Gravity Wave Research

SCOUR OF FLAT SAND BEACHES IN FRONT OF SEAWALLS

by

Stephen C. Ko

Fritz Engineering Laboratory Report No. 293.5
Project Report No. 53

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March 1967

Bethlehem, Pennsylvania

Fritz Engineering Laboratory Report No. 293.5
ACKNOWLEDGMENTS

The author wishes to express his gratitude for the advice and review of this manuscript by his adviser, Dr. John B. Herbich,* former Chairman of the Hydraulic and Sanitary Engineering Division, Fritz Engineering Laboratory, Civil Engineering Department at Lehigh University. Dr. L. S. Beedle is the Acting Chairman of the Department and Director of the Laboratory.

Financial support from the Institute of Research, Professor G. R. Jenkins, Director; is acknowledged.

A special note of thanks is also given to Miss Linda Nuss for typing this manuscript.

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# Table of Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acknowledgments</td>
<td>iii</td>
</tr>
<tr>
<td>Table of Contents</td>
<td>iv</td>
</tr>
<tr>
<td>List of Figures</td>
<td>vi</td>
</tr>
<tr>
<td>List of Tables</td>
<td>vii</td>
</tr>
<tr>
<td>List of Symbols</td>
<td>viii</td>
</tr>
<tr>
<td>Abstract</td>
<td>1</td>
</tr>
<tr>
<td>1. Introduction</td>
<td>2</td>
</tr>
<tr>
<td>2. Theoretical Consideration</td>
<td>5</td>
</tr>
<tr>
<td>2.1 Equation of Motion and Equation of Continuity</td>
<td>5</td>
</tr>
<tr>
<td>2.2 Boundary Conditions</td>
<td>6</td>
</tr>
<tr>
<td>2.3 Shallow Water Wave Theories</td>
<td>9</td>
</tr>
<tr>
<td>2.3.1 Stokes' Wave with Finite Amplitude</td>
<td>9</td>
</tr>
<tr>
<td>2.3.2 Cnoidal Theory</td>
<td>11</td>
</tr>
<tr>
<td>2.4 The Mechanics of Sediment Movement</td>
<td>13</td>
</tr>
<tr>
<td>2.5 Boundary Layer Along a Flat Sand Bed</td>
<td>15</td>
</tr>
<tr>
<td>2.6 A Theoretical Equation for Wave Scour</td>
<td>18</td>
</tr>
<tr>
<td>3. Dimensional Analysis</td>
<td>22</td>
</tr>
<tr>
<td>3.1 Significant Variables</td>
<td>22</td>
</tr>
<tr>
<td>3.2 Dimensionless Terms</td>
<td>24</td>
</tr>
</tbody>
</table>
# Table of Contents (continued)

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4. EXPERIMENTAL APPARATUS AND PROCEDURE</td>
<td>26</td>
</tr>
<tr>
<td>4.1 Apparatus</td>
<td>26</td>
</tr>
<tr>
<td>4.2 Geometric Configuration</td>
<td>28</td>
</tr>
<tr>
<td>4.3 Experimental Procedure</td>
<td>32</td>
</tr>
<tr>
<td>5. PRESENTATION AND DISCUSSION OF RESULTS</td>
<td>34</td>
</tr>
<tr>
<td>5.1 Observations</td>
<td>34</td>
</tr>
<tr>
<td>5.2 Relationship Between $\frac{S}{K}$ and $\frac{T}{t}$</td>
<td>36</td>
</tr>
<tr>
<td>5.3 Relationship Between $\frac{S}{K}$ and $C_r$</td>
<td>37</td>
</tr>
<tr>
<td>5.4 Relationship Between $\frac{S}{K}$ and $\lambda$</td>
<td>37</td>
</tr>
<tr>
<td>5.5 A Comparison Between Calculated Values and Experimental Results</td>
<td>37</td>
</tr>
<tr>
<td>6. SUMMARY AND CONCLUSIONS</td>
<td>44</td>
</tr>
<tr>
<td>7. APPENDIX</td>
<td>46</td>
</tr>
<tr>
<td>8. BIBLIOGRAPHY</td>
<td>49</td>
</tr>
</tbody>
</table>
### LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Schematic explanation of terminology</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>Geometry of wave motion</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>Free surface of wave motion</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>Comparison of horizontal components of water particle velocity with shallow wave theory</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>Initial movement of a sand particle</td>
<td>13</td>
</tr>
<tr>
<td>6</td>
<td>Drag coefficient of a sphere as a function of Reynolds Number</td>
<td>14</td>
</tr>
<tr>
<td>7</td>
<td>Envelope of wave motion</td>
<td>18</td>
</tr>
<tr>
<td>8</td>
<td>Geometry of sand scour</td>
<td>20</td>
</tr>
<tr>
<td>9</td>
<td>Schematic diagram of capacitance-type wave recording</td>
<td>27</td>
</tr>
<tr>
<td>10</td>
<td>Wave channel</td>
<td>29</td>
</tr>
<tr>
<td>11</td>
<td>Experimental facility</td>
<td>30</td>
</tr>
<tr>
<td>12</td>
<td>Grain size distribution</td>
<td>31</td>
</tr>
<tr>
<td>13</td>
<td>Sand scour and envelope</td>
<td>35</td>
</tr>
<tr>
<td>14</td>
<td>Relationship between $\frac{S}{K}$ and $\frac{T}{t}$</td>
<td>38</td>
</tr>
<tr>
<td>15</td>
<td>Relationship between $\frac{S}{K}$ and $C_r$</td>
<td>39</td>
</tr>
<tr>
<td>16</td>
<td>Fluctuation of $H$, $C_r$ as a function of $\frac{T}{t}$</td>
<td>42</td>
</tr>
<tr>
<td>17</td>
<td>Relationship between $\frac{S}{H}$ and $\frac{T}{t}$</td>
<td>43</td>
</tr>
</tbody>
</table>
# List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>The function of $f'(\eta)$ for boundary layer along a flat plate at zero incidence.</td>
<td>17</td>
</tr>
<tr>
<td>2.</td>
<td>Significant variables</td>
<td>23</td>
</tr>
<tr>
<td>3.</td>
<td>Experimental Program</td>
<td>32</td>
</tr>
<tr>
<td>4.</td>
<td>A comparison between calculated values and experimental results.</td>
<td>40</td>
</tr>
</tbody>
</table>
# List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$H_I + H_R$</td>
<td>L</td>
</tr>
<tr>
<td>B</td>
<td>$H_I - H_R$</td>
<td>L</td>
</tr>
<tr>
<td>$C_D$</td>
<td>Drag coefficient</td>
<td></td>
</tr>
<tr>
<td>$c_n$</td>
<td>Jacobian Function</td>
<td></td>
</tr>
<tr>
<td>$C_r$</td>
<td>Reflection Coefficient</td>
<td></td>
</tr>
<tr>
<td>$D$</td>
<td>Still Water Depth</td>
<td>L</td>
</tr>
<tr>
<td>$d$</td>
<td>Effective Sand Diameter 50 per cent finer</td>
<td>L</td>
</tr>
<tr>
<td>$\vec{F}_0$</td>
<td>Drag force, vector</td>
<td>F</td>
</tr>
<tr>
<td>$F_x, F_y, F_z$</td>
<td>Force Component</td>
<td>F</td>
</tr>
<tr>
<td>$g$</td>
<td>Acceleration due to gravity</td>
<td>L/T^2</td>
</tr>
<tr>
<td>$h$</td>
<td>$\xi - D$, Water particle under free surface</td>
<td>L</td>
</tr>
<tr>
<td>$H$</td>
<td>Wave Height</td>
<td>L</td>
</tr>
<tr>
<td>$H_I$</td>
<td>Incident wave height</td>
<td>L</td>
</tr>
<tr>
<td>$H_R$</td>
<td>Reflected wave height</td>
<td>L</td>
</tr>
<tr>
<td>$K$</td>
<td>$D - \frac{1}{2} A$, refer to Fig. 1</td>
<td>L</td>
</tr>
<tr>
<td>$K(k)$</td>
<td>Elliptical integral, first kind</td>
<td></td>
</tr>
<tr>
<td>$L$</td>
<td>Wave length</td>
<td>L</td>
</tr>
<tr>
<td>$p$</td>
<td>Pressure</td>
<td>F/L^2</td>
</tr>
<tr>
<td>$S$</td>
<td>Scour depth</td>
<td>L</td>
</tr>
<tr>
<td>$\overline{S}$</td>
<td>Average scour depth in front of a seawall</td>
<td>L</td>
</tr>
<tr>
<td>$t$</td>
<td>Wave period</td>
<td>T</td>
</tr>
<tr>
<td>$T$</td>
<td>Time elapse</td>
<td>T</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Dimension</td>
</tr>
<tr>
<td>--------</td>
<td>-------------------------------------------------------</td>
<td>-------------</td>
</tr>
<tr>
<td>$u_*$</td>
<td>Horizontal velocity within boundary layer</td>
<td>L/T</td>
</tr>
<tr>
<td>$U$</td>
<td>Horizontal velocity of wave motion</td>
<td>L/T</td>
</tr>
<tr>
<td>$V$</td>
<td>Velocity Vector</td>
<td>L/T</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Water Density</td>
<td>$F/L^3$</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>Sand Density</td>
<td>$F/L^3$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Angle of repose</td>
<td>degrees</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Velocity potential function</td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Specific weight of water</td>
<td>$F/L^3$</td>
</tr>
<tr>
<td>$\gamma_s$</td>
<td>Specific weight of sand</td>
<td>$F/L^3$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Dynamic viscosity</td>
<td>$FT/L^2$</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Kinematic viscosity</td>
<td>$L^2/T$</td>
</tr>
<tr>
<td>$Re$</td>
<td>Reynolds Number = $\frac{KV}{\nu}$</td>
<td></td>
</tr>
<tr>
<td>$Fr$</td>
<td>Froude Number = $\frac{V^2}{gK}$</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Slope of seawall</td>
<td>degrees</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Scour wave length</td>
<td>L</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Free water surface</td>
<td></td>
</tr>
<tr>
<td>$\eta$</td>
<td>Dimensionless co-ordinate of boundary layer</td>
<td></td>
</tr>
</tbody>
</table>
ABSTRACT

The nature of scour of a flat sand beach in front of a seawall was studied. Theoretical development made use of shallow water wave theory and boundary layer equations and a mathematical model was developed for the ultimate scour depth in front of a seawall. It was assumed that the drag coefficient for a spherical particle moving along a flat plate in an infinite fluid field is the same as the drag coefficient for a spherical particle moving through an infinite fluid field. The agreement between the mathematical model and the dimensional analysis is considered good and indicates that the assumptions made are reasonable.

Laboratory experiments indicated that the depth of the scour depends to a large extent on the wave characteristics. It is also found that the scour length is independent of time but is a function of incident wave length. The comparison between the theoretical calculated values and experimental results shows reasonable quantitative equality.
1. INTRODUCTION

The problem of beach erosion has been studied very extensively during the past few years. However, most of the investigations were confined to beach erosion due to "breaking" waves. Very few studies have been conducted in the area of beach erosion due to "non-breaking" waves. Several prior studies on this particular topic had been made in Fritz Engineering Laboratory of Lehigh University, Bethlehem, Pennsylvania.

The primary study of beach erosion due to "non-breaking" waves was conducted in 1964 dealing with the stability of a horizontal sand bed deposited in shallow water in front of an impervious seawall.\(^{(2)}\) In 1965 similar study was carried further and several conclusions were reached.\(^{(2)}\)

The present study has been confined to the non-breaking shallow water waves progressing toward a seawall. In such a case, waves may or may not break on the seawall. It has been the object of the study to investigate the nature of scour of a flat sand beach in front of a seawall due to wave action. The materials and construction of the seawall were not considered.

* Numbers in parenthesis refer to references on pages 46 and 47.
When a system of incident waves progresses toward the seawall and hits the seawall, another system of waves is formed due to the reflection from the seawall. These two wave systems form a third wave system as the reflected wave is superimposed on the incident wave. The velocity components of the new wave system are given by the summation of the velocity component vectors of the incident and reflected waves. A schematic explanation of the terminology is shown in Figure 1.

The theoretical development is based on the continuity equation and boundary layer theory equations. A mathematical model which describes the ultimate scour depth has been obtained. The agreement between the theoretical derivation and dimensional analysis gave considerable assurance of the correctness of the assumptions made and the theoretical concepts employed.
Fig. 1 Schematic explanation of terminology

1. Incident Wave
2. Wave Length
3. Wave Height
4. Wave Crest
5. Wave Trough
6. Water Depth
7. Ripples
8. Envelope
9. Loop of Envelope
10. Node of Envelope
11. Sand Scour Length
12. Sand Scour Depth
13. Reflected Wave
14. Seawall
15. Angle of Seawall
2. THEORETICAL CONSIDERATIONS

2.1 Equation of Motion and Equation of Continuity

Assume that water wave motion is generated from rest by a horizontal force, and the fluid pattern is irrotational and satisfies the velocity potential requirements. Under these assumptions two equations must be satisfied, namely, Euler's equation of motion and Laplace's equation of continuity.

Euler's equation of motion can be obtained from the concept of Newton's law of conservation of momentum, as we have

\[ d\bar{F} = D(dm \bar{V}) \]

\[ = dm \left( u \frac{\partial \bar{V}}{\partial x} + v \frac{\partial \bar{V}}{\partial y} + w \frac{\partial \bar{V}}{\partial z} + \frac{\partial \bar{V}}{\partial t} \right) \]

where \( dm \) is the mass of a particle in the velocity field \( \bar{V} \) \((x, y, z, t)\), and \( x, y, z \) are coordinate and \( t \) is time. The surface force on a fluid particle is due to pressure while the body force is due to gravity. Equation (2.1) can be written as:

\[ - \frac{1}{\rho} \text{grad} \ p + \bar{F} = \frac{\partial \bar{V}}{\partial t} \]

(2.2)

and it is usually referred to as the Euler's equation of motion.

In Euler's equation, the gravity force \( \bar{F} \) \((\bar{F}_x', \bar{F}_y', \bar{F}_z)\) is in terms of "gravity force potential". \( \bar{F}_x', \bar{F}_y', \bar{F}_z \) is a function of \( x, y, z, \)
and when differentiated with respect to any direction it yields the negative component of gravity force per unit mass in that direction.

The velocity potential is given by the line integral

\[ \bar{\psi}(x, z, t) = \int_0^x (u \, dx + v \, dy + w \, dz) \]  
(2.3)

From the continuity equation, \( \text{div} \, \vec{V} = 0 \), Equation (2.3) becomes a solution of the Laplace Equation

\[ \nabla^2 \bar{\psi} = \frac{2}{\bar{\psi}_x} + \frac{2}{\bar{\psi}_y} + \frac{2}{\bar{\psi}_z} \]  
(2.4)

or in two-dimensional cartesian coordinate form is:

\[ \frac{\partial^2 \bar{\psi}}{\partial x^2} + \frac{\partial^2 \bar{\psi}}{\partial y^2} = 0 \]  
(2.5)

2.2 Boundary Conditions

Referring to Figure 2, the following boundary conditions must be satisfied

\[ \xi(\alpha, y, t, t) = 0 \]

Fig. 2. Geometry of wave motion

\[ \frac{2}{\bar{\psi}_x^2} \]  
means the second differentiation with respect to \( x \).
The free surface of the water in contact with air can be defined by:

$$\xi (x,y,z,t) = 0 \quad (2.6)$$

Along the surface the pressure must be zero, so that,

$$- \frac{1}{\rho} \nabla p = 0 \quad (2.7)$$

Along the bottom, where \( y = -D \), the water particles must remain in contact with it,

$$\frac{\partial \phi}{\partial y} = - \frac{\partial \phi}{\partial x} = 0 \text{ at } y = -D \quad (2.8)$$

In two-dimensions, Equation (2.2) can be written as

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = F_x - \frac{1}{\rho} \frac{\partial p}{\partial x} \quad (2.9)$$

or

$$\frac{1}{\rho} \frac{\partial p}{\partial x} = - \frac{\partial \Omega}{\partial x} - \frac{\partial u}{\partial t} - u \frac{\partial u}{\partial x} - v \frac{\partial u}{\partial y}$$

$$= - \frac{\partial \Omega}{\partial x} - \frac{\partial^2 \phi}{\partial t \partial x} - \frac{1}{2} \left[ \frac{\partial}{\partial x} \left( \frac{\partial \phi}{\partial x} \right)^2 \right] - \frac{\partial^2 \phi}{\partial y \partial x} \frac{\partial^2 \phi}{\partial x \partial y}$$

$$= - \frac{\partial \Omega}{\partial x} - \frac{\partial^2 \phi}{\partial t \partial x} - \frac{1}{2} \left[ \frac{\partial}{\partial x} \left( \frac{\partial \phi}{\partial x} \right)^2 \right] + \frac{\partial}{\partial x} \left( \frac{\partial \phi}{\partial y} \right)^2 \quad (2.10)$$

where \( \Omega = g y \) (positive for upward direction). Integrating Equation (2.10), we have

$$\frac{F}{\rho} = -gh - \frac{\partial \phi}{\partial t} - \frac{1}{2} \left[ \left( \frac{\partial \phi}{\partial x} \right)^2 + \left( \frac{\partial \phi}{\partial y} \right)^2 \right] \quad (2.11)$$
From Figure 3, consider a particle $K$ on the surface. After an infinitesimal time $\delta t$, the particle will move to $x + \delta x$.

$$u \delta t = \delta x$$

$$\frac{\partial \phi}{\partial x} = \frac{\delta x}{\delta t}$$ \hspace{1cm} (2.12)

The corresponding pressure change will be

$$\delta P = \frac{\partial P}{\partial t} \delta t + \frac{\partial P}{\partial x} \delta x + \frac{\partial P}{\partial y} \delta y$$ \hspace{1cm} (2.13)

From Equation (2.12) and (2.13), we will have the condition of

$$\frac{\delta P}{\delta t} \frac{\partial \phi}{\partial t} + \frac{\partial \phi}{\partial x} \frac{\partial P}{\partial x} + \frac{\partial \phi}{\partial y} \frac{\partial P}{\partial y} = 0, \text{ along free surface}$$ \hspace{1cm} (2.14)

From equation (2.11) and Equation (2.14), neglecting the second order of small values, we have

$$g \frac{\partial \phi}{\partial y} + \frac{\partial^2 \phi}{\partial t^2} = 0, \text{ when } y = 0$$ \hspace{1cm} (2.15)

Equation (2.7), (2.8), (2.11), or (2.15) represent the boundary conditions for the wave motion.
2.3 Shallow Water Wave Theories

2.3.1 Stokes' Wave with Finite Amplitude

Shallow water waves were first studied by Stokes (1880). (5) His solution was obtained by successive approximations by expanding the velocity potential about the still water level. In this theory, it is not necessary to assume that amplitude and steepness are small. The final results are presented as nonlinear equations.

As stated in previous sections, the boundary conditions must be satisfied, i.e. Equation (2.7), (2.8), (2.11), or (2.15).

From Equation (2.6), the hydrostatic pressure of any water particle is

\[ P = \rho g (\xi - y) \]  \hspace{1cm} (2.16)

where \( \rho \) is the density of water, \( g \) is the gravitational acceleration. The differentiation of Equation (2.16) is

\[ \frac{\partial P}{\partial x} = \rho g \frac{\partial \xi}{\partial x} \]  \hspace{1cm} (2.17)

From Equation (2.17), it is obvious that \( u \), horizontal component of velocity, is independent of \( y \).

The second order differentiation of Equation (2.17) will yield the horizontal component of acceleration which, again, is independent of \( y \).
Up to this point in using the theoretical hydrodynamic concepts, no approximations have been made. It is obvious, that the important question is how to define the stream function \( \psi \) and free surface \( \xi \).

Fourier's Theorem, Jacobian Elliptical Function and Complete Elliptic Integral have been used by many investigators as approximative approaches. In 1952, Biesel developed a second order approximation of potential function as

\[
\psi = \frac{HL}{2T} \cosh \frac{2\pi(y + D)}{L} \sin 2\pi \left( \frac{x - \frac{T}{T}}{L} \right) + \\
\frac{3\pi^2}{16T} \cosh \frac{4\pi(y + D)}{L} \sin 4\pi \left( \frac{x - \frac{T}{T}}{L} \right) 
\]

(2.18)

By differentiating Equation (2.18) with respect to \( x \) and \( y \), the velocity components of any particle are obtained

\[
u = \frac{\partial \psi}{\partial x} = \frac{HL}{T} \cosh \frac{2\pi(y + D)}{L} \cos 2\pi \left( \frac{x}{L} - \frac{T}{T} \right) + \\
\frac{3}{4} \left( \frac{\pi^2 H^2}{TL} \right) \cosh \frac{4\pi(y + D)}{L} \sin 4\pi \left( \frac{x}{L} - \frac{T}{T} \right) 
\]

(2.19)

and,

\[
v = \frac{\partial \psi}{\partial y} = \frac{HL}{T} \sinh \frac{2\pi(y + D)}{L} \sin 2\pi \left( \frac{x}{L} - \frac{T}{T} \right) + \\
\frac{3}{4} \left( \frac{\pi^2 H^2}{TL} \right) \sinh \frac{4\pi(y + D)}{L} \sin 4\pi \left( \frac{x}{L} - \frac{T}{T} \right) 
\]

(2.20)
The free surface will be

\[ \xi = \frac{H}{2} \cos 2\pi \left( \frac{x}{L} - \frac{t}{T} \right) \frac{\pi H^2}{4L} + \frac{\pi H^2}{4L} \]

\[ \left[ 1 + \frac{3}{2 \sinh^2 \left( \frac{2\pi D}{L} \right)} \coth \frac{2\pi D}{L} \cos 4\pi \left( \frac{x}{L} - \frac{t}{T} \right) \right] \] (2.21)

2.3.2 **Cnoidal Theory**

In 1844, Scott Russell observed a different kind of wave called the "solitary wave", which represents a single disturbance, propagated essentially unaltered in form over long distance at a constant velocity. A few years later, J. Boussinesq (1871)(12) and L. Rayleigh (1876)(12) developed mathematical equations for its profile and velocity. In 1895, D. J. Korteweg and G. deVries modified Rayleigh's theory in such a way as to obtain waves that are periodic in profile and which tend to the solitary wave in the limiting case of long wave length. This is called the cnoidal theory.

The velocity components of any water particle in the water can be determined from

\[ u = \sqrt{gD} \left[ \frac{h}{D} - \frac{h^2}{4D^2} + \left( \frac{D}{3} - \frac{y^2}{2D} \right) \frac{\partial^2 h}{\partial x^2} \right] \] (2.22)

\[ v = \sqrt{gD} y \left[ \left( \frac{1}{D} - \frac{h}{2D^2} \right) \frac{\partial h}{\partial x} + \frac{1}{3} \left( D - \frac{y^2}{2D} \right) \frac{\partial^3 h}{\partial x^3} \right] \] (2.23)

where

\[ h = -D + y + H \cn^2 \left[ 2K(k) \left( \frac{x}{L} - \frac{t}{T} \right), k \right] \] (2.24)
where $K(k)$ is the first kind of complete elliptical integral, defined as

$$K(k) = \int_0^{\pi/2} \frac{d\tilde{\phi}}{(1 - k^2 \sin^2 \tilde{\phi})^{1/2}} \quad 0 \leq k \leq 1$$

The free surface profile is

$$\xi = y_t + H \cn^2 \left[ 2K(k) \left( \frac{x}{L} - \frac{T}{T} \right), k \right]$$

(2.25)

A comparison of horizontal velocity components of water particles among the Linear Theory, Stokes Second Order Theory and Cnoidal Theory had been made by Wiegel (1960). (12)

![Graph](image)

$H = 0.105$ ft.
$T = 1.62$ Sec.
$D = 0.242$ ft.
$L = 5.10$ ft.

$$(u), \text{ FEET/SECOND}$$

- • Experiment Points: Morison and Crooke, 1953
  - • $u$, Under Trough
  - • $u$, Under Crest
  — Linear Theory
  —— Stokes Second Order
  —— Cnoidal Theory

Fig. 4 Comparison of horizontal components of water particle velocity with shallow wave theory (after Wiegel, Reference 12).
2.4 The Mechanics of Sediment Movement

If the mean diameter of a sediment particle (50 per cent finer by weight) is \( d \), and \( u^* \) is the local fluid velocity parallel to the bottom, then the drag force just sufficient to initiate movement of a particle in the bed is

\[
F_D = C_D \rho \frac{u^*}{2} \frac{\pi d^2}{4}
\]

(2.26)

where \( C_D \) is the coefficient of drag, \( \rho \) is the density of the fluid.

![Diagram](image)

**Fig. 5 Initial Movement of a Sand Particle**

Consider a particle \( P \) as shown in Fig. 5; \( \theta \) is the angle of repose. The moment which is just sufficient to initiate movement of the particle about \( M \) must equal the moment of its own weight about point \( M \).
\[ F_D = C_D \rho \frac{u^2}{2} \frac{n^2}{4} = \frac{\pi}{6} d^3 (\gamma_S - \gamma) \tan \theta \]  \hspace{1cm} (2.27)

where \( \gamma_S \) is the specific weight of the sediment, \( \gamma \) is the specific weight of the fluid.

\( C_D \) is the coefficient of drag, which is a function of Reynolds Number and can be found from Fig. 6.

---

**Fig. 6** Drag Coefficient of Spheres as a Function of Reynolds Number (after Rouse, Ref. 8)
ERRATA

1. Page 6, Section 2.2, Figure 2 - \( \xi(\alpha, y, t, t) = 0 \)
   should be \( \xi(x, y, z, t) = 0 \)

2. Page 7, 4th line from the bottom, equation 2.10
   should be
   \[
   \frac{1}{\rho} \frac{\partial \rho}{\partial x} = - \frac{\partial \Omega}{\partial x} - \frac{\partial^2 \Phi}{\partial t \partial x} - \frac{1}{2} \left( \frac{\partial}{\partial x} \left( \frac{\partial \Phi}{\partial x} \right)^2 + \frac{\partial}{\partial y} \left( \frac{\partial \Phi}{\partial y} \right)^2 \right)
   \]

3. Page 8, equation (2.13)
   should be
   \[
   \delta P = \frac{\partial P}{\partial t} \delta t + \frac{\partial P}{\partial x} \delta x + \frac{\partial P}{\partial y} \delta y
   \]

4. Page 8, equation (2.14)
   should be
   \[
   \frac{\partial P}{\partial t} = \frac{\partial P}{\partial t} + \frac{\partial \Phi}{\partial x} \frac{\partial P}{\partial x} + \frac{\partial P}{\partial y} \frac{\partial P}{\partial y}
   \]

5. Page 11, equation (2.21)
   should be
   \[
   \xi = \frac{H}{2} \cos 2\pi \left( \frac{x}{L} - \frac{t}{T} \right) \frac{T^2}{4L} + \\
   \frac{T^2}{4L} \left( 1 + \frac{3}{2 \sinh^2 \left( \frac{2\pi D}{L} \right)} \right) \coth \frac{2\pi D}{L} \cos 4\pi \left( \frac{x}{L} - \frac{t}{T} \right)
   \]
ERRATA (continued)

6. Page 11, equation (2.22) and (2.23) - (\sqrt{gd})
   should be (\sqrt{gd})

7. Page 13, Figure 5, \( F_D = C_D \rho \frac{u^2}{2} \frac{nd^2}{4} \)
   should be
   \[ F_D = C_D \rho \frac{u^*^2}{2} \frac{nd^2}{4}, \]

   \( (\gamma_s - \gamma) \)
   should be \( (\gamma_s - \gamma) \)

8. Page 14, Figure 6, Strokes should be Stokes

9. Page 44, line 7, \( C \) should be \( C_D \)
2.5 **Boundary Layer Along a Flat Sand Bed**

The mechanics of sediment movement were discussed in the previous section and \( u^* \) was defined as the local horizontal velocity parallel to the bottom. However, the sand particle is so small that the boundary layer effect must be taken into consideration. Fortunately the boundary layer along a plate is the simplest example of the application of Prandtl's Boundary Layer Theory.

Based upon the cnoidal wave theory and laboratory observations (Fig. 3), it is reasonable to assume that the fluid flow pattern between the sand bed and water surface (within the scour wave length) is uniform and steady, or that \( \frac{\partial v}{\partial y} = \text{constant} \). The Prandtl's boundary layer equation can be simplified and presented in the following form:

\[
\begin{align*}
\frac{u^*}{\partial x} + \frac{v^*}{\partial y} &= \nu \frac{\partial^2 u^*}{\partial y^2} \\

\frac{\partial u^*}{\partial x} + \frac{\partial v^*}{\partial y} &= 0
\end{align*}
\]

Where \( y = 0, \ u^* = v^* = 0 \) and \( y = \infty, \ u^* = U, \) and

\( \nu \) is the kinematic viscosity.

If \( \delta \) is the thickness of the boundary layer, the relation of \( \delta \approx (\nu t)^{\frac{1}{2}} \) is used in the case of a suddenly accelerated plate (where \( t \) is the time elapsed after the movement starts). If a particle located at \( x = 0 \) and \( y > > \delta \) at time \( t = 0 \), then after a time interval of \( t \), the particle must be at \( x = Ut \), or \( t = x/U \). Substituting this value into the above relation we have:

\[
\delta \approx \left(\frac{\nu x}{U}\right)^{\frac{1}{2}}
\]
or \[ \eta = y \left( \frac{U}{v_x} \right)^{\frac{1}{2}} \] (2.31a)

where \( \eta = y/\delta \) is defined as a dimensionless term.

From the definition of stream function, we have

\[ u^* = \frac{\partial \Psi}{\partial y} = \frac{\partial \Psi}{\partial \eta} \cdot \frac{\partial \eta}{\partial y} = U f' (\eta) \] or

\[ \frac{u^*}{U} = f' (\eta) \] (2.32a)

where \( f' (\eta) \) is the first order differentiation of the dimensionless stream function. Similarly an equation for \( v^* \) - component in the form of new dimensionless coordinate may be obtained. This equation together with equation (2.32) by substituting into equation (2.28) and (2.29) will form a non-linear differential equation of the third order.

\[ ff'' + 2 f''' = 0 \] (2.32b)

The solution of this equation is very difficult and beyond the scope of the present study. In 1938, L. Howarth\(^{(2)}\) gave a very accurate solution of this differential equation.

The following table was taken from Howarth. Only the terms used in the present study are listed.
**TABLE 1**

**THE FUNCTION OF \( f(\eta) \) FOR THE BOUNDARY LAYER ALONG A FLAT PLATE AT ZERO INCIDENCE, (AFTER L. HOWARTH)**

\[
\begin{align*}
\eta &= y \left( \frac{U}{\sqrt{x}} \right)^{\frac{1}{2}} \\
\frac{u^*}{U} &= \\
0 &\quad 0 \\
0.2 &\quad 0.06641 \\
0.4 &\quad 0.13277 \\
0.6 &\quad 0.19894 \\
0.8 &\quad 0.26471 \\
1.0 &\quad 0.32979 \\
1.2 &\quad 0.39378 \\
1.4 &\quad 0.45627 \\
1.6 &\quad 0.51676 \\
1.8 &\quad 0.57677 \\
2.0 &\quad 0.62977 \\
2.2 &\quad 0.68132 \\
2.4 &\quad 0.72899 \\
2.6 &\quad 0.77246 \\
2.8 &\quad 0.81152 \\
3.0 &\quad 0.84605 \\
3.2 &\quad 0.87609 \\
3.4 &\quad 0.90177 \\
3.6 &\quad 0.92333 \\
3.8 &\quad 0.94112 \\
4.0 &\quad 0.95552
\end{align*}
\]
2.6 A Theoretical Equation for Wave Scour

Partial or complete wave reflection will occur from a seawall depending on the angle of the seawall. The incident and superimposed waves will form an envelope as indicated in Fig. 7.

![Diagram of Wave Motion](image)

**Fig. 7 Envelope of Wave Motion**

It was observed that the surface of the sand bed first becomes rippled under the nodes of the envelope. (Point N in Fig. 7) A few minutes later the rippled surface extend to cover the entire sand bed. Soon after the formation of these ripples the actual scour formations
appear. The crests and troughs of the sand formations correspond to the loops and nodes of the envelope.

The mechanics of the scouring process may be explained as follows: When the experiment is started with a flat sand bed, the horizontal velocity component under the node is affected more than the horizontal velocity component under the loop, so that the primary scour occurs under the nodes of the envelope. A few hours later (usually 1 to 3 hours) the crests of the sand formation move under the nodes of the envelope. This relative position will normally last throughout the duration of the experiment.

From equation (2.27), for a particular grain size of sand, the most important is \( u^* \). In other words, \( u^* \), the local velocity parallel to the bottom, is the main factor determining the depth of scour. Since the wave is generated from rest, the equation of continuity is valid in this case. It is then logical to say, that when scour depth increases, the local velocity must decrease until a certain point where the ultimate scour depth is reached. This does not imply that scour and sediment transfer come to a stop, it is only a limit which is approached asymptotically.

Figure 8 shows a side elevation of sand bed. Section "a" represents the initial condition (before the scour) and section "b" represents the condition when the ultimate scour is reached.
Fig. 8 Geometry of Sand Scour

Continuity equation can be written between section "a" and "b".

\[ U_a \left(D - \frac{1}{2} A\right) = U_b \left(D - \frac{1}{2} A + S\right) \]  

or \[ U_b = U_a \frac{D \frac{1}{2} A}{D \frac{1}{2} A + S} \]  

(2.33) \hspace{1cm} (2.34)

This equation should be modified to take account of reflection. The reflected wave height is a function of the slope of the seawall. Where the two wave systems approach from opposite directions and are superimposed, the velocity components may be added vectorially. Introducing equation (2.32), we have

\[ f'(\eta) \left(1 - C_r\right) U_b = (1 - C_r) f'(\eta) U_a \left(D - \frac{1}{2} A \right) \frac{D \frac{1}{2} A}{D \frac{1}{2} A + S} \]  

(2.35)

where \( C_r \) is reflection coefficient, defined as:

\[ C_r = \frac{A - B}{A + B} = \frac{H_R}{H_I} \]
Substituting Equation (3.5) into Equation (2.27) and simplifying, the following expression is obtained:

\[ S = (D - \frac{1}{2} A) \left[ (1 - C_r) u^* \left( \frac{3}{4} C_D \rho \frac{\cot \theta}{d (\gamma_s - \gamma)} \right)^{\frac{1}{2}} - 1 \right] \] (2.36)

where \[ A = H_I + H_R \]
3. DIMENSIONAL ANALYSIS

3.1 Significant Variables

The following may be considered to be the significant variables:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Units</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>Still Water Depth</td>
<td>ft.</td>
<td>L</td>
</tr>
<tr>
<td>H</td>
<td>Incident Wave Height</td>
<td>ft.</td>
<td>L</td>
</tr>
<tr>
<td>K</td>
<td>Wave Trough to Bottom</td>
<td>ft.</td>
<td>L</td>
</tr>
<tr>
<td>L</td>
<td>Wave Length</td>
<td>ft.</td>
<td>L</td>
</tr>
<tr>
<td>S</td>
<td>Scour Depth</td>
<td>ft.</td>
<td>L</td>
</tr>
<tr>
<td>λ</td>
<td>Sand Wave Length</td>
<td>ft.</td>
<td>L</td>
</tr>
<tr>
<td>t</td>
<td>Wave Period</td>
<td>sec.</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>Time Elapsed</td>
<td>sec.</td>
<td>T</td>
</tr>
<tr>
<td>V</td>
<td>Water Particle Velocity</td>
<td>ft./sec.</td>
<td>L/T</td>
</tr>
<tr>
<td>μ</td>
<td>Viscosity of Water</td>
<td>lb-sec./ft.²</td>
<td>FT/L²</td>
</tr>
<tr>
<td>g</td>
<td>Acceleration due to Gravity</td>
<td>ft./sec.²</td>
<td>L/T²</td>
</tr>
<tr>
<td>ρ</td>
<td>Density of Water</td>
<td>lb-sec.²/ft.⁴</td>
<td>FT²/L⁴</td>
</tr>
<tr>
<td>γ</td>
<td>Specific Weight of Water</td>
<td>lb/ft.³</td>
<td>F/L³</td>
</tr>
<tr>
<td>γ_s</td>
<td>Specific Weight of Sand</td>
<td>lb/ft.³</td>
<td>F/L³</td>
</tr>
<tr>
<td>d</td>
<td>Mean Diameter of Sand Particle, 50% finer</td>
<td>ft.</td>
<td>L</td>
</tr>
<tr>
<td>α</td>
<td>Slope of Seawall</td>
<td>degree</td>
<td>o</td>
</tr>
<tr>
<td>C_r</td>
<td>Reflection Coefficient</td>
<td>%</td>
<td></td>
</tr>
<tr>
<td>θ</td>
<td>Angle of Repose of Sand in Water</td>
<td>degree</td>
<td>o</td>
</tr>
</tbody>
</table>

It is obvious that K is a function of D, H.
So, for simplicity $K$ is used instead of $H$ and $D$.

$$f(K, L, S, \lambda, t, T, V, \mu, \rho, \gamma, \gamma_s, d, \alpha, C_r, \theta) = 0 \quad (3.1)$$

Using for Force-Length-Time System (FLT), the variables may be listed as follows:

<table>
<thead>
<tr>
<th>TABLE 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SIGNIFICANT VARIABLES</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variables</th>
<th>$F$</th>
<th>$L$</th>
<th>$T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K$</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$L$</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$S$</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$t$</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$T$</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$V$</td>
<td>0</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>$\mu$</td>
<td>1</td>
<td>-2</td>
<td>1</td>
</tr>
<tr>
<td>$g$</td>
<td>0</td>
<td>1</td>
<td>-2</td>
</tr>
<tr>
<td>$\rho$</td>
<td>1</td>
<td>-4</td>
<td>2</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1</td>
<td>-3</td>
<td>0</td>
</tr>
<tr>
<td>$\gamma_s$</td>
<td>1</td>
<td>-3</td>
<td>0</td>
</tr>
<tr>
<td>$d$</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$C_r$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
3.2 Dimensionless Terms

Employing the Buckingham's π Theorem, K, V, ρ, were selected as the fundamental variables. The following dimensionless terms were set up.

\[
\pi_1 = K^{x_1} \cdot V^{y_1} \cdot \rho^{z_1} \cdot L
\]
\[
\pi_2 = K^{x_2} \cdot V^{y_2} \cdot \rho^{z_2} \cdot S
\]
\[
\pi_3 = K^{x_3} \cdot V^{y_3} \cdot \rho^{z_3} \cdot \lambda
\]
\[
\pi_4 = K^{x_4} \cdot V^{y_4} \cdot \rho^{z_4} \cdot t
\]
\[
\pi_5 = K^{x_5} \cdot V^{y_5} \cdot \rho^{z_5} \cdot T
\]
\[
\pi_6 = K^{x_6} \cdot V^{y_6} \cdot \rho^{z_6} \cdot \mu
\]
\[
\pi_7 = K^{x_7} \cdot V^{y_7} \cdot \rho^{z_7} \cdot g
\]
\[
\pi_8 = K^{x_8} \cdot V^{y_8} \cdot \rho^{z_8} \cdot \gamma
\]
\[
\pi_9 = K^{x_9} \cdot V^{y_9} \cdot \rho^{z_9} \cdot \gamma_s
\]
\[
\pi_{10} = K^{x_{10}} \cdot V^{y_{10}} \cdot \rho^{z_{10}} \cdot d
\]
\[
\pi_{11} = K^{x_{11}} \cdot V^{y_{11}} \cdot \rho^{z_{11}} \cdot \alpha
\]
\[
\pi_{12} = K^{x_{12}} \cdot V^{y_{12}} \cdot \rho^{z_{12}} \cdot c_r
\]
\[
\pi_{13} = K^{x_{13}} \cdot V^{y_{13}} \cdot \rho^{z_{13}} \cdot \theta
\]

(3.2)
Solving the previous equations, the following π terms can be obtained.

\[ f \left( \frac{L}{K}, \frac{S}{K}, \frac{\lambda}{K}, \frac{V_t}{K}, \frac{V_T}{K}, \frac{\mu}{K}, \frac{gK}{R_e}, \frac{\gamma K}{V^2}, \frac{\gamma_s K}{V^2}, \frac{d}{K}, \alpha, C_r, \Theta \right) = 0 \] (3.2a)

\[ \pi_1, \pi_3, \pi_6, \text{ and } \pi_5, \text{ and } \pi_7 \text{ and } \pi_8 \text{ and } \pi_9 \text{ can be combined into the following } \pi \text{ terms:} \]

\[ \pi_{14} = \pi_3/\pi_1 = \lambda/L \]

\[ \pi_{15} = \pi_5/\pi_4 = T/t \]

\[ \pi_{16} = \frac{\pi_{10}}{\pi_9 - \pi_8} = \frac{V^2}{d(\gamma_s - \gamma)} \]

Equation (3.1) can be written as:

\[ f \left( \frac{S}{K}, \frac{\lambda}{L}, \frac{T}{t}, \frac{1}{R_e}, \frac{V^2}{d(\gamma_s - \gamma)}, \frac{1}{F_r}, \Theta, \alpha, C_r \right) = 0 \] (3.3)

or

\[ S = f' \left( \frac{\lambda}{L}, \frac{T}{t}, \frac{1}{R_e}, \frac{V^2}{d(\gamma_s - \gamma)}, \frac{1}{F_r}, \Theta, \alpha, C_r \right) \] (3.4)

For the sake of comparison, we can rewrite equation (2.36), which was obtained theoretically, in the following form:

\[ \frac{S}{K} = \left( 1 - C_r \right) \frac{3}{4} \frac{C_D}{C_D} \frac{1}{d(\gamma_s - \gamma)} \frac{V^2}{\left( \cot \Theta \right)^2} - 1 \] (3.5)

It is obvious that equations (3.4) and (3.5) are very similar. However, it must be pointed out that equation (2.36) is used only for the "ultimate" scour depth.
4. EXPERIMENTAL APPARATUS AND PROCEDURE

4.1 Apparatus

The wave channel is two feet wide, two feet deep, and 67 feet long, glass walled with absorbers at both ends. (Fig. 10). The absorbers were built with four thin perforated aluminum sheets. These sheets were bolted together with a 1/4 inch spacing between them, and for downstream absorber these sheets were installed on a 5/16 inch aluminum impermeable plate. The plate is inclined at 15 degrees to the horizontal. (The absorber, which is behind the generator, is inclined at 45 degrees.) It should be noted that the downstream absorber was not used in the study. There are rails along the entire channel to support a movable carriage. An adjustable wave probe is mounted on the carriage.

The generator is of pendulum type, the unit consists of an oscillating plate, Vickers transmission, and a 3/4 HP Westinghouse AC electric motor, which operates at a maximum speed of 1725 rpm on 9.4 amps and 115 volts. The frequency of the generator can vary from zero to 2.1 cycles per second. The stroke, period and movement of oscillating plate of the generator is adjustable so that the desired wave height, wave length and wave period may be obtained.
The wave recorder consists of a Sanborn Twin-Viso Recorder and a Sanborn Strain Gage Amplifier. The Sanborn Twin-Viso Recorder is a two-channel graphic recording system. Direct-writing recording in true rectilinear coordinates is accomplished by passing heated styli over the plastic-coated surface of the recording paper as it passes over a knife-edge plate. A third stylus records either one second timing pulses or marker traces to identify phenomena of particular interest. A choice of paper speeds between 0.5 and 100 milimeters per second is available.

A simplified diagram of recording system using a strain gage amplifier is shown in Figure 9.

Fig. 9 Schematic diagram of capacitance-type wave recording
The transducer is essentially a part of a capacitance bridge, which acts as a capacitor whose capacity varies directly with the depth of submergence.

4.2 Geometric Configuration

A false bottom of five inches deep and a bulk-head of plywood were placed under the generator and extended toward seawall of a distance of about 15 feet. Sand bed five inches deep extended throughout the remainder of the tank. The seawalls, also made of plywood, were placed in front of the downstream absorber. Wire-mesh filter (about 5 feet thick and 2 feet wide) was placed about 5 feet in front of the generator in an attempt to reduce the reflected-waves hitting the paddle and causing re-reflected waves. Unfortunately, it is very difficult (if not impossible) to cut down the reflected-waves completely. Schematic diagram of the experimental set-up is shown in Figure 11. Grain size distribution curve for the sand is shown in Figure 12.
1. Hydraulic Power Unit
2. Motor
3. Upstream Wave Absorber
   (45° With Horizontal)
4. Wave Generator Paddle
5. Perforated Plate Layers
6. 15° With Horizontal
7. Motor and Hydraulic Transmission
8. Wave Paddle
9. Probe
10. Probe Carriage

Fig. 10 Wave channel
Fig. 11 Experimental facility

1. Plywood Seawall
2. False Bed
3. Sand Bed
4. Wire Mesh Filter
Fig. 12 Grain size distribution
4.3 Experimental Procedure

The experimental program was conducted according to the following table:

**TABLE 3**

<table>
<thead>
<tr>
<th>Experiment No.</th>
<th>H/L</th>
<th>H/D</th>
<th>L/D</th>
<th>T (sec)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.03314</td>
<td>0.38400</td>
<td>10.500</td>
<td>1.265</td>
<td>15 degree</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Seawall</td>
</tr>
<tr>
<td>A2</td>
<td>0.04012</td>
<td>0.42125</td>
<td>10.500</td>
<td>1.600</td>
<td>&quot;</td>
</tr>
<tr>
<td>A3</td>
<td>0.03291</td>
<td>0.34556</td>
<td>10.500</td>
<td>1.695</td>
<td>&quot;</td>
</tr>
<tr>
<td>A4</td>
<td>0.02343</td>
<td>0.24600</td>
<td>10.500</td>
<td>1.790</td>
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<tr>
<td>B5</td>
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<td>12.500</td>
<td>2.080</td>
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<tr>
<td>C9</td>
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<td>12.656</td>
<td>2.000</td>
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<tr>
<td>C10</td>
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<td>0.33552</td>
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<td>1.500</td>
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Before each experiment, the sand bed was carefully leveled, wave recorder calibrated, water depth checked, and wave period and wave length adjusted. The reflection coefficients and wave heights were taken at a location between 10 to 20 feet away from the seawall. The
scour depths were taken upstream of the seawall over a distance of 20 feet. All experiments were conducted until the scour depths became fairly constant with time.
5. PRESENTATION AND DISCUSSION OF RESULTS

5.1 Observations

A few minutes after the wave motion started, ripples were observed under the nodes of the wave envelope, as stated in section 2.5. However, the ripples were not exactly under the nodes of the envelope and there was a space lag between the "ripple clusters" and the nodes of the envelope. This phenomena was also observed by other investigators. The ripple "clusters" soon extended toward both directions and in about thirty minutes the ripples covered the entire bed. However, the rate of ripple formation was faster in the landward direction than in the seaward direction. The sand wave length was approximately constant throughout the whole test. The slopes of the ripples were flatter on the seaward side and steeper on landward side. It is interesting to note that the angle of repose was much greater when the water was in motion than when the water was still. (A maximum angle of repose of 42 degrees had been found when the water was in motion, and 37 degrees for the still water ripples sand). A few hours later, usually in about one to three hours, sand-scoured formations (sand bars) began to appear among the ripples. By this time the crests of the sand scour formations developed under the nodes of the envelope and the troughs of the scour formations were under the loops of the envelope. Again the crests and
Fig. 13 Sand scour and envelope
troughs of the scour were not exactly under the nodes and loops of the envelope. (An actual measurement of this phenomena is presented in Figure 13). This relative position lasted throughout the remainder of the test. The sand bars moved back and forth but the above relative position remained the same. No definite explanation of the above phenomenon has been reached by the author. The probable answer may be due to the fact that the mass transfer was affected to some extent by the closed system. (Mass transport equation involving viscosity term had been developed by Launquet - Higgins in 1953). The scour length (sand wave length) remains the same from the first formation until the sand scour length is not a function of scour depth and time. It is more likely a function of wave length. This will be further discussed in the following section. When the scour depth remained nearly constant for several hours, the experiment was terminated. Usually this took more than twenty four hours. One experiment had been run for fifty hours and there was no indication of any sudden change or unusual phenomena.

5.2 Relationship Between $\frac{S}{K}$ and $\frac{T}{t}$

All the experiments indicate that there is a limit of scour depth which will be approached asymptotically. In other words, this limit will be reached at infinite time. The scour depth increases very swiftly in the first few hours to a certain value and then the scouring process slows down and reaches a state of what other investigators called "ultimate" scour depth. It is theoretically wrong to say there will be no scour when the ultimate scour depth is reached. Since the
scour limit is approached asymptotically, no matter how small the
increase in scour there is always a net small rate of scour. Figure 14
is a plot of $\frac{S}{K}$ versus $T/t$. $\overline{S}$ is the average scour depth taken within a
range of 15 feet in front of the seawall. The term $T/t$ is actually the
number of waves acting on the bed.

5.3 Relationship Between $\frac{S}{K}$ and $C_r$

Figure 15 is a plot of $\frac{S}{K}$ versus $C_r$ for a 15 degree seawall.
Figure 16 is a plot of $\overline{S}$ versus $C_r$ for a 45° seawall. It appears that
the scour depth is only a random function of the reflection coefficient.
This is probably due to the fact that the reflection coefficient depends
on the wave characteristics, seawall slope and kinematic behavior upon
hitting the seawall. There is a great deal of difference between the
reflection coefficients of a non-breaking and breaking wave on the sea-
wall for the same wave characteristics.

5.4 Relationship Between $\frac{S}{K}$ and $\frac{\lambda}{L}$

In 1964, Murphy concluded that the scour length is not at
all influenced by wave height, water depth, seawall slope and reflection
coefficient. The same results have been obtained by the author. The
scour length is about half the wave length.

5.5 A Comparison Between Calculated Values and Experimental Results

Table 4 shows the scour depths computed by equation 2.36 and
the experimental results. $\overline{S}$ is the average value at the ultimate
condition and $S_{\text{max}}$ is the maximum value recorded during the test.
Fig. 14 Relationship between $\frac{\bar{S}}{K}$ and $\frac{T}{t} \times 10^{-3}$
Fig. 15 Relationship between $\frac{S}{K}$ and $C_r$

$S = 15^\circ$

$\frac{H}{L} = 0.0234$

$\frac{H}{D} = 0.246$

$\frac{L}{D} = 10.5$
# Table 4

A comparison between calculated values and experimental results

<table>
<thead>
<tr>
<th>Experimental No.</th>
<th>Calculated Value from Eq. 2.36*</th>
<th>$\bar{S}$</th>
<th>$S_{max}$</th>
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<tr>
<td></td>
<td>in.</td>
<td>in.</td>
<td>in.</td>
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<td>$y = \frac{1}{2} \delta$</td>
<td>$y = 5d$</td>
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<tr>
<td>A1</td>
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<td>1.22</td>
<td>1.950</td>
<td>1.170</td>
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<td>B5</td>
<td>1.35</td>
<td>1.272</td>
<td>1.300</td>
</tr>
<tr>
<td>B6</td>
<td>1.09</td>
<td>1.170</td>
<td>1.420</td>
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<tr>
<td>B7</td>
<td>1.21</td>
<td>2.380</td>
<td>1.373</td>
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</table>

* The displacement boundary layer thickness is found from $\delta = 1.72 \left( \frac{ux}{U} \right)^{\frac{1}{2}}$. The above calculation is based on the assumption of $y = \frac{1}{2} \delta$ and $\eta = y \left( \frac{U}{L/2} \right)^{\frac{1}{2}}$. A calculation of the results based on the assumption of $y = 5d$ is also listed in column 3. The assumption of $y = 5d$ has been made because of the sand diameter being so small and also because the possibility that several sand particles might pile together.
It is now appropriate to discuss the experimental accuracy. Several difficulties were encountered during the experimental phase of the study.

1. As stated in section 4.2, the reflected waves were not completely attenuated by the wire mesh filter placed just in front of the generator. Figure 16 is a plot of wave height, and reflection coefficient versus $\frac{T}{t}$. The fluctuation was due to the reflected waves and re-reflected waves. (An average wave height is used in all the calculations and plots.)

2. It was very difficult to run the test continuously over a period of 20 to 30 hours. Some tests had to be stopped several times before the ultimate condition had been reached.

3. During the test, a maximum angle of repose of 42 degrees had been observed (generally about 39, 40, 41 degrees). Another measurement of the angle of repose was made just before the test was restarted and the readings were only 30, 32, and 34 degrees. (A maximum of 35 degrees had been observed). The dates and the corresponded changes of scour are indicated in Figure 17.

The reasons stated above, small scale experimentation, difficulty in determining the drag coefficients and water particle velocity, probably account for differences between calculated and experimental values.
Fig. 16 Fluctuation of $H$, $C_r$ as a function of $\frac{T}{t}$

- $T = 1.79$ SEC
- $D = 10.1$ IN
- $\alpha = 15^\circ$
- $L/D = 10.5$
- $H/D = 0.246$
Fig. 17 Relationship between $\frac{s}{H}$ and $\frac{T}{t}$

- $\alpha = 0^\circ$
- $T = 2.00$ SEC.
- $D = 8.375$ IN.
- $H = 2.81$ IN.
6. SUMMARY AND CONCLUSIONS

A mathematical model to describe the scour in front of a seawall is presented. The model was developed from the hydromechanical concepts, i.e. equation of continuity and boundary layer.

\[
S = K (1 - C_r) \left( \frac{u^*}{\left( \frac{3 C D \rho \cot \theta}{4d (\gamma_s - \gamma)} \right)^{\frac{1}{2}}} \right) - K
\]

where \( K = (D - \frac{1}{2} A) \) \( (6.2) \)

where \( S \) is the scour in front of a seawall, \( D \), the still water depth, \( A \) defined as \( H_I + H_R \), \( C_r \) is the reflection coefficient, \( C \) is the drag coefficient, \( \rho \) is the density of water, \( \theta \) is angle of repose \( d \) is effective diameter of sand with 50 per cent finer, \( \gamma_s \) and \( \gamma \) are the unit weight of sand and water respectively.

From the theoretical analysis and experimental observation, the following conclusions were reached:

1. The ripples first start under the nodes of the envelope and extend toward both sides. However, the rate of ripple formation toward the landward side is faster.

2. In considering the mathematical model the most important factors which affect the ripple formation are water velocity and sand diameter.
3. The scour length is independent of time and only a function of wave length.

4. There appears to be a limit for scour depth but this limit is only approached asymptotically.

5. The scour depth appears to be a random function of reflection coefficient which is a function of wave characteristics, seawall slopes, and kinematic configuration upon hitting the seawall.

6. The main factor controlling the scour is the boundary layer velocity.

7. A theoretical model can be obtained on the basis of the hydrodynamic concepts.
7. APPENDIX

<table>
<thead>
<tr>
<th>Photo</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>Photo A</td>
<td>Before experiment was started</td>
</tr>
<tr>
<td>Photo B</td>
<td>One minute after the experiment was started</td>
</tr>
<tr>
<td>Photo C</td>
<td>One hour after the experiment was started</td>
</tr>
<tr>
<td>Photo D</td>
<td>Five hours after the experiment was started</td>
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