Columns Reinforced by Welding Cover Plates

COLUMNS REINFORCED UNDER LOAD

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April 1962

Fritz Laboratory Report No. 286.1
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and
Lambert Tall

This work has been carried out as a part of an investigation sponsored by the American Institute of Steel Construction.

Fritz Engineering Laboratory
Department of Civil Engineering
Lehigh University
Bethlehem, Pennsylvania

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ABSTRACT

A summary is presented of an experimental investigation of the effect of welding cover plates to a WF column under load.

Columns in existing structures carry some load at the time of reinforcing by welding cover plates. Welding creates residual stresses which may or may not be helpful. The test program consisted of the determination of residual stress distribution, coupon strength, stub column tests and pin-end column tests on an unreinforced column and on columns reinforced under load and under no load. A short theoretical analysis of the column strength is appended. The columns tested showed no detrimental effect due to reinforcing by welding cover plates.
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1. INTRODUCTION

Purpose and Scope

Often a structural engineer finds it necessary to strengthen steel structures while they are in use, many years after construction. Some examples are the strengthening of a bridge to carry heavier traffic and the strengthening of the bottom columns of a multistory building to withstand further addition of floors.

Many times it is either impracticable or uneconomical to relieve the entire load during the alteration. If the member is reinforced by welding, the material properties are severely affected by the heat of welding. Thus some parts of the structure may be weaker at the time of welding than they would be under an overload. With the advent of high strength steel it is possible to strengthen a member without occupying much extra space.

As a preliminary to the study of the strength of members reinforced by high strength steels, an investigation was carried out on the strength of WF columns of A7 steel reinforced by welding cover plates of the same material.
Influencing Factors

The most important influencing factor is the welding operation. Welding creates very high temperatures at which the material properties are affected greatly. The immediate consequence of welding is either the occurrence of residual stresses in a shape which is free from residual stresses or else a change in the existing residual stress distribution. The geometry of the shape and of the plates, the type of welding, the speed of welding and the rate of cooling all influence the residual stress magnitude.\textsuperscript{1,2,3,4}

Previous Research

Very little research has been conducted in this field in the past. Though many structures were often strengthened, the processes were presumed to be safe. Also no special observations were made.

Davis has described the addition of four stories to the six story Rose Building in Cleveland, Ohio.\textsuperscript{5} The columns in the lower stories were strengthened by adding sections made up of angles inserted in the hollow centers of the existing columns.

Cook made a nine story addition to a building in Indianapolis.\textsuperscript{6} The existing H-columns were strengthened by
welding steel plates across the flanges; no special observations were reported.

In 1935 Wilson and Brown conducted tests on the strengthening of columns of a viaduct in Girard, Pennsylvania. Cover plates were welded to the existing sections. They measured longitudinal thermal stresses of the order of 5 to 15 ksi; and further, they concluded that in some cases this thermal stress may equal the yield point. They also found ultimate stress in the reinforced section was the same as that in the original section, provided the reinforced section remained straight after welding. Square-end columns having a slenderness ratio of about 65 were used in their tests. They noticed no great change in the slenderness ratio after welding.

In 1943 Spraragen and Grapnel reviewed all the literature on structures reinforced under load. They reported that some structures failed during reinforcing because of the weakening of the structure due to heat of welding, shrinkage stresses developed during cooling, defective welding, fatigue and to unexpected undesirable redistribution of loads. A summary of the results of the tests reviewed by them is given below:
1. Stresses in the strengthened and original parts agreed closely with values calculated on the basis of the section moduli.

2. Residual stresses, though high, did not seriously affect the ultimate strength.

3. Many of the investigators did not fully conceive the phenomena involved. There was confusion about elastic stress and plastic flow.

4. Some of the researchers were deceived by the redistribution of the load.

5. Distortion can be controlled.

6. Heat of welding in compression members might cause local buckling.

It would appear that subsequent to 1943 nothing further has been reported on the subject.
Test Program

Tests were planned to determine the strength characteristics during and after the welding of 7" x 3/8" cover plates to the flanges of an 8WF31 shape. This was a pilot investigation and was not planned to include all possible combinations of the factors involved. The 8WF31 shape was selected since it has one of the lowest shape factors and b/t ratios (which leads to early local buckling) of any of the sections rolled.\(^9\) The results of the tests would be conservative for other cross sections.

The material was confined to ASTM A7 structural steel of one rolled shape and one plate stock size. Welding methods conformed to ASCE-AWS standards. However, to prevent the weld size from becoming a factor in this investigation, the size of the weld was reduced so that the ratio of weld size to section size simulated that of thick plates welded to heavy WF shapes.

Although for the particular slenderness ratio of the test column the design stress was 16 ksi, the load at the time of reinforcement of a similar column in a building would be expected to be lower, since no live load would be present. Thus the load at the time of welding was fixed to give a stress of 10 ksi.
Welding sequence plays an important role in the formation of residual stresses as well as distortions. There are many ways of welding; in this investigation two methods were used as shown in Fig. 1, welding each flange one after another, stage by stage or as in Fig. 2, welding two diagonally opposite flanges simultaneously. Sequence No. 1 has been used in most of the tests as it appeared to be commonly used in practice. In one test, sequence No. 2 was used to study the effect of welding sequence on residual stress formation.

Table 1 shows the various tests conducted. The tests of Group 1 were exploratory in nature and are explained below:

**Test T-1**

This test consisted of welding cover plates to a fixed-end column of slenderness ratio of about 25 \( L = 4 \mbox{\,ft} \). The column was loaded to its working load (91 kips or 10 ksi) and keeping the deformation constant the cover plates were welded; for this purpose the test was conducted in a mechanical type testing machine. The welding sequence was sequence No. 1. The cover plates were \( 1/16\mbox{\,in} \) shorter than the column. The variation of the load as the welding progressed was recorded. SR-4 A-1 type and AB-3 type (bakelite)
strain gages were fixed in tiers at two unequal distances from the ends as in Fig. 3. With one welding rod a length of 12" could be welded. A selected few gages of both types were read continuously during the test. The remaining gages were read in series, commencing at the same time as the welding of each stage. The movement of the cross-head was also recorded continuously. After the specimen cooled to room temperature it was tested to failure.

Test T-2

This test was identical to test T-1 in all respects except that the load was kept constant throughout the welding operation and during cooling as well. This test was conducted in a hydraulic testing machine.

An actual column in a framework can be assumed to have both the load and the length varying when cover plates are welded on to it. To study the effect of load and length separately, the length was held constant in test T-1 and the load was held constant in test T-2.

Test T-18

This test was to observe if there would be local buckling or lateral buckling when the cover plates were welded to the WF shape under a constant high load of 225 kips (25 ksi).
The remaining tests fall into four broad categories - determination of residual stress distribution, coupon strength, stub column and pin-end column tests; they are also classified into three groups. In Group 2, the above tests were conducted on the as-received material; the results of these tests would serve as a basis for comparing the effect of strengthening obtained in Groups 3 and 4. From the results of tests T-1 and T-2, it was inferred that the welding of cover plates keeping the load constant was a simulation of the actual conditions and in subsequent tests the columns were reinforced under constant load. In Group 3, cover plates were welded to the WF shape under constant load; after the specimen cooled it was unloaded and portions of the length were used to determine the residual stress distribution, coupon strength and for stub column test. Similarly a pin-end column was reinforced under load and then, directly tested to failure; for convenience in testing and to compare with the results of the other columns, this column was reinforced with the length held constant. In Group 4, the WF shape was reinforced under no load and the following tests were conducted - determination of residual stress distribution, stub column and pin-end column tests.

Residual Stress Measurements

A knowledge of residual stress distribution is essential for the prediction of the strength of a column. All of the
shapes used in this investigation were from the same heat and the same rolling. The same was true for the plate material. It was assumed that the residual stress distribution would be the same for all the plates and for all the shapes. Representative portions were selected at random and the longitudinal residual stress was determined by the method of sectioning. Residual stress measurements were made on a 10" gage length both ends of which were at a distance from the cut edge more than the depth of the WF shape or the width of the plate.

**Coupon Tests**

Coupon tests were conducted on tension coupons taken from the WF shape and from the plates. Coupons were taken from the web as well as the flanges to include the variation of material properties across the whole cross section. The dimensions of the coupons were in accordance with the ASTM specifications. All coupons were tested in a 120,000 lb. screw type universal testing machine with an electronically operated load indicator and automatic recorder. Figure 4 shows a typical lay-out of coupons from the WF and plates.
Stub Column Tests

Stub column tests were conducted to obtain an average stress-strain curve of the cross section of the shape. Such a stress-strain curve includes the effect of residual stress and may be helpful in the prediction of column strength by the tangent modulus method. The length of the test specimen was 2'9" and was such that the stub column would not fail by buckling laterally and was long enough that the residual stress distribution measured was undisturbed and representative of the column. Strains were measured by 1/10,000" dial gages over a 10" gage length; two gages placed on opposite sides of the cross section were used to compensate for possible uneven deformation. Alignment of the columns was carried out with the help of four 1/1000" dial gages fixed at the four corners; alignment was considered satisfactory when the maximum deviation of any corner gage was less than 5% of the average deviation. Whitewash was used to help observe yield lines on the specimen under load.

In test T-12, a 5'6" long 8WF31 shape was reinforced under a constant load. When the column cooled the load was removed and a stub column was cut from this piece. A stub column taken from a 12' long 8WF31 shape reinforced under no load was used in test T-16. The remaining portion was used for residual stress measurement and for the pin-end column test.
Pin-end Column Tests

It was mentioned above that an actual column in a framework has both load and length varying at the time of reinforcing. Although this is an important factor, very little is known about it. In a framework there is restraint at the ends of a column from adjacent members. This restraint would resist elongation of the column during reinforcing. At the same time such an elongation would cause redistribution of loads on that column and adjacent members.

However, since this investigation was preliminary in nature, for convenience in testing and in comparison of the results, all the three pin-end columns were tested in an 800,000 lb. mechanical testing machine for weak-axis bending. Axial load was applied through special fixtures so that the columns bent about their weak axes. The length of each column was 8'0" giving a slenderness ratio of about 48. Strains were measured by means of SR-4 A-1 type strain gages of 1" gage length, placed at the mid-height of the column and near both ends. They were used also to align the columns. The columns had a negligible initial out-of-straightness. The reinforced columns had an out-of-straightness of 0.02" and 0.03", whereas the unreinforced column had an out-of-straightness of 0.17". The centerline deflection was measured by a 1/1000" dial gage at mid-height and by 1/100" scales and transit. The rotations
at the ends of the columns were measured by a 1/10000" dial gage and level bar.

In test T-13 cover plates were welded maintaining the length constant at a load of 91 kip. After the column cooled the test was resumed. There was no change in the slenderness ratio due to the addition of cover plates.
3. TEST RESULTS & DISCUSSIONS

The exploratory tests T-1 and T-2 lead to some useful observations. In test T-1 with the column length held constant, the load on the 4'0" column increased by about 10% when the cover plates were being welded. This was due to the elongation of the column. Once the welding was over, the column cooled down and tried to contract to a length less than its original length; consequently tension developed in the cover plates and the load decreased by about 30% over a period of four hours and remained at that level. A graph of load vs. time is shown in Fig. 5. The column was not unloaded. Later when the column was loaded to failure, it was found that its behavior was normal and the expected ultimate load was reached. The failure occurred due to the cracking of the welds at a strain about 19 ε_y (Fig. 6). As indicated by whitewash flaking the flanges of the WF shape yielded long before the reinforced plates showed any signs of yielding.

In test T-2 where a similar column was tested in a hydraulic testing machine, the load was held constant by adjusting the hydraulic pressure. Thus the cross-head had up-and-down movements as indicated in Fig. 7. This curve is quite similar to the load-time curve of test T-1. When tested to failure the behavior was almost the same as in
test T-1. In this case also, failure occurred due to the cracking of the weld after the ultimate load had been reached, but at a strain of \( \epsilon_y \) (Fig. 6).

In these two tests SR-4 A-1 and AB-3 type strain gages were used and both of them proved to be unaffected by the heat of the welding. Therefore, in view of the higher cost of AB-3 type gages, only A-1 type gages were used in the remaining tests.

Figure 8 shows the strains recorded by two typical SR-4 gages, one of each type. When the weld was nearest to this pair of gages they recorded very high strains. It is not known if the peak values indicated by the gages are correct. Except for that region, the readings are reliable elsewhere. Following the peak, there is an almost constant difference between the strains at any time, which is indicative of a permanent deformation in the strain gages.

In test T-18 cover plates were welded to a 4'0" long 8WF31 shape under a load of 225 kips, or 25 ksi stress. Local yielding was evidenced by the flaking of mill scale. There was neither local buckling nor lateral buckling nor overall failure. The high temperature due to welding is confined to a very small area in the vicinity of the weld and the material properties are not affected in a major portion of the cross section.
Residual Stress Measurements

Since the residual stress distribution is one of the most important factors in column strength, it was measured for the four conditions mentioned in the previous section; unreinforced 6WF31 shape, shape reinforced under no load, shape reinforced under load, with the plates welded according to sequence No. 1 and No. 2 (Figs. 1 and 2). Fig. 9a shows the distribution in the WF shape and the plate that were used in this investigation. For the WF shape the residual stress distribution was as expected from the results of earlier tests, with a compressive stress of 12-14 ksi at the flange tips and a tensile stress of 6-8 ksi at the flange centers. In the web the residual stress was completely compressive and of average value of about 6 ksi. The plate had been cold straightened and the yield lines were visible on the mill scale on the surface. The portions used for residual stress measurement were free from the effect of cold-straightening. The residual stress magnitude varied from 5 ksi tension at the center to 11 ksi compression at the edges of the plates.

The process of welding creates tensile residual stress in the vicinity of the weld, since that part cools last. Figure 9b shows the residual stress distribution in a WF shape reinforced under no load, whereas in Figs. 10a and 10b the shapes were reinforced under a constant load of
91 kips. For the shape in Fig. 10b (Test T-3) the two flanges were reinforced simultaneously according to the welding sequence in Fig. 2. The residual stress distribution is similar to that of test T-14 (Fig. 9b).

It can be noticed that the residual stress distribution in the flange has reversed after welding and at the flange tips the magnitude is 25-30 ksi tensile and the compressive residual stress, no longer at the flange tips, is as low as 7 ksi. From this type of residual stress distribution the section could be expected to be much stronger than the corresponding rolled shape.

**Coupon Tests**

Coupon test results are given in Table 2. There is a significant difference in the yield stress of the web, flange and plate material. The weighted mean yield stress increased slightly from 37.0 ksi to 37.4 ksi after the welding.

**Stub Column Tests**

The results of the stub column tests are given in Table 3. The stress-strain curves are shown in Fig. 11. From Table 3 it can be seen that the yield strength of the sections is the same for all the cases. The yield stress from the stub column tests, 37.2 ksi, are in good agreement
with those from the coupon tests 37.4 ksi. From the stub column curve the compressive residual stress $\sigma_{rc}$ can be calculated using

$$\sigma_{rc} = \sigma_y - \sigma_p$$

in which $\sigma_p$ is the stress at the proportional limit and $\sigma_y$ denotes the yield stress level. Although there is considerable variation in the residual compressive stress the stub column tests do not indicate this. However, there was good agreement between the expected limit of proportionality and the actual value, because of the presence of tensile residual stress over a considerable area of the section.

The stub columns from the unreinforced section exhibited local buckling at the yield load. For the 8WF31 shape, $b/t > 17$; hence the occurrence of local buckling was not unexpected. In the case of reinforced sections $b/t$ was reduced to 10; local buckling occurred at a strain of about 6 $\varepsilon_y$ prior to failure of the stub columns due to the cracking of welds. One of the reinforced sections was strained into the strain-hardening range before it failed; the one welded under constant load failed much before strain-hardening - at a strain of 10 $\varepsilon_y$; this concurs with the observation on test column T-2, which was also reinforced under constant load.
Pin-end Column Tests

A theoretical tangent modulus load curve for the reinforced section based on a modified residual stress distribution as shown in Fig. 12 was developed. The detailed calculations are given in the appendix. For purposes of estimating the column strength the yield stress was taken to be 37 ksi. In the same figure are shown the CRC column curve and the column curve for unreinforced 8WF31, weak axis bending. The CRC curve is an average curve for bending about both axes. The curve obtained for reinforced 8WF31 is above the CRC curve, indicating the higher strength of reinforced sections. The test results are plotted at the corresponding slenderness ratio of 48. It is interesting to note that the reinforced sections have a higher ultimate stress than the original section and have not lost any strength due to welding. In general the reinforcing improves the strength characteristics of a WF column about its weak axis.

The load-deflection curves for the three tests on pin-end columns are shown in Fig. 13. During the test there were small deflections from the straight configuration before the ultimate load was reached. This is due to the slight imperfection in alignment and the initial out-of-straightness of the columns. The reinforced column T-13 was fairly
straight \( (e = 0.02\)\) and the alignment was almost perfect. Thus the expected ultimate load was reached when the deflection was less than \( 1/200\); also 98\% of the yield load was reached.
4. SUMMARY

This investigation was confined to the welding of 7" x 3/8 " cover plates to 8WF31 columns, both of which were of ASTM-A7 steel. The applicability of these results to other sizes and to other grades of steel is yet to be verified, although it is expected that they are indicative of the behavior of reinforced columns of medium size.

Some of the important observations are:

1. The welding operation has a great effect on the length of a loaded column in that the length increases during welding and decreases during cooling, when the load is held constant. When the length is held constant, there is a corresponding change in the load. (Fig. 7).

2. Welding changes the residual stress distribution. The compressive residual stress at the flange tips before welding is changed to high tensile residual stress after welding. (Figs. 9 & 10).

3. The stub column tests and the pin-end column tests showed that the ultimate stress was not reduced by welding. The pin-end column tests showed that the strengthened sections had a higher ultimate load carrying capacity (Figs. 12 & 13). One of the pin-end columns tested reached 98% of its yield strength. (Test T-13)
4. The influence of welding is confined to a very small area in the vicinity of the weld. The material properties in the major portion of the section are not affected enough to reduce the strength of the section. This is verified by tests where the applied loads at the time of welding were both 10 ksi and 25 ksi.

In extending this investigation the following facts need consideration.

1. In heavier sections, the welds may not change the residual stress distribution to an advantage.

2. In the present investigation the working load was low. In old structures under modern loading conditions, this may be different.

3. Column tests were conducted at a commonly used slenderness ratio. For greater lengths of columns the possibility of the creation of large out-of-straightness may be a factor.

4. Welding cover plates of high strength steel may introduce the problem of brittle fracture due to a combination of higher carbon content and possible poor joint preparation.

The residual stress distribution that has been obtained in this investigation is completely different from that used
in studies of the strength of beam columns and of other frame components. Since reinforcement by welding is extensively used, it may be necessary to incorporate the reversed residual stress distribution in strength analysis.
5. NOMENCLATURE AND DEFINITIONS

\begin{align*}
A &= \text{cross-sectional area} \\
A_e &= \text{elastic part of cross-sectional area} \\
b &= \text{width of flange of WF shape} \\
E &= \text{Young's modulus of elasticity} \\
E_t &= \text{tangent modulus} \\
e &= \text{out-of-straightness at mid-height of column} \\
I &= \text{moment of inertia} \\
I_e &= \text{moment of inertia of the elastic part} \\
L &= \text{total length of a pin-end column} \\
L/r &= \text{slenderness ratio} \\
kL/r &= \text{effective slenderness ratio} \\
P &= \text{load on a column} \\
P_{cr} &= \text{critical load on a column} \\
r &= \text{radius of gyration} \\
t &= \text{flange thickness} \\
x &= \text{distance from the center of flange area} \\
x_o &= \text{distance from the center of flange area to the boundary of yielded area} \\
\varepsilon_y &= \text{strain at yield point in an idealized stress-strain curve} \\
\sigma &= \frac{P}{A} = \text{stress} \\
\sigma_{av} &= \text{average stress} \\
\sigma_{cr} &= \text{applied average maximum stress on a column}
\end{align*}
\[\sigma_p = \text{stress at proportional limit}\]
\[\sigma_{rc} = \text{compressive residual stress}\]
\[\sigma_{rx} = \text{residual stress at distance } x\]
\[\sigma_y = \text{yield stress level}\]

**DEFINITIONS**

**Critical Load** - The maximum load a column will carry. It is not coincident with the buckling load for an axially loaded column.

**Stage** - The length of the weld in one continuous sequence.

**Ultimate Stress** - The average stress in the cross section of a shape at ultimate (critical) load.

**Yield Point** - The first stress in a material, less than the maximum attainable stress, at which an increase in strain occurs without an increase in stress.

**Yield Stress** - The stress at which a material exhibits a specified limiting deviation from the proportionality of stress to strain.

**Yield Stress Level** - The average stress during actual yielding in the plastic range. It remains fairly constant for structural steel provided the strain rate remains constant.
6. **ACKNOWLEDGEMENTS**

This report summarizes the theoretical and experimental studies made during the course of a pilot investigation into the strength of columns reinforced by welding cover plates. The research was conducted at the Fritz Engineering Laboratory, Lehigh University, Bethlehem, Pennsylvania.

A Column Research Council Task Group under the Chairmanship of John A. Gilligan provided valuable guidance. The American Institute of Steel Construction sponsored the research.

Acknowledgement is also due to F. R. Estuar, J. A. Yura, and the other research assistants who assisted in the experimental work.
Column Strength

The strength of a column may be defined by its bifurcation (or buckling load) and by its ultimate load. The buckling load is the load at which a theoretically straight column is indifferent to its deflected shape. The ultimate load is the maximum load a column can carry. Euler investigated the buckling of columns where the cross section was considered to remain elastic throughout the loading. The inelastic column strength may be defined by three theories - 1) the tangent modulus theory, 2) the reduced modulus theory, and 3) the Shanley theory. The tangent modulus and reduced modulus theories give the lower and upper limits for column strength respectively. The tangent modulus concept assumes that no strain reversal takes place on the convex side of the bent column when it passes from the straight form to the deflected form. The reduced modulus concept assumes that strain reversal will take place on the convex side of the bent column when it passes from the straight form to the bent configuration. The Shanley theory shows that the ultimate strength of a column is between the values given by the tangent modulus and reduced modulus strengths. Further, Shanley showed that an initially straight column will start to bend at the
tangent modulus load. For many cases, such as rolled shapes, the column strengths given by the three theories are the same, for practical considerations. The tangent modulus load is a conservative definition for the buckling load of a centrally loaded and perfectly straight column\textsuperscript{16} and is given by the equation.

\[
\frac{P_{cr}}{A} = \sigma_{cr} = \frac{\pi^2 E_t}{(kL)^2}
\]

Residual stress distribution influences column strength significantly\textsuperscript{17,18} A few modifications are necessary to the above formula to incorporate the effect of residual stress. In a loaded column whose cross section contains residual stress, the portion having compressive residual stress yields before the rest of the section. The cross section is no longer homogeneous and the above general equation is not valid. A theoretical solution utilizing the tangent modulus concept is very complicated with a non-linear stress strain relationship. However, this difficulty is obviated by assuming that each fiber has an idealized elastic-plastic stress-strain relation such that

\[
E = E \quad \text{for} \quad \sigma < \sigma_y
\]

\[
E = 0 \quad \text{for} \quad \sigma = \sigma_y
\]
With this relation the tangent modulus load for a column with residual stress distribution is given by\(^{19}\)

\[
\sigma_{cr} = \frac{\pi^2 E I_e}{(kL/j)^2}
\]

The solution of this equation is possible only with a knowledge of the relation between \(\sigma_{cr}\) and \(I_e\). This can be obtained by two methods: 1) based on the assumed or measured residual stress distribution and 2) from the stress-strain curve of the stub column test.

**Tangent Modulus Load**

In this section the tangent modulus load is calculated for weak axis bending of the reinforced 8WF31. From the residual stress distributions in Figs. 8b, 9a and 9b, a simplified residual stress distribution as shown in the figure below is obtained. The residual stress at any point in the flange is given by the equations

\[
\sigma_{rx} = -0.2 \sigma_y \quad 0 \leq x \leq 0.265b
\]

\[
\sigma_{rx} = \left\{ \frac{4.29x}{b} - 1.34 \right\} \sigma_y \quad 0.265b \leq x \leq 0.475b
\]

where \(b = \text{flange width of the unreinforced 8WF31 shape.}\)
The reinforced WF is assumed to have an average flange width = 0.95b and flange thickness = 1.9t, giving \( A_f = 3.6bt \); hence \( A_w = 0.5bt \).

It is assumed that the web has no residual stress and that the yield stress level \( \sigma_Y \) is the same for the entire cross section.

**Buckling Cases**

a) **Elastic**

\[
\sigma_{cr} \leq 0.8 \sigma_Y
\]

\[
\sigma_{cr} = \frac{\pi^2 E}{(L/r)^2}
\]

b) **Load at Proportional Limit (start of yielding)**

\[
\sigma_{cr} = 0.8 \sigma_Y
\]

\[
I = 0.271 b^3 t \quad \frac{I_e}{I} = 0.806
\]

\[
\sigma_{cr} = \frac{\pi^2 E t}{(L/r)^2} = \frac{\pi^2 E}{(L/r)^2} \frac{I_e}{I}
\]

\[
\sigma_{cr} = 0.806 \frac{\pi^2 E}{(L/r)^2}
\]

c) **Load above Proportional Limit**

\[
\sigma_{cr} > 0.8 \sigma_Y
\]
Average Stress for Yielded Cross Section:

\[ 0.265b \leq x_0 \leq 0.312b \]

\[ \sigma_{av} = \sigma_{cr} = \sigma_Y - \frac{Ae}{A} \sigma_{rxo} - \frac{7.6t}{A} \int_0^{x_0} \sigma_{rx} \, dx \]

\[ 0.312b \leq x_0 \leq 0.475b \]

\[ \sigma_{av} = \sigma_{cr} = \sigma_Y + \sigma_{rxo} - \frac{1}{A} \left( \text{hatched area} \right) - \frac{Ae}{A} \sigma_{rxo} \]

Expression for \( \frac{I_e}{I} \):

\[ \frac{I_e}{I} = 1 - 9.3 \left( \frac{x_0}{b} \right)^3 \]

Summary:

a) Elastic:

\[ \sigma_{cr} = \frac{\pi^2 E}{(L/\gamma)^2} \leq 0.8 \sigma_Y \]

b) Load at Proportional Limit:

\[ \sigma_{cr} = 0.827 \frac{\pi^2 E}{(L/\gamma)^2} = 0.8 \sigma_Y \]
c) Load at various $x_0$

<table>
<thead>
<tr>
<th>$x_0$</th>
<th>$\sigma_{cr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.312</td>
<td>0.717 $\frac{\pi^2 E}{(L/r)^2} = 0.893 \sigma_y$</td>
</tr>
<tr>
<td>0.350</td>
<td>0.601 $\frac{\pi^2 E}{(L/r)^2} = 0.936 \sigma_y$</td>
</tr>
<tr>
<td>0.400</td>
<td>0.405 $\frac{\pi^2 E}{(L/r)^2} = 0.976 \sigma_y$</td>
</tr>
<tr>
<td>0.450</td>
<td>0.152 $\frac{\pi^2 E}{(L/r)^2} = 0.994 \sigma_y$</td>
</tr>
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**Slenderness Ratio**

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<th>$x_0$</th>
<th>$I_e/I$</th>
<th>$\sigma_{cr}/\sigma_y$</th>
<th>$L/r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.265</td>
<td>1.000</td>
<td>0.800</td>
<td>100</td>
</tr>
<tr>
<td>0.312</td>
<td>0.717</td>
<td>0.893</td>
<td>80</td>
</tr>
<tr>
<td>0.350</td>
<td>0.601</td>
<td>0.936</td>
<td>72</td>
</tr>
<tr>
<td>0.400</td>
<td>0.405</td>
<td>0.976</td>
<td>58</td>
</tr>
<tr>
<td>0.450</td>
<td>0.152</td>
<td>0.994</td>
<td>35</td>
</tr>
<tr>
<td>0.475</td>
<td>0.000</td>
<td>1.000</td>
<td>0</td>
</tr>
</tbody>
</table>

**Slenderness Ratio for 8WF31 (from Reference 9)**

<table>
<thead>
<tr>
<th>$\sigma_{cr}/\sigma_y$</th>
<th>$L/r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000</td>
<td>0</td>
</tr>
<tr>
<td>0.923</td>
<td>26</td>
</tr>
<tr>
<td>0.846</td>
<td>61</td>
</tr>
<tr>
<td>0.718</td>
<td>102</td>
</tr>
</tbody>
</table>

For $\sigma_{cr}/\sigma_y = 0.718$, the Euler curve defines $(L/r)$
8. TABLES AND FIGURES
Table 1  Scheme of Tests

<table>
<thead>
<tr>
<th>Group</th>
<th>Details</th>
<th>Material</th>
<th>Residual Stress</th>
<th>Coupon Strength</th>
<th>Stub Column</th>
<th>Pin-end Column</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>T-1, T-2, T-18 (Exploratory)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Plain material before reinforcing</td>
<td>T-4, T-6</td>
<td>T-5, T-7</td>
<td>T-8</td>
<td>T-9</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Cover plates welded under load</td>
<td>T-3, T-10</td>
<td>T-11</td>
<td>T-12</td>
<td>T-13</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Cover plates welded under no load</td>
<td>T-14</td>
<td>-</td>
<td>T-16</td>
<td>T-17</td>
<td></td>
</tr>
</tbody>
</table>
### Table 1 (cont'd)

<table>
<thead>
<tr>
<th>Group</th>
<th>Test No.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Exploratory</strong></td>
<td>1</td>
<td>short column welded under load, deformation constant, welding sequence No. 1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>short column welded under load, load constant, welding sequence No. 1</td>
</tr>
<tr>
<td></td>
<td>18</td>
<td>short column welded under constant load of 225k - 25 ksi.</td>
</tr>
<tr>
<td><strong>Plain material</strong></td>
<td>4</td>
<td>residual stress measurements on shape.</td>
</tr>
<tr>
<td>Before reinforcing</td>
<td>5</td>
<td>tension coupon tests on shape.</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>residual stress measurements on plate.</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>tension coupon tests on plate.</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>stub column test on shape.</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>column test on shape, $L/r \approx 50$.</td>
</tr>
<tr>
<td><strong>Cover plates</strong></td>
<td>3</td>
<td>short column welded under load, both flanges welded simultaneously and residual stresses measured.</td>
</tr>
<tr>
<td>welded under load</td>
<td>10</td>
<td>residual stress measurements on reinforced shape.</td>
</tr>
<tr>
<td></td>
<td>11</td>
<td>tension coupon tests on reinforced shape.</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>stub column test on reinforced shape.</td>
</tr>
<tr>
<td></td>
<td>13</td>
<td>column test on reinforced shape.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$L/r \approx 50$</td>
</tr>
<tr>
<td><strong>Cover plates</strong></td>
<td>14</td>
<td>residual stress measurements on reinforced shape.</td>
</tr>
<tr>
<td>welded under no load</td>
<td>16</td>
<td>stub column test on reinforced shape.</td>
</tr>
<tr>
<td></td>
<td>17</td>
<td>column test on reinforced shape.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$L/r \approx 50$</td>
</tr>
</tbody>
</table>
Table 2. Results of Coupon Tests - Yield Stress $\sigma_y$ in Ksi
Tests T-5,7 (Before welding)

<table>
<thead>
<tr>
<th>No.</th>
<th>WF Flange</th>
<th>WF Web</th>
<th>WF Weighted Mean</th>
<th>Plate Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>37.4</td>
<td>40.0</td>
<td></td>
<td>35.9</td>
</tr>
<tr>
<td>2</td>
<td>36.7</td>
<td></td>
<td></td>
<td>35.9</td>
</tr>
<tr>
<td>3</td>
<td>36.8</td>
<td></td>
<td>37.0</td>
<td>36.3</td>
</tr>
<tr>
<td>4</td>
<td>36.2</td>
<td></td>
<td></td>
<td>37.2</td>
</tr>
<tr>
<td>5</td>
<td>35.7</td>
<td>40.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>37.3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Test T-11 (After welding)

<table>
<thead>
<tr>
<th>No.</th>
<th>WF Flange</th>
<th>WF Web</th>
<th>Plate</th>
<th>Weighted mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>36.7</td>
<td></td>
<td>35.9</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>37.6</td>
<td>40.0</td>
<td>35.2</td>
<td>37.4</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td>37.3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td>35.5</td>
<td></td>
</tr>
</tbody>
</table>

Table 3. Results of Stub Column Tests

<table>
<thead>
<tr>
<th>Test No.</th>
<th>$E$ (Ksi)</th>
<th>$\sigma_p$ (Ksi)</th>
<th>$\sigma_y$ (Ksi)</th>
<th>$\sigma_{rc}$ (Ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T-8</td>
<td>29.5</td>
<td>27</td>
<td>37.4</td>
<td>10.4</td>
</tr>
<tr>
<td>T-12</td>
<td>30.8</td>
<td>28</td>
<td>37.4</td>
<td>9.4</td>
</tr>
<tr>
<td>T-16</td>
<td>30.4</td>
<td>27</td>
<td>36.9</td>
<td>9.9</td>
</tr>
</tbody>
</table>
ONE FLANGE WELDED FIRST, THEN THE OTHER ACCORDING TO NUMBERS SHOWN

BOTH FLANGES WELDED SIMULTANEOUSLY ACCORDING TO NUMBERS SHOWN

FIG. 1 WELDING SEQUENCE NO. 1

FIG. 2 WELDING SEQUENCE NO. 2
Odd Nos. A-I Gages
Even Nos. AB-3 (Bakelite) Gages

FIG. 3 LOCATION OF STRAIN GAGES (TESTS T-I & T-2)

FIG. 4 LAY-OUT OF COUPONS (TEST T-II)
FIG. 5 LOAD VS. TIME (TEST T-I)
Reinforced Under Constant Deformation $P_y = 540^k$ (T-1)

Reinforced Under Constant Load $P_y = 528^k$ (T-2)

$8WF31+2R, 7\times 3/8''$, $A = 14.37\text{ in}^2$

$L = 4'0''$, $L/r = 24$

FIG. 6 STRESS-STRAIN CURVE
FIG. 7 CROSS HEAD MOVEMENT VS. TIME (TEST T-2)
TEST T-1

SR-4 STRAIN GAGES: Odd Nos. A-I Type
Even Nos. AB-3 Type
(Bakelite)

Numbers in Circles are Welding Sequence

FIG. 8 STRAIN GAGE READING DURING REINFORCING (Test T-1)
FIG. 9a RESIDUAL STRESS DISTRIBUTION IN 8WF31 AND 7"x3/8" PL BEFORE WELDING

FIG. 9b RESIDUAL STRESS DISTRIBUTION IN 8WF31 REINFORCED UNDER NO LOAD
FIG. 10a RESIDUAL STRESS DISTRIBUTION IN 8WF31 SHAPE REINFORCED UNDER LOAD

FIG. 10b RESIDUAL STRESS DISTRIBUTION IN 8WF31 SHAPE REINFORCED UNDER LOAD, SIMULTANEOUS WELDING ON THE TWO FLANGES
FIG. 11 STRESS-STRAIN CURVES FOR STUB COLUMNS

- ○ 8WF31 UNREINFORCED (T-8)
- △ 8WF31 REINFORCED UNDER LOAD (T-12)
- □ 8WF31 REINFORCED UNDER NO LOAD (T-16)
- × FAILURE DUE TO CRACKING OF WELD
- ▼ LOCAL BUCKLING

$\frac{\sigma}{\sigma_y}$ vs. STRAIN IN IN/IN

STRAIN IN IN/IN

$\sigma_y = 16 \times 10^{-3}$

STRESS-STRAIN CURVES FOR STUB COLUMNS
FIG. 12 COLUMN CURVE FOR WEAK AXIS BENDING
FIG. 13 LOAD-DEFLECTION CURVES FOR PIN-ENDED COLUMNS

- ○ 8WF31 UNREINFORCED (T-9)
- △ 8WF31 REINFORCED UNDER LOAD (T-13)
- □ 8WF31 REINFORCED UNDER NO LOAD (T-17)

\[ P_{\text{max}} = 0.98 P_y \]
\[ P_{\text{max}} = 0.96 P_y \]
\[ P_{\text{max}} = 0.91 P_y \]
9. REFERENCES

1. Huber, A. W.
   RESIDUAL STRESSES IN WIDE-FLANGE BEAMS AND COLUMNS,
   Fritz Laboratory Report 220A.25, Lehigh University,
   July 1959.

2. Beedle, L. S. and Tall, L.
   BASIC COLUMN STRENGTH, ASCE Proc. Paper 2555, 86(ST7)
   p 139, July 1960.

3. Feder, D. K. and Lee, G. C.
   RESIDUAL STRESSES IN HIGH STRENGTH STEEL, Fritz
   Laboratory Report 269.2, Lehigh University,
   April 1959.

4. Tall, L.
   RESIDUAL STRESSES IN WELDED PLATES - A THEORETICAL
   STUDY, Fritz Laboratory Report 249.11, Lehigh
   University, July 1961.

5. Davis, A. F.
   TENANTS UNDISTURBED BY HALF MILLION DOLLAR ALTERATIONS
   TO OLD BUILDING, Journal, American Welding Society,
   7(7) p 55, July 1928.

6. Cook, W. T.
   ARC WELDING USED IN NINE-STORY ADDITION TO DEPARTMENT
   STORE BUILDING, Journal, American Welding Society

7. Wilson, W. M. and Brown, R. C.
   THE EFFECT OF RESIDUAL LONGITUDINAL STRESSES UPON
   THE LOAD-CARRYING CAPACITY OF STEEL COLUMNS, University
   of Illinois Engineering Experiment Station Bulletin
   No. 280, November 1935.

8. Spraragen, W. and Grapnel, L.
   REINFORCING STRUCTURES UNDER LOAD, Journal, American
   Welding Society, 23(2) p 65-S, February 1944.
9. Huber, A. W. and Beedle, L. S.

10. ASTM SPECIFICATION, No. A370-55T

11. Huber, A. W.

12. WRC and ASCE

13. Column Research Council
GUIDE TO DESIGN CRITERIA FOR METAL COMPRESSION MEMBERS, Engineering Foundation, 1960.

14. Bleich, F.

15. Shanley, F. R.

16. Column Research Council

17. Fujita, Y.
BUILT-UP COLUMN STRENGTH, PhD Dissertation, Lehigh University, August 1950.

18. Huber, A. W.
THE INFLUENCE OF RESIDUAL STRESS ON THE INSTABILITY OF COLUMNS, PhD Dissertation, Lehigh University, May 1956.