Welded Continuous Frames and Their Components

ROTATION CAPACITY OF BEAMS

by

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SYNOPSIS

An analysis of a three span beam is used to illustrate the effect of yield strength on rotation capacity. The analysis consists of:
(1) determination of the rotation capacity required to permit the formation of a mechanism (2) determination of the rotation capacity available (3) determination of limiting values of length to depth and unbraced length to ensure that adequate rotation capacity is available.
The limiting values of length to depth ratio resulting from the rotation capacity analysis are compared with those required to meet a working load deflection criterion.
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INTRODUCTION

Plastic design results in an efficient use of steel and a uniform factor of safety in statically indeterminate steel structures. The use of plastic design is, however, restricted at present (1965) to steels with yield stresses between 30 and 36 ksi. Consideration is now being given to the extension of plastic design methods to higher strength steels with yield stresses of up to 50 ksi. Before such steels can be used in plastically designed structures, careful consideration must be given to the requirements imposed on members and the extent to which high strength steel members meet these requirements.

The concept of plastic design is based on the ability of structures to redistribute internal forces as their components yield. If the redistribution of moments required by plastic design is to occur, the first regions to yield must be capable of deforming at or near the plastic moment, $M_p$, as the more rigid elastic portion of the structure continues to accept additional load.

The results of beam tests indicate that the ability of beams to deform at $M_p$ is terminated at the occurrence of a combination of large out-of-plane deformations of the beam between lateral supports and severe local buckling of the plate elements of the beam.\(^{(1,2,3)}\) An analysis, in which the extent of yielding and consequent reduction in stiffness of a member are considered, can be performed to determine the hinge rotations at which out-of-plane deformations begin to occur in an initially straight member.\(^{(4,5,6,7,8)}\) For the purposes of this discussion, rotation capacity shall be considered to be the inelastic rotation at which either of these out-of-
plane effects, lateral torsional buckling or local buckling, occurs.

To illustrate the effects of using high strength steels on the requirements imposed on members, an analysis of the three span beam shown in Fig. 1 has been performed. The beam is of uniform cross section over the three spans and symmetric with respect to the center line. Loads are applied at the quarter points in all three spans. The factor of proportionality between loads in the side and center spans is $\beta$. The ratio of side span to center span length is $\alpha$.

This simple structure was selected as an illustrative example because by the variation of only two parameters, $\alpha$ and $\beta$, the effects of variations in the distribution of loads and in the structural geometry may be considered. In addition, part of the structure is under moment gradient and part is under uniform moment. Local buckling occurs first in the former case and lateral torsional buckling in the latter.

The analysis consists of (1) the determination of the rotation capacities required to permit the formation of a mechanism (2) the determination of the rotation capacities that can be delivered and (3) evaluation of the effect upon these quantities of pertinent variables. The effects of relevant material properties are discussed with special emphasis given to the effect of variation of yield stress. Expressions for rotation capacity required are equated to expressions for rotation capacities available to obtain limiting values of length-to-depth and slenderness ratios for selected values of $\alpha$ and $\beta$. The length-to-depth ratio limitations imposed to ensure adequate rotation capacity are compared with those imposed to meet a working load deflection criterion.
REVIEW OF PAST WORK

A brief review of the behavior of beams loaded in the plane of the web will help to illustrate the nature of the problem under consideration. For the purposes of this discussion, a distinction is made between two types of loading. The first type of loading results in the formation of the first plastic hinges in a region of uniform moment. The second type of loading results in the formation of the first plastic hinge in a region of moment gradient.

The beam shown in Fig. 2 represents the uniform moment case. When the beam is loaded as shown, a plastified region forms between the point loads. Due to the discontinuous stress-strain relationship of steel, yielding occurs at discrete points in the region of uniform moment. This fact is characterized by the occurrence of yield lines at intervals along the region of uniform moment as sketched in Fig. 2 (b). Additional deformation results in additional yield lines, again at discrete points as shown in Fig. 2 (c). The occurrence of yielding at discrete points results in discontinuities in curvature in the region of uniform moment as is shown in Fig. 2 (d). Some portions of the region of uniform moment are at the curvature corresponding to first yield $\phi_y$ and others are at the curvature corresponding to complete yield (strain hardening) $\phi_{ST}$.($4,6$)

When the applied load is plotted as a function of the rotation occurring between the load points, the curve shown in Fig. 3 results. The solid curve represents the behavior of the beam as idealized in plastic analysis. The dashed curve shows the influence of residual stresses, the shape factor and ultimately the effect of out-of-plane
Between points (c) and (d) of Fig. 3, a large rotation occurs with little increase in load. At point (d) the beam unloads. Beyond this point additional deformations results in a reduction in load. Unloading is accompanied by large out of plane deformations of the beam and of the individual plate elements of which the beam is composed.

The rotation capacity of the critical beam, which in this case is the segment between the two loads, is the difference in rotation between points (a) and (d) of Fig. 3. The rotation capacity is the integrated effect of the portions of the curvature function in which $\phi_y$ is exceeded in the region of uniform moment. The rotation capacity is represented graphically by the summation of the areas under the projections on the curvature diagram shown in Fig. 2(d). The rotation capacity required to permit the formation of a mechanism is the rotation that occurs between points (a) and (c) of Fig. 3. The minimum requirement imposed upon a beam that is to be used in plastic design is that points (c) and (d) coincide. That is, the rotation capacity of the beam must be equal to or greater than that required to permit the formation of a mechanism.

The beam shown in Fig. 4 represents the moment gradient case. The behavior of the beam under moment gradient differs significantly from that of the beam in the uniform moment case. In the moment gradient case, the discontinuity of the stress-strain relationship results in discontinuities of the curvature function only at the boundaries of the yielded zone as shown in Fig. 4(d). This behavior is evidenced by the concentration of yield lines shown in Figs. 4(b) and (c). The strains and consequently curvature are at or in excess of the strain hardening value over the entire yielded region.
In Fig. 5 the load is plotted as a function of the rotation occurring between the end points of the yield zone $T_L$ and $T_R$ shown in Fig. 4 (c). The solid curve in Fig. 5 represents the behavior as idealized in plastic analysis. The dashed curve shows the influence of residual stresses, the shape factor, strain hardening and ultimately out of plane deformations. As can be seen in Fig. 5, the ultimate load predicted by simple plastic theory can be exceeded. This occurs because the plastic moment is exceeded due to the effects of strain hardening. As in the uniform moment case, the beam unloads at some point (d). Unloading is accompanied by large out-of-plane deformations of the beam itself and the plate elements of which the beam is composed. The rotation capacity, in this case, is the change in rotation between yielding and unloading, points (a) and (d) of Fig. 5. This angle is represented graphically by the shaded portion of the curvature function shown in Fig. 4(d). The required rotation capacity is the change in rotation which occurs between yielding and formation of a mechanism, points (a) and (c) of Fig. 5. The minimum acceptable performance is the case in which points (c) and (d) coincide. That is, unloading must not occur before the load predicted by plastic analysis is attained.

In both the uniform moment and the moment gradient cases, the rotation capacity required to permit the formation of a mechanism can be approximated by the change in angle between points $(a')$ and $(c')$ of Figs. 3 and 5. That is, the true plastic hinge angle required to permit the formation of a mechanism can be approximated by performing an elastic analysis of a beam in which all but the last plastic hinges to form are replaced by true hinges loaded by couples equal to $M_p^{(9,10,11)}$. The
rotations that occur at the hinges in the beams of Fig. 6 can be used to approximate the plastic hinge angles of the beams in Figs. 2 and 4.

In both the moment gradient and the uniform moment case, the rotation capacity is defined as the inelastic rotation at which unloading occurs. A lateral torsional buckling analysis can be performed to determine the rotations at which out of plane deformations of the beam become large. A local buckling analysis can be performed to determine the rotation at which out of plane deformations of the plate elements become large.

The rotation capacity expressions presented in Refs. 6, 7, and 8 are used. There are two expressions, one applicable to the moment gradient case, the other applicable to the uniform moment case. The expression for rotation capacity in the moment gradient case is:

\[
\theta_H = \left[ \frac{\epsilon_{ST}}{\epsilon_y} - 1 \right] \epsilon_y \frac{\theta \sqrt{d}}{t} \left[ 1.42 \frac{tb}{w} \frac{A_w}{A_f} \left( 1 + \frac{V_1}{V_2} \right) \right]
\]

(1)

The meanings of the terms are as follows:

- \( \theta_H \) is the rotation capacity in radians,
- \( \epsilon_{ST} \) is the strain hardening strain,
- \( \epsilon_y \) is the yield strain,
- \( d \) is the section depth,
- \( t \) is the flange thickness,
- \( b \) is the flange width,
- \( A_w \) and \( A_f \) are the web and flange areas and
- \( V_1 \) and \( V_2 \) are the lesser and greater shears respectively on opposite sides of the point of maximum moment. The quotient \( V_1/V_2 \) must be less than or equal to unity. It is suggested in Ref. 7 that the expression for rotation capacity for the moment gradient case be used for values of \( \rho \), shown in Fig. 7, (where \( \rho \) is the ratio of end moments on the segment to be braced) of less than or equal to 0.7. For values of \( \rho \) greater than 0.7 the uniform moment expression should be used. The expression is applicable only to "compact"
sections. In Ref. 15 compact sections are defined as sections in which

\[ \frac{b}{t} \leq 2 \left( \sqrt{\frac{\sigma_y}{\sigma_y}} \left( \frac{4}{\frac{3}{\sigma_y}} + \frac{\sigma_y}{\sigma_u} \right) \right) \]  

(2)

In this expression, \( \sigma_y \) is the yield stress, \( \sigma_u \) the ultimate stress, and

\[ G_{ST} = G \left( \frac{2}{1 + \frac{E}{4E_{ST}(1 + \nu)}} \right) \]  

(3)

\( G \) is the shear modulus, \( E \) is Young's modulus, \( E_{ST} \) is the strain hardening modulus and \( \nu \) is Poisson's ratio. The resulting values given in Ref. 15 are \( \frac{b}{t} \leq 17.1 \) for A36 steel and 13.4 for A441. In Ref. 6 it is shown that the expression is applicable when the unbraced length to slenderness ratio \( \frac{L_b}{r_y} \leq \frac{0.7 \pi}{\sqrt{\varepsilon_y}} \) in which \( L_b \) is the unbraced length and \( r_y \) is the weak axis radius of gyration. These values are \( \frac{L_b}{r_y} \leq 65 \) for A36 and \( \frac{L_b}{r_y} \leq 55 \) for A441. These values do not take advantage of any restraint available from adjacent spans. If these restrictions on bracing spacing and flange geometry are met, then the rotation defined by Eq. 1 can be reached without unloading due to local and/or lateral-torsional buckling.

The expression for rotation capacity in the uniform moment case is:

\[ \theta_H = \frac{2H_L}{d} \left[ \frac{E_{ST}}{\varepsilon_y} - 1 - \frac{\pi^2}{\frac{kL_b}{r_y}^2} - \varepsilon_y \right] \]  

(4)

\( H_L \) is the hinge length, \( \frac{kL_b}{r_y} \) is the effective unbraced length and the other terms are as defined earlier. The expression is applicable when the \( \rho \) of Fig. 7 is less than or equal to 1 and greater than 0.7. The effective length factor \( k \) is 0.54 when the bracing is applied at the ends of the region of uniform moment. That is, when the spans beyond the
braces are elastic. The value of $k$ is 0.8 when the bracing is applied inside the region of uniform moment, that is, the spans outside the braces are plastified. (6) The expression is applicable only to compact sections. The expression is applicable when

$$\frac{kL_b}{r_y} \geq \frac{\pi}{\sqrt{\epsilon_y(1 + 0.56 \frac{E}{E_{ST}})}}$$

In Ref. 8 this is termed the optimum bracing spacing since more closely spaced bracing does not increase rotation capacity (because of local buckling). The optimum support spacing for A36 steel ($\epsilon_y = 0.00122$, $E/E_{ST} = 33$) is $\frac{kL_b}{r_y} = 20.2$. The optimum support spacing for A441 steel ($\epsilon_y = 0.00172$, $E/E_{ST} = 45$) is $\frac{kL_b}{r_y} = 14.9$. For the case in which the adjacent spans offer the maximum restraint ($k = 0.54$), the optimum bracing spacing is 37.5 for A36 and 27.5 for A441 beams.
REQUIRED ROTATION CAPACITY

The rotation capacity required to permit the formation of a mechanism is determined as shown in Refs. 9 and 10. A plastic analysis is performed first to determine at which of the possible hinge locations hinges will form. Then an elastic analysis is performed to determine the location of the maximum moment. The first hinge forms at this location and thus the hinge rotation is calculated at this point. The beam considered is shown in Fig. 1. It is symmetric and twice redundant.

When a plastic analysis is performed, it is found that one of the mechanisms shown in Fig. 8 will occur. For the side span mechanism shown in Fig. 8(a) the ultimate load \( P_{us} \) is \( \frac{5M}{\beta \alpha L} \). For the center span mechanism shown in Fig. 8(b), it is found that the ultimate load \( P_{us} \) is \( \frac{8M}{P L} \).

The expression for the side span mechanism ultimate load can be equated to that of the center span mechanism to determine the relationship between \( \alpha \) and \( \beta \) required to obtain the combined mechanism shown in Fig. 8(c). The relationship is:

\[ \beta = \frac{5}{8 \alpha} \]  

(6)

If this relationship is expressed as an inequality, it can be used to determine which mechanism will occur for a given combination of \( \alpha \) and \( \beta \).

When the load required to cause the side span mechanism is set equal to or greater than the load to cause the center span mechanism, the following inequality results.

\[ \beta \leq \frac{5}{8 \alpha} \]  

(7)

When this inequality is satisfied, the center span mechanism occurs.

Equation 6 is plotted as curve A in Fig. 9. Inequality 7 shows that for
points above curve A the side span mechanism governs, for points below curve A the center span mechanism governs and on curve A the mechanisms occur together.

Next, the order of formation of the hinges must be determined for each mechanism. An elastic analysis is performed to determine expressions for moment at each of the possible points of maximum moment. The resulting expressions are:

\[ M_B = PL \left( \frac{\beta a}{4} - \frac{3}{128} \left( 1 + \beta a \right) \right) \]  \hfill (8)

\[ M_C = PL \left( \frac{\beta a}{4} - \frac{9}{128} \left( 1 + \beta a \right) \right) \]  \hfill (9)

\[ M_D = \frac{3PL}{32} \left( \frac{1 + \beta a}{a + \frac{1}{2}} \right)^2 \]  \hfill (10)

\[ M_E = PL \left( \frac{1}{4} - \frac{3}{32} \left( 1 + \beta a \right) \right) \]  \hfill (11)

The subscripts indicate the location of the point at which the moment is calculated. The locations of the points are shown in Fig. 1.

An inequality is then written which states that the magnitude of the moment at point B is equal to or greater than the magnitude of the moment at point D. This results in an inequality in \( \alpha \) and \( \beta \) which can be used to determine whether the first hinge forms at point B or point D when the side span mechanism occurs. The inequality is:

\[ \beta \geq \frac{90}{96 \alpha - 26 \alpha^2} \]  \hfill (12)

When this inequality is satisfied, the first hinge forms at point B in the
side span. Equation 12 is shown as curve B in Fig. 9. Combinations of \( \alpha \) and \( \beta \) falling in region II, above curve B, cause the first hinge to form at point B of Fig. 1. Combinations of \( \alpha \) and \( \beta \) falling in region I, between curves A and B, cause the first hinge to form at point D of Fig. 1. On curve B, between regions I and II, the hinges form simultaneously.

Another inequality is written which states that the magnitude of the moment at point D is greater than that at point E. The resulting inequality in \( \alpha \) and \( \beta \) can be used to determine whether the first hinge forms at point D or E when the center span mechanism governs. The inequality is:

\[
\beta \geq \frac{8\alpha - 6}{18\alpha^2}
\]  

When this inequality is satisfied, the first hinge in the center span mechanism occurs at point D. Equation 13 is shown as curve C in Fig. 9. Combinations of \( \alpha \) and \( \beta \) falling in region III, between curves C and A, of Fig. 9 cause the first hinge of the center span mechanism to occur at point D. Combinations of \( \alpha \) and \( \beta \) falling in region IV beneath curve C of Fig. 9 cause the first plastic hinge to occur over the region of uniform moment at the center of the center span.

The inequalities shown in Eqs. 7, 12, 13 or Fig. 9 can be used to determine which mechanism will occur and which hinge forms first within the mechanism for any combination of \( \alpha \) and \( \beta \). When the location and order of formation of the hinges are known, the rotation capacity required to permit the formation of a mechanism can be determined. The rotation that must occur at a hinge is evaluated by performing an elastic analysis on the statically determinate beam that exists as the last hinge forms. (9,10,11)
The beam shown in Fig. 10(a) is analyzed to determine the rotation capacity required to permit the side span mechanism to occur when the first hinge forms over the support. The resulting expression, applicable for combinations of $\alpha$ and $\beta$ which fall in region I of Fig. 9, is:

$$\theta_{DI} = \frac{2\varepsilon yfL}{d} \left[ \frac{13\alpha}{96} + \frac{15}{32}\beta - \frac{1}{2} \right]$$

Equation 14 is plotted in dimensionless form in Fig. 11.

The beam shown in Fig. 10(b) is analyzed to determine the rotation capacity required to permit the side span mechanism to occur when the first hinge forms under the load at point B. This hinge forms first for combinations of $\alpha$ and $\beta$ falling in region II of Fig. 9. The expression for required rotation capacity is:

$$\theta_B = \frac{2\varepsilon yfL}{d} \left[ \frac{26\alpha}{48} + \frac{15}{8}\beta - 2 \right]$$

Equation 15 is plotted in dimensionless form in Fig. 12.

The beam shown in Fig. 10(a) is analyzed to determine the rotation capacity required to permit the center span mechanism to occur when the first hinge forms over the support. This hinge forms first for combinations of $\alpha$ and $\beta$ falling in region III of Fig. 9. The expression for required rotation capacity is:

$$\theta_{DIII} = \frac{2\varepsilon yfL}{d} \left[ \frac{3\beta^2}{4} - \frac{\alpha}{3} + \frac{1}{4} \right]$$

Equation 16 is plotted in dimensionless form in Fig. 13.

The beam shown in Fig. 10(c) is analyzed to determine the
rotation capacity required to permit the center span mechanism to occur when the first plastic hinge forms over the region of uniform moment. The inelastic rotation occurring between the centerline of the beam and the load point is calculated. This hinge forms first for combinations of $\alpha$ and $\beta$ falling in region IV of Fig. 9. The expression for required rotation capacity is:

$$\theta_E = \frac{2\varepsilon_y f_L}{d} \left[ \frac{2\beta\alpha^2}{3} - \frac{\alpha}{4} + \frac{1}{4} \right] (17)$$

Equation 17 is plotted in dimensionless form in Fig. 14.

Equations 14 through 17 were placed in the form shown by replacing the term $\frac{M_p L}{EI}$ with the term $\frac{2f\varepsilon_y L}{d}$. The rotation capacity required to permit the formation of a mechanism is directly proportional to the yield strain and to the length-to-depth ratio of the beam considered. Thus, for a given beam, an A441 member requires 50/36 times as much rotation capacity as does an A36 member.

High strength steels yield greater plastic moments for a given cross section. Therefore, an A441 member selected to carry a given load is lighter than an A36 member selected to carry the same load. If the reduction in weight is accomplished by selection of a shallower high strength member, the difference in rotation capacity requirement is even greater than the difference in yield stress would indicate.
The rotation capacities of the beams in the structure of Fig. 1 are evaluated by means of the expressions developed in Refs. 6, 7, and 8. Equations 7, 12, and 13 or Fig. 9 are used to determine which hinge forms first. Equation 1 can be used to determine the rotation capacity if the first hinge forms in a region of moment gradient. Equation 4 can be used if the first hinge forms in the region of uniform moment.

**Beams Under Moment Gradient**

The case in which the first hinge forms in a region of moment gradient is discussed first. Equation (1), the rotation capacity of a beam under moment gradient, is a function of material properties, a cross sectional property, and the shears on opposite sides of the point of maximum moment. The effects of these variables are considered separately to permit a comparison of the relative magnitude of the influence of each on the rotation capacity.

When the last plastic hinge begins to form in the structure of Fig. 1, there are as many plastic hinges present as there were redundants originally. Therefore, the equations of static equilibrium may be used to evaluate the moments and shears in the remainder of the beam. For example, Fig. 9 shows that for a beam with equal spans and equal loads the side span mechanism governs and the first hinge forms at the supports. Therefore, when determining the rotation capacity available, the ratio of shears at the inner support must be known as the beam reaches its ultimate load. The ultimate load for this case is \( \frac{5M_D}{L} \). The plastic moment is known to occur at points B and D. By equating the sum of the forces normal to the
beam and the moments about one support equal to zero for the center span, it is determined that the shear at the support in the center span is \( \frac{5M}{L} \).

By equating the sum of moments about the exterior support equal to zero, it is determined that the shear at \( D \) in the outer span is \( \frac{6MP}{L} \). The quotient \( \frac{V_1}{V_2} \) is equal to \( \frac{5}{6} \) in this case.

A calculation similar to the foregoing can be performed in general terms to permit an evaluation of the effects of \( \alpha \) and \( \beta \) on the quotient \( \frac{V_1}{V_2} \). The equations of equilibrium are used to determine the shears \( V_L \) and \( V_R \) on the left and right of the point of maximum moment. \( V_L \) and \( V_R \) are related by means of an inequality of the form \( \frac{V_L}{V_R} \leq 1 \). The resulting inequality in \( \alpha \) and \( \beta \) can be used to determine which shear is larger for a given combination of \( \alpha \) and \( \beta \). \( \frac{V_1}{V_2} \) is then determined as a function of \( \alpha \) and \( \beta \). At the same time, \( \rho \) is checked to determine that the moment gradient expression for rotation capacity is applicable.

When the above calculation is performed, the following results are obtained. For case I, the side span mechanism with the first hinge at the support, the shear in the side span at support \( D \) is greater when

\[
\beta \leq \frac{5}{6} \quad \text{and:} \quad \left[ 1 + \frac{V_1}{V_2} \right] = \left[ 1 + \frac{6\beta}{5} \right] \tag{18}
\]

For \( \beta \geq \frac{5}{6} \) the shear in the center span is greater and:

\[
\left[ 1 + \frac{V_1}{V_2} \right] = \left[ 1 + \frac{5}{6\beta} \right] \tag{19}
\]

Equations 18 and 19 are plotted as the straight line and hyperbolic portion of the curve shown in Fig. 15. The quotient \( \frac{V_1}{V_2} \), and thus the rotation capacity, is independent of the relative length of spans \( \alpha \) and, as can be seen in Fig. 15, is sensitive to changes in the relative magnitude of
For case II, the side span mechanism with the first hinge at point B, the shear between points A and B is always four times greater than between points B and C. Therefore, for all $\alpha$ and $\beta$:

$$\left[1 + \frac{V_1}{V_2}\right] = \frac{5}{4} \quad (20)$$

For this case, the quotient $\frac{V_1}{V_2}$ and thus the rotation capacity is independent of $\alpha$ and $\beta$. This fact indicates that when the first hinge forms within the span, the rotation capacity is independent of conditions in the adjacent span.

For case III, the center span mechanism with the first hinge at the support, the shear at D is greatest in the center span for

$\beta \geq \left[1 - \frac{1}{8\alpha}\right]$ and:

$$\left[1 + \frac{V_1}{V_2}\right] = \left[1 + \beta + \frac{1}{8\alpha}\right] \quad (21)$$

For $\beta \geq \left[1 - \frac{1}{8\alpha}\right]$ the shear in the side span is greater and:

$$\left[1 + \frac{V_1}{V_2}\right] = \left[1 + \frac{8}{8\beta + \frac{1}{\alpha}}\right] \quad (22)$$

Equations 21 and 22 are plotted in Fig. 16. The curves are terminated at combinations of $\alpha$ and $\beta$ for which mechanism III does not occur. The quotient $\frac{V_1}{V_2}$ is a function of both $\alpha$ and $\beta$ in this case.

The maximum rotation capacity can only be twice as great as the minimum for a given cross section and steel since $\frac{V_1}{V_2}$ can vary only between 0 and 1.
The variation in the effect of cross sectional properties is roughly of the same order of magnitude. The cross sectional property \( \frac{tb}{wd} \sqrt{\frac{A_w}{A_F}} \) has been calculated for the first and last section in each of the groups of sections listed by the A.I.S.C. as beam sections. These values are shown under the heading K in Table 1. The minimum value is 0.48, the maximum 0.96 and a typical value is 0.7.

The rotation capacity of a beam under moment gradient is directly proportional to the yield strain \( \varepsilon_y \) and the ratio of strain hardening strain to yield strain \( \frac{\varepsilon_{ST}}{\varepsilon_y} \). In Reference 16 a value of 12 is suggested for \( \frac{\varepsilon_{ST}}{\varepsilon_y} \) for A36 steel. In Reference 17 a value of 11.7 is presented for A441 steel. In the computations which follow 11 is used as a conservatively representative value of \( \frac{\varepsilon_{ST}}{\varepsilon_y} \) for both steels. When this value of \( \frac{\varepsilon_{ST}}{\varepsilon_y} \) is used, the rotation capacity for compact sections is affected by using a high strength steel in the same way as the rotation capacity required to permit the formation of a mechanism.

In the beam used as an example, (Fig. 1), \( \alpha \) and \( \beta \) do not influence the rotation capacity when the first hinge forms in the region of uniform moment. In this case the rotation capacity is a function of the unbraced slenderness ratio \( \frac{kL_b}{r_y} \) and material properties \( \varepsilon_y, \frac{\varepsilon_{ST}}{\varepsilon_y} \) and \( \frac{E}{E_{ST}} \). The ratio of Young's modulus to the strain hardening modulus \( \frac{E}{E_{ST}} \) is somewhat different for A36 and A441 steel. In Ref. 6 values of \( \frac{E}{E_{ST}} \) are reported as 33 for A36 steel and 45 for A441. A consideration of Eq. 4 shows that the rotation capacity of high strength steel beams is less for beams under uniform moment because \( \Theta_H \) is inversely proportional to \( \frac{E}{E_{ST}} \).

Equations 18 through 22 can be used in Eq. 1 to determine...
the rotation capacity available when the first hinge forms in a region of moment gradient. The adequacy of the beam, rather than the rotation capacity of the beam, is of interest. The expression for rotation capacity required is a function of $\frac{L}{d}$ (Eqs. 14 - 17). The expression for rotation capacity contains a cross sectional property but is independent of $\frac{L}{d}$.

Therefore the expressions for rotation capacity can be related to the expressions for rotation capacity required to determine limiting values of $\frac{L}{d}$ in terms of the cross sectional properties, $\frac{\epsilon_{ST}}{\epsilon_y}$, $\alpha$ and $\beta$ to ensure that the beam has sufficient rotation capacity.

The expression for rotation capacity is related to that for rotation capacity required by means of the inequality $\theta_{\text{poss}} \geq \theta_{\text{req'd}}$. There are two expressions for $\frac{L}{d}$ for cases I and III since there are two expressions for $\frac{V_1}{V_2}$ in these cases.

For case I, side span mechanism with first hinge at the support, the maximum value of $\frac{L}{d}$ is determined for $\frac{5}{8} \alpha \leq \beta \leq \frac{5}{6}$ as:

\[
\frac{L}{d} \leq \frac{1.42 \left[ \frac{\epsilon_{ST}}{\epsilon_y} - 1 \right]}{f \left[ \frac{13 \alpha}{96} + \frac{15}{32} \alpha \beta - \frac{1}{2} \right]} K \left[ 1 + \frac{6}{5} \beta \right]
\]  
(23)

and for $\beta \geq \frac{5}{6}$ as:

\[
\frac{L}{d} \leq \frac{1.42 \left[ \frac{\epsilon_{ST}}{\epsilon_y} - 1 \right]}{f \left[ \frac{13 \alpha}{96} + \frac{15}{32} \alpha \beta - \frac{1}{2} \right]} K \left[ 1 + \frac{5}{6} \beta \right]
\]  
(24)

in which $K = \frac{tb}{wd} \sqrt{\frac{A_y}{A_F}}$.  
(25)
For case II, the side span mechanism with first hinge at B, the maximum value of \( \frac{L}{d} \) is determined as:

\[
\frac{L}{d} \leq \frac{1.42 \left( \frac{\varepsilon_{ST}}{\varepsilon_y} - 1 \right) K \left[ \frac{5}{4} \right]}{f \left[ \frac{26\alpha + 15}{48} + \frac{15}{8\beta\alpha} - 2 \right]}
\]  

(26)

For case III, the center span mechanism with the first hinge at the support, the maximum value of \( \frac{L}{d} \) is for \( \beta \leq 1 - \frac{1}{8\alpha} \):

\[
\frac{L}{d} \leq \frac{\left( \frac{\varepsilon_{ST}}{\varepsilon_y} - 1 \right) K \left[ 1 + \beta + \frac{1}{8\alpha} \right]}{f \left[ \frac{3\beta\alpha^2}{4} - \frac{\alpha}{3} + \frac{1}{4} \right]}
\]  

(27)

and for \( \beta \geq 1 - \frac{1}{8\alpha} \) is:

\[
\frac{L}{d} \leq \frac{\left( \frac{\varepsilon_{ST}}{\varepsilon_y} - 1 \right) K \left[ \frac{8}{1 + 8\beta + \frac{1}{\alpha}} \right]}{f \left[ \frac{3\beta\alpha^2}{4} - \frac{\alpha}{3} + \frac{1}{4} \right]}
\]  

(28)

The quotient \( \frac{K}{f} \) has been calculated for the first and last of each group of wide-flange shapes listed as beams in the A.I.S.C. Steel Construction Manual. These values are shown in Table 1. The maximum and minimum values for compact sections are 0.42 and 0.86. A typical value is 0.64.

Both the rotation capacity required to permit the formation of a mechanism and the rotation capacity are directly proportional to \( \varepsilon_y \). Therefore, when the two are equated to obtain limiting values of \( \frac{L}{d} \), the resulting expression does not contain \( \varepsilon_y \). The only material property appearing in the \( \frac{L}{d} \) limitation is the quotient \( \frac{\varepsilon_{ST}}{\varepsilon_y} \). This indicates that
for a given section in a combination of geometry and loading that has been found to result in adequate rotation capacity for A36 steel, rotation capacity should be adequate if a higher strength steel with the same \( \frac{\varepsilon_{ST}}{\varepsilon_y} \) is used.

For \( \frac{\varepsilon_{ST}}{\varepsilon_y} \) equal to 11, limiting values of \( \frac{L}{d} \) have been determined for the maximum, minimum and typical value of \( \frac{k}{f} \) shown in Table 1. The resulting limiting values of \( \frac{L}{d} \) are shown in Table 2 for selected values of \( \alpha \) and \( \beta \). It can be seen that the combinations of \( \alpha \) and \( \beta \) for which the \( \frac{L}{d} \) limitation is severe fall in region II of Fig. 9. That is, the rotation capacity is apt to be inadequate only when the first hinge forms at point B. A consideration of Fig. 12 and Eq. 20 explains this. As can be seen in Fig. 12, the rotation capacity required increases with \( \beta \). As can be seen in Eq. 20, the rotation capacity is independent of \( \alpha \) and \( \beta \) for this case. Therefore, with increasing \( \beta \) the maximum allowable \( \frac{L}{d} \) is reduced.

Beams Under Uniform Moment

Both the rotation capacity required and the rotation capacity are directly proportional to \( \frac{L}{d} \) when hinge forms in the region of uniform moment. Therefore, when the two are equated, no limitation is imposed on the length to depth ratio. An expression is obtained which relates the slenderness ratio \( kL_b/r_y \) to a function of \( \alpha \), \( \beta \), and \( \frac{\varepsilon_{ST}}{\varepsilon_y} \). The expression is

\[
kL_b \leq \frac{r}{\pi} \left[ \frac{\frac{\varepsilon_{ST}}{\varepsilon_y}}{1} \right]^{2} \left[ \frac{\frac{\varepsilon_{ST}}{\varepsilon_y}}{1} \right] \varepsilon_y + 1.4 \frac{E}{\varepsilon_{ST}} f \varepsilon_y \left[ \frac{3 \beta \frac{a}{2}}{4} - \frac{a}{3} + \frac{1}{4} \right]
\]

As can be seen in Fig. 9, the first hinge forms in the region of uniform
moment only for small values of $\beta$. As can be seen in Fig. 14 the rotation capacity required is greatest when $\beta$ is equal to zero. Equation 29 is plotted for $\beta$ equal to zero in Fig. 17 for A36 and A441 steels. The unbraced slenderness ratio limitation is inversely proportional to $\varepsilon_y$ and contains the quotient $\frac{E}{E_{ST}}$ in the denominator. Therefore, bracing must be much more closely spaced to permit the formation of a mechanism when high strength steel is used. The value $\frac{kL_b}{r_y}$ resulting from equation 29 is subject to the same limitation as is the $\frac{kL_b}{r_y}$ in equation 4. That is, the expression is applicable only when the optimum bracing spacing is exceeded.

Working Load Deflection Limitation

Working load deflections must be restricted in some designs. A maximum permissible value of $\frac{L}{d}$ can be determined to ensure that a deflection requirement is met. This has been done for the beam analyzed as an example. The calculation was performed for a load factor of 1.70 and a maximum permissible working load deflection of $\frac{1}{360}$ of the span length. For a load factor of 1.70 the beam was found to be elastic at working loads for all $\beta$ values and for all $\alpha \leq 6$. The value of $\alpha$ can be expected to be less than 3 in general. Therefore the deflection calculation is an elastic analysis with the load equal to $\frac{P_{ult}}{1.70}$. The results of the deflection analysis are plotted in Fig. 18 and tabulated for A7 and A441 steels for the values of $\alpha$ and $\beta$ considered in the rotation capacity analysis in Table 3. A comparison of the limiting values of $\frac{L}{d}$ shown in Tables 2 and 3 indicates that for values of $\alpha$ and $\beta$ between 0.5 and 3 the deflection limitation is much more stringent than is the rotation capacity limitation. The deflection limitation imposed is one that is usually used for live load alone. If it is assumed that live load is only half the total load, the deflection ...
limitation is less restrictive. The allowable $\frac{L}{d}$ based on a consideration of working load deflections is lower for higher strength steels. For example, an A441 member can have an $\frac{L}{d}$ of only $\frac{36}{50}$ times that permitted for an A36 member. Therefore, deflection limitations are much more stringent for high strength steel members when an arbitrarily selected value is used for permissible deflections.
SUMMARY

A three span beam has been analyzed to determine the requirements that must be satisfied to ensure that the beam exhibits sufficient rotation capacity to permit the formation of a mechanism. The influence of material properties, especially the yield stress, has been considered to permit some conclusions to be drawn concerning the use of high strength steel beams in plastic design. The beam was first analyzed to determine the rotation capacity required to permit the formation of a mechanism and then analyzed to determine its rotation capacity. Algebraic expressions for required rotation capacity and available rotation capacity resulting from these analyses were equated to obtain expressions which can be used to determine whether a given cross section, made of a given steel, will exhibit adequate rotation capacity when used in a specified structure.

When the first hinge to form occurs in a region of moment gradient, a maximum length to depth ratio of the center span can be specified in terms of a cross sectional property, $K_f$, a material property $\frac{\varepsilon_{ST}}{\varepsilon_y}$, and the relative lengths $\alpha$ and magnitudes of loads $\beta$ of the center and side span. The $\frac{L}{d}$ limitation resulting from this analysis are shown in Table 2 for selected values of $\alpha$ and $\beta$ for a high, low, and typical value of the cross section property $K_f$. A value of 11, typical of A36 and A441 steels, was used for the material property $\frac{\varepsilon_{ST}}{\varepsilon_y}$. This is the only material property that enters the $\frac{L}{d}$ limitations for compact sections. Therefore, the results of the analysis can be used conservatively for steels with higher values of $\frac{\varepsilon_{ST}}{\varepsilon_y}$. The $\frac{L}{d}$ limitation is not stringent in most cases. As can be seen in Table 2, the $\frac{L}{d}$ limitation becomes less than the normally encountered value of approximately 20 only for relatively high side span loads.
The beam was found to be elastic at working loads, therefore an elastic analysis was performed to determine working load deflections. A limitation on $L/d$ was determined to ensure that working load deflections due to dead and live load were less than $\frac{L}{360}$ the span length. The results of the deflection analysis are plotted in Fig. 18 and tabulated in Table 3 for the values of $a$ and $\beta$ shown in the rotation capacity analysis. The $L/d$ limitation imposed to meet the working load deflection requirement is more stringent than that imposed to ensure adequate rotation capacity for normally encountered values of $\beta$. Since the $L/d$ limitation imposed to meet a working working load deflection requirement is inversely proportional to the yield strain, the disparity between the two limits is even greater for high strength steels.

When the first hinge to form occurs in a region of uniform moment, a maximum unbraced slenderness ratio $\frac{kLb}{r_y}$ can be specified as a function of $e_y$, $\frac{e_y}{E_T}$, $f$, and a function of $\alpha$ and $\beta$. The resulting limitations on $\frac{kLb}{r_y}$ are plotted for $\beta = 0$ for A36 and A441 steels in Fig. 17. For the range of $\alpha$ considered, $\frac{kLb}{r_y}$ was always equal or greater than the optimum value. That is, no case was encountered in which adequate rotation capacity could not be obtained by the use of properly spaced beacing.

The results of the analysis indicate that compact properly braced A441 sections can be expected to have sufficient rotation capacity in structures in which the same sections have been found to have adequate rotation capacity when A36 steel was used. Working load deflection limitations present more difficulties when using high strength steels, but rotation capacity should not if properly braced compact sections are used.
ACKNOWLEDGEMENTS

This study is part of a general investigation "Plastic Design in High Strength Steel" currently being carried out at Fritz Engineering Laboratory, Department of Civil Engineering, Lehigh University. Professor W. J. Eney is Head of the Civil Engineering Department and Professor L. S. Beedle is Director of the Laboratory. The investigation is sponsored jointly by the Welding Research Council, and the Department of the Navy with funds furnished by the American Institute of Steel Construction, the American Iron and Steel Institute, Lehigh University Institute of Research, the Bureau of Ships and the Bureau of Yards and Docks. The Column Research Council acts in an advisory capacity.

The author wishes to express his appreciation to Professor Theodore V. Galambos under whose direction the work was performed and whose many suggestions have been most helpful. The author wishes to express his appreciation to Peter F. Adams whose comments on the original manuscript were found to be very useful. Thanks are due to Miss Nancy Turner who typed the report with care and to Herb Izquierdo who prepared the drawings.
**NOMENCLATURE**

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$P$ magnitude of point load

$P_{uc}$ ultimate load when center span mechanism occurs

$P_{us}$ ultimate load when side span mechanism occurs

$r_y$ weak axis radius of gyration

$t$ flange thickness

$V_1$ lesser shear at point of maximum moment

$V_2$ greater shear at point of maximum moment

$w$ web thickness

$\alpha$ side span length/center span length

$\beta$ side span load/center span load

$\epsilon_y$ yield strain

$\epsilon_{ST}$ strain hardening strain

$\theta_{B,C,E}$ required rotation capacity at $B,D$ or $E$

$\theta_H$ rotation capacity

$\theta_{HF}$ total rotation at plastic hinge

$\theta_{HL}$ angle at hinge assumed at left end of region of uniform moment

$\theta_{HR}$ angle at hinge assumed at right end of region of uniform moment

$\theta_{HY}$ elastic portion of total rotation at a plastic hinge
\( \nu \)  Poisson's ratio

\( \rho \)  ratio of moments at adjacent parts in moment diagram

\( \sigma_{\text{ULT}} \)  ultimate stress

\( \sigma_y \)  yield stress

\( \phi_{\text{ST}} \)  strain hardening curvature

\( \phi_y \)  yield curvature
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Table 1 - continued
\[ K_f = 0.42, 0.64, 0.86 \] respectively for first, second and last value shown in each group.

Table 2 MAXIMUM \( \frac{L}{d} \) WHEN ROTATION CAPACITY GOVERNS

(Beams under moment gradient)
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* A7 Steel
* A441 Steel

Table 3 MAXIMUM $\frac{L}{d}$ WHEN DEFLECTIONS GOVERN
Fig. 1 Three Span Beam
Fig. 2 Beam With Portion Under Uniform Moment
Simple Plastic Theory

Effects of Residual Stresses
Shape Factor, and Strain Hardening

\[ \frac{\theta_B d}{2\varepsilon_y L_f} \]

Fig. 3 Load vs. Joint Rotation
Fig. 4  Beam Under Moment Gradient
PLM
Put. from Plastic Analysis

Fig. 5  Load vs. Joint Rotation
Fig. 6    Idealization of Hinge Angles
Fig. 7  Moment Diagram
(a) Side Span Mechanism

(b) Center Span Mechanism

(c) Combined Mechanism

Fig. 8 Possible Mechanisms
Fig. 9  Location and Order of Occurrence of Hinges
Fig. 10  Hinge Angles
Fig. 11  Inelastic Rotation at D (Case I)
Fig. 12 Inelastic Rotation at B (Case II)
Fig. 13  Inelastic Rotation at D (Case III)
Fig. 14  Inelastic Rotation Between E and \( \xi \) (Case IV)
Fig. 15 \[ 1 + \frac{V_1}{V_2} \] for Case I
Fig. 16  \( 1 + \frac{V_1}{V_2} \) for Case III
Fig. 17 Unbraced Lengths when $\beta = 0$
Fig. 18  Deflection Limitation on $\frac{2 \epsilon_y L}{d}$
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