High-Strength Steels for Plastic Design

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Current design specifications permit the use of plastic design procedures for low carbon structural steels only.1 This restriction is justified because the research on which the specifications were based was performed mainly on steels of this family.²

Since 1962 a project has been in existence at Lehigh University which has as its aim the extension of plastic design procedures to include the high-strength steels having yield stresses up to 50 ksi. This group of steels includes the following ASTM designations: A440, A441 and A242.

The object of this paper is to compare the behavior of members of high-strength steel (in this case ASTM-A441) with that of low-carbon structural steel members (A36 or A7) and to draw conclusions concerning the suitability of the high-strength steels for plastic design. Since the low carbon steels, henceforth called structural steels, have been used for many years in plastically designed structures, it will be assumed that properly designed members of these steels will behave in a satisfactory manner.³ The high-strength steel members will then be evaluated using the behavior of the structural steel members as a reference.

MATERIAL PROPERTIES

The initial portions of the stress-strain (σ-ε) curves of the A36 and A441 steels are shown in Fig. 1. These curves are plotted from average values taken from tensile tests performed in connection with various projects.⁴, ⁵ The most obvious difference in the two curves is the increased yield stress, σₚ, shown by the A441 specimen. The length of the inelastic plateau for both specimens is approximately 12ε₀ where ε₀ represents the yield strain (ε₀ = σ₀/E). The other factor of importance is the lower strain-hardening modulus, E_SH, for the A441 material. Table 1 summarizes the properties of the curves shown in Fig. 1, and in addition includes typical values for other pertinent properties. The modulus of elasticity, E, for both materials has been taken as 29,500 ksi. The strain at the onset of strain-hardening is ε_SH. The properties not shown in Fig. 1, but listed in Table 1, are σ_U, the ultimate stress and the percent elongation at failure.

It is apparent that any differences in behavior between members of the two types of steel must be credited to the changes in yield stress and strain-hardening modulus. It has been shown that the variation in properties exhibited by the steels in the structural group considered will not influence the in-plane behavior of these steels.⁷ In other words, members constructed of any of these steels will successfully redistribute bending moments after the formation of plastic hinges and will fail by the

Table 1

<table>
<thead>
<tr>
<th></th>
<th>σ₀</th>
<th>ε₀</th>
<th>E_SH</th>
<th>E_H</th>
<th>σ_U</th>
<th>% Elongation</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASTM-A36</td>
<td>36</td>
<td>0.0012</td>
<td>0.014</td>
<td>900</td>
<td>65</td>
<td>25</td>
</tr>
<tr>
<td>ASTM-A441</td>
<td>50</td>
<td>0.0017</td>
<td>0.021</td>
<td>700</td>
<td>75</td>
<td>23</td>
</tr>
</tbody>
</table>

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formation of a mechanism. For both the structural and high-strength steels termination of the in-plane rotation will be marked by local and lateral buckling. In the case of beam-columns this may be preceeded by unloading of the member due to excessive bending.

**CROSS-SECTIONAL PROPERTIES**

The residual stress distribution in a member caused by rolling and subsequent cooling has been carefully documented. A typical distribution is shown in Fig. 2a for a section of A36 steel. The average compressive residual stress at the flange tips is 12.4 ksi or 0.34 $\sigma_y$. The distribution for a section of A441 steel is shown in Fig. 2b. In this case the average compressive residual stress at the flange tips is 11.6 ksi or 0.23 $\sigma_y$. It has been noted in other investigations that the magnitude of the compressive residual stress appears to be independent of the yield stress level. The residual stress distributions shown in Fig. 2 are due to the rolling process; recent investigations indicate that the continuous cold straightening process used for the smaller sections, may alter the distribution shown in Fig. 2 and may, in fact, alter the measured material properties. For the present however, the distributions shown may be considered as typical.

One other factor that may be considered as a property of the cross-section is the local buckling strength of the member. For a section to be admissible under currently used design specifications it is required that the flange plate be capable of sustaining strains up to strain-hardening before local buckling occurs. For A36 steel the flange width-to-thickness ratio, $b/t$, must be less than 17 for the section to be admissible; for A441 the corresponding limitation is 14. In general terms, the expression for the limiting value is

$$\frac{b}{t} = 2 \sqrt{\frac{1}{\sigma_y} \left( \frac{G_{SH}}{E} \right) \left( \frac{4}{3 + \frac{\sigma_{ULT}}{\sigma_y}} \right)} \quad (1)$$

where $G_{SH}$ is the torsional rigidity of the material in the strain-hardening range. $G_{SH}$ is directly proportional to $E_{SH}$ and thus the local buckling limit depends not only on $\sigma_y$ but also on the strain-hardening modulus.

The importance of the plate geometry is illustrated clearly by the results of a concentric stub column test. A typical specimen is shown ready for testing in Fig. 3. In this test the specimen is subjected to a compressive strain which is applied uniformly over the cross-sections. The specimen is loaded until the material has yielded. Further deformation occurs with little change in load until local buckling of the plate elements is observed. At this stage the specimen is unable to maintain the load and gradual unloading proceeds with additional deformation. Figure 4 shows the load-strain $P$-$\epsilon$ curves for two specimens. In Fig. 4 the load has been non-dimensionalized as $P/P_y$ where $P_y$ is the load which would cause the member to yield ($P_y = A\sigma_y$). The strain has been non-dimensionalized as $\epsilon/\epsilon_y$. The first specimen is of A7 steel and has a $b/t$ ratio of 17. Thus it is considered acceptable under the plastic design specifications. Local buckling was not observed until the average strain was 13 $\epsilon_y$. The second specimen was of A441 material and it also had a $b/t$ ratio of 17. For this material the $b/t$ ratio was above the limit given by equation (1) and therefore the section could not be...
used in a plastically designed structure. Local buckling was observed at an average strain of 4.0 $\varepsilon_y$ for this specimen. The conclusion of the test is shown in Fig. 5. The distortion of the cross-section is evident in this photograph.

For most cases, the development of severe local buckling marks the initiation of unloading. Since portions of members used in plastically designed structures must be strained into the strain-hardening range—to allow redistribution of the bending moment—the plate elements of such members must be stocky. This is to ensure that local buckling will not occur before the strain in the material reaches the strain-hardening. In the cross-sections which follow, it will be assumed that the selected members meet the requirements of equation (1). Similar requirements must meet web geometry.$^2$

MEMBER BEHAVIOR

A member designed according to plastic theory must deliver the full plastic moment, $M_p$, and maintain this moment while the hinge rotates a sufficient amount to ensure the development of a mechanism. The ability to deform inelastically is a function of the adequacy of the bracing system together with the material properties.

BEAMS UNDER UNIFORM MOMENT

A beam under uniform moment has been the subject of much research performed in order to study the behavior of members in the inelastic range.$^3, 4, 13$ This loading condition was selected because in the elastic range it represents one of the more critical loading conditions and is perhaps deceptively simple to analyze.

A typical test arrangement is shown in Fig. 6a. The beam is suspended at its third points by high tensile steel rods which resist the reactions, $R$, and is loaded at each end with a load, $P$, applied by means of hydraulic jacks. These jacks are operated off a common pressure circuit to ensure equal loads. The specimen is braced laterally at the ends and third points by knife-edge braces which allow deformations in the plane of the applied loads but prevent any out-of-plane movement. This is shown in Fig. 6b. The bending moment distribution is shown in Fig. 6c. The central span, under the uniform moment, is critical with respect to lateral buckling. The adjacent spans act as elastic restraints. A photograph of a test in progress is shown in Fig. 7.

The beam is deformed until the moment in the central critical span reaches $M_p$. Under further in-plane deformation, the yielding process decreases the stiffness of the critical span thus magnifying any initial lateral
deflections present in the compression flange. This action continues until the large lateral deflections, produced by the in-plane deformations, lead to local buckling of the compression flange. This is usually followed by a drop off in moment capacity.\(^8\) The local buckled area of a typical test specimen is shown in Fig. 8.

In Fig. 9 the moment rotation \((M-\theta)\) curves are given for two specimens, one of A7 material and the second of A441 steel.\(^4\) \(^13\) Both specimens were 10WF25 shapes with an unbraced length of \(35r_v\), where \(r_v\) is the weak axis radius of gyration of the section. In Fig. 9 the moment is nondimensionalized as \(M/M_p\) and the rotation as \(\theta/\theta_p\), where \(\theta_p\) is the rotation at the end of the central span. It is assumed that the material is ideally elastic \((\theta_p = M_pL/2EI)\). Local buckling occurred in the A36 member at a rotation of 14.4 \(\theta_p\) and for the A441 member at a rotation of 5.6 \(\theta_p\). Again the difference in behavior is due to the change in yield strength and the difference in the strain-hardening moduli of the two materials.

The inelastic rotation which a given section is capable of delivering can be related to the material properties and the unbraced length. The optimum capacity is obtained by spacing the lateral bracing so that the compression flange of the member will buckle locally. For this type of buckling, the results occur only when the applied in-plane strains are at the same rotation that will produce large lateral deflections and subsequent local buckling as described above.\(^8\) This will be termed the optimum bracing spacing and is given by:

\[
\frac{L}{r_v} = \frac{\pi}{K\varepsilon_y \sqrt{1 + \frac{0.56E}{E_{SH}}}}
\]  

In equation (2), \(K\) is the effective length and is assumed equal to 0.54 if the adjacent spans are elastic, and 0.8 if the adjacent spans are fully yielded. For beams braced at spacings less than that given by equation (2), unloading will be triggered by local buckling, much in the manner as beams under moment gradient. This will be discussed in the following section.

**BEAMS UNDER MOMENT GRADIENT**

The behavior of a beam under moment gradient will be discussed with respect to the simply supported beam shown in Fig. 10a. The bending moment distribution is shown in Fig. 10b, and the corresponding curvature distribution in Fig. 10c.

For a beam loaded in this manner, the maximum moment under the load point may be greater than \(M_p\) due to the influence of strain-hardening. As the load is increased, the member deforms inelastically, causing the yielded zone to spread on either side of the load point. The extent of the yielded zone at a particular stage in loading is shown by the heavy line in Fig. 10b.
The curvatures within this yielded zone, and thus the strains, are greatly increased over the elastic values and the distribution may be approximated by that shown in Fig. 10c.

For a section just meeting the requirements of equation (1) the member would deform inelastically until the yielded length reaches a value sufficient to allow a local buckle to form. The formation of the local buckles is shown in Fig. 10d. At this point the response of the member would fall below that predicted by in-plane considerations and the member would unload gradually. A photograph of a specimen showing the formation of a local buckle under the load point is given in Fig. 11. The moment-rotation curve is shown in Fig. 12. The increase in moment above $M_p$ due to the strain-hardening capacity of the material has not been considered in the theoretical prediction shown by the dashed line.

The behavior characterized by the response curve of Fig. 12 is probably the best that can be obtained and is independent of the bracing spacing. If the unbraced length is excessive, the member will buckle laterally before the formation of a local buckle. To prevent this, it is recommended that the unbraced length be limited to:

$$\frac{L}{r_y} = \frac{0.7\pi}{\sqrt{\epsilon_y}}$$  \hspace{1cm} (3)

It should be noted that although test results are not available for structural carbon steel members which are comparable with the behavior of the A441 member, the test results are compared with a theory that has been checked against the behavior of structural carbon steel members in other similar situations, and are found to be satisfactory. This will also be the case for the test results which follow.

**BEAM-COLUMNS**

The early research on plastic design concentrated on low, rigid frame structures in which the axial forces were relatively small.\(^2\) The beam under its various loading and restraint conditions was the primary research topic. In a multi-story frame, however, the behavior of the beam-columns is of great importance and in recent years experimental and theoretical investigations have treated this subject.\(^9\, 14, 15\)

For convenience, the beam-columns considered will be subjected to a constant axial force and an applied end-moment. The latter is the variable load parameter. Unloading of the member occurs when the applied end-moment drops off with increasing deformation.

Three representative loading conditions, to which the beam-column may be subjected, are shown in Fig.
Although the behavior of individual members has been well documented, it is only recently that investigations have been oriented toward the behavior of large frames loaded into the inelastic range.\(^7\) Design methods for such frames have been established, however, using as a basis the behavior of subassemblages or groups of members which are characteristic of the structure con-

13. The response of the member under the various conditions, as characterized by its \(M-\theta\) curve, is also shown in this figure.

For loading conditions other than that producing symmetrical single curvature, the response of the member is very similar to that of a beam. As the member is deformed the end-moment increases until it reaches \(M_{pc}\). At this point further deformation may occur at a constant moment. Unloading occurs only when the yielded length (at the point of maximum moment) reaches a length sufficient for the formation of a local buckle. To attain this optimum response the bracing rules used for the beam under moment gradient may be applied. For most columns used in multi-story frames this behavior is typical. For more slender or highly loaded members the end-moment may not reach \(M_{pc}\).

The symmetrical, single curvature case shown in Fig. 13 represents the critical loading condition for a beam-column. In this case the secondary moments, produced by the axial load acting through the in-plane deflection of the member, will reduce the bending capacity of the section, so that the maximum moment will be less than \(M_{pc}\). Fortunately this severe case is uncommon in multi-story frames, but may arise where checkerboard loading is considered.

Recent experiments on high-strength beam-columns have dealt with this loading case.\(^8\) Figure 14 shows the test setup. The beam-column is erected in the testing machine and the axial load is applied through the head of the machine. The end-moments, which deform the member in a symmetrical single curvature mode, are applied by means of an independent hydraulic jack acting through stub beams, which deliver the jack forces eccentrically to the specimen.

Figure 15 presents the \(M-\theta\) curve of a high-strength beam-column. The response, experimentally determined, is predicted adequately by a theory based on the material properties. This is typical of the structural carbon steels and accounts for the increased strength of the A441 material. In order to obtain the optimum response of the member, the bracing provisions recommended for beams subjected to uniform moment were used.

SUBASSEMBLAGES FOR BRACED FRAMES

Although the behavior of individual members has been well documented, it is only recently that investigations have been oriented toward the behavior of large frames loaded into the inelastic range.\(^7\) Design methods for such frames have been established, however, using as a basis the behavior of subassemblages or groups of members which are characteristic of the structure con-
Considered. For example, consider the braced frame shown in Fig. 16. The frame is subjected to a uniformly distributed live load on alternate bays and stories so that the columns are deformed into a symmetrical single curvature mode.

A characteristic subassemblage for this frame is shown in Fig. 17. It consists of the column AB restrained by the beam and column segments which frame into the top and bottom joints. The column is deformed by the unbalanced moments due to the checkerboard loading on the beams acting together with the axial load from the stories above.

For simplicity in testing, the subassemblage is modified to that shown in Fig. 18. The head of the testing machine is used to apply the axial load $P$ and the end-moments are applied by a hydraulic jack which produces a force, $F$. This is delivered to the beam-columns through stub beams, producing the deformed shape shown by the dashed lines.

A photograph of a typical subassemblage under test is shown in Fig. 19. In this photograph the three sets of lateral braces on the column can be seen, as well as the knife edge braces used to hold the restraining beams against lateral movement. The tower used to provide reaction points for the restraining beams is seen in the right side of the photograph. This tower is bolted to the laboratory floor. The hydraulic jack, which acts through the stub beams is shown on the left side of the figure. This is connected in series with a dynamometer used to measure the jack force.

The applied moment $M_j$ is resisted partly by the column, with a resisting moment $M_c$, and partly by the restraining beam. The condition at a braced joint is shown in Fig. 20a. The resisting moment of the beam is denoted as $M_B$ and the joint rotation as $\theta$. To maintain equilibrium, the applied moment must be balanced by
the sum of the resisting moments for a given joint rotation, $\theta$. Compatibility at the joint requires that the beam and column-end rotations be the same as that of the joint. Thus the response of the structure may be taken as the sum of the responses of the column and the restraining beam as shown in Fig. 20b.

In the design method proposed for braced multi-story frames, it is desirable that the beam reach its maximum moment at a smaller rotation than the column. Thus the unloading of the joint would be precipitated by the unloading of the column.

With this in mind, the subassemblage shown in Fig. 21 was tested. The column was a 6WF25 section of A441 steel while the restraining beams were 12B16.5 sections of A36 steel. The dimensions of the subassemblage were chosen for correlation with a full scale braced frame test. The axial load on the column was held at $0.6 P_u$.

The results of the subassemblage test are shown in Fig. 22. This figure shows the $M$-$\theta$ response of the beam, the column and finally the total joint response. The responses of the subassemblage components were predicted by methods developed to predict the behavior of structural carbon steel structures, modified to account for the increased strength of the A441 beam-column. The agreement is excellent.

Figure 23 is a photograph which shows the subassemblage after test. Unloading of the subassemblage was precipitated by the unloading of the column due to excessive bending. The restraining beams continued to accept end-moment even after local buckling had
SUMMARY AND CONCLUSIONS
The main points of this paper are summarized as follows:
1. Differences in behavior between members of high-strength steel and members of carbon structural steel were chosen so that the frame would exhibit significant sway effects.

The theoretical prediction for the behavior of the frame is shown by the dashed lines in Fig. 25. This prediction accounts for 1. the secondary moments produced by the sway of the frame, 2. the reduction in stiffness due to the formation of plastic hinges, and 3. the increase in strength due to strain-hardening after the formation of a mechanism.

The experimental results are shown by the data points connected by the solid lines in Fig. 25. The agreement between theory and experiment is excellent.

The frame failed in a combined mechanism, forming two hinges in the leeward column, one at the base of the windward column and one in the beam at the windward load point. The localized behavior at one of the hinge locations in the high-strength column is shown in the form of a load-moment (H-M) curve in Fig. 26. Again the agreement between experiment and theory is exceedingly good.

Figure 27 shows the frame after testing. It was in excellent condition even after considerable inelastic deformation. The column hinge area is shown in Fig. 28. The extent of yielding is considerable, but due to stocky plate geometry, local buckling did not occur.

SUMMARY AND CONCLUSIONS
The main points of this paper are summarized as follows:
1. Differences in behavior between members of high-strength steel and members of carbon structural steel...
are caused by the increased yield point and the reduced strain-hardening modulus of the high-strength steel. If these factors are taken into account, the inelastic behavior of high-strength steel members may be predicted. This investigation was restricted to structural steels having yield stress levels up to 50 ksi and yielding in the same manner as the structural low carbon steels.

2. The magnitude of the residual stresses due to the rolling process appears to be independent of the yield stress of the material. Solutions based on a maximum compressive residual stress of 0.3 \( \sigma_y \) (which has been accepted for the A36 steels) would in general be conservative when applied to steels having higher yield stresses.

3. In designing for the higher yield stress level and reduced strain-hardening modulus, flange plates of members, used in plastically designed structures, should meet the requirements of equation (1). Corresponding limitations are being developed for web plates.

4. Lateral bracing for members under uniform moment and for those under moment gradient may be spaced according to equations (2) and (3).

5. Beams under uniform moment and moment gradient have been tested and the results of these tests have adequately predicted theory. (The members were of A441 steel.)

6. Beam-columns of A441 steel have been tested alone and as segments of subassemblies under both non-sway and sway conditions. In all cases the results have been predicted successfully by a theory which is based on the same principles as those used in the design of low-carbon steel members.

7. It appears certain that, in the near future, high-strength steels will be used in plastically designed structures. The problems that remain to be solved before this advance can be taken, are those associated with web buckling, connection behavior and overall frame stability. Work is in progress in these areas.

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