Plastic Design in High Strength Steel

THE DUCTILITY OF STEEL STRUCTURES

by
Theodore V. Galambos
Maxwell G. Lay

Fritz Engineering Laboratory Report No. 297.5
MEMBERS OF LEHIGH PROJECT SUBCOMMITTEE (WRC)

Higgins, T. R.  Gilligan, J.  Kreidler, C.
Amirikian, A.  Grover, L.  Newmark, N. M.
Beedle, L. S.  Jameson, W. H.  Perrone, N.
Dill, F. H.  Johnston, B. G.  Pisetzner, E.
Epstein, S.  Kavanagh, T. C.  Stuchell, R. M.
Fox, G.  Ketter, R. L.  Vasta, J.

Gentlemen:

March 16, 1964

RE: THE DUCTILITY OF STEEL STRUCTURES
Fritz Laboratory Report No. 297.5

Enclosed you will find a report entitled "The Ductility of Steel Structures" by T. V. Galambos and M. G. Lay (Fritz Laboratory Report No. 297.5). This report is the written version of a talk under the same title delivered at the ASCE Structural Engineering Conference in San Francisco (October 1963). The contents of the report represent a summary of our present experimental and theoretical knowledge on the static, unidirectional load-deformation behavior of beams, beam-columns and beam-and-column subassemblages.

The report is sent to you for review and evaluation for possible publication. We hope to submit the report to the ASCE Structural Division Journal for publication. Please indicate your opinion about publication on the enclosed postcard. If you have further comments to make, please write us. We would appreciate if you would return your communication by April 15, 1964.

Sincerely yours,

Theodore V. Galambos
Project Director

TVG:ps

Encs.

cc: G. C. Driscoll, Jr.
Plastic Design in High Strength Steel

THE DUCTILITY OF STEEL STRUCTURES

by

Theodore V. Galambos

and

Maxwell G. Lay

This work has been carried out as part of an investigation sponsored jointly by the Welding Research Council and the Department of the Navy with funds furnished by the following:

American Institute of Steel Construction
American Iron and Steel Institute
Institute of Research, Lehigh University
Column Research Council (Advisory)
Bureau of Ships (Nobs 90041)
Bureau of Yards and Docks (NBY-5310)
Welding Research Council

Reproduction of this report in whole or in part is permitted for any purpose of the United States Government.

March 1964

Fritz Engineering Laboratory
Department of Civil Engineering
Lehigh University
Bethlehem, Pennsylvania

Fritz Engineering Laboratory Report No. 297.5
SYNOPSIS

Part of the energy of a severe earthquake shock is absorbed by the inelastic deformation of the structural frame. In order to assess the inelastic deformability of a steel frame, it is necessary to know the load-deformation behavior of its components. The inelastic load-deformation response of beams, beam-columns and beam-and-column sub-assemblages which are subjected to static uni-directional loading is examined. The response criterion for a member is the end moment-end rotation relationship. It is shown, both from experimental and theoretical considerations, that the inelastic load-deformation behavior of wide-flange steel frames and frame components can be predicted and that therefore the energy absorption capacity of these frames can be assessed.
## CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>SYNOPSIS</td>
<td>1</td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2. INELASTIC DEFORMATION CAPACITY OF BEAMS</td>
<td>3</td>
</tr>
<tr>
<td>Beams Under Uniform Moment</td>
<td>3</td>
</tr>
<tr>
<td>Beams Under Moment Gradient</td>
<td>8</td>
</tr>
<tr>
<td>3. INELASTIC DEFORMATION CAPACITY OF BEAM-COLUMNS</td>
<td>10</td>
</tr>
<tr>
<td>4. BEHAVIOR OF RIGIDLY JOINTED FRAMES</td>
<td>14</td>
</tr>
<tr>
<td>CONCLUSIONS</td>
<td>20</td>
</tr>
<tr>
<td>6. ACKNOWLEDGEMENTS</td>
<td>23</td>
</tr>
<tr>
<td>7. NOMENCLATURE</td>
<td>24</td>
</tr>
<tr>
<td>8. FIGURES</td>
<td>26</td>
</tr>
<tr>
<td>9. REFERENCES</td>
<td>48</td>
</tr>
</tbody>
</table>
1. INTRODUCTION

Studies on the behavior of rigid frame buildings during earthquakes have led to the conclusion that for an efficient design part of the energy of a severe shock should be absorbed by inelastic deformation of the structural frame\(^{(1)}\). There are at least two ways recommended for the inclusion of inelastic deformability into the earthquake design of rigid frame multi-story buildings:

1. It can be specified that each primary load-carrying member (beam or beam-column), and the connections between these members, be able to deform a certain minimum amount beyond the initiation of yielding without a reduction of the load carrying capacity\(^{(1)}\). In Fig. 1 is shown a typical load-deformation curve for a member or a connection. The inelastic deformability is characterized by the "ductility factor" \(\mu\), which is defined by

\[
\mu = \frac{\nu_u}{\nu_y}
\]

In Eq. 1 \(\nu_y\) is the elastic deflection corresponding to the maximum load \(P_u\), and \(\nu_u\) is the deformation at the start of unloading.

2. A more complicated method is the "Reserve Energy Technique" which uses the area under the load deformation curve of the whole frame (or a meaningful portion of the frame, such as a whole story) as a measure of the energy absorption capacity\(^{(1)}\) (Fig. 2).

These two methods of earthquake design have been cited to illustrate that in order to apply them it is necessary to know the complete load-deformation characteristics of the beams, beam-columns and connections.
which make up the frame. From this knowledge both the "ductility factor" and the energy absorption capacity of the whole frame can be deduced.

The two methods, at least in the form in which they were presented in Ref. 1, both utilize the static uni-directional load response of members, despite the fact that earthquakes produce dynamic reversible loads. Partial justification for this approach has been given on the basis of computer studies on ideally elastic-plastic structures which were subjected to the strong motion accelerations recorded for the El Centro earthquake\(^1\). These arguments are not very convincing, and future studies must necessarily include the dynamic load-deformation behavior of members and connections under load reversal. Information is quite scant on this at present (1964), and so the methods utilizing static uni-directional member response represent a useful first step toward a rational earthquake design. Therefore, it seems worth while to examine present knowledge about the static uni-directional load-deformation response of frames and frame components. This examination would serve as a step toward formulating further studies on the more realistic behavior encountered in earthquakes.

The purpose of this paper is to present a review of experimental and theoretical results on the inelastic deformation capacity of steel beams and beam-columns which are subjected to static uni-directional loads. It will be shown that the limit of inelastic deformability of these members can be predicted with adequate certainty. It will also be illustrated how the load-deformation relationship for the whole frame can be developed from the moment-rotation curves of the members. The discussion of frame
Components will be limited to beams and beam-columns, connections having been treated elsewhere (2)(3). Connections are usually stiffer than the members which they connect, and thus the major portion of the inelastic action takes place in the members.

2. Inelastic Deformation Capacity of Beams

The post-elastic behavior of as-rolled structural steel wide-flange beams has been studied extensively both experimentally and theoretically (Refs. 4 through 8). This research has shown that the deformation capacity before the start of unloading of such beams is affected by the following factors: (1) moment gradient, (2) the spacing of the lateral bracing, (3) the end restraint provided by spans adjacent to the inelastic span, (4) the properties of the lateral bracing, (5) the occurrence of local buckling, (6) the material and cross-sectional properties of the beam and (7) the occurrence of lateral-torsional buckling.

Beams Under Uniform Moment

A typical experimental static load-deflection curve for an ASTM A36 steel 10WF25 rolled beam is shown in Fig. 3(4). The beam was loaded and supported as shown in the sketch in this figure. Lateral bracing was provided at the ends and at the third points (see sketch in Fig. 4) such that at these points the member remained in the vertical plane while it was able to both deflect and rotate vertically in addition to being able to rotate also horizontally. The curve in Fig. 3 gives the relationship between uniform moment in the central third of the beam and the resulting
vertical deflection at the center. This relationship is presented non-dimensionally, and thus an identical curve would have resulted if the non-dimensional end rotation or the center curvature were plotted as the abscissa.

The P-v_o curve in Fig. 3 exhibits an initially elastic portion which is followed by a "knee" in the curve (constrained plastic flow), after which it flattens out to form a "plastic hinge plateau". A slight increase of the local occurs just prior to the start of unloading which is triggered by local buckling at a deflection of 13.7 v_o y. Since v_o y is the deflection corresponding to the theoretical initiation of yielding at a moment PL = M_y = S σ_y (where S is the section modulus and σ_y is the static yield point of the flange material), the value of 13.7 v_o y is also approximately equal to the "ductility factor" as defined in Fig. 1.

The curve in Fig. 3 shows the behavior of the beam in the plane in which the loads act, and this is the relationship which is of interest when the deformation capacity in an earthquake analysis is desired. However, the behavior in this plane does not completely describe the whole deformation history of the member. Deformations also occur in the lateral direction, and these are shown in Fig. 4. Here one curve gives the lateral deflection of the compression flange and the other curve gives the lateral deflection of the tension flange at the center of the middle span. These curves illustrate that while the tension flange deforms very little in the lateral direction during the loading history of the beam, the compression flange deflects considerably. The deflection in this case is about 1.5 in. in a span of 46 in. at the outset of local buckling. The
beam, in fact, starts to deflect laterally from the very start of loading, and this deflection becomes quite pronounced after the compression flange has fully yielded. The member however continues to carry the full plastic moment until finally local buckling occurs, despite the severe lateral buckling type deflections perpendicular to the plane of loading. Thus the assumption of in-plane deformation after yielding, which underlies "simple plastic theory", can indeed not predict the complete post-elastic behavior of steel beams. The actual behavior can be explained by a model which considers the discontinuous yielding process of structural steel as well as the lateral deformations, and with this model the complete load-deflection history of the beam, including the onset of failure due to local or lateral-torsional buckling, has been satisfactorily predicted(8).

In an earthquake analysis it is necessary to know under what conditions a plastic hinge will develop, and for what amount of deformation this hinge will be maintained before failure occurs. One of the principal factors affecting the amount of inelastic rotation is the spacing of the lateral bracing (L in Figs. 3 and 4). The P-versus-\(v_0\) curves of four tests are given in Fig. 5 to illustrate the effect of this variable. These tests were identical except for the unbraced span L which was varied from 35 \(r_y\) to 50 \(r_y\). A comparison of the curves indicates that as the span L is decreased, the rotation capacity increases.

In a theoretical study of the load-deformation behavior of steel wide flange beams the following expressions have been developed, relating the ductility factor \(\mu\), and the various parameters affecting beam behavior for beam segments under uniform moment when failure is due to local
buckling\(^{(8)}\).

For \((\mu - 1) \leq 0.8(s-1)\)

\[
\lambda = \frac{1}{\sqrt{1 + 0.7\left(\frac{h}{s}\right)(\mu - 1)}} \tag{2}
\]

for \(0.8(s-1) < (\mu - 1) < (s-1)\)

\[
\lambda = \frac{1}{\sqrt{1 + 0.56h}} \tag{3}
\]

and for \((\mu - 1) > (s-1)\)

\[
\lambda = \frac{1}{\sqrt{(1+0.56h)(1+\frac{\mu - s}{h})}} \tag{4}
\]

The various terms in Eqs. 2 through 4 have the following meaning:

\[
h = \frac{E}{E_{SH}} \tag{5}
\]

where \(E\) is the elastic modulus, and \(E_{SH}\) is the strain-hardening modulus, as measured on a tensile coupon of the material.

\[
s' = \frac{\varepsilon_{SH}}{\varepsilon_y} \tag{6}
\]

where \(\varepsilon_{SH}\) is the strain at the initiation of strain-hardening, and \(\varepsilon_y\) is the strain at the initiation of yielding. For ASTM A36 steel, \(h = 33\) and \(s = 11.5(9)(10)\); for A441 material these quantities are \(h = 45\) and \(s = 10.5(8)\).

The term \(\mu\) is as defined in Fig. 1 and Eq. 1 as the ratio of the deformation (end rotation or vertical deflection) at the start of pronounced unloading to the elastic deformation when the moment equals \(M_p\), the plastic moment of the section.

The term \(\lambda\) is the non-dimensionalized slenderness ratio, and it is defined by
\[ \lambda = \frac{KL}{r_y} \left( \frac{E_y}{F} \right) \]  

In Equation 7, \( L/r_y \) is the weak axis slenderness ratio of the unbraced span, and \( K \) is an "effective length factor" which takes into account the restraint offered by the adjacent spans. For the usual case of elastic adjacent spans (Fig. 3) \( K \approx 0.54 \), and for yielded adjacent spans \( K \approx 0.8 \). \(^{(8)}\)

A comparison between the theoretical predictions of Equations 2 through 4 with experimentally observed ductility factors (Refs. 4 through 8) is given in Fig. 6, where the ratios \( \frac{h_f}{s} - 1 \) and \( \lambda \) are the coordinates. Since the actual start of unloading, as well as the observation of the start of local buckling, is gradual and not well defined, the value of \( \mu \) was taken arbitrarily at 0.95 (PL)\(_{\text{max}}\) on the unloading portion of the experimental load-deformation curves.

The correlation in Fig. 6 is quite good, and thus one may use formulas (2) through (4) to obtain the required bracing spacing for a desired ductility factor, or in reverse, determine the ductility factor for a given spacing. For example if the spacing of the bracing is 35 \( r_y \) for an A36 wide-flange beam having elastic adjacent spans (\( K = 0.54 \), \( s = 11.5 ; \ h = 33 \)), the ductility factor is approximately 17. For this same beam the required spacing of the lateral bracing is 60 \( r_y \) for a required ductility factor of \( \mu = 4.0 \).

The discussion above pertains to the relationship between the ductility factor and the spacing of the lateral bracing. In order that these relationships can be realized, it is necessary that additional conditions be met. These are: (1) the bracing members must frame into the
compression flange of the beam to be braced(5), (2) the cross-sectional area of the bracing member must be at least 2.5% of the area of the compression flange of the braced beam(8), (3) the depth-to-length ratio of the bracing members should be no larger than 30(5), (4) the ratio of the depth of the beam to the depth of the brace should not exceed 3(5), (5) a vertical stiffener covering at least the compression side of the web should be provided at each bracing location(5), and (6) the width-thickness ratio of the compression flange should not exceed(8)(9)(10)

\[
\frac{b}{t} = 17\sqrt{\frac{36}{G_0}}
\]  

(8)

At a first glance these restrictions would seem to impose rather impressive limitations; however, it should be realized that they represent practical conditions in common use, and thus they do not inflict new and costly qualifications.

**BEAMS UNDER MOMENT GRADIENT**

Beams under purely uniform moment occur rarely in practice. In a rigid frame only the positive moment regions within the span of the beams under distributed load could be idealized to be under uniform moment, and thus the discussion above would in general not apply to the ends of beams. However, in reconstructing the behavior of the whole frame from the response of its component members to an end moment, it is important to be able to develop a moment-end rotation curve when there is a pronounced change in the moment along the length of the beam.
Such a situation is illustrated in Fig. 7a, where a beam under a central concentrated load is shown. The moment diagram (Fig. 7b) shows that the extent of yielding is concentrated in a relatively small region in the center. As a result, most of the post-yielding deformation will take place in this region, and a hinge type movement will result (Fig. 7c). Because of the steep curvature gradient over a relatively short portion of the beam, strain hardening effects will be more dominant than in the case of uniform moment, and the moment-versus-center rotation curve will continue to rise after $M_p$ is reached (Fig. 7d) until unloading is initiated by local buckling. Local buckling will occur when the average strain in the yielded part of the compression flange has reached strain-hardening and when the yielded zone has extended far enough to permit the development of a buckling wave. Theoretical studies have shown that the inelastic rotation for this case will be approximately 0.07 radian before unloading starts. This amount is nearly independent of the moment gradient, except that it does not apply for extremely steep gradients—where shear deformations enter the picture—and for extremely shallow gradients—which approximate the case of uniform moment. To obtain this inelastic deformation it is necessary that premature local buckling is prevented by limiting the width-thickness ratio of the compression flange (Eq. 8), and that lateral buckling is prevented by bracing. It has also been shown that a lateral brace at the hinge-location and lateral bracing at a distance of at most $60 \, r_y$ to either side of the hinge will assure the type of behavior idealized in Fig. 7d. An examination of all available tests has provided verification of this theory.

The determination of the ductility factor of beams subjected to
intra-panel loads and an end moment is illustrated by the example of a uniformly loaded beam in Fig. 8a. The problem is to find the amount of end rotation corresponding to an applied end moment $M_a = M_p$ when the distributed load $w = \frac{4 M_p}{L^2}$. The moment diagram is shown in Fig. 8b, and Fig. 8c gives the development of a formula for the ductility factor $\mu$. The curve in Fig. 8d illustrates that, for a given material, $\mu$ is only a function of the $L/d$ ratio of the beam. A value of 0.035 radian has been used in Fig. 8 for the hinge rotation, since the conditions here simulate only one half of the situation shown in Fig. 7.

Curves similar to that given in Fig. 8 could be developed for other loading cases. It should be realized that such curves are conservative since unloading after local buckling is gradual. They do indicate, however, when local buckling starts and can be used to check the adequacy of a given design.

3. INELASTIC DEFORMATION CAPACITY OF BEAM-COLUMNS

The load-deformation history of beam-columns is similar to that of beams (Fig. 3), except that in addition to the local and lateral torsional instability phenomenon there is the occurrence of overall member instability. This new effect is the result of the combined influence of secondary moments introduced by the axial force and the reduction of stiffness due to yielding. Overall instability will cause an unloading of the load-deflection curve even in the absence of local and lateral-torsional buckling.
A great deal of work has been published on the post-elastic performance of as-rolled steel wide-flange beam-columns, and it is possible to make a very reliable prediction of the ultimate strength of such members if bending is restricted to one of the principal planes. The theoretical work, as well as the experimental verification, has been well documented (see for example Refs. 11 through 14) and, therefore, the following discussion will deal only with the inelastic deformability rather than the ultimate strength.

Load-deformation curves for beam-columns, as governed by overall instability and exclusive of local and lateral-torsional buckling (hereafter termed "in-plane" behavior), can be developed with the aid of Column Deflection Curves\(^\text{(15)}\). The deformation history is usually represented by curves relating the end moment and the end rotation for specified values of the axial load, the lengths, and the ratio of the moments at the two ends of the member. It was found that a non-dimensional representation of these curves is nearly independent of the size of the wide-flange shape\(^\text{(12)}\). Nomographs to facilitate the construction of such M-θ curves, as well as many groups of M-θ curves for both strong and weak axis bending are available\(^\text{(16)}\).

Typical M-θ curves for strong axis bending are shown in Fig. 9 for the case of \(L/r_x = 30\) and \(P/P_y = 0.3\), \(r_x\) being the major radius of gyration and \(P_y = A \sigma_y\) the yield load (\(A\) is the area of the cross section). The three curves are for the loading cases shown by the sketches. These loadings represent the two possible extremes (equal end moments causing single and double curvature deflection) and an intermediate
situation (one end moment only). It is seen that in the presence of this axial load the full value of \( M_p \) is not reached. At best a reduced moment \( M_{pc} \) (13), with a plastic hinge plateau, is developed (two of the three curves in Fig. 9). In the third instance the ultimate moment is below \( M_{pc} \) and unloading occurs soon after this moment is reached.

Many beam-columns of relatively short length and low axial load will behave as beams rather than beam-columns. The maximum moments will occur at the ends of the member and plastic hinges will form there. Therefore, the ductility of such beam-columns can be predicted by the methods used for beams under moment gradient. The upper two curves in Fig. 9 represent such a situation. The situation obviously cannot occur for a beam-column with equal and opposite end moments for which the maximum moment will always be at midheight.

The rotation capacity, or ductility, of individual beam-columns can be found from the \( M-\Theta \) curves by noting the point at which unloading commences. Since this point is poorly defined (see Fig. 9), the rotation corresponding to 95% of the maximum moment is used arbitrarily hereafter to define the ductility factor (see Fig. 10).

With the definition of inelastic rotation capacity as noted above, and by considering many \( M-\Theta \) curves and the above mentioned limits of beam-column behavior, relationships such as those shown in Figs. 11 and 12 have been developed(8)(17). The curves give the variation of the ductility factor \( \mu \) with the axial load ratio \( P/P_y \) for constant values of \( L/r_x \) for the two cases of loading shown in the insets. In-plane ductility increases for decreases in the axial load and the
slenderness ratio. For situations commonly occurring in structures
designed primarily for lateral loads (as is the case with earthquakes)
the slenderness ratios and the axial load ratios seldom exceed 50 and
0.3, and the lateral loading means that the single curvature loading
case (Fig. 12) does not occur very frequently. Such beam-columns
behave primarily as beams and will deliver end-rotations of the order
of 0.05 radian before local buckling occurs. The calculation of a
particular ductility factor then depends on the initial elastic
behavior of the member. From Fig. 11 it can be seen that beam-columns
with slenderness ratios of 30 or less never have ductility factors
below four.

The local buckling behavior discussed for beams also applies
to beam-columns. A buckle will not form until a region long enough
to permit the formation of one buckling wave has yielded, provided the
flange b/t ratio is not more than the limit specified in Eq. 8 and the
member depth-to-web thickness ratio is less than or equal to\(\frac{d}{w} = \frac{3k}{\sqrt{\gamma_0}} \left[ \frac{70 - 100}{P/P_y} \right] \geq 43 \frac{3k}{\sqrt{\gamma_0}}\) \(\text{(9)}\)

A graphical search of the Column Deflection Curves for the
critical strain situations necessary for local buckling resulted in
the dashed cut-off curves in Figs. 11 and 12. Obviously, there is
little difference between the point of local buckling and the point at
which a hinge forms in the beam-column. It is seen that the probable
maximum conditions in laterally loaded multi-story frames described
above are such that the hinges and, therefore, local buckling will
occur at the ends of the member.
An empirical estimate of the reduction of the rotation capacity due to lateral-torsional buckling has been made\(^\text{17}\), but as this estimate is not based on theoretical post-lateral buckling M-Θ curves and since it is dependent on the size of the wide-flange shape, curves are not reproduced here. However, this study\(^\text{17}\), and also observed test observation\(^\text{14}\), shows that not a great deal of inelastic deformability is likely to be present after lateral-torsional buckling has occurred. Thus for optimum performance it is desirable that this type of instability be not the governing case. Studies similar to those which led to Eqs. 2 through 4 for beams have indicated that bracing spaced according to Figs. 13 and 14 will effectively prevent lateral-torsional buckling from reducing the rotation capacity.

Because of the many variables involved it is not possible to make a comparison of theoretically computed and experimentally observed rotation capacities for beam-columns in as compact a manner as was done in Fig. 6 for beams. Extensive comparisons were made, however, and they showed reasonable agreement\(^\text{17}\). Such a comparison between theoretical and experimental M-Θ curves is given in Fig. 15 for three tests. These beam-columns were braced and failure in each case was due to in-plane instability. The correlation is satisfactory and it is typical of the other comparisons\(^\text{17}\).

4. BEHAVIOR OF RIGIDLY JOINTED FRAMES

The preceding discussion indicated that the post-elastic load-deformation behavior of individual steel beams, beam-columns and connections
can be quite reliably predicted if the loads are static and proportional. Furthermore, it was shown that for geometric configurations of practical importance in multi-story frames the beams and beam-columns can be assumed to develop plastic hinges of predictable magnitude and rotation capacity, provided certain reasonable conditions of lateral bracing and cross-sectional width-thickness ratios are fulfilled. The inelastic deformability was represented by the ratio \( \mu \) ("ductility factor"), and curves were presented for both beams (Fig. 6) and beam-columns (Figs. 11 and 12) for the ready determination of this ratio.

Knowledge of the plastic moment (as reduced for axial load where appropriate) and the length of the plastic hinge plateau enables one to perform the analysis of many types of problems by the relatively uncomplicated methods of simple plastic theory, where the ultimate condition corresponds to a mechanism rotation. An example of such an analysis follows.

The structure is shown on the top of Fig. 16. It is a single-story three-bay bent having equal member sizes throughout. The axial load is assumed to remain constant while the horizontal force \( H \) varies from zero to its ultimate value.

Since all member sizes are equal and since the \( M_p \) value of the beams is not reduced by axial force, hinges will form at the column tops. For simplicity it is assumed that the axial loads in the columns remain 0.3 \( P_y \) and thus the \( M-\Theta \) curve shown in Fig. 9 (the curve for the case of one end moment) is used.* To further simplify the problem,

*The neglect of the variation of the axial load due to the horizontal force was found to influence the ultimate load very little.
this M-θ curve is assumed to consist of two straight lines: an elastic portion from zero to $M_{pc}$, and a plastic hinge portion from $θ = 0.0088$ radian to $θ = 0.046$ radian. This latter rotation corresponds to the onset of local buckling (Fig. 9).

For the geometric and material conditions given on the top of Fig. 16 three types of $H-Δ$ curves were computed. These curves are shown in the lower portion of Fig. 16. One curve represents the case where the analysis neglects the additional moments introduced by the axial force times the panel deflection ("first order" analysis). Hinges form first at the top of the interior columns (at $H = 0.085 \, P_y$) and a side-sway mechanism develops when hinges form at the top of the exterior columns also (at $H = 0.102 \, P_y$). After the formation of the mechanism the frame continues to deform without a drop in load until local buckling develops in the interior columns at $Δ = 0.055 \, L$. The second curve shows the relationship between $H$ and $Δ$ when the effect of $PΔ$ is included ("second order" analysis without a brace). The sequence of hinge formation is the same as for the first order analysis, but the deformations corresponding to a given value of $H$ are larger, and a considerably lower ultimate load is reached ($0.078 \, P_y$ versus $0.102 \, P_y$ for the first order analysis). After the peak load is reached, unloading sets in due to the increased influence of the $PΔ$ moment. Local buckling is again reached at $Δ = 0.055 \, L$.

In order to counter the detrimental effect of the second order moments a brace is introduced to take up the difference in carrying capacity$^{(19)}$. The brace is assumed to be elastic-plastic, and only the brace in tension is assumed to act. The required bracing area to allow
the original carrying capacity to be reached was found to be 2.69% of the area of one column, and the resulting H-Δ relationship is shown as the third curve in Fig. 16 ("second order" analysis with brace). Yielding of the brace occurs at H = 0.04 P_y, the first two hinges form at the top of the interior columns at H = 0.094 P_y, and the ultimate load is reached at H = 0.102 P_y. After the ultimate load the curve unloads rapidly because of the PΔ effect until finally local buckling sets in.

The simple example in Fig. 16 illustrates how a load-deformation curve can be constructed. The area under such a curve would represent the energy absorbed by the frame, and so provide one of the steps in the "reserve energy technique" for designing against earthquake loads(1).

In some situations, arising primarily in connection with the plastic design of braced multi-story frames under vertical loads, it is possible that beam-columns possess very little rotation capacity. In this case it is possible that members with large rotation capacity, the beams for example, carry the relatively non-ductile members with them.

Such a situation would arise in a frame such as that shown in Fig. 17 where all the beams form simple beam-mechanisms. This condition determines the size of the beams. The most critical condition for the design of the beam-columns in the lower part of the building arises when all the beams except two, which frame into the column to be designed at opposite ends from opposite directions, are fully loaded. The two beams are only subjected to dead load, resulting in a "checker-board" type loading and thus subjecting the column to single curvature bending. (See area circumscribed by the dashed line in Fig. 17). The column is
under nearly the full axial load, and its ends support moments producing the most unfavorable deflection (see Fig. 9).

The "subassemblage" containing the column to be analyzed can be idealized by the structure shown in Fig. 18. The two beams represent the restraint provided by the members framing into the ends of the column (Fig. 17), the moments \( M \) are due to the plastic hinges, and \( P \) is the axial load transmitted from the stories above.

For such a subassemblage (Fig. 18) simple plastic theory is not valid because the beam-column no longer delivers a hinge, as shown by the \( M-\theta \) curve in Fig. 19 (curve marked "column"). However, the strength of the subassemblage can be defined by superimposing the \( M-\theta \) curves of the beam and the beam-column, assuming compatible rotations at the joints (upper curve in Fig. 19). Thus if the \( M-\theta \) curves of both components are known, the load-deformation relationship of the whole subassemblage should be fully defined.

The validity of this statement was checked by a series of tests performed on frames similar to that shown in Fig. 18(20). Axial load was applied independently from the end moments, and it was kept constant while the end moments varied from zero to their ultimate value. Bending was about the strong axis of the members, and lateral bracing was provided for the beams and the column(20).

Typical results of two tests are given in Figs. 20 and 21. In each of these figures there are two sets of curves: one representing theory and the other experiment. Moment-rotation curves are given for
the beam, the column, and the joint. This latter curve, which defines the behavior of the whole frame, was measured independently from the M-θ curves of the beams and the column, and each experiment conclusively proved that the total frame behavior was indeed closely predictable from the behavior of the components (20). Experiment and theory is seen to be also in close agreement (Figs. 20 and 21), especially up to the peak of the maximum moment of the frame. In the unloading range the test performance proved to be somewhat stiffer than the theoretical prediction. This was due to a variety of reasons, which are not relevant to this discussion and explained fully elsewhere (20). Especially good agreement was found to exist between theory and experiment with regard to the ultimate moment of the frame.

The two tests given by the curves in Figs. 20 and 21 illustrate two situations. In Fig. 20 the column is more flexible than the beam, and the maximum moment of the frame is equal to the sum of the maximum moments delivered by the beam and the beam-column. This is the condition when simple plastic theory can be used. A second situation, illustrated by the test of Fig. 21, is when the beam-column reaches its ultimate moment before the beam attains its plastic moment. This frame did not, however, fail when the column began to unload. Because of the ability of the beam to sustain additional moment, it carried the weaker beam-column along with itself, and failure of the frame occurred when the beam-column had already considerably unloaded.

The test of Fig. 21 thus emphasizes the fact that failure of a relatively non-ductile member does not necessarily cause the failure of
the whole frame. The maximum moment of the frame, however, is not the sum of the maximum loads of the components. It is less than this sum, and it must be determined from a compatibility analysis involving knowledge of the M-θ curves of all the components. Such an analysis is used to design braced frames under vertical loads (19).

5. CONCLUSIONS

The ability of steel frames to deform into the inelastic range is the basis for both plastic design under static loads and for certain proposed design methods against earthquake loads. The load-deformation history of a whole frame can be fully defined if the moment-rotation characteristics of the components of the frame are known. The load-deformation curve of the frame furnishes the ultimate load and deflections for plastic design, and the area under this curve provides a measure of the energy absorption capacity for earthquake design.

In the first portions of this report the inelastic deformability of beams and beam-columns was examined from both a theoretical and an experimental viewpoint. The "ductility factor" \( \mu \) (Eq. 1 and Fig. 1) was used to define this inelastic deformability, or "rotation capacity".

It was shown that wide-flange beams fulfilling certain reasonable conditions of geometry and bracing fail by local buckling of the compression flange. Theoretical equations (Eqs. 2 through 4), curves (Fig. 6) and examples (Fig. 8) were provided for determining the ductility factor. Numerous experimental results were cited to prove the validity
of the theoretical predictions. For example, it was shown that the rotation capacity of beams under uniform moment depends on the material properties (especially the strain-hardening characteristics of the steel) and on the spacing of the bracing (Figs. 5 and 6). In this case, particularly good correlation was found to exist between theory and experiment (Fig. 6).

The inelastic deformation behavior of laterally braced beam-columns was shown to be not only dependent on the occurrence of local buckling but also on the overall instability of the member in the plane of the applied loads. Sample moment-rotation curves (Fig. 9), curves relating the ductility factor to the length, the axial load, and the loading condition (Figs. 11 and 12), and spacing rules for lateral bracing (Figs. 13 and 14) were given for A7 steel wide-flange members. Again, good correlation was shown to exist between experimental results and theoretical predictions (Fig. 15).

The principal conclusion from the discussion of member behavior was that if the conditions of loading and geometry, as well as the material properties and the bracing conditions, are known, then the complete moment-rotation curve from zero load to failure, and thus also the ductility factor, can be determined. Conversely, if the requirements of ductility are prescribed, it is possible to design a member which will fulfill these requirements. Properly braced beams and beam-columns having practical proportions and loads were shown to form plastic hinges with considerable rotation capacity.

In the final section of this report the information obtained from
the member behavior was used to determine the load-deformation history of entire frames. An example was presented in which simple plastic theory was applied to obtain the panel deflection-versus-horizontal load curve of a one-story, three-bay bent. It was found that secondary effects due to the $P\Delta$ moment reduced the ultimate load, but that this reduction could be taken up by a diagonal brace. Local buckling was shown to occur only after considerable unloading had already taken place from the $P\Delta$ moments—an effect having nothing to do with the plastic hinge behavior of the member.

It was further illustrated by citing experimental results from frame tests that the behavior of the frame can indeed be predicted from a knowledge of the $M-\Theta$ curves of the individual frame components, and that in some instances non-ductile members are supported by ductile members.

From the work presented or referred to herein it is clear that sufficient information is now available to know how frame components and thus entire frames behave beyond the elastic limit, provided the loads are static and non-reversible. Future work on component behavior should concentrate on assessing the effects of dynamic loading and load reversal. Experimental and analytical techniques evolved by previous research on static response will need to be modified for these new conditions, but the experience gained should serve as an effective guide in the new work.
6. **ACKNOWLEDGEMENTS**

This study is part of a general investigation "Plastic Design in High Strength Steel" currently being carried out at Fritz Engineering Laboratory, Department of Civil Engineering. Professor W. J. Eney is Head of the Civil Engineering Department and Professor L. S. Beedle is Director of the Laboratory. The investigation is sponsored jointly by the Welding Research Council, and the Department of the Navy, with funds furnished by the American Institute of Steel Construction, the American Iron and Steel Institute, Lehigh University Institute of Research, the Bureau of Ships and the Bureau of Yards and Docks. The Column Research Council acts in an advisory capacity.

Mrs. P. Steitz very carefully typed this report and Mr. H. Izquierdo did the drawings.
7. NOMENCLATURE

b  breadth of a beam

d  depth of a beam

f  shape factor of a beam

h  ratio E/E_{SH}

r_x, r_y  radii of gyration

s  ratio \frac{S_H}{f_Y}, span of a beam

t  thickness of a beam

u  lateral deflection

v  vertical deflection

v_o  midspan deflection

v_{oy}  midspan deflection at M_p (elastic behavior)

v_u  deflection at unloading

v_y  elastic deflection corresponding to ultimate load

w  web thickness

A  area

A_b  brace area

A_c  column area

E  modulus of elasticity

E_{SH}  strain hardening modulus

H  horizontal force

K  effective length factor

L  length

M  moment

M_{max}  maximum moment

M_p  plastic moment
\( M_{pc} \) plastic moment reduced by axial load
\( M_y \) yield moment
\( P \) force
\( P_u \) ultimate force
\( P_y = A \sigma_y \) yield moment
\( R \) reaction
\( S \) section modulus
\( \mu \) ductility factor
\( \sigma \) stress
\( \sigma_y \) yield stress (static)
\( \sigma_{yf} \) yield stress of flange
\( \lambda \) slenderness factor, eq. (7)
\( \varepsilon \) strain
\( \varepsilon_y = \sigma_y/E \)
\( \varepsilon_{SH} \) strain at strain-hardening
\( \Theta \) rotation
\( \Theta_p \) elastic rotation corresponding to ultimate load
\( \Theta_u \) rotation at unloading
\( \Delta \) horizontal deflection
8. FIGURES
FIG. 1  LOAD-DEFORMATION BEHAVIOR OF FRAME COMPONENTS
FIG. 2 LOAD-DEFORMATION BEHAVIOR OF FRAMES
FIG. 3 LOAD-DEFORMATION CURVE FOR A BRACED BEAM

Local Buckling

Ductility Factor \( \mu = 13.7 \)

\[
\frac{P_L}{M_p} = \frac{V_o}{V_{oy}}
\]

A7 10 W 25
L = 35\( r_y \)
FIG. 4 LATERAL DEFORMATIONS OF A BRACED BEAM
FIG. 5 INFLUENCE OF BRACING SPACING ON DEFORMATION CAPACITY
FIG. 6  COMPARISON OF BEAM TEST RESULTS WITH THEORY
FIG. 7 BEAM BEHAVIOR UNDER NON-UNIFORM MOMENT

a) Yielded Region

b) $M_{\text{MAX}}$

c) Approx. $0.07$ radians

d) Local Buckling

Approx. $0.07$ radians
FIG. 8 INELASTIC DEFORMABILITY OF A UNIFORMLY LOADED BEAM
Local Buckling

FIG. 9  TYPICAL BEAM-COLUMN MOMENT ROTATION CURVES
FIG. 10 DEFINITION OF DUCTILITY FOR BEAM-COLUMNS
FIG. 11  DUCTILITY FACTORS FOR BEAM-COLUMNS WITH ONE END-MOMENT (STRONG AXIS BENDING)
FIG. 12 DUCTILITY FACTORS FOR BEAM-COLUMNS WITH EQUAL END-MOMENT (STONG AXIS BENDING)
FIG. 13 REQUIRED BRACING SPACING FOR BEAM-COLUMNS WITH ONE END MOMENT
FIG. 14 REQUIRED BRACING SPACING FOR BEAM-COLUMNS WITH EQUAL END MOMENTS
FIG. 15 COMPARISON OF THEORETICAL AND EXPERIMENTAL M-Θ CURVES
All Members are the Same Size
\( P = 0.3 \, P_y \quad L/r = 30 \quad A_b = 0.0269 \, A_c \)
\( \sigma_y = 33 \, \text{ksi} \quad r/d = 0.43 \)
\( E = 30,000 \, \text{ksi} \quad f = 1.10 \)

**FIG. 16 PLASTIC ANALYSIS OF A THREE BAY BENT**
FIG. 17  MULTI-STORY FRAME UNDER VERTICAL LOADS
FIG. 18  CRITICAL BEAM-COLUMN SUBASSEMBLAGE
FIG. 19  SUBASSEMBLAGE ANALYSIS
FIG. 20  TEST RESULT OF STANDARD CASE
FIG. 21 TEST RESULT OF CASE WHERE COMPATIBILITY IS IMPORTANT
REFERENCES

1. Blume, J. A., Newmark, N. M., Corning, L. H.
   DESIGN OF MULTISTORY REINFORCED CONCRETE BUILDINGS FOR EARTHQUAKE MOTIONS, Portland Cement Association, 1961

2. Beedle, L. S., Christopher, R.
   TESTS OF STEEL MOMENT CONNECTIONS, Fritz Laboratory Report No. 205.79, October 1963

3. ASCE and WRC
   COMMENTARY ON PLASTIC DESIGN IN STEEL, Chapt. 8 "CONNECTIONS"
   ASCE Manual No. 41, 1961

4. Lee, G. C., Galambos, T. V.

   EXPERIMENTS ON BRACED WIDE-FLANGE BEAMS, Fritz Laboratory Report No. 205H.6, March 1963

6. Prasad, J., Galambos, T. V.

7. Lee, G. C.
   INELASTIC LATERAL INSTABILITY OF BEAMS AND THEIR BRACING REQUIREMENTS, Ph.D. Dissertation, Lehigh University, 1960

8. Lay, M. G.
   THE STATIC LOAD-DEFORMATION BEHAVIOR OF PLANAR STEEL STRUCTURES
   Ph.D. Dissertation, Lehigh University, 1964

9. Haaijer, G.

10. Haaijer, G., Thürlimann, B.
    INELASTIC BUCKLING IN STEEL, Trans. ASCE, Vol. 125, p. 308, 1960

11. W. J. Austin

13. ASCE and WRC
COMMENTARY ON PLASTIC DESIGN IN STEEL, Chapt. 7, "COMPRESSION MEMBERS", ASCE Manual No. 41, 1961

BEAM-COLUMN EXPERIMENTS, Fritz Laboratory Report No. 205.30, June 1961

15. Ojalvo, M.

16. Ojalvo, M., Fukumoto, Y.
NOMOGRAPHS FOR THE SOLUTION OF BEAM-COLUMN PROBLEMS, WRC Bulletin No. 78, June 1962

17. Lay, M. G., Galambos, T. V.

18. Galambos, T. V.
INELASTIC LATERAL TORSIONAL BUCKLING OF ECCENTRICALLY LOADED WIDE FLANGE COLUMNS, Ph.D. Dissertation, Lehigh University, 1959

19. Levi, V.
MULTI-STORY FRAMES II; Chapt. 22 in "Structural Steel Design" Ronald Press, New York, 1964

TESTING TECHNIQUES FOR RESTRAINED BEAM-COLUMNS, Fritz Laboratory Report No. 278.7, 1963