EFFECT OF INITIAL OUT-OF-STRAIGHTNESS

ON THE STRENGTH OF COLUMNS

by

Ching-Kuo Yu

A THESIS

Presented to the Graduate Faculty

of Lehigh University

in Candidacy for the Degree of

Master of Science

Lehigh University

1964
This thesis is accepted and approved in partial fulfillment of the requirements for the degree of Master of Science.

Jan. 6, 1967
(Date)

Dr. Lambert Tall
Professor in Charge

Professor W. J. Eney, Head
Department of Civil Engineering
ACKNOWLEDGEMENTS

The work described in this thesis was conducted in Fritz Engineering Laboratory of the Department of Civil Engineering, Lehigh University, Bethlehem, Pennsylvania. Professor W. J. Eney is Head of the Department of Civil Engineering and Dr. L. S. Beedle is the Director of Fritz Engineering Laboratory.

The author wishes to acknowledge his gratitude to Professor Lambert Tall, thesis supervisor, for his continued advice, suggestions, and help.

The assistance extended by the author's associates in Fritz Laboratory, especially Messrs. Fiorello R. Estuar and Fumio Nishino, is gratefully acknowledged. Miss Grace Mann typed the manuscript and her patience and cooperation are appreciated.
# TABLE OF CONTENTS

1. INTRODUCTION  
   1  

2. THEORETICAL ANALYSIS  
   2.1 Assumptions  
   2  
   2.2 H-shaped columns bent about the strong axis  
   2.2.1 No Yielding in Both Flanges  
   2.2.2 Yielding Only in One Flange  
   2.2.3 Yielding on Both Flanges  
   2.2.4 One Flange Totally Yielded  
   4  
   2.3 H-shaped Columns Bent About the Weak Axis  
   2.3.1 No Yielding in Both Flanges  
   2.3.2 Flange Tips on Only One Side Yielded  
   2.3.3 Flange Tips of Both Sides Yielded  
   12  
   2.4 Graphical Method  
   17  
   2.5 Hot-rolled Box-section  
   2.5.1 In the Elastic Range  
   2.5.2 Boundary of Elastic Zone  
   2.5.3 Elastic-plastic Zone  
   19  

3. EXPERIMENTAL INVESTIGATION  
   3.1 Preliminary Tests  
   3.1.1 Tensile Coupon Tests  
   3.1.2 Residual Stress Measurements  
   3.1.3 Stub Column Tests  
   26
3.2 Column Tests

3.2.1 Preparation of the Column Before Testing

3.2.2 Alignment

3.2.3 Column Test Procedure

3.2.4 Column Test Results

4. CONCLUSIONS

5. NOMENCLATURE

6. TABLES AND FIGURES

7. REFERENCES

8. VITA
This thesis presents a theoretical and experimental study of the significance of the combined effect of residual stresses and initial out-of-straightness on the strength of H-shaped and rolled box-shaped columns. The reduction in column strength due to these two factors is determined theoretically for pinned-end idealized H-sections and for box-shaped columns.

A description of the experimental investigation conducted on rolled box columns made of A36 steel is included in this thesis. A comparison between the results of the theoretical analysis and the test results is given. The difference in column strength between the H-section and the box-shaped column is also compared theoretically.
1. INTRODUCTION

It is known that residual stresses and initial out-of-straightness set up in columns due to manufacturing processes are unavoidable. Their influence may be considerable for the axially loaded steel column, depending on the geometry of the section, the slenderness ratio, the pattern and magnitude of the residual stresses, and the amount of initial out-of-straightness.

The present study was made to determine the significance of the combined effect of residual stresses and initial out-of-straightness on the strength of H-shaped and rolled Box shaped columns. The reduction in column strength due to these two factors is determined theoretically for pinned-end idealized H-sections and for Box-shaped columns. Equations relating load and lateral deflection are obtained and a graphical method of solving the equations is presented.

A description of the experimental investigation conducted on rolled box columns made of A-36 steel is presented. A comparison between the theoretical analysis and the test results is given. The difference in column strength between the H-section and the Box-shaped column also is presented theoretically.
2. THEORETICAL ANALYSIS

The purpose of the theoretical analysis is to find the theoretical expressions for the load versus deflection relations of initially crooked columns. Particular interest is given to the H-shaped and the Box-shaped columns. Their behavior in the elastic, elastic-plastic and plastic regions are investigated.

Due to the difficulties in obtaining the ultimate load of the column by directly differentiating the load versus deflection equation, the relation is separated into two equations, one of load versus curvature and the other of curvature versus deflection, and a graphical method is used to obtain the solution. The graphical method will be explained in Section 2.4. By this method the ultimate strength of a column can be determined directly from the curvature versus deflection curves once the slenderness ratio (L/r) and initial out-of-straightness (e) of the column are known.

A set of simplifying assumptions similar to those in the simple elastic theory of bending is adopted here as the basis for the theoretical analysis.\(^{(1,2,3)}\)

2.1 Assumptions

The following assumptions are made in the theoretical analysis:
(1) The stress-strain relationship for each longitudinal fiber in the cross section is the same and is idealized as shown in Fig. 1.

(2) The material is homogeneous.

(3) Plane cross section remains plane before and after deformation.

(4) The residual stress distribution is assumed constant along the length of the column.

(5) Both the cross section and the residual stress distribution have axial symmetry.

(6) An idealized H-section is assumed which consists of two thin flanges of equal area separated by a web of infinite shear stiffness and negligible area.

(7) Both initial shape and deflection curves of the column are assumed to be sinusoidal as shown in Fig. 2.

In the following sections, a derivation of either the two fundamental relationships (load versus curvature and curvature versus deflection) or the load versus deflection relationship which define the behavior of an initially crooked column is given. The following cases are treated in detail:

(1) H-shape bent about the strong axis.
(2) H-shape bent about the weak axis.

(3) The Box-shape.

A graphical method developed by the author is used to obtain ultimate load carrying capacity of the columns.

2.2 H-shaped Columns Bent About the Strong Axis

For the H-sections a residual stress distribution which is piecewise linear and symmetrical is assumed. This is shown in Fig. 3a.

As the column goes through its deformation five states of stress conditions may be experienced, namely

(1) Cross section is fully elastic
(2) Yielding in one flange
(3) Yielding in both flanges
(4) One flange is totally yielded
(5) Both flanges are totally yielded.

For the column with residual stresses and initial out-of-straightness case (5) can only happen to very short columns since the condition of moment equilibrium must be satisfied. For usual columns of symmetrical cross-section case (4) is the extreme stress condition. Therefore, case (5) will not be discussed here and only case (1) to case (4) are treated in this report.
In deriving the load versus deflection relationship the following conditions have to be fulfilled at the mid-height of the column:

1. Static equilibrium of internal and external forces and moments.
2. Geometric compatibility of curvature and deflection.
3. Compatibility of stress and strain.

The processes of derivation of equations are shown as follows.

### 2.2.1 No yielding in both flanges

At the beginning of loading, column behaves totally elastically, that is $\varepsilon_2 < \varepsilon_1 < \varepsilon_p$; $\varepsilon_1$ and $\varepsilon_2$ are strains in higher and lower compressed flanges, respectively. There is no effect of residual stresses at this stage. The column behaves as if no residual stress existed and

\[
\sigma_1 = E\varepsilon_1 \tag{2.1}
\]
\[
\sigma_2 = E\varepsilon_2 \tag{2.2}
\]

$\sigma_1$ and $\sigma_2$ are average stresses in the higher and lower compressed flanges, respectively, $E$ is the Modulus of Elasticity. By applying the condition of vertical force equilibrium

\[
P = \sigma A = \frac{A}{2} \sigma_1 + \frac{A}{2} \sigma_2
\]

or

\[
\sigma_1 + \sigma_2 = 2\sigma \tag{2.3}
\]
where A is the total area of an idealized H-shaped column, P is the total force applied and \( \sigma \) is the average stress in the cross-section.

By the condition of moment equilibrium:

\[
\frac{A}{2} \sigma_1 \cdot \frac{d}{2} - \frac{A}{2} \sigma_2 \cdot \frac{d}{2} = Pu = \sigma A u
\]

or

\[
\sigma_1 - \sigma_2 = 4\sigma \frac{u}{d} = 4\sigma f
\]  

(2.4)

d is the depth, \( u \) and \( f \) are the dimensional and non-dimensional total lateral deflections of the column at mid-height of the column, respectively. Solving Eqs. (2.3) and (2.4)

\[
\sigma_1 = \sigma (1 + 2f)
\]  

(2.5)

and

\[
\sigma_2 = \sigma (1 - 2f)
\]  

(2.6)

The relationship between curvature \( \varphi \) and deflection \( y \) is

\[
\varphi = -\frac{d^2}{dx^2} (y - y_0)
\]

\( y_0 \) is the initial deflection at mid-height of column.

\[
\varphi = \frac{\varepsilon_1 - \varepsilon_2}{d} = -\frac{d^2}{dx^2} (y - y_0) \quad x = \frac{L}{2}
\]

Referring to Fig. 2, by the assumption of sinusoidal curve for deflection shape

\[
\varphi = \frac{\varepsilon_1 - \varepsilon_2}{d} = -\frac{d^2}{dx^2} \left(u \sin \frac{\pi x}{L} - u_0 \sin \frac{\pi x}{L}\right) \quad x = \frac{L}{2}
\]
Thus, the mid-height strains and deflections are related in the following manner.

\[ \phi = \frac{e_1 - e_2}{d} = \frac{\pi^2}{L^4} (u - u_o) \]  
(2.7)

For idealized H-shaped columns,

\[ I = \frac{A}{4} d^2 \]

or

\[ r^2 = \frac{I}{A} = \frac{d^2}{4} \]

Thus, from Eq. (2.7)

\[ E \phi d = E(e_1 - e_2) = 4 \frac{E \pi^2}{(L/r)^2} (f - e) \]  
(2.8)

\( e \) (or \( u_o/d \)) is the dimensionless initial deflection of column at mid-height before loading.

By substituting Eqs. (2.5) and (2.6) into Eqs. (2.1) and (2.2), \( \sigma_1 \) and \( \sigma_2 \) can be eliminated, and

\[ E \sigma_1 = \sigma (1 + 2f) \]
\[ E \sigma_2 = \sigma (1 - 2f) \]

Substituting into Eq. (2.8), the stress versus curvature relation can be gotten as

\[ E \phi d = 4 \sigma f = 4 \frac{E \pi^2}{(L/r)^2} (f - e) \]

or

\[ \sigma \frac{\sigma}{\sigma_y} = \frac{\pi^2 E}{(L/r)^2 \sigma_y} (1 - \frac{e}{f}) \]  
(2.9)
If the ultimate load is obtained during this case

\[
\frac{\partial (\sigma /\sigma_y)}{\partial f} = 0 \quad f = \infty
\]

Thus

\[
\frac{\sigma_{\text{ult}}}{\sigma_y} = \frac{\pi^2 E/(L/r)^2}{f}
\]

or

\[
P_{\text{ult}} = \frac{\pi^2 EI}{L^2}
\]

This is the same as Euler's load. (11)

2.2.2 Yielding only in one flange

For the case when the average stress in the higher compressed flange is beyond the proportional limit stress and the average stress in the lower compressed flange is still within the proportional limit stress, Eq. (2.9) is not applicable any more. A new equation must be derived. Referring to Fig. 3, the higher stressed flange of the idealized section is assumed to carry a load \( P_1 \) which is a function of strain \( \varepsilon_1 \) and a linear residual stress pattern.

\[
P_1 = \frac{A}{2E\psi b} \left[ (E\varepsilon_1)(E\psi b) - (E\varepsilon_1)^2 + (E\varepsilon_1)\sigma_p - \sigma_p^2 \right]
\]

or

\[
\sigma'_{1} = \frac{2P_1}{A} = \frac{1}{E\psi b} \left[ -(E\varepsilon_1)^2 + (2\sigma_p + E\psi b)(E\varepsilon_1) - \sigma_p^2 \right] \quad (2.10)
\]

\( \psi \) is the rate of residual stress variation. Solving Eq. (2.10), the stress-strain relation for the higher compressed flange is obtained as: (3)
For the lower compressed flange

$$E\varepsilon_2 = \sigma_2$$

By the same process as shown in subsection 2.2.1 and simplifying, the following stress versus deflection equation can be obtained.

$$\frac{E\Psi b}{\sigma_Y} + 2\left[ \frac{\sigma_p}{\sigma_Y} - \frac{\sigma_l}{\sigma_Y} (1 - 2 f) \right] - \sqrt{\left( \frac{E\Psi b}{\sigma_Y} \right) \left( \frac{E\Psi b}{\sigma_Y} + \frac{4 \sigma_p}{\sigma_Y} - \frac{4 \sigma_l}{\sigma_Y} (1 + 2 f) \right)}$$

$$= 8 \frac{E\pi^2}{(L_f)^2} (f - e) \quad (2.11)$$

However, it is very tedious to get the ultimate load by directly differentiating Eq. (2.11). Equation (2.11) is separated into two equations of stress versus curvature and deflection versus curvature relations. These are

$$\frac{E\Psi b}{\sigma_Y} + 2\left[ \frac{\sigma_p}{\sigma_Y} - \frac{\sigma_l}{\sigma_Y} (1 - 2 f) \right] - \sqrt{\left( \frac{E\Psi b}{\sigma_Y} \right) \left( \frac{E\Psi b}{\sigma_Y} + \frac{4 \sigma_p}{\sigma_Y} - \frac{4 \sigma_l}{\sigma_Y} (1 + 2 f) \right)}$$

$$= \frac{2E\Phi_d d}{\sigma_Y} \quad (2.12.)$$

and

$$\frac{4E\pi^2}{(L_f)^2} (f - e) = \frac{E\Phi_d d}{\sigma_Y} \quad (2.13)$$
A graphical method which will be described later is used to obtain the ultimate load.

2.2.3 Yielding in both flanges

Referring to Fig. 4 the higher stressed flange of the idealized section is assumed to carry a load \( P_1 \).

\[
P_1 = \frac{A}{2E\psi b} \left[ -(E\varepsilon_1)^2 + (2\sigma_p + E\psi b)(E\varepsilon_1) - \sigma_p^2 \right]
\]

or

\[
\sigma_1 = \frac{1}{E\psi b} \left[ -(E\varepsilon_1)^2 + (2\sigma_p + E\psi b)(E\varepsilon_1) - \sigma_p^2 \right] \quad (2.14)
\]

Solving Eq. (2.14) for \( \varepsilon_1 \):

\[
E\varepsilon_1 = \frac{2\sigma_p + E\psi b}{2} - \frac{1}{2} \sqrt{E\psi b [E\psi b + 4(\sigma_p - \sigma_1)]} \quad (2.15)
\]

For the same reason, on the lower stressed flange

\[
E\varepsilon_2 = \frac{2\sigma_p + E\psi b}{2} - \frac{1}{2} \sqrt{E\psi b [E\psi b + 4(\sigma_p - \sigma_1)]} \quad (2.16)
\]

Imposing the strain versus deflection equation (Eq. 2.8) the following equation is obtained.

\[
\sqrt{E\psi b} \left( \sqrt{E\psi b + 4\sigma_p - 4\sigma(1-2f)} - \sqrt{E\psi b + 4\sigma_p - 4\sigma(1+2f)} \right) = 2E \phi_{x} d \quad (2.17)
\]
Dividing both sides by $\sigma_y$, the stress-curvature relation in dimensionless form can be expressed as:

$$\sqrt{\frac{E\psi b}{\sigma_y}} \left( \sqrt{\frac{E\psi b}{\sigma_y} + \frac{\sigma_p}{\sigma_y} - \frac{4\varepsilon}{\sigma_y}} - \sqrt{\frac{E\psi b}{\sigma_y} + \frac{4\sigma_p}{\sigma_y} - \frac{4\varepsilon}{\sigma_y}} \right) = 2\varepsilon \frac{\phi d}{\sigma_y} \quad (2.18)$$

Also, from Eq. (2.9)

$$\frac{4\pi^2 \varepsilon}{(b_f)^2 \sigma_y} (f - e) = \frac{E\phi d}{\sigma_y} \quad (2.19)$$

The graphical method is also used to get ultimate load in this case.

2.2.4 One flange totally yielded

The stress-deflection relation can be obtained as;

$$\sigma_1 + \sigma_2 = \sigma_y + \sigma_2 = 2\sigma$$

$$\sigma_1 - \sigma_2 = \sigma_y - \sigma_2 = 4\sigma$$

Combining the above equations
There is no initial out-of-straightness \( e \) in Eq. (2.26). This shows that the stress-deflection curves of all the columns go along one parabolic curve after one flange is totally yielded no matter what the initial out-of-straightness of the column is. \(^{(1)}\)

\[
\frac{\sigma}{\sigma_y} = \frac{1}{1 + 2f}
\]  

(2.20)

2.3 H-shaped Column Bent About the Weak Axis

For weak axis bending, the following cases are to be considered (see Fig. 5):

1. No yielding in any of the flanges.
2. Yielding on one side of the flange only.
3. Yielding on both sides of the flanges.

2.3.1 No yielding in both flanges

As discussed in subsection 2.2.1, when the maximum fiber stress still is within the proportional limit the residual stress does not effect the column. By the same approach as subsection 2.2.1 the same expression of \( \sigma/\sigma_y \) versus \( f \) can be derived as

\[
\frac{\sigma'}{\sigma_y'} = \frac{\pi^2 E}{(L/r)^2 \sigma_y} \left(1 - \frac{e}{f}\right)
\]  

(2.21)
2.3.2 Flange tips on only one side yielded

Referring to Fig. 5b, the equations for axial thrust and moment equilibrium at the mid-height of the columns are as follows:

\[
\frac{P}{a} = \int_{x_1}^{b_2} \left[ \sigma_p + E\psi \left( \frac{b}{2} - x \right) \right] dx
\]

\[
+ \int_{-b_2}^{x_1} \left[ \sigma_p + \frac{E\psi b}{2} - E x_1 (\psi + \phi) + E\phi x \right] dx
\]

where \( p \) is the total applied load from which

\[
x_1 + x_1 b - \frac{2\sigma_p b + E\psi b^2}{E(\psi + \phi)} + \frac{b^2}{4} + \frac{2P}{Ea} \frac{1}{\psi + \phi} = 0
\]

Solving for \( x_1 \)

\[
x_1 = -\frac{b}{2} + \sqrt{\frac{2\sigma_p b + E\psi b^2 - 2P/a}{E(\psi + \phi)}}
\]

(2.22)

The moment \( M_o \) at the center of the column equals

\[
\frac{M_o}{a} = \frac{Pu}{a} = \int_{x_1}^{b_2} \left[ (\sigma_p + \frac{E\psi b}{2}) x - E\psi x^2 \right] dx
\]

\[
+ \int_{-b_2}^{x_1} \left[ (\sigma_p + \frac{E\psi b}{2} - E x_1 (\psi + \phi) x + E\phi x^2 \right] dx
\]

(2.23)
Substituting Eq. (2.22) into Eq. (2.24), simplifying and dividing each term by \( \sigma_y \), the load versus curvature relation in non-dimensional form is obtained as:

\[
\frac{\chi^3 (\psi + \phi) - \frac{3}{4} \chi b^2 (\psi + \phi) + \frac{b^2}{4} (\psi - \phi)}{\frac{6P\mu}{Ea}} = (2.24)
\]

\[
\frac{\left( \frac{2}{\sigma_y} + \frac{E\psi b}{\sigma_y} - \frac{2\sigma}{\sigma_y} \right)}{\left( \frac{3}{\sigma_y} + \frac{E\psi b}{\sigma_y} - \frac{3\sigma}{\sigma_y}(1+2f) \right)^2} - \frac{E\psi b}{\sigma_y} = \frac{E \phi^d_d}{E_y}
\]

(2.25)

Using the same approach as mentioned in subsection 2.2.1, the curvature versus deflection equation is

\[
\frac{E \phi_d}{\sigma_y} = \frac{12 \pi^2 \phi}{(f)^2 \sigma_y} (f - e)
\]

(2.26)

### 2.3.3 Flange tips of both sides yielded

Referring to Fig. 5c axial equilibrium requires that at the center of the column the following equation must be satisfied.
\[
\frac{P}{a} = \int_{x_1}^{x_2} \left[ \sigma_f + E \psi (b/2 - x) \right] \, dx \\
+ \int_{x_2}^{x_1} \left[ E_p + \frac{1}{2} E \psi b - E x_1 (\psi + \phi) + E \psi x \right] \, dx \\
+ \int_{-x_2}^{-x_1} \left[ \sigma_f + E \psi (b/2 + x) \right] \, dx
\]  
\text{(2.27)}

When integrated and simplified, this becomes:

\[ -(\psi + \phi) x_1^2 - (\psi - \phi) x_2^2 + \frac{b}{E} \left( 2 \sigma_f + \frac{1}{2} E \psi b \right) - \frac{2P}{Ea} = 0 \]  
\text{(2.28)}

\[ x_1 \text{ and } x \text{ are related by geometry} \]
\[ x_1 (\psi + \phi) = x_2 (\psi - \phi) \]  
\text{(2.29)}

Solving Eqs. (2.28) and (2.29), we obtain;

\[ x_1 = \frac{\psi - \phi}{2 \psi (\psi + \phi)} \left[ \frac{b}{E} \left( 2 \sigma_f + \frac{1}{2} E \psi b \right) - \frac{2P}{Ea} \right] \]  
\text{(2.30)}

\[ x_2 = \frac{\psi + \phi}{2 \psi (\psi - \phi)} \left[ \frac{b}{E} \left( 2 \sigma_f + \frac{1}{2} E \psi b \right) - \frac{2P}{Ea} \right] \]

Moment equilibrium at the center section requires that
\[
\frac{M_0}{a} = \frac{P u}{a} = \int_{x_1}^{b/2} \left[ \left( \sigma_p + \frac{1}{2} E \psi b \right) x - E \psi x^2 \right] dx
\]

\[
+ \int_{x_1}^{x} \left[ \left( \sigma_p + \frac{1}{2} E \psi b - E x_1 \left( \psi + \phi \right) \right) x + E \phi x^2 \right] dx
\]

\[
+ \int_{-b/2}^{x_1} \left[ \left( \sigma_p + \frac{1}{2} E \psi b \right) x + E \psi x^2 \right] dx
\]

or

\[
- \frac{6 P u}{\alpha E} - x_1^3 \left( \psi + \phi \right) + \left( \psi - \phi \right) x_2 = 0 \quad (2.31)
\]

By the same process as the preceding case the load versus deflection relation is

\[
\frac{E \psi b}{\sigma_y} = \frac{E \phi b}{\sigma_y} = \frac{E \phi_d b}{\sigma_y} \quad (2.32)
\]

and also the deflection versus curvature relation is

\[
\frac{E \phi_d b}{\sigma_y} = 12 \frac{\pi^2 E}{\left( \frac{\sigma_y}{\sigma_p} \right)^2} \left( f - e \right) \quad (2.33)
\]
Both cases described in subsections 3.3.2 and 3.3.3 are too complicated to find the ultimate load by direct differentiation. The graphical method described in section 2.4 is used.

To simplify the process of obtaining the ultimate load of columns, a graphical method is developed to obtain the solution. The method is illustrated below as used in obtaining the ultimate load when the method of directly differentiating the load versus deflection equation is too complicated.

2.4 Graphical Method

In Eqs. (2.12), (2.18), (2.25), and (2.32), the relationship between $\sigma/\sigma_y$, $f$, and $E\sigma_d/\sigma_y$ has been presented under different cases of stress behavior. By using $E\sigma_d/\sigma_y$ as the ordinate and $f$ as abscissa, assuming different values of $\sigma/\sigma_y$ a set of curves can be drawn. Figures 6, 7, 8, and 9 show the curves which represent Eqs. (2.12), (2.18), (2.25) and (2.32) respectively. Residual stress on flange tip is assumed to be 5 ksi compression and that on flange center as 5 ksi tension and A36 steel ($\sigma_y = 36$ ksi) is also assumed.

$$\frac{E\sigma_d}{\sigma_y} = K \frac{2E}{(L/r)^2} \frac{(f - e)}{y}$$

For a column of given slenderness ratio and initial out-of-straightness, the relationship between $E\sigma_d/\sigma_y$ versus $f$ can be represented by
a straight line with a slope equal to $\frac{n^2E}{(L/r)^2\sigma_y}$ and passing through the point $(0,e)$. The intersections of the straight line and the curves (see Figs. 6 and 7) give a set of points which represent the $\sigma/\sigma_y$ and $f$ relation of this column for the case for which the graph is drawn. Thus, the strength versus deflection curve can be obtained (when the column is behaving elastically the $\sigma/\sigma_y$ and $f$ relation can be gotten directly from the $\sigma/\sigma_y$ and $f$ equations). Figures 10 and 11 show the $\sigma/\sigma_y$ versus $f$ curves of columns bending about the strong axis with slenderness ratio 57 and 33 and initial out-of-straightness varying from 0 to 0.05.

Figures 7, 8, and 9 can be used also to find the required slenderness ratio for a certain value of ultimate load, that is to get the column strength curves. For an initial out-of-straightness $e$, a set of straight lines all starting at point $(0,e)$, each tangent to one curve, can be drawn as shown in Fig. 8. Since the slope of each straight line $S$ is a function of slenderness ratio,

$$S = \frac{K \pi^2 E}{(L/r)^2 \sigma_y}$$

the required slenderness ratio of the column which has the same ultimate strength as the strength represented by the curve tangential to this straight line can be determined immediately.

By applying the method mentioned above, the column curves for columns with different initial out-of-straightness can be obtained.
This is shown in Figs. 12 and 13, in which Fig. 12 represents the column curves for an idealized H-shaped column bending about the strong axis and Fig. 13 for those bending about the weak axis. For columns bent about the strong axis, Fig. 11 shows that columns of low slenderness ratios and columns of medium slenderness ratios and small initial out-of-straightness, the ultimate strength is attained with both flanges yielded (Eq. 2.8). This means that the residual stress has more influence than the initial out-of-straightness. For columns of higher slenderness ratio or medium slenderness ratio with large initial out-of-straightness, ultimate strength is attained with only one flange yielded.

For H-shaped columns bending about the weak axis, only the case of the initially perfect column attains its ultimate strength with the flange tips on both sides of the bending axis yielded (Eq. (2.30)). All columns with initial out-of-straightness reach their ultimate loads with the tips of the concave side yielded.

Compared with the H-section bent about the strong axis, (see Fig. 12) the strength of a column bending about the weak axis (see Fig. 13), is approximately 15% less.

2.5 Hot-rolled Box-section

Due to the absence of any considerable residual stress in hot-
rolled Box-sections made of A-36 steel, the effect of residual stress upon column strength is significantly small and may be neglected for purposes of analysis.\(^{(5)}\)

Columns without residual stresses and with initial out-of-straightness may fail in either of two ways: (1) when the cross section is still fully elastic (elastic case), or (2) when part of the cross section is yielded (elastic-plastic case). Failure within the elastic range is a relatively simple problem and is discussed in section 2.5.1. When the column fails beyond the elastic range, the fiber at the concave side starts to yield and the effective area of the section is reduced. Due to this phenomenon, a column with initial geometric imperfection attains an ultimate strength less than that given by Euler's formula, except for very long columns where the ultimate strength is almost the same as predicted by Euler's formula.\(^{(9)}\)

To simplify the theoretical analysis, an idealized Box-section of very small \(t/b\) and \(t/d\) ratios is assumed. Therefore, the fiber stresses on either wall which is parallel to the bending axis can be assumed the same across the whole thickness of that wall. Also the area of the cross-section can be represented simply by

\[ A = 2(b + d)t. \]

The theoretical relationship between stress and deflection is discussed as follows.
2.5.1 **In the elastic range**

The load-deflection relation can be derived by applying the conditions of static equilibrium of vertical forces and moments. Referring to Fig. 14a, we have:

\[ P = \int \varepsilon \, dA = \sigma A \]

or

\[ \sigma_1 + \sigma_2 = 2 \sigma \]  \hspace{1cm} (2.34)

and

\[ M_o = pu = \sigma' Au \]

or

\[ \sigma'_{1/2} = \frac{12}{3b + d} \left( \frac{b + d}{3b + d} \right) f \]  \hspace{1cm} (2.35)

curvature at mid-height of column

\[ \phi_{\xi} = \frac{1}{d} \left( \frac{2}{L^2} \right) (y - y_o) \bigg|_{x = \frac{L}{2}} \]

or

\[ \phi_{\xi} = \frac{1}{d} \left( \frac{2}{L^2} \right) \left( u - u_o \right) \]

in the elastic range

\[ \sigma_1 = E \varepsilon_1 \]

\[ \sigma_2 = E \varepsilon_2 \]

thus

\[ E \phi_{\xi} d = \sigma_1 - \sigma_2 = \frac{2E}{(L/d)^2} (f - e) \]  \hspace{1cm} (2.36)
By eliminating $\delta_1 - \delta_2$ from Eqs. (2.35) and (2.36) and substituting the radius of gyration, $r$,

$$ r^2 = \frac{(3b + d) d^2}{12(b + d)} $$

thus

$$ \sigma = \frac{G_E}{\delta_y} (1 - \frac{e}{f}) $$
or

$$ f = \frac{1 - \frac{e}{\sigma/\delta_E}}{1 - \frac{\sigma}{\sigma E}} \quad (2.37) $$

when $\sigma_E$ = Euler's average stress, which is equal to $\frac{\pi^2E}{(L/x)^2}$

2.5.2 Boundary of elastic zone

Referring to Fig. 14b when $\delta_1 = \delta_y$ the extreme fibers at the concave side starts to yield.

Thus

$$ E\delta d = \delta_y' - \delta_2 = \frac{E \pi^2}{(L/d)^2} (f - e) \quad (2.38) $$

and

$$ \delta_y + \delta_2 = 2 \sigma $$

or

$$ \delta_2 = 2 \sigma - \delta_y \quad (2.39) $$

From Eq. (2.37)

$$ \sigma' = \delta_E (1 - \frac{\delta}{f}) \quad (2.40) $$

thus

$$ \delta_2' = 2 \delta_E (1 - \frac{\delta}{f}) - \delta_y $$
substituting into Eq. (2.38)

\[ 2 \sigma y - 2 \left(1 - \frac{e}{f}\right) \sigma E - \frac{12(b + d)}{3b + d} \sigma E (f - e) = 0 \quad (2.41) \]

For the square Box-section b=d Eq. (2.4) becomes

\[ 3f^2 - (3e + \frac{\sigma y}{\sigma E} - 1) f - e = 0 \]

thus

\[ f = \frac{1}{6} \left[ (3e + \frac{\sigma y}{\sigma E} - 1) + \sqrt{(3e + \frac{\sigma y}{\sigma E} - 1)^2 + 12e} \right] \quad (2.42) \]

By Eqs. (2.40) and (2.42), the boundary stress and deflection of elastic zone for a given column can be computed.

2.5.3 Elastic-plastic zone

Beyond the elastic boundary, the section starts to yield.

Columns with initial out-of-straightness reach ultimate strength within this zone.

Referring to Fig. 14c, by the equilibrium condition of external and internal forces

\[ \varepsilon (E \gamma + \xi z) bt + 2Ed + \varepsilon y - \frac{E (E \gamma - \xi z)^2}{2 \phi} = 2(b + d) t \delta^* = P \]
For the square Box-section \( b=d \) we get

\[
\varepsilon_2^2 - 2\varepsilon_2 (\varepsilon_y + \phi d) + (\varepsilon_y^2 - 6\phi d \varepsilon_y + 8\phi d \frac{\sigma}{E}) = 0
\]

or

\[
\varepsilon_2 = \varepsilon_y + \phi d - \sqrt{\phi d (8\varepsilon_y - 8 \frac{\sigma}{E} + \phi d)}
\]

(2.43)

Applying moment equilibrium

\[
E (\varepsilon_y - \varepsilon_2) \cdot \frac{b d t}{2} + \frac{E (\varepsilon_y - \varepsilon_2)^2 (3\phi d - 2\varepsilon_y + 2\varepsilon_2)}{3 \phi^2} = 2 \frac{E}{b} (b + d) t u
\]

For the square Box-section

\[
(\varepsilon_y - \varepsilon_2) [3\phi^2 d^2 + (\varepsilon_y - \varepsilon_2) (3\phi d - 2\varepsilon_y + 2\varepsilon_2)] = \frac{24\sigma}{E} (\phi d)^2
\]

(2.44)

Substituting Eq. (2.43) into Eq. (2.44) and changing the equations into dimensionless form, we obtain

\[
72 \left(1 - \frac{\sigma}{6\gamma}\right) + \frac{11 E \phi d}{6\gamma} - 24 \frac{\sigma}{6\gamma} f
\]

\[
= \left[16 \left(1 - \frac{\sigma}{6\gamma}\right) + \frac{11 E \phi d}{6\gamma}\right] \sqrt{\frac{6\gamma}{E \phi d}} \left[8 \left(1 - \frac{\sigma}{6\gamma}\right) + \frac{E \phi d}{6\gamma}\right]
\]

(2.45a)
Since

\[
\frac{E \phi d}{\gamma} = 6 \frac{\sigma_e}{\gamma} (f - e) \quad (2.45 \, b)
\]

for square Box-section, by using the graphical method as described in Section 2.4 for the elastic-plastic zone. A \( \frac{E \phi d}{\gamma} \) vs. \( f \) diagram was drawn by Eq. (2.45) as shown in Fig. 15. The load versus deflection curve of a given column and column curves can be obtained. Figure 16 shows the column curves of square box-shaped columns with initial out-of-straightness \( e \) ranging from 0 to 0.05.
3. EXPERIMENTAL INVESTIGATION

The purpose of the research is to study the behavior of columns made of hot-rolled structural tubing of A36 steel. Special attention is given to the effect of initial out-of-straightness on the strength of the column since it seems to be playing a major role on the reduction of column strength.

The test program includes the test of ten columns with slenderness ratios varying from 30 to 100. Preliminary tests consist of stub column tests, residual stress measurements and tensile coupon tests. The details of the program are summarized in Table 1.

3.1 Preliminary Tests

3.1.1 Tensile coupon test

The dimensions of the tensile coupon were selected according to ASTM standards; full thickness of material and 1-1/2 in. width over an 8-in. gage length.\(^{(5)}\) A total of 88 tensile coupons were tested in a 120 kip mechanical screw-type testing machine. The strain was recorded and plotted automatically. Fig. 17 shows the typical stress-strain relation for the coupons tested. The average values of static yield stress, ultimate stress and Young's modulus are 37.8 ksi and 66.1 ksi and 29.7 ksi, respectively. The data obtained from
the tensile coupon tests give a check on the static yield level of the material used for the sections and verify the assumption of idealized elastic-plastic stress-strain relationship for longitudinal fibers.

3.1.2 Residual stress measurements

The "sectioning method"(5) was adopted for the measurement of residual stress. Residual stresses on the inside of the Box-section were measured for pieces AA and EE (see Table 1). An indirect method was used to find the residual stress distribution at the inside face of the column. This indirect method for measuring the residual stresses is explained as follows:

A piece of column 4'-3' length is prepared as shown in Fig. 18a; the center portion of 11" is used for residual stress measurement. Before cutting, holes are drilled 10" apart and 1/2" from the adjoining ones and numbers are marked beside the holes in sequence as shown in Fig. 18b. The specimen was cut into 2-L shape pieces after the first Whittemore gage readings (Fig. 18c). Then, additional gage holes were laid out on the inside face of the L-shapes (Fig. 18d), corresponding to those holes on the outside face. The second Whittemore gage readings on the inside face were taken prior to final cutting the section into strips.

Figure 19 shows the typical residual stress distribution. It shows only a slight variation in the magnitude of residual stresses measured on the outside face and on the inside face. The average residual stress is always less than that on the outside face and the difference
between them is small, thus, for pieces other than pieces AA and EE only the residual stresses on the outside face were measured. Figures 20b, 21b, 22b, 23b and 24b show the magnitude and distribution of residual stresses in the sections. The magnitudes of the residual stresses varying from -10 ksi to -10 ksi, but were more predominantly within +5 ksi. Due to the random type of the residual stress distribution and their small magnitudes, the effect of residual stress to column strength is significantly small and may be neglected. (5)

The measurement of residual stress on two different sections of a column showed that there was no significant difference in the residual stress distribution at different positions along the length of a column. The measurement of residual stresses on one section at least b distant from the ends (where b is the largest dimension of the cross section) as sufficient to represent the residual stress along the column.

3.1.3 Stub column tests

The setup of the stub column test is shown in Fig. 30. The four gages were used for alignment. The alignment of the specimen was made at loads not exceeding one half of the expected yield stress level. The alignment was considered satisfactory if the deviation of any of the corner gage readings did not exceed 5% of the average value at the maximum alignment load.
The stub-column test gives a stress-strain curve showing the effect of residual stress and also furnishes data about the proportional limit, the static yield stress level, and the elastic and the elastic-plastic moduli. Those data are necessary for the prediction of column strength.

Figures 25b, 26b, 27b, 28b and 29b give the stress versus strain diagrams for the different stub-column tests conducted. It can be observed that the proportional limit of the section approaches its yield load. These results verify the presence of low residual stresses in the section and its effect can be neglected for practical purposes. (5)

3.2 Column Tests

A total of 10 full scale column tests were conducted (refer to Table 1). All the columns except two were tested in an 800,000 lb. screw-type testing machine. Columns C7 and C8 were tested in a 5,000,000 lb. hydraulic-type machine.

The columns were tested with pinned-end supports. The end fixtures used were standard fixtures at Fritz Laboratory. (7)
3.2.1 Preparation of the column before testing

The following preparations were made on the column before testing.

1. The external dimensions of column were measured and checked for variation within the acceptable tolerance of 0.05 in.

2. The column was whitewashed with hydrated lime. The flaking of whitewash gives an indication of the extent and location of yielding during the test.

3. Strip scales about 12 in. long were attached to column at 1/4-points or 1/6 points. The initial out-of-straightness of the column with respect to its neutral axis was determined by the readings of the strip scales by a theodolite. The scales were also read with a theodolite during the testing to obtain a measurement of lateral deflection along the length of the column. As an added precaution, a short strip scale was attached to the fixed crosshead of the testing machine to check lateral movement of testing machine and a floor standard was used to check any disturbances of the theodolite setting.

Mid-height lateral deflection of the column was also measured by a 1/1000 in. dial gage fixed to the testing
machine with its plunger attached with taut thin wire to a small screw tapped-in at the centerline of the column width. The setup of the strip scales and the dial gage is shown in Figs. 31 and 32.

(4) Four SR-4 gages were attached at four corners at each end and eight SR-4 gages at the mid-height level; 4 at corners and one at centerline of each face for the measuring of strain during the testing.

(5) The rotation about the test axis was measured by a level bar mounted on support brackets welded to the base plate and the top plate of the column.

3.2.2 Alignment

For the purpose of centralizing the load with respect to the specimen during the column test the column was first centered geometrically in the testing machine as the first position. Then, the column was loaded in increments up to a load not exceeding about one-half of the estimated maximum load. The alignment was based on four corner gages at each end and at the mid-height. Alignments were made by relative movement of column bases with respect to the base fixtures. The column was considered satisfactorily aligned when the maximum deviation of any of the four gage readings from the average value did not exceed 5% at each load level.
3.2.3 Column test procedure

The testing of the column specimen was conducted as follows:

1. Start the test with an initial load of about 30 kips.

2. Record the readings of all SR-4 gages, strip scales, mid-height gage and level bars and measure 3 pair of 10 in. gage holes at mid-height of column with a Whitemore type mechanical strain gage.

3. Plot the point in the load versus strain and load versus deflection diagram of mid-height after each loading level. The plots showed the value of proportional limit and occurrence of the first yield.

4. Add the loading by an increment of 20 to 50 kips depending upon the size of the test column and also the plot of loading versus deflection curve. Within the proportional limit loading can be applied with bigger increment but when close to the ultimate loading increases in a smaller amount. Repeat step 3.

5. Above the proportional limit, a load relaxation diagram (as Fig. 33) was plotted for each load. Be sure the ultimate load of the column would be clearly defined in the load-deflection diagram.

6. After ultimate load, continue loading until deflection is judged too great to be able to test safely.
3.2.4 Column test results

The results of the column tests are summarized in Table 2. Figures 25a, 26a, 27a, 28a, and 29a show the load-deflection relationship of the columns. The data given in Table 2 include slenderness ratio, initial out-of-straightness, experimental column strength and estimated column strength by theoretical analysis of each of the ten columns tested in this program. The initial out-of-straightness ranged from a minimum eccentricity ratio, e, of 0 to a maximum 0.05. Figures 20a, 21a, 22a, 23a and 24a show the variation of the initial out-of-straightness along the length of the columns.

It indicates that columns C1, C2, C3, C4 and C8 obtained their ultimate load at larger deflection than those of others. This verify the theoretical analysis that the mid-height deflection at ultimate load is a function of initial out-of-straightness and the slenderness ratio of column. The larger the initial out-of-straightness and the slenderness ratio, the larger the mid-height deflection when the ultimate load is attained.

It also shows columns with higher initial out-of-straightness and slenderness ratios (Columns C1, C2, C4 and C8), the close of the unloading curve is slight whereas for columns with small initial out-of-straightness the rate of drop in load is very pronounced after the ultimate strength is attained. Due to this kind of column behavior, column with small initial out-of-straightness, as columns C5, C6, C9, and C10, attained their ultimate load suddenly and without any
appreciable yielding of the cross section prior to attaining the ultimate load. Furthermore, the load drops very fast. Thus, it is very hard to define clearly the ultimate load point. When the predicted ultimate load is nearly reached, a close watch of the load and the mid-height dial gage readings is necessary.

Figure 34 gives typical stress-strain curve at the mid-height section of the extreme fibers and the fibers at the centerline. Part of the stress-strain curve of the stub-column is also plotted. The divergence of the stress-strain curves is due to the initial out-of-straightness of the column. If the column was perfectly straight and homogeneous, the curves of the three fibers would coincide up to the point of bifurcation.
4. CONCLUSIONS

Summarizing the test results and theoretical analysis, the following conclusions are made:

1. From the practical measurement of residual stresses on both inside and outside face of a Box-shaped column, it shows no significant variation of residual stress distribution across the thickness up to thickness equal to 1/4 in.

2. It has been shown in Table 2 and Fig. 35 that the test results are in good agreement with the theoretical ultimate load analysis based on a column with initial out-of-straightness \( e \), varying from 0 to 0.05 and without considering residual stress.

3. There is no significant error introduced in the theoretical analysis of the strength of columns if residual stresses of random values and low magnitude are neglected. This is shown as the good agreement between the theoretical predicted column strength and experimental results.

4. From both theoretical analysis and experimental investigation, it was shown that the initial out-of-straightness has a more pronounced influence on column strength for columns with medium slenderness ratio than for short and long columns.

5. In Fig. 35 when considering the initial out-of-straightness of the column \( e \), to be 0.05, the CRC Basic Column Curve and the theoretical curve are approximately the same for \( L/r \) greater than 80. There-
fore, for columns with initial out-of-straightness less than 0.05, the CRC Basic Column Curve is applicable for longer columns, while for short columns with initial out-of-straightness bigger than 0.03 the theoretical column curves are more conservative.

6. Comparing Fig. 35 with Fig. 12, it shows that the rolled Box-shape column of high L/r has higher strength than that of rolled H-section of the same L/r. But for short columns they are nearly the same. Therefore, to use the CRC formula for the H-section for the Box-shape column design is on the safe side.
5. NOMENCLATURE

\( A \) Cross-section area of column

\( a \) Twice flange thickness

\( b \) Flange width or width of Box-section

\( d \) Depth of H-section or Box-section

\( E \) Young's modulus

\( e \) Dimensionless initial lateral deflection of column at mid-height before loading, \( u_0/d \) or \( u_0/b \)

\( f \) Dimensionless total lateral deflection of column at mid-height after loading \( u/d \) or \( u/b \)

\( I \) Moment of inertia

\( L \) Total length of a pin-ended column

\( L/r \) Slenderness ratio

\( M \) Moment

\( M_0 \) Moment at mid-height of column

\( P \) Column load, \( A \)

\( r \) Radius of gyration in the plane of bending

\( t \) Flange thickness

\( u_0 \) Initial lateral deflection of column at mid-height before loading

\( u \) Total lateral deflection of column at mid-height after loading

\( x_1 \) Distance from the center of the cross-section to the beginning of yielded area

\( x_2 \) Distance from the center of the cross-section to the beginning of yielded area

\( \varepsilon_1 \) Compressive strain in concave flange at mid-height of column

\( \varepsilon_2 \) Compressive or tensile strain in convex flange at mid-height of column

\( \varepsilon_y \) Strain corresponding to yield point
\( \sigma \) Average cross-sectional stress of column at mid-height  
\( \sigma_1 \) Average stress in concave flange at mid-height of column  
\( \sigma_2 \) Average stress in convex flange at mid-height of column  
\( \sigma_p \) Proportional limit stress  
\( \sigma_E \) Euler buckling stress  
\( \sigma_{rc} \) Residual stress at flange edges  
\( \sigma_{ro} \) Residual stress at flange centers  
\( \sigma_y \) Yield stress level; average stress in plastic range  
\( \psi \) Rate of residual stress variation  
\( \phi \) Curvature at mid-height of column at mid-point of column
6. TABLES AND FIGURES
<table>
<thead>
<tr>
<th>PIECE DESIGN</th>
<th>CROSS SECTION</th>
<th>LENGTH</th>
<th>COL. NO.</th>
<th>L/r</th>
<th>SPECIMENS</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA 3 1/2 x 3 1/2</td>
<td>36' 4&quot;</td>
<td>1</td>
<td>2</td>
<td>80</td>
<td>8' 6&quot; column</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>100</td>
<td>10' 8&quot; column</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>coupons (4 sets)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>res. stress (2 sets)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>stub column</td>
</tr>
<tr>
<td>BB 3 1/2 x 3 1/2</td>
<td>36' 3&quot;</td>
<td></td>
<td></td>
<td></td>
<td>Coupons (2 sets)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>res. stress</td>
</tr>
<tr>
<td>CC 4 x 4 x 3/16</td>
<td>39' 8&quot;</td>
<td>3</td>
<td>4</td>
<td>67.6</td>
<td>8' 8&quot; column</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>11' 7&quot; column</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>coupons (4 sets)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>res. stress</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>stub column</td>
</tr>
<tr>
<td>DD 6 x 6 x 1/4</td>
<td>39' 5&quot;</td>
<td>5</td>
<td>6</td>
<td>32</td>
<td>6' 2&quot; column</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>9' 10&quot; column</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>coupons (4 sets)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>res. stress</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>stub column</td>
</tr>
<tr>
<td>EE 10 x 10 x 1/2</td>
<td>2 of 42' 0&quot;</td>
<td>7</td>
<td>8</td>
<td>60</td>
<td>19' 0&quot; column</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>28' 6&quot; column</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>coupons (4 sets)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>res. stress (3 sets)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>stub column (2 sets)</td>
</tr>
<tr>
<td>FF 10 x 10 x 1/4</td>
<td>12' 0&quot;</td>
<td></td>
<td></td>
<td></td>
<td>coupons (2 sets)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>res. stress</td>
</tr>
<tr>
<td>GG 6 x 4 x 1/4</td>
<td>36' 6&quot;</td>
<td>9</td>
<td>10</td>
<td>50</td>
<td>6' 7&quot; column</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10' 7&quot; column</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>coupons (4 sets)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>res. stress</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>stub column</td>
</tr>
</tbody>
</table>

**TABLE 1. HOLLOW STRUCTURAL TUBING**
**SCHEDULE OF SPECIMENS**
**Table 2. Comparison of Column Strength Between Theoretical Analysis and Experimental Results for Rolled Box-Shaped Columns**

<table>
<thead>
<tr>
<th>COLUMN NUMBER</th>
<th>INITIAL OUT OF STRAIGHTNESS</th>
<th>L/(\bar{e}) SLENDERNESS RATIO</th>
<th>(P/P_y) EXPERIMENTAL RESULTS</th>
<th>(P/P_y) BASIC CURVE</th>
<th>(P/P_y) THEORETICAL ANALYSIS</th>
<th>((6) - (4))% ERROR</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>0.034</td>
<td>80</td>
<td>0.80</td>
<td>0.80</td>
<td>0.82</td>
<td>1.25</td>
</tr>
<tr>
<td>C2</td>
<td>0.046</td>
<td>100</td>
<td>0.67</td>
<td>0.69</td>
<td>0.68</td>
<td>1.45</td>
</tr>
<tr>
<td>C3</td>
<td>0.020</td>
<td>67.6</td>
<td>0.87</td>
<td>0.86</td>
<td>0.91</td>
<td>4.39</td>
</tr>
<tr>
<td>C4</td>
<td>0.020</td>
<td>90</td>
<td>0.75</td>
<td>0.75</td>
<td>0.84</td>
<td>10.70</td>
</tr>
<tr>
<td>C5</td>
<td>0</td>
<td>32</td>
<td>0.94</td>
<td>0.97</td>
<td>1</td>
<td>6.00</td>
</tr>
<tr>
<td>C6</td>
<td>0</td>
<td>51</td>
<td>0.95</td>
<td>0.92</td>
<td>1</td>
<td>5.00</td>
</tr>
<tr>
<td>C7</td>
<td>0</td>
<td>60</td>
<td>1.00</td>
<td>0.89</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>C8</td>
<td>0.022</td>
<td>90</td>
<td>0.92</td>
<td>0.75</td>
<td>0.84</td>
<td>9.52</td>
</tr>
<tr>
<td>C9</td>
<td>0</td>
<td>50</td>
<td>0.99</td>
<td>0.93</td>
<td>1</td>
<td>1.00</td>
</tr>
<tr>
<td>C10</td>
<td>0.010</td>
<td>80</td>
<td>0.94</td>
<td>0.81</td>
<td>0.93</td>
<td>1.07</td>
</tr>
</tbody>
</table>
FIG. 1 IDEALIZED STRESS-STRAIN CURVE

\[ P = 0 \]

\[ y = u_0 \sin \frac{\pi x}{L} \]

FIG. 2 SINUSOIDAL DEFLECTION CURVE

\[ P \]

\[ y = u \sin \frac{\pi x}{L} \]
(a) RESIDUAL STRESS DISTRIBUTION IN FLANGES

(b) IDEALIZED H-SECTION

(c) FLANGE 1
(c) FLANGE 2

FIG. 3 STRESS DISTRIBUTION IN H-SECTION BENT ABOUT STRONG AXIS (YIELDING ONLY IN ONE FLANGE)

FIG. 4 STRESS DISTRIBUTION IN H-SECTION BENT ABOUT STRONG AXIS (BOTH FLANGES YIELDED)
FIG. 5 STRESS DISTRIBUTION IN H-SECTION BENT ABOUT WEAK AXIS
FIG. 6  CURVATURE DEFLECTION CURVE FOR H-SHAPED COLUMN
BENT ABOUT STRONG AXIS (YIELDING IN ONLY ONE FLANGE)
FIG. 7  CURVATURE DEFLECTION CURVE FOR H-SHAPED COLUMN BENT ABOUT STRONG AXIS. (YIELDING IN BOTH FLANGES)
FIG. 8 CURVATURE DEFLECTION CURVE FOR H-SHAPED COLUMN BENT ABOUT WEAK AXIS (ONLY ONE SIDE FLANGE TIP YIELDED)
FIG. 9  CURVATURE DEFLECTION CURVE FOR H-SHAPED COLUMNS
BENT ABOUT THE WEAK AXIS (BOTH SIDE FLANGE TIPS YIELDED)
FIG. 10 LOAD DEFLECTION CURVES FOR COLUMNS (H-SECTION) 
\((L/r = 33)\) BENT ABOUT THE STRONG AXIS
FIG. 11  LOAD DEFLECTION CURVES  \((L/r = 57)\)
FOR COLUMNS BENT ABOUT THE STRONG AXIS
Fig. 12 Theoretical Column Curves for Idealized Wide Flange Shape Bending about Strong Axis
FIG. 13 THEORETICAL COLUMN CURVES FOR IDEALIZED H-SHAPED COLUMNS BENT ABOUT WEAK AXIS
(a) ELASTIC RANGE

\[ \varepsilon_y > \varepsilon_1 > \varepsilon_2 \]

(b) BOUNDARY BETWEEN ELASTIC AND PLASTIC-ELASTIC RANGE

\[ \varepsilon_1 = \varepsilon_y > \varepsilon_2 \]

(c) ELASTIC-PLASTIC RANGE

\[ \varepsilon_1 > \varepsilon_y > \varepsilon_2 \]

FIG. 14 STRESS DISTRIBUTION IN BOX-SHAPED COLUMNS UNDER AXIAL LOAD
FIG. 15 CURVATURE DEFLECTION CURVE FOR BOX-SHAPED COLUMNS (IN THE ELASTIC PLASTIC RANGE)
FIG. 16  THEORETICAL COLUMN CURVES FOR SQUARE BOX-SECTION COLUMN
FIG. 17  TYPICAL STRESS-STRAIN CURVE FOR STANDARD COUPONS
(a) COLUMN FOR RESIDUAL STRESS MEASUREMENT

(b) POSITION OF HOLES DRILLING FOR WHITMORE GAGE READINGS

FIG. 18. DETAIL OF DRILLING AND SECTIONING FOR RESIDUAL STRESS MEASUREMENT
(c) CUTTING INTO L-SHAPES

(d) POSITION OF HOLES ON INSIDE FACE OF COLUMNS FOR SECOND WHITTEMORE GAGE READINGS

FIG. 18 DETAIL OF DRILLING AND SECTIONING FOR RESIDUAL STRESS MEASUREMENT (Continued)
Note: Stress plotted relative to outside face.

Compression -- inside

Tension -- -- outside

Average residual stress
Residual stress on outside
Residual stress on inside

FIG. 19 TYPICAL RESIDUAL STRESS DISTRIBUTION
Section: $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{5}{16}''$

Column C-1: $L/r = 80$
Column C-2: $L/r = 100$

**FIG. 20a**
INITIAL OUT-OF-StraIGHTNESS

**FIG. 20b**
RESIDUAL STRESS DISTRIBUTION.
Section: 4" x 4" x 3/16"
Column C-3: L/r = 67.6
Column C-4: L/r = 90

FIG. 21a INITIAL OUT-OF-StraIGHTNESS

FIG. 21b RESIDUAL STRESS DISTRIBUTION.
FIG. 22a  INITIAL OUT-OF-StraIGHTNESS

FIG. 22b  RESIDUAL STRESS DISTRIBUTION

Section: 6" x 6" x \(\frac{1}{4}\)"
Column C-5: \(L/r = 32\)
Column C-6: \(L/r = 51\)

HORIZONTAL SCALE

INCH

INCH

KSI
Section: 10" x 10" x 1/2"
Column C-7: L/r = 60
Column C-8: L/r = 90

FIG. 23a  INITIAL OUT-OF-STRAIGHTNESS

FIG. 23b  RESIDUAL STRESS DISTRIBUTION
Section: 6" x 4" x 1/4"
Column C-9: L/r = 50
Column C-10: L/r = 80

FIG. 24a  INITIAL OUT-OF-StraIGHTNESS

FIG. 24b  RESIDUAL STRESS DISTRIBUTION
FIG. 25a
LOAD-DEFLECTION CURVE

FIG. 25b
STRAIN ($10^{-3}$ in/in)

STUB COLUMN TEST RESULT
FIG. 26a  MID-HEIGHT DEFLECTION (inches)

LOAD-DEFLECTION CURVE

 Column C-3
 Column C-4

FIG. 26b  STUB COLUMN TEST RESULT
FIG. 27a  MID-HEIGHT DEFLECTION (inches)

LOAD-DEFLECTION CURVE

FIG. 27b  STUB COLUMN TEST RESULT
FIG. 28a LOAD-DEFLECTION CURVE

FIG. 28b STUB COLUMN TEST RESULT
FIG. 29a  LOAD-DEFLECTION CURVE

FIG. 29b  STUB COLUMN TEST RESULT
FIG. 30 INSTRUMENTATION OF STUB COLUMN TESTS
FIG. 31 INSTRUMENTATION OF COLUMN TEST
FIG. 32  SETUP OF STRIP SCALES AND MEASUREMENT OF LATERAL DEFLECTION
COLUMN TEST
COLUMN C5
LOAD NO. 16

FIG. 33  TYPICAL LOAD RELAXATION DIAGRAM
COLUMN NO. C1

△ FIBER AT CONVEX SIDE
○ FIBER AT CONCAVE SIDE
□ FIBER AT CENTERLINE
● AVERAGE

FIG. 34 STRAIN READING BY SR-4 GAGE
FIG. 35  COMPARISON BETWEEN THEORETICAL ANALYSIS AND TEST RESULTS
7. REFERENCES

1. Tall, L. THE STRENGTH OF WELDED BUILT-UP COLUMNS, Ph D. Dissertation, Lehigh University, May 1961

2. Wilder, T. W. and Brooks, W. A. THE EFFECT OF INITIAL CURVATURE ON THE STRENGTH OF AN INELASTIC COLUMN, NACA Technical Note 2872, January 1953

3. Huber, A. W. and Ketter, R. L. THE INFLUENCE OF RESIDUAL STRESS ON THE CARRYING CAPACITY OF ECCENTRICALLY LOADED COLUMNS, 18th Vol. of Publication

4. Nishino, F. and Tall, L. COLUMN TESTS ON LIPPED ANGLES OF COLD-FORMED STAINLESS STEEL, Nov. 1962, Fritz Laboratory Report No. 200.61.166.1


8. Column Research Council GUIDE TO DESIGN CRITERIA FOR METAL COMPRESSION MEMBERS, Column Research Council Publication, 1960


10. Gerald, G. INTRODUCTION TO STRUCTURAL STABILITY THEORY,
The author was born in Chhiang, Hunan, China, on October 31, 1938, the eldest son of Ching-Chih and Wo-Ming Yu. He entered Taiwan Provincial Cheng-Kung University in September, 1956 and was awarded a Bachelor of Civil Engineering Degree in June 1960.

In July, 1962 the author joined the staff at Fritz Engineering Laboratory, Lehigh University, as a research assistant. He has been associated with the research project on residual stress and column instability.