Plastic Design in High Strength Steel

EXPERIMENTAL AND ANALYTICAL BEHAVIOR OF A
HYBRID FRAME

by

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A test of a full scale, single story, single bay frame was performed as part of a program to investigate the behavior of multi-story frames. The test frame consisted of two A441 columns rigidly connected to an A36 beam and was tested under combined vertical and horizontal loading. The frame was so designed that the effect of the secondary moments produced by axial forces acting through sway displacements could be observed experimentally. The test also furnished important information concerning the behavior of the high strength steel columns.

The experimental results are compared with predicted values obtained from an elastic-plastic analysis. The prediction takes into account the effects of the secondary moments in the frame, the axial loads in the columns, and the extra strength provided by the beam-to-column connections. In addition, in predicting the post-mechanism behavior, the influence of strain hardening on the strength of the frame is included in a rational, yet simple manner. The theory closely predicted the behavior of the test frame. It is concluded that this type of analysis can be used to predict the behavior of frames in which the effect of axial load is significant.
1. INTRODUCTION

Extensive research has been conducted in recent years to investigate the behavior of multi-story frames and their component parts. Much of this work has been aimed at the development of a practical design procedure for multi-story frames by applying the principles of plastic design. One possible approach to the development of such a design procedure would be to perform laboratory experiments on full size frames and to develop rational design approximations based on the observed behavior. This approach is impractical for highly complex frames. A second approach would be to examine the behavior of frames through a theoretical analysis and then formulate design approximations based on the analytical results. This approach is prohibitive if all the factors affecting the behavior of multi-story frames are to be considered. Several authors have attempted such analyses, but the capacities of modern electronic computers are often exceeded when even a modest frame is analyzed.\textsuperscript{1,2,3} Some of the significant factors are: the spread of yielded zones, the overturning moment due to the gravity loads, and the effect of strain hardening.

The development of a rational design procedure should be based on both theoretical and experimental evidence. Therefore, the approach chosen to obtain some of the needed evidence was to test a simple frame in which secondary effects similar to those found in multi-story frames are significant. The test conditions for a simple frame can be accurately controlled and such a frame can be rigorously analyzed with relative ease.
In addition to safety, the cost of a structure is a prime consideration. In recent years high strength steels have been developed which have lower price to strength ratios than the structural carbon steels. High strength steel members may prove to be economical in multi-story frames, especially when used as columns of the lower stories. Experimental evidence of the behavior of high strength steel members under these conditions is needed. In addition, conventional methods of analysis must be tested to see whether they satisfactorily predict the behavior of high strength steel members.

1.1 OBJECTIVES

The objectives of the investigation discussed in this report are threefold. First, to test a simple frame which exhibits behavior similar to that of a multi-story frame, but which can be analyzed rigorously and tested under closely controlled conditions. Second, to establish approximations to account for the secondary factors which influence the behavior of the frame. In particular, the test program was designed to investigate the effects of axial loads on an unbraced structure, and the influence of strain hardening. Third, to examine the behavior of the high strength steel columns used with the structural carbon steel beam under sway conditions.

1.2 REVIEW OF PREVIOUS EXPERIMENTS

The experimental behavior of inelastic frames and members has been reasonably well documented in the literature. Several tests have been performed on full size frames, including the test of a two-story frame by Baker and Charlton. These tests, however, either
had sway prevented or were loaded so that the axial loads had no significant effect on the response of the frame. Therefore, the results of these tests cannot be applied directly to multi-story frames. Model tests of multi-story frames have been conducted which give only qualitative indications of the behavior of actual frames. The results of these studies may be correlated with test results of full size frames in the future. The experiments referred to above were conducted on, or modeled after, structural carbon steel frames. There is no experimental evidence related to the use of high strength steel members in multi-story frames.

1.3 SCOPE OF REPORT

This report will present the results of an experimental and analytical investigation of a full scale, single story, single bay portal frame subjected to combined horizontal and vertical loads. The details of the test setup, the material properties, and the instrumentation will be presented. The loading condition and the procedure followed during testing will also be discussed. The methods of analysis commonly used to predict the behavior of frames will be reviewed briefly. This description will be used as background for the development of the modifications used in the final analysis of the test frame. These modifications include the consideration of the shift in hinge locations and the effect of strain hardening on the response. Strain hardening has been the subject of many previous studies, but has received a somewhat unique treatment here. Finally, the results of the test will be presented and compared with the theoretical analysis.
2. DESCRIPTION OF TEST

2.1 TEST FRAME AND SETUP

The frame tested was a one bay, single story unbraced portal frame with fixed bases as shown in Fig. 1. The beam was a 10 I 25.4 structural shape of ASTM A36 steel and the columns were 5 W 18.5 structural shapes of ASTM A441 steel. The frame height, h, was 8'-9" measured from the base to the centerline of the beam and the center-to-center span length, L, was approximately 15'. The frame was designed so that the geometry and relative stiffness would be typical of the lower stories of multi-story frames. The frame loading was designed to avoid a beam mechanism under vertical load alone and to provide a combined mechanism at failure which would exhibit measurable instability effects.

The corner connections were designed to transfer ultimate moment, shear, and axial loads. Doubler plate stiffeners were chosen because of the confined space available. Figure 2 shows a detail of the beam-to-column connection. The columns were welded to 2-1/2" thick base plates which were prestressed to the test bed by two 3" threaded studs. This base detail can be seen in Fig. 3 which also shows the gages used for measuring rotations at the bases.

The possibility of premature local buckling was minimized by choosing beam and column sections with stocky flange plates. Out of plane movements were prevented by placing lateral braces at appropriate points on the frame. Figure 4 shows the location of the lateral braces.
These braces were designed to offer no restraint to in-plane deflections and can be seen acting during the test in Fig. 5.24

2.2 MATERIAL PROPERTIES

Tension tests were performed on specimens cut from the same lengths (thus the same heat) as the sections used to fabricate the test frame. These tests were performed to determine the stress-strain characteristics of the A36 and A441 steels. Typical residual stress patterns were obtained for the beam section which was mill-straightened by rotarizing, and for the column section which was straightened by gagging.25 Results of the tensile tests and the residual stress measurements are given in Table 1. In Table 1 $\sigma_y$ represents the yield stress, $\varepsilon_y$ is the yield strain, $\varepsilon_{st}$ is the strain hardening strain, $E_{st}$ is the strain hardening modulus and $\sigma_{ult}$ is the ultimate stress.

Cross-section measurements were made with micrometer and vernier calipers. The readings varied slightly along the length of the sections, however, the difference between the measured values and those found in the AISC Manual26 were negligible and the latter values were used in all computations. Two beam tests were performed which substantiated the values of moment capacity computed from the results of the tensile tests. Table 2 contains a summary of the member properties used in the theoretical analysis. The properties shown are $Z$, the plastic section modulus, $M_p$, the plastic moment capacity, $M_{pc}$, the plastic moment capacity of the column reduced for the effect of axial load, $r_x$ and $r_y$, the radii of gyration about the strong and weak axes. The cross-section dimensions are given as $b$, the width of the flange, $d$, the depth of the section, $t$, the thickness of the flange, and $w$, the thickness of the web.
2.3 LOADING ARRANGEMENT

For the purpose of testing, the loads were divided into three groups. All three sets of loads were applied by hydraulic jacks, controlled by a pump console, shown in Fig. 6, which allowed the load in each system to be adjusted independently. Figure 7 is an overall view of the test setup showing the test frame in white. The components of the loading system can be seen schematically in Fig. 8.

The vertical loads, W and 3W (Figure 1) were applied by three tension jacks, C in Fig. 8, each of which was attached to a gravity load simulator, B, placed beneath the frame and parallel to its plane of action. The gravity load simulators, B, were designed to apply vertical loads to the test frame and maintain the loads in a vertical line of action through large lateral sway movements, while offering no restraint to this lateral motion. The jacks were attached to the simulators at the bottom and to the spreader beams, G, at the top. Figure 9 shows the simulators and spreader beams. The middle spreader beam (in the plane of the test frame) was attached to the beam of the test frame, A, at two points, 40.5" on either side of the centerline by cold-rolled steel bars with pinned connections. Thus the vertical beam loads, W, were applied approximately at the quarter points (see Fig. 1). The outside spreader beams, forming the second loading system, were attached to a loading beam, E, in position atop the test frame column tops by loading arms, D. These forces simulate the loads from the stories above. The loading beam transfers these loads, 3W, to the columns through rollers at each end. These roller supports bear on 1/2" plates welded to the column tops. Figure 10 shows this part of the loading system.
2.4 LOADING SEQUENCE

The test frame was loaded non-proportionally. First, full vertical load was applied at the beam and column load points and maintained. Then the lateral load was applied in increments. Ostapenko, Vogel, and others have discussed the implications of different loading sequence on a given frame. Although the load and deformation at the failure mechanism are independent of the loading sequence, the overall response of the frame does depend on this sequence. It has not been said that a particular load sequence (for example, full vertical then gradual application of horizontal load) always yields a higher or lower maximum load, but only that different ultimate loads can be obtained by different load sequences. It is therefore important to either investigate all possible sequences or base the design on the most probable one.

The non-proportional loading sequence (full vertical load then gradual application of horizontal load) was chosen for this frame, believing it to be rational for multi-story frames. During the test, full vertical load was reached in two steps. Full beam load, $W = 20$ kips, was applied first and then full column top load with $3W = 60$ kips.
Application of full vertical load facilitated the checking of the instrumentation and loading systems as well as the symmetry of the frame. Lateral movement of the test frame under full vertical load was negligible and initial moments and rotations compared well with computed values. The frame was completely elastic at this stage. After full vertical load was reached, horizontal load was applied in convenient increments until the maximum load of $H = 16.9$ kips was reached. Deformation increments were used after the maximum load was reached. At the end of the test, determined by a maximum desired deflection of approximately 7", the frame was unloaded in three stages. First, the horizontal load was taken off in two increments, then the full vertical load was released. After completing this test program, the frame was tested under reversed horizontal loads. The object of this extension of the test program was to provide experimental evidence of the energy absorption capacity of frames under significant axial loads. This part of the investigation will not be presented here, but is the subject of a forthcoming report.²⁹

2.5 INSTRUMENTATION

The instrumentation of the test frame was chosen to provide a complete picture of the frame behavior under testing conditions. Since this test was performed as a demonstration during the 1965 Summer Conference on Plastic Design of Multi-Story Frames at Lehigh University, it was also necessary to consider the speed of reading and recording the instruments. The instrumentation is shown schematically in Fig. 11.

Deflection readings were taken to observe the vertical and horizontal movements of the frame. Lateral deflection readings were taken
near the top of both columns with Ames dial gages as shown in Fig. 11. A third gage, placed at the base of the center gravity load simulator provided an accurate and more convenient measure of the lateral deflection. Readings were also taken near the mid-height of both columns to provide data for the determination of the secondary column moments.

Vertical deflection readings were taken at the column tops, the beam loading points, and the beam centerline. These readings were taken by means of a surveyor's level and 1/100" scales, which were fixed in place at the five points of interest.

Strain readings were taken at two cross-sections on each member. At each cross-section four SR-4 strain gages were mounted, two on the outside of each flange, 1/2" from the flange tip. This configuration was chosen to minimize errors resulting from any twisting action that might occur during testing. The strain readings were used to compute the bending moments in the frame and the axial loads in the members.

Pressure readings from the pump console were used to compute the applied loads, W and H. The pressure dials were calibrated previously in a testing machine.

Rotations, θ, were recorded at six locations on the frame (see Fig. 11) by level bubble gages. Gages were located at the beam-to-column connections and load points as well as the column bases. As the test was designed with nominally fixed bases, the actual base rotations were of particular interest. Two rotation gages were placed at each base (as seen in Fig. 3). The far gage in this figure was welded to the column web at a distance "d", the depth of the column, up from the base plate.
The progress of yielding and other pertinent observations were recorded at each stage of the test.

The near gage was welded directly to the base plate and reflected the degree of fixity of the bases.
3. THEORETICAL ANALYSIS

Various methods of analysis exist which can be used to predict the load-deflection behavior of a structure. Each of these methods takes into account one or more characteristics of material behavior and loading. No single method can predict the entire response of the structure since the behavior changes continuously during deformation.

3.1 AVAILABLE THEORIES

The implications of the various analyses are best described by discussing the assumptions involved in formulating the equilibrium conditions and the end moment-end rotation (M - θ) characteristics of the members. These assumptions constitute attempts to approximate the characteristics of the behavior at different times during the loading process. It usually follows that more accurate assumptions result in a more cumbersome analysis. The degree of accuracy necessary is prescribed by the information desired from the analysis. In the present case, the maximum load capacity of the frame and its load-deflection behavior were required to allow a complete evaluation of the approximations made in the analysis.

Several methods of analysis will be discussed with reference to Fig. 12. This figure plots the relationship between the horizontal load, H, and the horizontal deflection, Δ, for a simple portal frame as predicted by these various analyses. The first order elastic analysis
results in a linear load-deflection curve. The material is assumed to be linearly elastic and equilibrium is formulated on the undeformed structure. In the second order elastic analysis the material is again assumed to be linearly elastic, but the equilibrium equations are formulated on the deformed structure. Thus, the overturning effect of the vertical loads (the P - Δ effect) in the frame is accounted for. In the case of non-proportional loading considered here, the second order elastic curve also becomes linear.

The elastic theories give no indication of the ultimate capacity of the frame or of the true behavior in the region of ultimate load. This is primarily because these theories do not account for the deterioration of the stiffness of the frame during loading.

In the plastic approach the material is assumed to be rigid-plastic. For the curves shown, the plastic moment capacity of the columns, $M_{pc}$, includes the reduction due to the axial loads in the columns. In the simple plastic analysis equilibrium is formulated on the undeformed structure. Under these assumptions, the structure does not deform until the ultimate load, $H_{pc}$, is attained. At this load, deformations can increase indefinitely.

The elastic buckling load, $W_e$, is sometimes used in conjunction with the simple plastic theory load to estimate the ultimate load of a structure. This approach, developed for framed structures by Merchant, is an attempt to include the influence of both strength and stiffness. However, Merchant's formula has no real significance for frames subjected to non-proportional loads.
In the second order plastic analysis the structure is assumed not to deform until the failure mechanism has formed. Beyond this point, however, equilibrium is formulated on the deformed mechanism. Thus, the overturning effect of the vertical loads is included and equilibrium can only be maintained if the applied load decreases as the sway deflection increases. Although the second order plastic analysis gives a good approximation of the ultimate load of the frame, this approach neglects the elastic deformations of the structure before the formation of the mechanism.

The second order elastic-plastic analysis predicts both the elastic and the plastic portions of the response of the structure. The material is assumed to be elastic-perfectly plastic and a second order elastic analysis of the structure is performed until the plastic moment capacity is reached at one location. A real hinge is placed in the structure at this point and a second order elastic analysis is carried out on the deteriorated structure under additional load. This procedure is repeated until a sufficient number of hinges have formed to produce a mechanism. Equilibrium requires that the mechanism forms at the intersection of the second order elastic-plastic and second order plastic curves. From this point the load-deflection curve follows the second order plastic curve. It should be noted that while the P-Δ effect is included, yielding is assumed to be restricted to the points of hinge formation. The effects of residual stresses and strain hardening are neglected.
3.2 THEORETICAL PREDICTION

The envelope formed by the second order elastic-plastic curve (including the pertinent portion of the second order plastic curve) provides the best estimate of the maximum capacity of the frame and its load-deflection response. This response includes the influence of the deflections which occur before the mechanism is formed as well as the gradual decrease in stiffness due to the formation of plastic hinges. This approach was used to obtain a final predicted load-deflection curve.

The application of a second order elastic-plastic analysis to the test frame was performed using slope deflection methods. As each hinge formed, the equations were modified to account for the reduction in stiffness. The resulting mechanism is shown in Fig. 13. This would also be the controlling mechanism obtained from a plastic analysis of the frame.

The response of the test frame was predicted on the basis of nominally fixed bases. Although a high degree of fixity was obtained, some rotation of the base plates did occur under horizontal load. In Fig. 14 the measured rotation, \( \Theta \), at each base plate is plotted versus the column moment, \( M \), at that base. These base rotations were accounted for in Fig. 15 which shows the elastic-plastic prediction of the frame behavior. In this figure the applied horizontal load, \( H \), is plotted against the horizontal deflection of the column top, \( \Delta \). The discontinuities in the curve occur where hinges have formed and the letters at these points identify the locations on the frame at which these hinges form.
The above analysis assumes that hinges form at the centers of the joints. Because of the increased strength in the area of the connection, the hinges will not form at the centerline of the joints, but will be forced some distance away from the joint. From previous tests it has been consistently observed that when two members are framed together a plastic hinge will form in the weaker member a distance "d" away from the face of the connection. The cause of this seeming increase in strength has not been conclusively established; however, it may result from the combined stress condition at the face of the joint. In the test frame, this would tend to stiffen the columns and therefore raise the ultimate load. To account for this behavior in the theoretical analysis, it was assumed that column hinges formed a distance \( d = 5" \) away from the faces of the connections. This modification raises the maximum load and slightly alters the post-mechanism behavior. In addition, the order of hinge formation is changed. Figure 16 shows the modified prediction of the load-deflection behavior. The effect of the shift in hinge location can be seen in Fig. 17 which superimposes the two solutions.

The material has been assumed to be ideally elastic-plastic. The steels used exhibit significant strain hardening characteristics. The influence of strain hardening on the ultimate load has been investigated by several authors. In most cases, the treatment has been rigorous and has resulted in a cumbersome analysis. It is the purpose here to develop a simple approximation which may easily be included in the analysis to account for the effect of strain hardening. The strain hardening characteristic of the material can be seen graphically in the idealized stress-strain relation shown in Fig. 18a.
Assuming a unit shape factor for the cross-sections, the moment-curvature relationship is shown in Fig. 18b, in which $s = \varepsilon / \varepsilon_y$. In this figure, $\phi_p$ is the curvature corresponding to the attainment of $M_p$, assuming ideally elastic behavior ($\phi_p = M_p / EI$).

The influence of strain hardening can be illustrated by considering the behavior of the cantilever beam shown in Fig. 19a. The concentrated load, $P$, is greater than that which makes the support moment equal to $M_p$. The moment diagram is shown in Fig. 19b, where $\tau L$, the yielded length, is the length over which $M > M_p$. The increment of rotation, $\Delta \theta$, which occurs after the hinge has formed, is required.

Figure 19c represents the curvature distribution in the beam obtained from the $M-\phi$ relation in Fig. 18b. This can be approximated by the curvature distribution shown in Fig. 19d. The increment of rotation is equal to the area under the curvature diagram over the length $\tau L$:

$$\Delta \theta = s \phi_p \tau L$$

In an indeterminate structure, strain hardening has no effect until the formation of the first hinge, after which it tends to slightly increase the load at which later hinges are formed. For the test frame, the effect of strain hardening is not significant before the mechanism forms. After the mechanism forms, however, all rotations occur at the hinges and strain hardening becomes significant.

In the second order plastic theory, equilibrium equations are formulated on the deformed mechanism. Using the approximation illustrated for the cantilever, the effect of strain hardening can be included in the predicted response of the frame. For given hinge rotations, $\theta$, 

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obtained from an arbitrary sway of the mechanism, the yielded length can be determined at each hinge point:

\[ \tau_L = \frac{\Delta \theta}{s \phi_p} \]

The increased moments at each hinge can then be obtained from the geometry of the bending moment distribution. Using these values in the equilibrium equations, the lateral load, \( H \), can be determined. By subjecting the mechanism to arbitrary increments of displacement, the post-mechanism branch of the load-deflection curve can be constructed. This modification was incorporated into the theoretical prediction and is shown in Fig. 20. This curve is based on the second order elastic-plastic theory as modified to account for the measured base rotations, the shift of hinge locations, and the effect of strain hardening and will be used as the final prediction of the overall response of the frame.
4. EXPERIMENTAL BEHAVIOR AND COMPARISON

A discussion of the experimental behavior of the frame is first given. The overall response of the frame will then be presented and compared with the theoretical prediction. To substantiate the assumptions made, the experimental behavior at several potential hinge locations will be compared with the behavior predicted by the theory described in Chapter 3.

The overall response of the frame, as characterized by the $H - \Delta$ curve, is shown in Fig. 21. $H$ has been used as the load parameter since the vertical loads remained constant throughout the test. In Fig. 21 and the figures that follow, solid lines joining full circles will be used to denote test results and dashed lines will denote the theoretical prediction. The numbers adjacent to the full circles indicate the stages at which data were taken during the test. All the plotted points, including those on the descending portions of the curve, represent points at which the frame was in static equilibrium with the loads.

Load Nos. 1 through 7 represent stages during which the vertical loads were applied and the instrumentation checked. From the initial application of horizontal load (Load No. 8) up to Load No. 11 the frame was completely elastic. Equilibrium was attained almost instantaneously in the elastic range. First yielding was observed under the left beam.
load and at the top of the right column. Once yielding had been observed, a waiting period of approximately five minutes was allowed at each loading stage for the frame and loading system to reach static equilibrium. Yielding progressed under further deformation and the pattern of yielding followed the predicted order of hinge formation (see Fig. 16). Maximum load was reached at Load No. 16. The portion of the response between Load No. 16 and Load No. 20 was characterized by the spread of yielding at the hinge locations. The extent of yielding near the top and bottom of the right column can be seen in Figs. 5 and 3. The frame was unloaded at a preset maximum deflection (Load No. 20). At this point the frame showed no sign of lateral or local buckling.

The correlation between the experimental and theoretical results is excellent throughout the entire response. The point of maximum load, which for the test frame corresponds with the formation of the failure mechanism, is in close agreement with the predicted value. The maximum load reached was approximately 20% below that predicted by simple plastic theory, indicating the importance of the instability effect.

Figure 22 shows the rotations, θ, at six locations on the frame plotted against the applied load, H. The locations of these measurements are shown schematically in the center of the figure. The correlation between theory and experiment is good. Each plot reflects the behavior of a localized section of the frame. The initial portion of each curve is linear. The peak indicates the point at which the failure mechanism formed and the gradually descending portion reflects the post-mechanism behavior of the frame. The final portion of each curve reflects the unloading of the test frame at the conclusion of
the test. The largest descrepancies occur at points D and E after the frame had deflected significantly. These two locations are points at which hinges formed early in the test and therefore these locations underwent extensive yielding. The discrepancies are probably due to the fact that rotations can only be measured at a point on the frame, whereas the actual change in curvature takes place over a finite and often significantly large distance.

Figure 23 shows the relation of moment, M, to load, H, at the bases and connections of the test frame. The moments at each location were computed from the strain data. The characteristics of the shape of these plots are the same as those of the H - θ curves. The location and direction of the moments plotted are shown in the schematic view near the vertical axis of each plot. The agreement between the theoretical and experimental results is satisfactory. In particular, the predictions of the maximum moments are excellent.

The strain readings indicated that the changes in axial load in the columns were relatively small, with a maximum change of 12 kips. The axial load in the beam varied up to a maximum of 10 kips. These changes had little effect on the behavior of the frame and were neglected.

The agreement between the theoretical predictions and the experimentally measured behavior supports the validity of the assumptions made in the analysis presented in Chapter 3. In particular, the close prediction of the overall behavior up to the formation of the failure mechanism supports the shift in hinge location concept. The post-mechanism behavior of the frame substantiates the significance of strain hardening as well as the manner of its inclusion in the theoretical prediction.
During the test the high strength steel columns were required to form plastic hinges and undergo large inelastic rotations. The test showed that high strength members can be used with confidence and that the behavior of such members can be predicted by methods conventionally used for structural carbon steel members.

5. SUMMARY AND CONCLUSIONS

A full-sized frame was tested which exhibited behavior believed to be characteristic of multi-story frames. The equipment used in testing ensured that in-plane behavior was maintained and allowed the effects of significant axial loads and strain hardening to be studied experimentally.

A second contribution of the investigation is the presentation of a second order elastic-plastic approach to the prediction of the load-deflection behavior of a simple portal frame. This prediction accounts for the secondary moments in the frame, the axial forces in the columns, and the extra strength provided by the connections. In addition, to predict the post-mechanism behavior, the influence of strain hardening on the strength of the frame is included in a rational, yet simple manner. The theory predicted the overall behavior of the frame in an excellent fashion. The theory was also able to closely predict the local behavior of the areas adjacent to the plastic hinge locations.

During the test the high strength steel columns were required to form plastic hinges and undergo large inelastic rotations. The test showed that high strength members can be used with confidence and that the behavior of such members can be predicted by methods conventionally used for structural carbon steel members.
This investigation has provided evidence related to the influence of secondary factors on the behavior of multi-story frames. Presently (1966) work is continuing on the development of a design method for multi-story frames by combining the responses of simple frames.
6. NOTATIONS

A  =  Cross-sectional Area
b  =  Flange Width
d  =  Depth of Section
E  =  Modulus of Elasticity of Steel (29,500 ksi)
E_{st}  =  Strain Hardening Modulus
H  =  Applied Lateral Load
h  =  Column Height from Top of Base Plate to Centerline of Beam
H_{pc}  =  Simple Plastic Theory Load (based on M_{pc})
I  =  Moment of Inertia
L  =  Span Length, Center to Center of Columns
M  =  Moment
M_{p}  =  Full Plastic Moment
M_{pc}  =  Plastic Moment, Reduced for Axial Load
r_{x}  =  Radius of Gyration About Strong Axis
r_{y}  =  Radius of Gyration About Weak Axis
s  =  \varepsilon_{st}/\varepsilon_{y}
t  =  Flange Thickness
w  =  Web Thickness
W  =  Applied Vertical Beam Load (20 kips)
W_{e}  =  Elastic Buckling Load
Z  =  Plastic Modulus
\alpha  =  Length Parameter (.27 for Test Frame)
\[ \Delta = \text{Horizontal Deflection of Column Tops} \]
\[ \varepsilon = \text{Strain} \]
\[ \varepsilon_y = \text{Yield Strain} \]
\[ \varepsilon_{st} = \text{Strain at Initiation of Strain Hardening} \]
\[ \rho = \text{Chord Rotation} \]
\[ \sigma = \text{Stress} \]
\[ \sigma_y = \text{Static Yield Stress} \]
\[ \sigma_{ult} = \text{Ultimate Stress} \]
\[ \tau L = \text{Yielded Length} \]
\[ \phi = \text{Curvature} \]
7. TABLES AND FIGURES
### Table 1. Material Properties

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<th>$\epsilon_y$</th>
<th>$\epsilon_{st}$ ksi</th>
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|          |              | 56.3           | .00191      | -                   | -                   | 83.5   | 20.9                          |
|          | 2            | 56.0           | 190         | .0194              | 677                 | 81.2   | 19.4                          |
|          | 3            | 56.0           | 160         | 172                | 594                 | 83.0   | 20.0                          |
|          | 4            | 56.6           | 192         | 177                | 896                 | 82.4   | 20.0                          |
|          | 5            | 55.8           | 189         | 176                | 648                 | 81.2   | 21.0                          |
|          | 6            | 56.3           | 191         | 195                | 718                 | 81.4   | 21.2                          |

### Table 2. Member Properties

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Fig. 1  Test Frame

Fig. 2  Beam to Column Connection
Fig. 3 Column Base Detail

Fig. 4 Lateral Brace Detail
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Fig. 6  Pump Console
Fig. 7  Overall View of Test Setup

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Fig. 18b  Moment - Curvature Relationship
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Fig. 23  Load - Moment Relationships
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