Welded Continuous Frames and Their Components

TESTING TECHNIQUES FOR RESTRAINED BEAM-COLUMNS

by

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I. INTRODUCTION

a) Background

In recent years considerable advances have been made toward a clear understanding of the behavior of structural frames. Analytical methods have been developed which allow the load-deformation behavior of beam-columns to be predicted throughout their deformation history.\(^1\) Experimental studies have confirmed the theoretical results and consequently the prediction of the behavior of individual beams-columns no longer presents a serious problem.\(^2,3\) The same techniques have been available for beams for a number of years,\(^4\) and recent experimental investigations have delineated the necessary requirements for ensuring that this behavior is obtained.\(^5,6\)

As the responses of the individual components of a steel frame are now well known, there exists a rational basis for the design of such frameworks. Recent theoretical investigations of frame behavior have utilized the member studies mentioned above,\(^7,8\) and it is now necessary to verify experimentally the resulting predictions of frame behavior.
This report will discuss a testing arrangement designed to investigate the load-deformation behavior of structural assemblages composed of beams and restrained beam-columns. The purpose of the tests is to verify the proposed frame analysis methods. The tests utilize frames of practical materials and dimensions.

b) Experimental Conditions

In a study of this nature it would be both inconvenient and irrelevant to test an entire structural frame or assemblage. Thus some typical subassemblage must be chosen such that it provides the required experimental conditions and yet is capable of direct extrapolation to practical structural assemblages. It has been shown theoretically that the structure in Fig. 1 represents a valid structural subassemblage. From this building block a structural framework can be analytically constructed, and the analytical behavior of the subassemblage can be predicted by the methods discussed above.

c) Theoretical Bases for Subassemblage Behavior

If the joints of a structure remain in their original
position throughout the loading process, a frame is considered to be perfectly braced (Fig. 2a). If the joints translate, as well as rotate, relative to each other, the term "unbraced frame" will be used in this report (Fig. 2b). Actually, any real frame would be an unbraced frame by this definition.

In a braced frame, the joint equilibrium equations formulated on the undeformed structure remain valid under all loading conditions. The end-moment end-rotation curves for beams and beam-columns may be obtained using the step-by-step methods of calculation throughout the elastic and elasto-plastic ranges.\(^1,4\) For a group of such members meeting at a joint of a braced frame, the joint resisting moment \(M_J\) at a given joint rotation, \(\theta_J\), is given by (Fig. 3a)

\[
M_J (\theta_J) = \sum_{\text{Joint}} M_M (\theta_J)
\]  

(1)

where \(M_M (\theta)\) is the end-moment in a member at an end-rotation \(\theta\). Equation (1) is based on the assumptions of compatibility and equilibrium. In Fig. 3b the top two graphs are typical \(M_M (\theta)\) curves while the bottom graph is derived using Eq. (1).

In an unbraced frame the situation is shown in Fig. 4. The end-rotations are now functions of the amount
of joint translation. If the joint translation angle is \( \alpha \) then the member rotations \( \theta \) are given by:

\[ \theta = \theta_J - \alpha \]  

(2)

In Fig. 4, \( \alpha \) is given as:

\[ \alpha = \frac{\Delta i+1 - \Delta i}{h_i} \]  

(3)

for the columns, and \( \alpha = 0 \) for the beams.

Therefore Eq. (1) for the unbraced frame becomes:

\[ M_J (\theta_J) = \sum_{\text{Joint}} M_M (\theta_J - \alpha) \]  

(4)

Solutions of Eq. (4) yield \( M_J \) as function of \( \theta_J \) and \( \alpha \). The maximum value of \( M_J \) is found by differentiation (when \( dM_J = 0 \)) and is determined by the relationship

\[ \sum_{\text{Joint}} \frac{\partial M_M(\theta)}{\partial \theta} \left( 1 - \frac{\partial \alpha}{\partial \theta_J} \right) = 0 \]  

(5)

For a braced frame this becomes

\[ \sum_{\text{Joint}} \frac{\partial M_M(\theta)}{\partial \theta} = 0 \]  

(6)

d) Aim of Tests

In section (b) it was stated that the aim of the tests was to verify the load-deformation behavior of structural subassemblages containing beams and restrained beam-
columns. The particular problems requiring verification are as follows:

(1) Are the available theoretical relationships between $M_M$ and $\theta$ valid throughout the loading range of the member?

(2) Does the compatibility-equilibrium equation (Eq. (4)) correctly predict the complete joint moment-rotation curve?

(3) Does the gradient equation (Eq. (5)) correctly predict the maximum moment capacity of a joint?

Problem (1) is to verify that the existing load-deformation curves are valid when the members involved are part of a real structure. The curves had previously been verified only for isolated members. Problems (2) and (3) depend to some extent on problem (1), however, there are additional ramifications. The use of Eq. (6) to predict load capacity would supersede a variety of criteria which are presently in use. These range from the simple use of the gradient criterion for an individual member

$$\frac{dM_M(\theta)}{d\theta} = 0$$  (7)
rather than the summations of Eqs. (5) and (6), to the more sophisticated procedures given in the current AISC Specifications. In the latter case there are provisions for calculating effective lengths and for considering column curvature configurations. The improved techniques provided by Eqs. (5) and (6) will normally result in some members being used into their unloading range in order that the summation of gradients can be zero. The question arises as to whether this range of behavior is inherently unstable and whether it can be used in a stable structure. If the range is stable there is no existing assurance that the present member curves adequately represent this range.

Finally, there is the question of the sensitivity of the failure criterion (Eq. (6)). While correct mathematically, it may be overshadowed by other effects in a real structure; for instance local or lateral buckling may become critical before it can be attained.

The subassemblage tests were designed to answer the above questions. To do this requires the selection of a testing arrangement of practical dimensions and construction and yet capable or providing the necessary experimental data.
II. DESIGN OF TESTING ARRANGEMENT

a) Selection of a Testing Arrangement

Figure 1 illustrates the basic structural subassemblage. The case being investigated is one in which the columns play a significant role and it is desirable to select a severe column loading condition. For a given axial load a suitable condition is provided by a checker-board load distribution (Fig. 5). In this situation the columns are in single curvature and this is the most severe condition from a stability aspect.3

The largest moment a beam can transmit is its fully plastic moment and with the requirements of a single curvature column and an applied beam moment in mind, the subassemblage in Fig. 1 has been modified to the form shown in Fig. 6. For simplicity it has been assumed that the restraints at the far ends of the beams (C and D in Fig. 6) and from the columns above and below (B and E), have been replaced by pins. The elimination of all restraints except those from the three members BC, EB and ED is a matter of testing convenience and also provides a better definition of the factors involved. If the theory is verified for the case shown in Fig. 6, then
there is no logical difficulty in accounting for the presence of further restraints.

To allow for more accurate measurement of beam moments, these members are left externally unloaded along their lengths (ED, CB) and are loaded only at the reactions C and D (Fig. 6). The applied moments M are more conveniently applied on the same side of the column BE. Hence the sub-assemblage is reoriented and the final testing layout is shown in Fig. 7. In this case the lower applied moment M (M = F x AB) is in the reverse direction to its building counterpart (Fig. 5). The effect of this change is eliminated by also changing the position of the lower restraining beam (BC).

The moments M are applied in such a manner that they remain in the ratio FE/AB (Fig. 7) throughout the test. The restraining beams (CB and ED) provide both elastic and elasto-plastic restraint, depending on the applied loads. The stub beams remain elastic throughout the test and so the prototype (Fig. 5) has been changed to the extent that all inelastic beam restraint has been concentrated in the two restraining beams (CB and ED). Fig. 8 shows the resulting bending moment diagram. The points D and C might thus be regarded as inflexion points in a real beam.
b) **Operation of the Testing Arrangement**

The variables introduced into the various test specimens are designed to produce various load-deformation characteristics (such as the M-θ curves in Fig. 3b) in the beam-column and the beams. For the beam-column this is achieved by varying the height of the column and also the applied axial load. The moment-rotation curves for the beams are altered by simply varying the lengths of the beams. These arrangements allow the beam and beam-column sections to be kept constant and so variations introduced by different cross sections are eliminated.

The length \( s \) of the beams is a useful test parameter. Fig. 9 shows how three different test conditions are obtained by holding all variables constant except for the length of the beams. The three conditions produced are:

1. A plastic hinge forms in the beams before the joint moment capacity is reached. Unloading of the joint is precipitated by unloading of the column. (Fig. 9c, \( s = s' \), short beam).

2. Beams and beam-column reach their peak
moments simultaneously. This is also the point of maximum joint moment.
(Fig. 9c, $s = s''$, medium beam).

(3) Beams remain elastic until after the joint has begun to unload. Unloading is precipitated by severe unloading of the beam-column
(Fig. 9c, $s = s'''$, long beam).

These three conditions embrace all practical cases and can easily be duplicated by the above testing arrangement.

c) Design Details of Testing Arrangement

An overall view of the test set-up is shown in the photograph in Fig. 10. The applied moments are introduced by a hydraulic jack which is pin connected between F and A (Fig. 7) by a 1 inch dia. rod. A dynamometer is coupled in series with the jack; the complete unit can be seen in Fig. 11.

The ends of the restraining beams (C and D in Fig. 7) are fixed vertically in a large vertical support made from two braced 8WF67 sections. The support is independently attached to the laboratory floor. The arrangement can be seen in the foregound of Fig. 10. Columns up to 20 ft. and beams up to
16 ft. in length can be accommodated. The reactions between the beams and the supports are carried by 1-1/2 in. diameter pins. These pins pass through a slotted hole in the stiffened web of the restraining beam (Fig. 12) and duplicate a roller type reaction.

The ends of the columns are carried on cylindrical end fixtures which are described elsewhere. Fig. 13 is a diagrammatic representation of the fixtures,* which provide a column in which the applied axial load always passes through two fixed points. The points are the centers of the cylindrical surfaces. The test columns are designed such that the centers of the surfaces are also the centers of the joint details.

One difficulty associated with these fixtures is that, as the column joints rotate about 0 (Fig. 13), the cylindrical surface will roll along the horizontal base B. Thus the specimen will translate in its own plane during the test. For this reason it is necessary to insure that the beam supports are on rollers, as described above.**

* The end fixtures can also be seen in the photograph in Fig. 15

** These rollers were omitted in test RC-1.
Channel-type lateral braces are provided for the beams and the beam-column to insure that lateral buckling does not become significant. The braces can be seen in Fig. 10. The beam lateral braces were described previously in Reference 5 and the column braces in Reference 2.

d) Design of Test Specimen

The sections and dimensions chosen depend on the aims of the particular test. In tests performed so far, the columns have been 8WF31 shapes with h/rₓ of 60, 40 and 30; the beams have been 5WF18.5 with s = 16', 12' and 18'. Fig. 14 is a photograph of a typical specimen (after test).

The connections are designed according to plastic design provisions with additional stiffeners provided to distribute the heavy axial load into the columns. Fig. 15 shows a typical connection detail. The base plate provides for two bolts to attach the specimen to the end fixtures (Fig. 13). These bolts are nominal as no shears are developed between the column and the end fixtures (Fig. 8).

The stub beams (FE and AB in Fig. 6) are designed to remain elastic throughout a test and required an 8WF40 section for the tests performed so far.
Three factors dictated the choice of the 8WF3l for the column section in these tests. Firstly, this section provides convenient testing lengths when used with h/r values between 30 and 60. Secondly, it has been used in the past for both column tests and column analyses. Thirdly, it is a section which is frequently used in commercial structures. Once a column section was chosen the beam s/d ratio had to be such as to produce the three cases shown in Fig. 9 for values of s which were practically limited to about 16 ft. This narrowed the choice of beam sections considerably and a d (depth) of five inches was most suitable. A column rather than a beam section was used as the larger value of r_y/r_x reduced the chance of undesirable lateral buckling. The final choice was the 5WF18.5. The ratio of beam to column stiffness thus represented a typical situation in the lower stories of a tall building.
III. MEASUREMENT OF FORCES AND DEFORMATIONS

The testing procedure which has been followed is to apply all axial load, $P$, (Fig. 8) initially and then to begin applying the end-moments to the stub beams $FE$ and $AB$. This does not directly reproduce structural reality as axial load and applied moment will generally be related. However this relation depends on the surrounding structure; in the upper columns of a tall building the axial load and moment will be almost proportional whereas in the lower columns they will be almost independent. The testing procedure adopted follows the latter situation and has the added advantage of being much simpler experimentally. Once the applied moments are added, reactions will develop in the stub beams ($F$) and in the restraining beams ($R$). Hence the total axial force in the columns, $Q$, is given by

$$Q = P + R + F$$ (8)

The force $Q$ can either be kept constant by decreasing $P$ as $R$ and $F$ increase, or $Q$ can be allowed to increase slightly, thus more precisely reproducing the conditions in the lower columns of a tall building.

The force $P$ is measured by the testing machine and the force $F$ is measured by the dynamometer between the stub
beams (Fig. 11). The applied moment is then the product of F and the stub beam length (a in Fig. 8).

The shear force in the restraining beams, R, is measured by three sets of strain gages placed in the elastic portions of the restraining beam. These gages are calibrated directly to read in terms of moment, and this calibration is described below. Four gages are used at each location (Fig. 16a), but they are wired so that only one reading is needed at each location. This reading is proportional to the curvature at that location. The wiring arrangement also has useful self-compensating properties.

Fig. 16b shows a schematic layout of the gage locations. M₁, M₂, and M₃ are three recorded moments from the strain gages and the moment at the pin is known to be zero. This gives three estimates of the reaction R and if weighted in proportion to their size the estimate of R is

\[ R = \frac{M_1 + M_2 + M_3}{\frac{1}{1} + \frac{1}{2} + \frac{1}{3}} \]  

Consequently the moment at the beam-column connection is M_B

where

\[ M_B = \frac{M_1 + M_2 + M_3}{\frac{1}{1} + \frac{1}{2} + \frac{1}{3}} \cdot s \]
To calibrate the gages the specimen is erected but the pins are removed from the restraining beams, leaving them as cantilevers. Known weights are then placed on the beam at the reaction points. The gages are read and gage factors are found from the known cantilever bending moments at each location. This calibration procedure is illustrated in Fig. 17 for test RC-2.* The variation in gage factors results mainly from the dependence of the factor on cross-sectional dimensions, material properties and gage properties.

Recorded moments for two loading stages of test RC-2 are shown in Fig. 18, which also illustrates the application of Eq. (10) to estimate the beam end-moment.

The ends of the columns remain elastic in most situations and gages are also placed close to the column ends. Moments $M_{SG}$, can be calculated from the gages and if the column deflection, $\delta$, (see sketch in Fig. 19) at the gage location is also measured, then the column end moment, $M_c(s)$, 

* The relevant dimensions of test RC-2 are given in the Appendix. This test will be used to illustrate various points in the remainder of this report.
is given by

\[ M_c(s) = M_{SG} - Q \times \phi \]  \hspace{1cm} (11)

Another estimate of the column end moment is given by

\[ M_c(J) = F_a - M_B \]  \hspace{1cm} (12)

Thus there is a cross-check on the two load estimates. The relationship between the two predictions of column end-moment are shown in Fig. 19 for test RC-2. It is seen that there is a variation of only +4% to -1% for the relevant region of the test. This variation is well within the limits of experimental accuracy, and confirms the adequacy of the measuring techniques.

Dial gages are used to measure deformations and rotations are measured by the level bar method described elsewhere.\(^2\)
IV. SECONDARY EFFECTS INTRODUCED BY THE TESTING ARRANGEMENT

In this section the influence of various secondary effects will be discussed. It will be shown that none of these exert any serious influence; consequently the specimen may be assumed to be sufficiently close to both the mathematical model and the practical structure which it represents.

a) Erection Stresses

As the structure is statically determinate during fabrication, it is not expected that any abnormal fabrication stresses will be developed. Stresses may be induced during erection due to the dead weight of the specimen and to the fact that only 1/8" tolerance is allowed for the distance DC (Fig. 6). This can not always be met and therefore the connecting pins at D and C may have to be forced into place. The erection stresses resulting from this process are readily measured in the following manner.

The test specimen is erected and readings are taken on the beam strain gages (Fig. 16b). The reaction pins C and D are then removed and the beam is allowed to hang as a cantilever from the column. The only stresses in the beam are
those due to its own weight per length, \( w \). If the pin is now inserted at the reaction, there will be a change in strain gage moments \( M_1, M_2, \) and \( M_3 \). These values may be substituted into Eq. (9) to estimate the force, \( R_o \), exerted by the pin. In the ideal case the beam would now be a propped cantilever with

\[
R_o = R_w = \frac{3}{8} ws
\]

(13)

Normally, \( R_o \neq R_w \), and the erection stresses are defined as \( R_e \) where

\[
R_e = R_o - R_w
\]

(14)

The total initial stresses in the frame are those due to the combination of \( R_o \) and the dead weight stresses with the beams hanging as cantilevers. Fig. 20 illustrates these measurements for test RC-2. It is seen that the total pin reaction is 160 lb. (\( 3/8 \) \( ws = 83 \) lb.) and that the maximum erection moments are only 1.5% of the beam plastic moment.

b) Alignment of Specimen

The specimen is constructed as a rigid frame and placed in a rigid testing frame. There is thus no specific provision for alignment once the specimen is erected. Reliance is placed on the accurate fabrication of the test speci-
men to eliminate any large eccentricities and to keep the existing eccentricities within the limits to be expected in practical structures.* The problem of alignment is not as serious in a restrained column test as it is in a pin-ended axially loaded column test; in a pin-ended column any moments due to eccentricity must be compared to zero applied moments whereas in a restrained column the applied moments have some finite magnitude.

The moments due to misalignment can be measured once the initial axial force is applied. Fig. 21a shows the measured moments due to misalignment in test RC-2; the magnitudes of these were typical of those obtained in all tests. It is seen that the misalignment moment is only 2.6% of the maximum joint moment. Some of this moment is due to the dead weight of the jack-dynamometer assemblage (between A and F in Fig. 7). It is estimated that this amounted to 25% of the initial moment in test RC-2, however, no distinction has been made between the two causes as it is only their combined effect which is of interest.

* However, lateral misalignment (out of the plane of the specimen) can be critical, but can also be easily corrected.
c) Effect of Axial Deformation

At the stage represented in Fig. 21a the frame was still completely elastic. Hence, it should be possible to distribute the moments at the column ends (22.4 kip-in. on the top, and 10.0 kip-in on the bottom) and obtain the bending moment distribution shown in the figure. However, when the process is carried out (considering the effect of axial force, \( \frac{P}{P_E} = 0.159 \)), the bending moment distribution in Fig. 21 (b) is obtained. The additional moments required to produce Fig. 21 (a) are of the same order of magnitude as the misalignment moments and are shown in Fig. 21 (c).

These latter moments result from the axial shortening of the column and the settlement of the end fixtures under axial load. In Fig. 7 it is seen that points D and C are fixed vertically, however, both points E and B will settle vertically. Point E will lower due to axial shortening of the column and settlement of the lower end support and point B due to the latter cause only.

The deflections at E and B can be calculated from the moment diagram in Fig. 21 (c). They are 0.125" at E (down) and 0.064" at B (down). Thus the deformation of the
column is 0.061" and the settlement of the bottom end the
settlement of the bottom end fixture is 0.064". Assuming
the top end fixture settled an equal amount the total cross-
head deformation would be 0.189". Now, both the axial strain
in the column and the movement of the top cross-head were
measured during the tests. For the case considered above the
column deformation was 0.000435 x h = 0.090" and the cross-
head movement was 0.159". The discrepancy between the two
sets of figures (0.064", 0.189" and 0.090") is not serious
considering the devious nature of the derivation of the first
set. In fact the agreement indicates the applicability of
the above reasoning.

Fig. 22 presents the axial deformation curves for
column and cross head as recorded in test RC-2. Although
the end fixtures will not settle further after axial load
has been applied, further settlement of the cross-head will
result from the increasing lateral deflections of the column.
This curvature shortening and hinge contraction will only
affect the top beam and thus will tend to cause a difference
between the top and bottom end moments and this situation
will worsen as the test progresses. The curvature values
plotted in Fig. 22 are found as the difference of the two
other sets of readings.

Fig. 22, which is typical of the tests, shows that the axial shortening does not become significant until the column load-deflection curve (Fig. 23) departs significantly from a linear curve. The total shortening in RC-2 during moment application is 0.513" (Fig. 22), and this is equivalent to relaxing the top beam moment by 57 kip in. (3 $E I \Delta/s$), or 14% of the top-beam plastic moment. The actual process involved is not precisely represented by this simple calculation. For each increment of rotation the increment of axial shortening relaxes the angle by some amount, (Fig. 24).

The relationship between the applied increment and the final increment is very similar to the sway equation, Eq. (2), and can be seen from Fig. 24 to be

$$\delta \overline{\theta} = \delta \theta - \delta \left( \frac{\Delta_{ax}}{s} \right)$$

(15)

where $\delta \overline{\theta}$ is the final, $\delta \theta$ the applied increment and $\delta \Delta_{ax}/s$ the increment of axial shortening. The ratio $\delta \overline{\theta}/\delta \theta$ is plotted for test RC-2 in Fig. 25. The effect on rotation is generally less than 10%. The effect on moment is further reduced when it is recalled that the end-moment of the beam
becomes independent of the end-rotation when the plastic moment is reached, provided the increment remains positive. Thus the increment of moment increases in the same ratio as the increment of rotation in the elastic range but does not increase at all with rotation after $M_p$ is sensibly attained. Using the known end-moment end-rotation curve for the beam the effect shown in Fig. 25 is transformed into a moment reduction effect and is illustrated in Fig. 26. The maximum effect is only 4% and the advent of rotations near the $M_p$ value will soon eliminate any cumulative differences.

The foregoing is not an analytical proof of the relative insignificance of axial shortening in the test procedure. However, it does illustrate the situation for the cases tested in this series in which the beams and column all reached their peak moment in the same rotation region. The effect will become more significant as the elastic range of the beam curves increases relative to that of the column load-rotation curves.

d) Applied Moment

The lever arm of the applied moment (a in Fig. 8) is reduced as the stub beams AB and FE rotate. This change
is very small and is given (as a first approximation) by where

\[ \Delta_a = a (1 - \cos \Theta) \approx a \theta_J^2 / 2 \]  

(16)

where \( \theta_J \) is the rotation of the joint. In practical arrangements \( \theta < 0.06 \) and hence \( \Delta a / a < 0.06^2 / 2 = 0.2\% \).

e) Translation of Test Specimen

Mention was made in Section II (a) of the fact that the specimen translates laterally as the joints roll on the cylindrical end fixtures. The translation can be measured directly or it can be calculated from the known joint rotation, \( \theta_J \), and the radius of the cylindrical surface, \( r \) (Fig. 13). In the latter case the translation, \( \Delta_t \), is given by

\[ \Delta_t = r \theta_J \]  

(17)

For \( r = 10 \text{ in} \ldots \Delta_t < 10 \times 0.06 = 0.6 \text{ in} \).

Fig. 27 shows a plot of the measured translation of the end of the lower beam and the translation as calculated from Eq. (17). The discrepancy between the two readings was generally within 5% but became more significant towards the end of the test when curvature shortening of the beam began to absorb some of the rolling translation.
V. SWAY TESTS

The problem of sway was discussed in Section I. It was shown that Eq. (1) must be modified by Eq. (2) in this situation, i.e. the angle at which the M-θ charts are entered is not the joint rotation angle but some other angle which is a function of the applied load. Now this is also the situation which has been discussed in Section IV (c) in connection with the effect of column shortening. It can be seen that Eq. (15) is identical to Eq. (2) with \( α = \frac{Δ_{ax}}{s} \). This point can also be illustrated by Fig. 28. Fig. 28a shows a swayed subassemblage developed from Fig. 4; if the sway behavior of only one joint is to be investigated the other may be restored to 90° and hence Fig. 28b results. The major difference between the effect of axial deformation and the swayed subassemblage is in the relative size of \( Δ_{ax}/s \) and the sway angle, \( α \). A test with larger values of \( Δ_{ax} \) would require a slight modification to the test set-up described earlier in the report. This is achieved by replacing the vertically fixed support at C by a mechanical screw jack. As the test structure is loaded C is raised by a predetermined amount \( Δ_{ax} \) giving an equivalent sway angle in the column of \( \frac{Δ_{ax}}{s} \). The column remains vertical under
this system. The testing technique requires that $\Delta_{ax}/s$ and $\theta_j$ be kept in a constant ratio.

Provisions must be made to carry the column shears. As these will be small (c 4 kip) they may be resisted by shear between the surfaces of the end fixtures (Fig. 13). For the tests conducted, the axial load was between 130 kip and 200 kip and the friction developed was adequate to carrying the sway shears.
VI. CONCLUSIONS

Although it is not the intent of this report to discuss actual test results, Fig. 23 is included as a typical test result obtained from the testing set-up. Table I presents a summary of tests so far performed.

The testing arrangement described in this report allows representative structural subassemblages to be tested under conditions of no-sway or sway. Thus it is possible to investigate the behavior of beams and columns under real rather than simulated conditions and furthermore the behavior of the subassemblage itself may be studied.

The critical member remains the column; however, it is now possible to measure the effect of elasto-plastic restraints and loads on the behavior of the column and to further utilize its basic strength and ductility.
VII. ACKNOWLEDGEMENTS

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The authors express their thanks to those whose theoretical work provided the basis for these tests, and especially to Dr. Victor Levi for many fruitful discussions. Particular thanks are due to Mr. K. Harpel and his staff whose patience and skill turned the design drawings into a working reality. Miss Valerie Austin very carefully typed the report and Mr. J. Szilagyi and R. Sopko did the drawings.
VIII. NOMENCLATURE

a
lever arm for moments

h
column height

\( d \)
depth of beam

\( l \)
length term

\( r \)
radius, \( r_x \) = radius of gyration, strong axis

\( r_x \)
radius of gyration, weak axis

s
beam length

\( w \)
uniformly distributed load, weight per length

\( w_d \)
uniformly distributed load due to dead load

\( E \)
elastic modulus

F
jack force

I
strong axis moment of inertia

\( M \)
moment

\( M_B \)
beam moment of connection

\( M_c(J) \)
column end moment from jack and beam

\( M_c(s) \)
column end moment from strain gages

\( M_J \)
joint resisting moment

\( M_M \)
member resisting moment

\( M_p \)
maximum beam moment

\( M_{SG} \)
moment from strain gages on column

\( M_u \)
maximum member moment

\( M_1, 2, 3 \)
strain gage moments on beam
P  applied axial load
Q  column axial load
R  beam reaction, R₀-initial, Rₚ-self weight, Rₑ-erection
  \( P_y \)  area times yield stress of column
  \( \alpha \)  joint translation angle
  \( \theta \)  member rotation angle
  \( \theta_j \)  joint rotation angle
  \( \bar{\theta} \)  rotation reduced by axial shortening
  \( \Delta \)  joint translation
  \( \Delta_a \)  shortening of stub beam
  \( \Delta_{ax} \)  axial shortening
  \( \delta \)  deflections
  \( \Delta_t \)  rolling translation of specimen
  \( \delta \theta \)  increments of rotation
IX. TABLES AND FIGURES
<table>
<thead>
<tr>
<th>Test No.</th>
<th>Axial Load ( (P/P_y) )</th>
<th>Column Slenderness Ratio</th>
<th>Beam Length -Depth Ratio</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>RC-1</td>
<td>0.4</td>
<td>60</td>
<td>37</td>
<td>Column Fails First</td>
</tr>
<tr>
<td>RC-2</td>
<td>0.4</td>
<td>60</td>
<td>27.8</td>
<td>Beam and Column fail Together</td>
</tr>
<tr>
<td>RC-3</td>
<td>0.4</td>
<td>60</td>
<td>18.5</td>
<td>Beam Hinges First</td>
</tr>
<tr>
<td>RC-4</td>
<td>0.6</td>
<td>40</td>
<td>37</td>
<td>Column Fails First</td>
</tr>
<tr>
<td>RC-5</td>
<td>0.6</td>
<td>40</td>
<td>27.8</td>
<td>Column Fails First</td>
</tr>
<tr>
<td>RC-6</td>
<td>0.8</td>
<td>30</td>
<td>27.8</td>
<td>High Axial Load</td>
</tr>
<tr>
<td>RC-7</td>
<td>0.4</td>
<td>60</td>
<td>27.8</td>
<td>Test with Sway</td>
</tr>
</tbody>
</table>

**TABLE OF TESTS PERFORMED**
Fig. 1  Basic subassemblage

Fig. 2  Braced and unbraced frames
Fig. 3 Braced joint
Fig. 4  Unbraced joint

\[ a = \frac{\Delta}{h} \]
Fig. 5 Checkerboard loading
Fig. 6 Subassemblage under load
Fig. 7 Test subassemblage

Fig. 8 Bending moment diagram
Fig. 9 Effect of design variables
Fig. 10  View of frame
Fig. 11 View of jack and dynamometer
Fig. 12  Beam end reaction detail
Fig. 13 Connection and end fixture
Fig. 14 Tested specimen
Fig. 15  View of connection and end fixture
Fig. 16 Measurement of beam moments
Moments @ R=150 lb (kip-in) \[\{\begin{array}{c}13.8 \\ 10.2 \\ 6.60 \end{array}\]

Gage reading: \[\{\begin{array}{c}89 \\ 62 \\ 42 \end{array}\]

Gage factor: (readings/kip-in) \[\{\begin{array}{c}6.45 \\ 6.08 \\ 6.36 \end{array}\]

Error ± gage reading = ± 0.173 kip-in
= ± 0.4 % \(M_p\)

Fig. 17 Calibration of beams
Fig. 18  Beam moment during test

\[ E_{eq}(10) \quad M_B^{(10)} = \frac{306}{17} \times 12 = 216 \text{ kip-in} \]

\[ M_B^{(15)} = \frac{633}{17} \times 12 = 447 \text{ kip-in} \]
Fig. 19 Joint moments

\[ M_{C(S)} = M_{SG} - Q \delta \]

\[ M_{C(J)} = M_J - M_B \]
Moments due to cantilever action under self-weight

Final Moments existing after erection

Moments applied during erection

\[ R_o = \frac{23.0}{144} = 0.160 \text{ kip} \]

\[ R_w = \frac{3}{8} \text{WS} = 0.083 \text{ kip} \]

\[ R_e = 0.160 - 0.083 = 77 \text{ lb} \]

Fig. 20 Initial moments in beam
Fig. 21  Initial moment in frame
Fig. 22  Vertical deflections of frame
Fig. 23  Test result for RC-2
Fig. 24 Effect of axial shortening
Fig. 25 Rotation increment effect

Fig. 26 Moment increment effect
Fig. 27 Translation of frame
(a) Swayed subassemblage

(b) Test simulating one swayed joint

Fig. 28 Sway behavior
X. APPENDIX

Results from Test RC-2 were used to illustrate effects discussed in Section IV. The following are the relevant dimensions:

RC-2  
Column:  \( h/r = 60 \)

Section:  8WF31

\( P/P_y = 0.40 \)
\( P = 139 \text{ kip} \)

Beam:  \( s/d = 27.8 \)

Section:  5WF18.5
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