text{ in}^4, \ a = 13.89 \text{ in} \))

In deflection computation assume constant composite cross section over the full length of girder.

Short term deflection

Loading: \( w + \frac{1}{2} W_L = 0.74 \text{ k/ft} \)

Section properties

\[
a = \sqrt{\left(\frac{9(7.05)}{50.0}\right)\left(\frac{9(7.05)}{50.0} + 13.89 + 2(5)\right) - \frac{9(7.05)}{50.0}} = 4.3 \text{ in} \cdot 5 \text{ in}.
\]

\[
I_{tr} = 242.8 + 50.0(4.3)^3 + 7.05 \left(\frac{13.89 + 5 - 4.3}{2}\right)^2 = 2300 \text{ in}^4
\]

Deflection

\[
\delta_{st} = \frac{1}{192} \cdot 0.74(18.80 \times 12)^4 \frac{4}{2} = 0.0123 \text{ in.}
\]

Live load deflection: \( \delta_L = 2 \delta_{st} = 2(0.148) = 0.296 \text{ in.} < \frac{18.30}{360} = 0.051 \text{ in.} \)

Long term deflection (creep)

Loading: \( w = W_d + \frac{1}{2} W_L = 2.88 + \frac{1}{2}(1.48) = 3.02 \text{ k/ft} \)

Section properties

\[
a = \sqrt{\left(\frac{27(7.05)}{50.0}\right)\left(\frac{27(7.05)}{50.0} + 13.89 + 2(5)\right) - \frac{27(7.05)}{50.0}} = 0.38 \text{ in} \cdot 5 \text{ in}
\]

Neutral axis is in the steel beam and the following equation is derived for \( I_{tr} \)

\[
I_{tr} = I_s + \frac{tb}{n} \left[ \frac{t^2}{12} + \left(\frac{d_s t}{2}\right)^2 \right] \frac{1}{1 + \frac{t}{h} / A_s}
\]
\[ I_{tr} = 242.6 + \frac{5(5G.6)}{27} \left[ \frac{3^2}{2} + \left( \frac{13.89+5}{2} \right)^2 \right] = 57.6 \text{ in}^4 \]

Deflection:
\[ \delta_{Le} = \frac{1}{192} \cdot \frac{9.02(18.86 \times 12)^4}{29(10)^4(57.6)/12} = 0.217 \text{ in.} \]

**Shrinkage deflection**

Use: \( n = 9 \)

\[ e = a \cdot \frac{L}{2} = 4.3 \cdot \frac{8}{2} = 1.8 \text{ in.} \]

\[ M = 56.6(5)(0.00010)(200)(1.8) = 261 \text{ k-in.} \]

For the given \( M \)-diagram,
\[ \delta_{sh} = \frac{M L g^2}{8 E_s I_{tr}} = \frac{261(18.86 \times 12)^2}{8(29)(10)^4(260)} = 0.0244 \text{ in.} \]

**Deflection due to shear**

Loading: \( w = W_0 + W_L = 4.3G \, 4/ft. \)

\[ G = \frac{E}{2(1+\nu)} = \frac{E}{2(1+0.3)} = \frac{E}{2.6} \]

\[ \text{Web area } A_w = 13.89(0.255) \]

\[ e = \frac{d}{2} \frac{L}{2} \]

\[ M \cdot e = \frac{M L g^2}{8 E_s I_{tr}} = \frac{261(18.86 \times 12)^2}{8(29)(10)^4(260)} = 0.0244 \text{ in.} \]

\[ \delta_{s} = \frac{4.36(18.86 \times 12)^2}{8(29)(10)^4(13.89)(0.255)(0.3)} = 0.059 \text{ in.} \]

**Maximum total deflection**

\[ \delta_{total} = \delta_{s} + \delta_{sh} + \delta_{s} + \delta_{s} = 0.023 + 0.217 + 0.0244 + 0.059 = 0.313 \text{ in.} \]

shear deflection contributes \((0.059/0.313) \cdot 100 = 19\% \)
Table A

<table>
<thead>
<tr>
<th>Formulas and Explanatory Notes</th>
<th>Details of Connections</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Corner Connections</strong></td>
<td></td>
</tr>
<tr>
<td>Required Web Thickness for Shear</td>
<td></td>
</tr>
<tr>
<td>$W = \frac{\sqrt{3} M_p}{\sigma_y d_b d_c}$</td>
<td>![Corner Connections Diagram]</td>
</tr>
<tr>
<td>Diagonal Web Stiffener</td>
<td></td>
</tr>
<tr>
<td>$A_s = \frac{1}{\cos \theta} \left[ \frac{M_p}{\sigma_y d_b} - \frac{W_{d_0}}{\sqrt{3}} \right]$</td>
<td>![Corner Connections Diagram]</td>
</tr>
<tr>
<td>Alternatively use Web Doubler Plates to Provide the Required Web Thickness</td>
<td></td>
</tr>
<tr>
<td><strong>Exterior Beam-to-Column Connections</strong></td>
<td></td>
</tr>
<tr>
<td>Required Web Thickness of Column to Prevent Web Crippling</td>
<td>![Exterior Beam-to-Column Connections Diagram]</td>
</tr>
<tr>
<td>$W = \left[ \frac{A_{bf}}{t_b + 5k_c} \right] \sigma_y b$</td>
<td></td>
</tr>
<tr>
<td>Flange Stiffener Thickness When Required for Web Crippling</td>
<td>![Exterior Beam-to-Column Connections Diagram]</td>
</tr>
<tr>
<td>$A_{st} = \left[ A_{bf} \cdot \phi_c (t_b + 5k_c) \right]$</td>
<td></td>
</tr>
<tr>
<td>Required Column Flange Thickness (Tension Distortion)</td>
<td>![Exterior Beam-to-Column Connections Diagram]</td>
</tr>
<tr>
<td>$t_c \geq 0.4 \sqrt{A_{bf} \cdot \sigma_y b}$</td>
<td></td>
</tr>
<tr>
<td>Formulas and Explanatory Notes</td>
<td>Details of Connections</td>
</tr>
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<td>--------------------------------</td>
<td>------------------------</td>
</tr>
<tr>
<td><strong>Exterior Beam-to-Column Connections (Cont.)</strong></td>
<td><img src="image1.png" alt="Diagram" /></td>
</tr>
<tr>
<td>Required Web Thickness for Shear [ W = \frac{\sqrt{3} M_3}{\sigma_y d_b d_c} ]</td>
<td></td>
</tr>
<tr>
<td>Diagonal Web Stiffener for Shear [ A_s = \frac{1}{\cos \theta} \left( \frac{M_3}{\sigma_y d_b} - W \frac{d_c}{\sqrt{3}} \right) ]</td>
<td></td>
</tr>
</tbody>
</table>

**Interior Beam-to-Column Connections**

| Required Web Thickness of Column to Prevent Web Crippling: \[ W = \frac{A_{bf} \sigma_{yb}}{t_b + 5k_c} \frac{d_c}{\sigma_{yc}} \] |
| Required Web Thickness to Prevent Column Web Buckling: \[ W \geq d_c / 30 \] |
| Flange Stiffener Thickness When Required for Web Crippling: \[ A_{st} = \left[ A_{bf} \cdot W_0 \left( t_b + 5k_c \right) \right] \] | ![Diagram](image2.png) |
Interior Beam-to-Column Connections (Cont.)

Required Column Flange Thickness (Tension Distortion)

\[ t_c \geq 0.4 \sqrt{A_{bf} \frac{\sigma_{yb}}{\sigma_{yc}}} \]

Required Web Thickness for Shear

\[ W = \sqrt{3} \frac{\Delta M}{\sigma_{yd_b} d_c} \]
\[ \Delta M = M_1 + M_2 \ (\text{+ Clockwise}) \]

Diagonal Web Stiffener for Shear

\[ A_s = \frac{1}{\cos \theta} \left[ \frac{\Delta M - W y_d}{\sigma_{yd_b} \sqrt{3}} \right] \]

Welds (Maximum Load)

Fillet Welds (Taken at Throat)

\[ \tau_{\text{max}} = 24.4 \text{ ksi E60 Electrodes} \]
\[ \tau_{\text{max}} = 26.9 \text{ ksi E70 Electrodes} \]

\[ \sigma = 1000 \text{ Lb/in.} / 16^{th} \]

\[ \sigma = 1200 \text{ Lb/in.} / 16^{th} \]

Tension on Butt Welds = \( \sigma_y (\text{Base Metal}) \)
### Formulas and Explanatory Notes

<table>
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<tr>
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<tr>
<td><strong>Bolts (Maximum Load)</strong></td>
</tr>
<tr>
<td>A 325 Bolts</td>
</tr>
<tr>
<td><strong>Shear (Friction-Type)</strong></td>
</tr>
<tr>
<td>$\tau_{\text{Max}} = 20$ ksi</td>
</tr>
<tr>
<td><strong>Shear (Bearing-Type)</strong></td>
</tr>
<tr>
<td>$\tau_{\text{Max}} = 68$ ksi</td>
</tr>
</tbody>
</table>

**Bearing \leq Tensile Strength**

Tension = Bolt Proof Load $\times 1.15$

Combined Tension and Shear (Bearing)

$(\text{Shear})^2 + \alpha^2 (\text{Tension})^2 = \alpha^2 (1.1 PL)^2$

- $\alpha = 0.8$ Shear Plane Through Shank
- $\alpha = 0.65$ Shear Plane Through the Threads

<table>
<thead>
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</thead>
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<tr>
<td><strong>Shear (Friction-Type)</strong></td>
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<td>$\tau_{\text{Max}} = 68$ ksi</td>
</tr>
</tbody>
</table>

Bearing $\leq$ Tensile Strength
### Formulas and Explanatory Notes

**Bolts (Continued)**

Tension = Bolt Proof Load

Combine Tension and Shear

\[(\text{Shear})^2 + \alpha^2(\text{Tension})^2 = \alpha^2(PL)^2\]

- \(\alpha = 0.75\) Shear Plane Through Shank
- \(\alpha = 0.55\) Shear Plane Through Threads
Fig. 1 Moment Diagram for Mechanism Forming in Clear Span of Girder

Fig. 2 Roof Girder Framing Into Weak Column
Fig. 3 A Three-Bay, n-story Braced Frame

Fig. 4 Axial Forces in a Brace
Fig. 5 Assumed Forces on Girder in Braced Bay

Fig. 6 Interaction Curve for Fixed-End Beam-Columns
\[ \gamma = \text{End-Moment Ratio} \]

**Fig. 7 Definition of End-Moment Ratio**

**Fig. 8 Bracing Details**
Fig. 9a. Strong Axis Buckling of a Column

b. Weak Axis Buckling of a Column
Fig. 10 In-Plane Bending of a Column

Fig. 11 Lateral-Torsional Buckling of a Column
Fig. 12 Effect of Lateral-Torsional Buckling

Fig. 13 Beam-Column Bent in Double Curvature
Fig. 14 Forces Acting on a Beam-to-Column Connection

Fig. 15 Influence of Axial Force, Slenderness Ratio and End Moment Ratio on the Strength of Beam-Columns
(a) Deformed configuration assumed in developing moment-rotation relationship

(b) Dead load on all girders

(c) After application of checkerboard load

(d) Pinned Moment = $M_p$

Fig. 16 Deformed Configuration of Girders

Fig. 17 Subassemblage for Use in Designing Interior Columns Under Checkerboard Loading
Fig. 18 Determination of Maximum Resisting Moment of a Joint

Fig. 19 Selection of $q$ values in design of Columns With Unequal Adjacent Bays
Fig. 20 Axial Forces Applied to Columns in Braced Bay by Lateral Loads

Fig. 21 Lateral Buckling Moment for Uniform Bending
Fig. 22 Brace Deformations

Fig. 23 Effect of Beam Shortening
Plastic and Allowable Stress Methods

**Fig. 25** Member Sizes of the Example Frame Designed by

<table>
<thead>
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<th>Allowable Stress Design</th>
<th>Plastic Design</th>
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<tbody>
<tr>
<td>14WF127</td>
<td>14WF19</td>
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<tr>
<td>14WF13</td>
<td>14WF195</td>
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<tr>
<td>14WF19</td>
<td>14WF74</td>
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<td>14WF68</td>
<td>14WF61</td>
</tr>
<tr>
<td>14WF48</td>
<td>14WF43</td>
</tr>
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**Fig. 24** Effect of Column Deformation
Fig. 26 Weight Comparison for the Example Frame

Fig. 27 Stress Distribution in Composite Beam at Plastic Moment
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