ON THE BEHAVIOR OF FASTENERS AND PLATES WITH HOLES

by

John W. Fisher

December, 1964

Fritz Engineering Laboratory Report No. 288.18
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This work was carried out as part of the Large Bolted Connections Project sponsored financially by the Pennsylvania Department of Highways, the Department of Commerce - Bureau of Public Roads, and the American Institute of Steel Construction. Technical guidance is provided by the Research Council on Riveted and Bolted Structural Joints.

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December, 1964

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1. **Abstract**

In this report the behavior of the individual components of bolted or riveted joints is discussed. General stress-strain relationships are developed for a plate with holes in the elastic range and beyond. The relationships are applicable to low alloy, low carbon steels such as A7, A36, A242, A440, and A441. They are able to accommodate various specimen geometries. The analytical models are compared with experimental results and show good agreement.

In addition, a load-deformation relationship is developed for mechanical fasteners in shear. The shape of the curve was observed to be governed by the ultimate shear strength and two empirical parameters. The analytical model for the shear-deformation relationship of mechanical fasteners was in excellent agreement with the test data.
2. **INTRODUCTION**

Several theoretical studies of the load partition in riveted and bolted joints in and beyond the elastic range were reported recently\(^{(1,2,3,4)}\). Solutions were achieved by establishing the relationship between deformation and load for the joint components throughout the elastic and inelastic regions.

Both Francis\(^{(1)}\) and Rumpf\(^{(2)}\) used actual load-deformation curves derived from tests of specimens which simulated components of the specimen proper. However, this approach has serious drawbacks. The semi-graphical construction used by Francis and Rumpf is convenient only for the analysis of short joints. Analysis of longer joints is extremely tedious and time-consuming, if not impossible. In addition, it is necessary to establish load-deformation curves for each geometrical specimen considered by testing plate specimens.

A more efficient means of solution was sought. It was considered desirable to develop analytical expressions to describe the load-deformation relationships of the component parts. These were intended to be adaptable to different material properties and geometric configurations. Such expressions would provide a means of extrapolating and interpolating to various geometric forms without requiring the extensive testing necessary with the previous method.

This report describes tests of the component parts of joints and the development of suitable mathematical models which can predict the load-deformation characteristics of the component parts. The study is confined to low alloy, low carbon steels such as A7, A36, A242, A440, or A441.
Although the mechanical fasteners considered are primarily A325 high-strength bolts, A141 steel rivets and A490 high-strength bolts are also studied.
3. TENSILE STRESS-STRAIN RELATIONSHIPS FOR PLATES WITH HOLES

1. PLATES IN TENSION

Any plate with one or more fastener holes in an integral part of a mechanically-fastened joint. As was noted in the introduction, the load partition and strength of any such joint can be determined only if the load-deformation relationships of the fasteners and plates are known. These relationships must be determined experimentally.

The "standard plate calibration coupon" which yields the load-deformation relationship for the connected plate is shown in Fig. 1. The plate calibration coupon should be cut from the same material as the test connections. Its geometrical properties should be similar: the thickness, gage, pitch, and hole diameter must be the same as those of the test or prototype connections.

If a ductile polycrystalline metal bar is loaded continuously and the resulting stresses are plotted as a function of the strain, the characteristic stress-strain as a function of the strain, the characteristic stress-strain relationship shown in Fig. 2 is observed.* This curve is characteristic of most structural steels. The material first stretches elastically until the load reaches a value at which permanent deformations start to develop. After a short transition curve from the elastic to the plastic range of strains, a relatively flat plateau is reached during which the bar continues to stretch without any appreciable change in load. When the strain is about ten times the yield strain the material begins to

* The data is plotted to two different longitudinal scales to more clearly describe the behavior in the plastic region.
strain-harden and additional strain results in an increase in load. This increase continues until the ultimate tensile strength is reached. Thereafter, the material begins to neck and finally ruptures.

When the "standard plate calibration coupon" is loaded continuously and the average stresses at the net section are plotted as a function of the average strain between the two holes, the stress-strain relationship shown in Fig. 3 is observed. The average strain between the two holes has been computed as the average change in length divided by the pitch, p. Also shown are the points at which the average static yield stress is reached on the net and gross sections.

The material first stretches elastically until the load reaches a certain value at which permanent deformation starts to develop around the holes. However, there is no yield plateau at which the material stretches without an appreciable change in load as there is for the plain bar. Strain-hardening begins almost immediately and additional strain is accompanied by an increase in load. This continues until the ultimate tensile strength of the material is reached at the net section. The specimen has necked considerably at the net section and ruptures almost invariably at the ultimate load. It can be seen that the yield plateau observed during the test of the standard bar coupon over an 8 in. gage length did not occur in the plate calibration coupon when yielding started at the net section around the holes, and the plateau did not appear when the yield level was reached in the gross section.

That the presence of holes in a steel plate influences the stress-strain relationship can be seen in the comparison in Fig. 4 of the results of the "standard plate calibration coupon test" with the stress-
strain relationship of the standard flat bar coupon. Elastic studies have
shown that the effect of the stress concentration at the holes is not uni-
formly distributed around the hole but occurs at discreet points on the
boundary of the hole\(^{(5)}\) as shown schematically in Fig. 5a. The contour
lines of radial stress computed according to elastic theory are shown. In
Fig. 5b the stresses perpendicular to line A-A are compared with the
stresses in the bar some distance from the hole. First yield begins at
the points of maximum stress concentration around the hole. As the tensile
stress is increased, yielding spreads and very soon tends to progress along
two comparatively narrow strips symmetrically situated with respect to the
axis of load and at angles of approximately 45 degrees with the direction
of the load\(^{(5,6)}\) as shown schematically in Fig. 6. This type of behavior
has been observed in both A7 and A440 steel specimens.

For many pitches and gages, the yield strips which form symmetri-
cally about adjacent holes will overlap as indicated in Fig. 6b and inter-
ference of the slip bands takes place. The photographs of typical yield
patterns shown in Fig. 7 clearly indicate that interference has occurred.
Because compatibility at grain boundaries is necessary, slip occurs in
several slip systems. This causes severe deformation of the crystal
lattice of each grain which results in the stress rising continuously with
increasing strain\(^{(6)}\).

A number of investigators have developed analytical models for
the plain plate. Hollomon\(^{(7)}\) developed an expression for the relationship
between true stress and natural strain. Nadai\(^{(8)}\) proposed an analytical
expression for the conventional stress-strain curve for use in studies of
plastic buckling. Later Ramberg and Osgood\(^{(9)}\) suggested a slightly
different analytical expression. Unfortunately, none of these are suit-
able for the "standard plate calibration coupon" because they are unable to account for the variations in material properties or plate geometry nor did they fit the test data.

The semi-graphical solution of Francis\(^{(1)}\) and Rumpf\(^{(2)}\) uses the actual stress-strain relationship for the "standard plate calibration coupon." Because the semi-graphical analysis of long joints is extremely tedious and time-consuming, if not impossible, there is a need to develop an analytical model which will describe the stress-strain behavior of the coupon throughout the elastic and inelastic ranges. Ideally this model should account for variations in both material properties and plate geometry.

2. DESCRIPTION OF TESTS

The special plate calibration coupon tests were conducted by testing a plate of the same material used in large joints\(^{(10,11)}\). The plates tested had a width equal to the gage distance \(g\), a thickness \(t\), and two holes drilled a distance \(p\) on center as shown in Fig. 1. The tension-elongation data was recorded for the material with the distance between the hole centers as gage length, which was equal to the pitch length in large joints.

The test specimens, described in Table 1, were flame cut and then milled to the desired width. The dimensions of each calibration specimen are listed along with the measured ultimate tensile strength of the plate and the tensile strength determined in the tests of standard bar coupons. The A7 steel plates for specimens A7-1 to A7-6 were 10\(\frac{3}{4}\) in. wide universal mill strips of thickness varying from 9/16 in. to 7/8 in. All strips were rolled from the same heat. The static yield points varied
from 32.0 to 34.0 ksi and the ultimate tensile strengths from 63.5 to 65.3 ksi. A7 steel test specimens 7031 to 709le were cut from 24 x 1 in. universal mill plates rolled from the same heat. Their mean static yield point was 28.2 ksi and the mean ultimate strength 60.0 ksi. A440 steel test specimens PE4la to PE16l were cut from 26 x 1 in. universal mill plates rolled from the same heat. The mean static yield point was 43 ksi and the mean ultimate strength 76 ksi. Additional details of the standard bar coupon tests are given in Refs. 10 and 11.

The specimens were tested in a 800,000 lb. screw-type testing machine. In the elastic range the cross-head separation was 0.055 in. per min., while in the inelastic range a speed of 0.40 in. per min. was used. In the elastic range equal load increments were applied and the elongation center-to-center of the holes was measured with a slide bar extensometer. Strain increments were used in the inelastic range. Elongations were measured on one edge of the specimen with the testing machine in motion. When the desired strain increment was reached, the cross head movement was stopped and the load was allowed to stabilize to a constant value. Elongation measurements and changes in hole diameter were then measured. This procedure was repeated until failure occurred.

3. DEVELOPMENT OF STRESS-STRAIN RELATIONSHIP

In order to establish the behavior of the plate element in a bolted joint, the stress-strain relationship of the material was determined from a "standard plate calibration coupon" as shown in Fig. 1. The data from such a test is plotted in Fig. 3. The response of the plate calibration coupon can be idealized as shown in Fig. 8. Under initial loading the material remains elastic and the strain increases linearly
with the applied stress.

The primary criterion in the choice of a suitable analytical model to define the stress-strain relationship of the plate calibration coupon is the degree of correlation between the observed test data and the corresponding values calculated from the analytical expression. If possible, it is desirable to obtain a single general relationship which will take into account all the physical and geometrical factors which influence the stress-strain relationship. The major variables influencing the stress-strain relationship of the plate calibration coupon are:

1. the width or gage of the plate \( g \),
2. the hole diameter \( d \),
3. the spacing or pitch of adjacent holes \( p \),
4. the yield point of the coupon \( \sigma_y \),
5. the ultimate tensile strength of the coupon \( \sigma_u \),
6. the type of steel, and
7. the speed of testing of the plate calibration coupon.

The ratio of the net plate area to the gross plate area, governed by the first two variables \( g \) and \( d \), influences the shape of the stress-strain curve (see Fig. 3). In a plate having a large width \( g \), the hole, if small, will have little influence on the average stress-strain relationship. However, with increasing plate width \( g \), the resistance to necking is greater.

When the hole spacing \( p \), the third variable, is close, interference occurs between the slip bands. As was pointed out earlier, this will also influence the stress-strain curve. When the holes are placed farther apart, their effect on the deformation occurring between the two
holes is probably lessened. Hence, the plate calibration coupon will approach the behavior of the standard coupon without holes.

In a bolted or riveted joint, the first three variables, g, d, and p, will be limited to practical ranges. Minimum gage and pitch distances are usually specified. If not, a relative minimum can be estimated from the dimensions of pitch and gage, which can be estimated from practical considerations if they are not specified.

The fourth variable, the yield point of the plate calibration coupon, is influenced by residual stresses and stress concentrations in the vicinity of the holes which cause non-linearity in the plate calibration coupon before it reaches the yield point of the standard bar coupon. The test data indicates that the point of deviation from linearity for the plate calibration coupon is approximately equal to the static yield point of the standard coupon test. Hence, the influence of residual stress concentrations can be accounted for by using this lower yield value.

The ultimate strength of the perforated plates at the net section, the fifth variable, is usually higher than the coupon ultimate strength of the main plate. It is well known from early experimental work that the ultimate strength of a cylindrical bar having a short circumferential groove is considerably higher than the ultimate strength of a round bar because normal necking is prevented in the constricted portion\(^6\). It is to be expected that a flat plate having a hole will behave in a similar manner. The free lateral contraction which must accompany an axial extension cannot develop and a higher ultimate strength results. This behavior was reported in Ref. 12 in work on riveted joints. It was found, particularly at the smaller gage or rivet spacings, that the strength was
greater than one would normally expect on the basis of the joint geometry. The increase attributed to the "reinforcement" or bi-axial stress effect created by the closely-spaced holes. As the gage is increased, this effect is less noticeable. Additional information on this behavior is given in Ref. 13 where the efficiency coefficient is discussed.

The type of steel, the sixth variable, is obviously important and is related to the fourth and fifth variables. Two different grades of steel, A7 and A440, were considered in this study. The ultimate strength of the plate calibration coupon is compared in Fig. 9 with the mean ultimate strength of the same material given by standard laboratory bar coupon tests. The ratio of these strengths is greater with the A440 steel. In general, the behavior of these test specimens was similar to the behavior reported by other investigators (6,12).

The seventh and last variable, the speed of testing, may also influence the stress-strain relationship. The dynamic stress-strain relationship was used because the analytical model was needed to aid in predicting the ultimate strength of the bolted joints. Generally, the bolted joints fail when load is being applied and thus the speed of testing probably affects the test results.

The following assumptions were made in the development of a suitable analytical relationship based in part upon the above-mentioned factors:

(1) stress is proportional to strain when the strain is less than the yield strain,

(2) the average computed elastic strain is based on the gross section area,

(3) the deviation from linearity in the plate calibration coupon can be approximated by considering the static yield point of the standard bar coupon,
the stress-strain relationship beyond the elastic limit is based on the dynamic stress-strain measurements of the plate calibration coupon,

at \( \varepsilon < \varepsilon_{y} \), the stress \( \sigma \) increases at a decreasing rate as the stress approaches the ultimate strength, and

the average computed stress beyond the elastic limit can be calculated using the initial net area as the reference point.

4. A GENERAL STRESS-STRAIN RELATIONSHIP

Hooke's law applies for the initial elastic region and can be expressed as

\[
\sigma = \varepsilon E
\]

where \( \varepsilon < \varepsilon_{y} \) and \( \sigma = \frac{P}{A} \)

In the elastic range, the effect of holes on stretch can usually be ignored. For loads in the elastic range, the washer or bolt head and nut reinforce the lap plate in the area around the hole and serve as a stress bypass. Because the net section area of the lap plate carries less stress, longitudinal deformations in the vicinity of the hole are reduced. Neglecting the non-uniformity of strain because of the holes, the average strain between the holes is approximately

\[
\varepsilon = \frac{\varepsilon}{\frac{p}{p}} = \frac{p}{A E}
\]

where \( p = \) pitch or distance between the centerline of holes,
\( A_{g} = \) gross cross-sectional area,
\( P = \) load, and
\( \varepsilon = \) total deformation between the holes

This expression is applicable until yielding commences in the net section area, at which time the stress is less than the yield point of the gross
area.

As yielding commences in the net section of the plate, the linear relationship between stress and strain is no longer valid. A relationship must be developed that follows a path from B to C as in Fig. 8. Furthermore, it should fit the boundary condition so that at

\[ \sigma = \sigma_y, \; \varepsilon_t = \varepsilon_y, \text{ and at } \sigma = \sigma_{\text{ult}}, \; \varepsilon = \varepsilon_{\text{ult}} \]

The following expression was selected:

\[ \sigma = \varepsilon_y E + K (1 - e^{-\alpha \varepsilon})^\beta \] (3)

where \( \varepsilon < \varepsilon < \varepsilon_{\text{ult}} \) and \( \sigma < \sigma_{\text{ult}} \)

\[ \sigma = \text{average stress on the net area,} \]
\[ \alpha, \beta, K = \text{empirical parameters,} \]
\[ e = \text{base of natural logarithm, and} \]
\[ \varepsilon = \text{inelastic strain} = \varepsilon_t - \varepsilon_y = \frac{\varepsilon_t - \varepsilon_y}{p} \]

Equation 3 was selected after investigation of several other analytical models, including those reported in Refs. 7, 8, and 9. This equation exhibits the following characteristics:

1. as the inelastic strain approaches zero, the stress approaches the yield stress, \( \sigma_y \), which is the limit of Eq. 1.

2. with appropriate values of \( \alpha \) and \( \beta \), the term \( e^{-\alpha \varepsilon} \) approaches zero as the strain approaches the ultimate strain \( \varepsilon_{\text{ult}} \).

3. the equation satisfies the experimentally observed behavior in that the stress \( \sigma \) increases at a decreasing rate as the ultimate stress, \( \sigma_{\text{ult}} \), is approached.

Taken together, Eqs. 1 and 3 describe the complete relationship between stress and strain from Point A to Point C in Fig. 8.
5. EVALUATION OF THE PARAMETERS WHICH INFLUENCE THE STRESS STRAIN RELATIONSHIP

For Eq. 3 to be general and represent the stress-strain relationship for steels of different yield points and ultimate strengths, it is to be expected that the parameters \( \alpha, \beta, \) and \( K \) may be functions of these values. As was discussed earlier, other variables which may influence the stress-strain relationship are the hole diameter and the gage or plate width. In effect, they relate the net area at the hole to the gross plate area.

The parameters \( \alpha, \beta, \) and \( K \) were initially evaluated by regression analysis. Equation 3 was first linearized as

\[
\log \sigma = \log \epsilon \cdot E + \log K + \beta \log (1-e^{-\alpha \epsilon})
\]

\[
= \log \gamma + \beta \log (1-e^{-\alpha \epsilon}) \quad (4)
\]

where

\[
\log \gamma = \log \epsilon \cdot E + \log K
\]

The least square normal equations that will minimize the sum of squared residuals for \( N \) sets of data (values of stress and strain from the plate calibration coupon) are:

\[
\Sigma \log \sigma = N \log \gamma + \beta \Sigma \log (1-e^{-\alpha \epsilon}) \quad (5)
\]

\[
\Sigma (\log \sigma) (\log (1-e^{-\alpha \epsilon})) = \log \gamma \Sigma \log (1-e^{-\alpha \epsilon}) + \beta \Sigma (\log (1-e^{-\alpha \epsilon}))^2 \quad (6)
\]

The coefficients \( \log \gamma \) and \( \beta \) were determined by simultaneous solution of Eqs. 5 and 6. Several values of \( \alpha \) were assumed for the analyses made with the data from each plate calibration coupon. A comparison of the test data for each coupon (Fig. 10) indicates that this parameter will differ from coupon to coupon. Because the parameter \( \alpha \) cannot be arrived at explicitly from the regression analysis, it was necessary to repeat the analysis for several values of \( \alpha \) until the best correlation was obtained.
After obtaining the values of $\alpha$, $\beta$, and $K$ from the test data for A7 and A440 steel specimens with large differences in plate width, the final analysis was made by evaluating these coefficients in terms of the known boundary conditions.

The coefficient $K$ was evaluated from the boundary condition at $\varepsilon = \varepsilon_{ult}$, $\sigma = \sigma_{ult}$. Therefore, from Eq. 3

$$\sigma = \sigma_{ult} = \sigma_y + K(1-e^{-\alpha \varepsilon})^\beta$$

which yields

$$\sigma_u = \sigma_y + K$$

Therefore $K = \sigma_u - \sigma_y$ where $\sigma_u$ = ultimate tensile strength at the net section of a perforated plate and $\sigma_y$ = yield point at the net section.

The parameter $\alpha$ which varied from plate to plate was finally evaluated as a function of the geometry and material properties. It was evaluated from the regression coefficient as

$$\alpha = (\sigma_u - \sigma_y) \left(\frac{g}{g-d}\right)$$

where $g$ = the width of the specimen
$d$ = the diameter of the hole.

The ratio $g/(g-d)$ is in effect a ratio of the gross area to the net area, and $\alpha$ could be written as $(\sigma_u - \sigma_y) A_g / A_n$.

The parameter $\beta$ was found to be a constant common to all materials and conditions. It was evaluated from the regression analysis as

$$\beta = 3/2$$

The final general relationship for stress-strain applicable to both A7 and A440 steel and various specimen geometries was found to have the form:
\[ \sigma = \sigma_y + (\sigma_u - \sigma_y) \left[ 1 - e^{-\left(\sigma_u - \sigma_y\right)(g/(g-d))e/p} \right]^{3/2} \]  

(11)

where

\[ p = \text{pitch or distance center to center of the holes} \]
\[ e = \text{total deformation in the pitch after yielding on net section} \]
\[ e/p = \varepsilon_p = \text{plastic strain}. \]

This equation is applicable for values

\[ \sigma_y < \sigma < \sigma_{\text{ult}} \]

For stresses lower than the yield point, Eq. 1 is applicable. Equation 11 takes into account variations in material properties (\(\sigma_u\), \(\sigma_y\)) and geometrical configuration of the plate calibration coupon (\(g\), \(p\), \(d\)).

6. COMPARISON OF THEORY AND EXPERIMENTAL DATA

The test data for the plate calibration specimens of different thickness are plotted in Fig. 11. The load acting on the specimen is plotted as a function of the measured elongations, \(e\), from center to center of the holes. Also shown in Fig. 11 are the computed load-deformation curves based on Eqs. 1 and 11. The agreement between the computed and experimental results clearly shows the applicability of the mathematical models.

Equations 1 and 11 are further compared with the test data for several 1 in. A7 steel plate in Fig. 12. The average stress on the net section is plotted as a function of the average strain for the material between the hole centers. Each plot corresponds to a different plate calibration test. The principal difference between the different specimens was the gage width \(g\). Also shown in each plot is the static yield
point, $\sigma_y$, determined from the standard bar coupon tests. This value is reached at the net section ($\sigma_y$)net. In addition, the average stress on the net section at which the static yield value is reached on the gross section is indicated as ($\sigma_y$) gross. The standard coupon ultimate strength is indicated as ($\sigma_u$)coup. For all cases, the strength of the perforated plate was higher than the coupon ultimate strength. A direct comparison of these values is given in Table 1. The static yield point, $\sigma_y$, was determined from coupon tests as 28.2 ksi.

In the A7 steel tests $d$ was maintained constant at 0.94 in. and the thickness at 1 in. Gage $g$ varied from 2.92 to 6.88 in. The theoretical line is in excellent agreement with the test data.

A similar comparison is made with A440 steel test data in Fig. 13. The static yield point of the A440 material was 43.0 ksi. The gage $g$ varied from 3.32 to 6.94 in. and the hole diameter was again constant at 0.94 in. Again the computed line is in excellent agreement with the test data.

In the comparisons made in Figs. 12 and 13, the pitch $p$ was constant at 3.5 in. A special series of tests performed for an earlier study was used to evaluate the effect of pitch. The pitch $p$ was varied from 2.5 to 6 in. while the hole diameter and gage width were constant. The computed lines are compared in Fig. 14 with the test data for the 2.5 in. and 6 in. pitch specimens. The agreement in all cases is good.

Figure 14 indicates that a "yield plateau" is approached as the pitch between the holes is increased. Figure 15 is a schematic of the elastic and initial plastic region for the plate calibration coupon. The smooth transition curve between the initial yield on the net section,
(σ_y)net, and the onset of yielding on the gross section, (σ_y)gross, is to be expected as discussed earlier. For the larger pitches, the holes should have a less influence on the average strain and one would expect a yield plateau similar to those encountered with the standard bar coupon. As the distance between adjacent holes is decreased, the slip line interference will become more pronounced with a consequent decrease in the length of the yield plateau for the gross section area between the holes. An examination of Fig. 14 indicates that this was the case.
4. SHEAR DEFORMATION RELATIONSHIP FOR MECHANICAL FASTENERS

1. THE BEHAVIOR OF MECHANICAL FASTENERS

In the development of load-deformation relationships for mechanical fasteners it is generally assumed that the deformation of the fastener will involve the effects of shearing, bending, and bearing of the fastener as well as the localized deformation of the main and lap plates.

If a single fastener joint is loaded as shown in Fig. 16, the relative movement of points a and b is influenced by the shear, bending, and bearing of the fastener. Fig. 17 shows a deformed bolt illustrating this behavior. The connected members will also deform and the relative movement of a, and b, if measured at the edges of the plate, will be greater as a result of the compression of the members behind the fastener. For the elastic case, Coker has shown that the longitudinal compressive stress in the plate dies away at a distance of about twice the hole diameter from the edge of the hole\(^{(16)}\). Hence, the bearing deformations in the plate are localized. In the side view of the joint they are indicated by the dark edges. In measuring the relative movement of a and b, the deformation of the fastener and plate are combined because there is no reason to separate them.

Two types of control tests can be conducted with coupons to determine the load-deformation relationship. In one type the bolts are subjected to double shear by plates loaded in tension as indicated in Fig. 18. In the other control test the bolts are subjected to double shear by applying a compressive load to the plates (Fig. 18). As long as the shear jig plate is reasonably stiff and nothing other than local yield-
ing due to bearing occurs, any plate elongations other than those due to bearing are negligible.

The load-deformation relationship for the two control tests are shown in Fig. 18 for a typical A325 bolt lot. Extensive calibration tests have shown that single bolts tested in plates loaded in tension had approximately 5 to 10% less shear strength than bolts loaded in plates under compression (17). The relative merits of the two types of control tests are discussed in greater detail in Ref. 17.

The results of the shear tests were used to develop the required form of the fastener load-deformation (shear-deformation) relationship shown schematically in Fig. 19. The relationship is essentially linear until inelastic deformations occur. Thereafter it becomes non-linear. The ultimate shear strength is assumed to be approached in an asymptotic manner, and the reduction in strength observed at final fracture is neglected.

No analytical expressions are known to have been developed for the elastic-inelastic load-deformation relationship of a fastener. For the elastic region a linear relationship is usually assumed such as

$$R = \overline{K} \Delta$$

The elastic constant $\overline{K}$ has usually been determined from experimental data. Reference 18 gives a solution for the coefficient $\overline{K}$ by assuming the fastener to be a fixed-end beam. It is noted that such an analysis violates several basic assumptions underlying conventional beam theory.

The deflection caused by shear, bending, and bearing was determined separately. Deflection was measured relative to a line passing through the centroids of the end cross sections of fasteners, and shearing,
and bending deflections were found at the center of the span. The bolt bearing deformation was defined as a percentage of the bolt diameter. For shear it was found that

$$ K_s = \frac{t + t'}{3G_bA_b} $$  \hspace{1cm} (13)

for bending,

$$ K_b = \frac{(t')^3 + 4t(t')^2}{192EI_b} $$ \hspace{1cm} (14)

for bearing,

$$ K_{br} = \frac{2(t + t')}{Et t'} $$  \hspace{1cm} (15)

The localized bearing effect of the fastener on the plate was found to be the same as Eq. 15. Hence, the constant $\bar{K}$ in Eq. 12 was evaluated as

$$ \bar{K} = \frac{2}{K_s + K_b + 2K_{br}} $$  \hspace{1cm} (16)

where $E$ = modulus of elasticity,

$G_b$ = shear modulus,

$A_b$ = fastener area = $\pi d^2/4$,

$I_b$ = moment of inertia = $d^4/64$,

$t'$ = thickness of lap plates, and

$t$ = thickness of main plate.

Reference 19 suggested a means of approximating the load-deformation relationship for rivets by considering the load-deformation relationship to be represented by two straight lines. The displacement $\Delta$, between the main and lap plates was computed as being due to bending and shear. The bearing deformations in the plate and rivet were approximated. The total deformation is given by
\[
\Delta = R \left[ \frac{1.125(t')^3 + 3.75t(t')^2 + 5t't^2 + 2t^3}{11.78d^4E} + \frac{0.3(t+t')}{d^2E} \right] + \frac{0.375}{dE} + \frac{1.3}{E} \left( \frac{1}{t'} + \frac{1}{t} \right)
\]  

(17)

Hence, \( \bar{K} \) is given by bracketed term in Eq. 17. This is assumed to hold for large diameter rivets which are stiff and do not bend appreciably. For small diameter fasteners the deformation is influenced by large bending deformations. It is expressed as

\[
\Delta = R \left( \frac{3.6g+6.8g^3}{Ed} \right)
\]  

(18)

where \( g \) is an empirical parameter. In order to obtain a coefficient giving the correct order of magnitude of the deformation, an approximate relationship is given as

\[
\Delta = \frac{R}{E} \left[ \frac{6.5 + 0.8}{t} + \frac{2.5}{d} \right]
\]  

(19)

The parameters relating load and deformation are determined empirically. However, the expression takes into account the geometrical properties of the fastener and connected material. The relationship is valid only below the limit of proportionality. The slope of the line representing the elastic behavior is assumed to be 4 times as great as the line representing the inelastic behavior.

Equations 16 and 17 were used to make an initial approximation of the elastic constant \( \bar{K} \) in Eq. 12. This in turn was used to help evaluate the parameters for the analytical model developed.

2. ASSUMPTIONS

The criteria in the choice of the analytical expression describing the load-deformation relationship of a bolt in double shear are
the boundary conditions and the known experimental data. A number of vari-
ables are known to influence the load-deformation relationship of the bolt
control test. Among these are: (1) the diameter of the bolt; (2) the
thickness of the lap plates; (3) the thickness of the main plate; (4) the
type or grade of steel plates; and, (5) the type of bolt. Reference 17
discusses each of these variables in detail.

The following assumptions are made for the analytical relation-
ship developed herein. They are based in part on the behavior observed in
Fig. 18.

1. At zero loads the deformation is zero.

2. For small values of deformation the relationship between
load and deformation is approximately linear.

3. As \( \Delta \) approaches \( \Delta_{\text{ult}} \), the bolt force increases at a de-
creasing rate.

4. The deformation \( \Delta \) contains the components due to shear,
bending, and bearing of the fastener as well as the
shearing deformation of the plates.

The following expression is selected because it satisfies these conditions
and because only one continuous function was necessary

\[
R = \tau [1 - e^{-\mu \Delta}]^\lambda \tag{20}
\]

where

\[ \Delta_{\text{ult}} > R > 0, 0 < \Delta < \Delta_{\text{ult}}. \]

\( \Delta \) = total deformation of bolt and bearing deformation
of the connected material,

\( \tau, \mu, \lambda \) = regression coefficients, and

\( e \) = base of natural logarithm.

Equation 20 satisfies the boundary condition that requires the load to be
zero at a zero deformation.

If the function described by Eq. 20 is expanded in a Maclaurin's
series there is obtained if \( \lambda \) is unity

\[
f(\Delta) = \mu \tau \Delta - \mu \frac{\Delta^2}{r^2} + \ldots + (-1)^{n+1} \frac{\mu^n \tau^n}{n!} \tag{21}
\]

This series is convergent as long as \( \mu \Delta < 1 \). For small values of \( \Delta \) this condition is satisfied and an approximate solution is obtained by considering only the first term. Hence,

\[ R = \mu \tau \Delta \tag{22} \]

This is directly analogous to Eq. 12 and the expressions used in Refs. 18 and 19. It also shows that Eq. 20 satisfies assumption 2.

The equation satisfies the experimentally observed behavior shown in Fig. 18 because it allows the bolts force \( R \) to increase at a decreasing rate as the ultimate shear strength of the bolt is approached.

3. EVALUATION OF PARAMETERS

The parameters \( \tau, \mu, \) and \( \lambda \) were evaluated by regression analysis and the boundary conditions. Equation 20 was first linearized as

\[ \log R = \log \tau + \lambda \log[1 - e^{-\mu \Delta}] \tag{23} \]

The coefficients \( \log \tau \) and \( \lambda \) were determined by the solution of the simultaneous least squares normal equations for the linear function given as Eq. 23. It was necessary to assume several values of \( \mu \) for the analysis made on each type of control specimen. Actual values of measured load and the corresponding deformation as reported in Refs. 10 and 17 were used in the analysis. An initial estimate of \( \mu \) could be determined using Eq. 16, 17, and 22. A best fit was obtained when the squared residuals were minimized and the boundary condition \( R = R_{\text{ult}} \) was satisfied. Hence, the coefficient \( \tau \) was found to be
\[ \tau = R_{ult} \]  

(24)

The parameter \( \mu \) varied for the different fasteners investigated. For 7/8 in. A325 bolts tested in one-inch A7 steel plates, the value is approximately 18. For 7/8 in. A325 bolts tested in one-inch A440 steel plates, the value is approximately 23. These values appear to be the same for bolts tested in plates loaded in tension as well as plates loaded in compression.

The parameter \( \lambda \), almost constant for the 7/8 in. A325 bolts and A7 or A440 connected material, is approximately unity.

The final relationship for load-deformation or shear-deformation is

\[ R = R_{ult} \left[ 1-e^{-\mu \lambda \gamma} \right] \]  

(25)

where \( R_{ult} \) = Ultimate shear Strength.

The average values of \( R_{ult} \), \( \mu \) and \( \lambda \) are tabulated in Table 2 for typical lots of bolts and rivets and compared to Eq. 25 in the next section.

The total deformation capacity \( \Delta_{ult} \) for a given bolt and connected material is a function of the shear, bending, and bearing of the bolt and the bearing deformation of the plates. As might be expected, this will vary with the type of calibration test, the type of connected steel, and the thickness of the gripped material. Values of \( \Delta_{ult} \) are also tabulated in Table 2.

4. COMPARISON OF COMPUTED AND EXPERIMENTAL RESULTS FOR SINGLE BOLTS

The two types of control shear tests are described briefly in the previous articles. Additional information on the test methods and a detailed description of the test specimens and test data are given in Ref.
The test data for both types of control tests on A325 bolts in A440 steel are plotted in Fig. 20 for the same bolt lot. Usually, three different specimens were tested for each type of test made for each bolt lot. The load-deformation data for 7/8 in. A325 bolts in A7 steel is given in Fig. 21. The type of calibration test had little effect on the parameters $\mu$ and $\lambda$. The predicted line is in good agreement with the test data in Figs. 20 and 21.

The actual values of $\mu$, $R_{ult}$, and $\lambda$ for several bolt and rivet lots are given in Table 2. The exponent $\lambda$ is affected only slightly by the variations in the connected material properties and the specimen geometry for 7/8 in. A325 bolts. The type of control test had little influence on the parameters $\mu$ and $\lambda$. Only the ultimate strength $R_{ult}$ was affected as described earlier. Apparently the coefficient $\mu$ was mostly affected by the type of connected material.

It is believed that the parameters $\mu$ and $\lambda$ can be related to the physical and geometrical properties of the plate and bolt. Thus additional studies are desirable if a generalized expression is to be developed.

The total deformation capacity of the fasteners is less in the higher strength steels because the bearing deformation in the plate is less. However, this disadvantage is offset by the more favorable redistribution of the joint load which occurs among the A325 bolts in higher strength steels (3).
5. **SUMMARY**

Analytical expressions for the stress-strain relationship of a plate with holes and for the shear-deformation relationship of rivets and high-strength bolts have been developed. Both expressions are necessarily applicable to the elastic and inelastic regions.

The analytical expressions for the plate with holes can be adapted to changes in the geometrical configuration as well as differences in the yield point and ultimate strength. The analytical model was compared with tests of plate specimens having two drilled holes. Among the variables checked were plate width, pitch or distance between the centers of the holes, plate thickness, and grade of steel. The analytical model adequately responded to changes in geometry and material properties.

A continuous function was used to represent the load-deformation characteristics of a single bolt in shear. The shape of the curve was governed by the ultimate shear strength and two empirical parameters. These parameters were found to vary for different fasteners and different types of connected material.
6. ACKNOWLEDGEMENTS

This study has been carried out as a part of the research project on "Large Bolted Connections" being conducted at Fritz Engineering Laboratory, Department of Civil Engineering, Lehigh University. Professor W. J. Eney is head of the Department and Laboratory and Dr. L. S. Beedle is Director of the Laboratory. The project is sponsored by the Pennsylvania Department of Highways, the U. S. Department of Commerce - Bureau of Public Roads, and the American Institute of Steel Construction.

The author is appreciative of the supervision and encouragement of Dr. L. S. Beedle during the preparation of this report. Thanks are also extended to Miss Valerie Austin who typed the manuscript; to H. A. Izquierdo who prepared the drawings; to W. H. Dígel who reviewed the manuscript; and to the guiding committee of the Research Council on Riveted and Bolted Structural Joints for many helpful suggestions.
## Table 1.

**GEOMETRY AND TEST RESULTS OF PLATE CALIBRATION SPECIMENS**

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Fig. 1  Schematic of Plate Calibration Coupon

Fig. 2  Typical Stress-Strain Diagram for Standard Bar Coupon
Fig. 3  Typical Stress-Strain Diagram for Plate Calibration Coupon

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Fig. 6  Slip Lines on the Plate Calibration Coupon
Fig. 7  Yield Lines Indicated by Whitewash Coating
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Fig. 9  Effect of Gage on Ultimate Strength of Plate Calibration Coupon

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A7 = Steel
P = 3.5 in.
g = 4.94 in.
d = 6.94 in.

Fig. 11 Summary of Plate Calibration Test for A7 Steel of Variable Thickness
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Fig. 15  Schematic Stress-Strain Relationship
Fig. 16 Deformation of a Single Bolt

Fig. 17 Sawed Section of a Single Bolt
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Fig. 20  Comparison of Computed and Experimental Results for A325 Bolts in A440 Steel

Fig. 21  Comparison of Computed and Experimental Results for A325 Bolts in A7 Steel
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